Calculation of droplets impinging on a plane surface: Accurate calculations of interfacial curvature with the level-set method.

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Outline

Background and motivation

The level-set method

The curvature problem

Solution strategy

Conclusion and final remarks

The main objectives

Hypothesis

A thorough understanding of the processes and phenomena occurring at a small-scale level in the heat exchanger is necessary to obtain an improved understanding of the heat exchanger, and its design.

Long term goal

Improve initial design accuracy and reduce full-scale testing and oversizing.

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- Study micro-scale phenomena, e.g. droplet-film coalescence, both with experiments and with simulations

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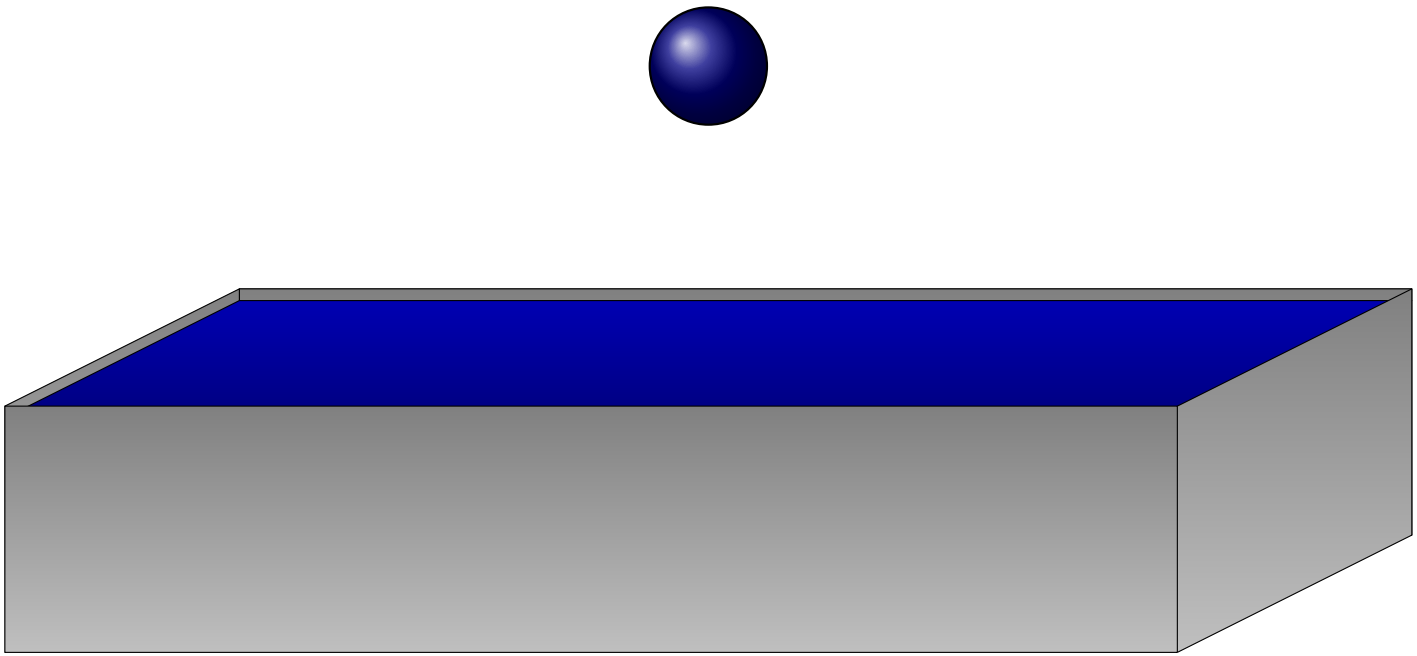

Numerical approach

We use

- the Navier-Stokes equations to solve the two-phase fluid flow
- equidistant, orthogonal Cartesian grids and finite-difference discretizations
- the level-set method to represent the interface

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The level-set method

- The level-set method is an interface-capturing method
- The interface is represented by the zeroth level set of the level-set function, which is often denoted by $\varphi(x)$
- Standard choice of the level-set function: Signed distance-function

$$|\varphi(\boldsymbol{x})| = \mathrm{d}(\boldsymbol{x}, \Gamma)$$

- The level-set advection-equation

$$\frac{\partial \varphi}{\partial t} + \boldsymbol{u} \cdot \nabla \varphi = 0$$

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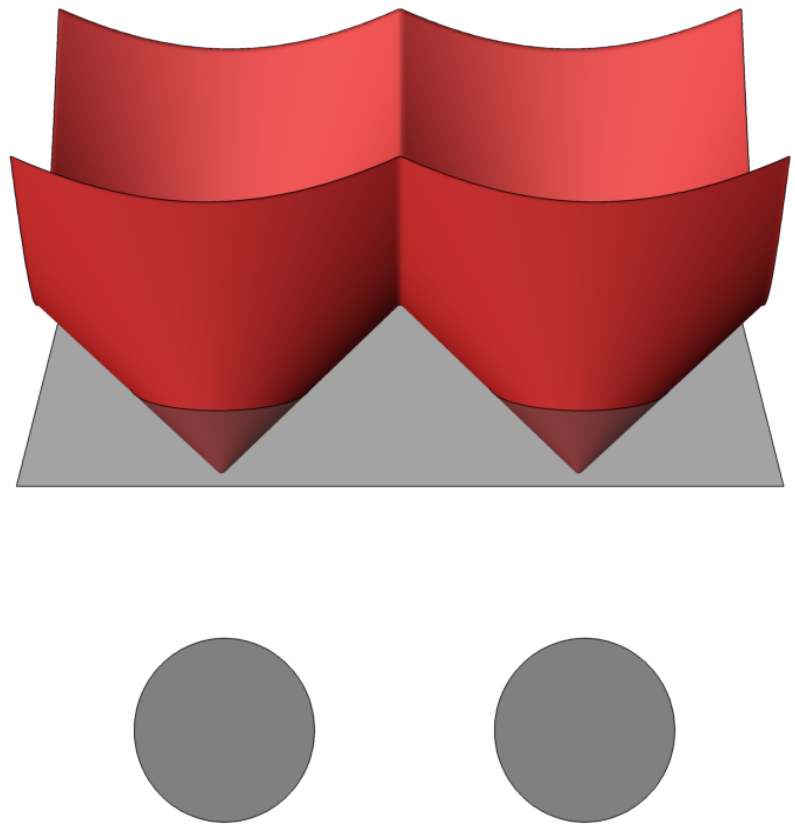
A level-set function that captures two 2-dimensional droplets:



Interface geometries

One of the advantages of the level-set method: Interface geometries are “easy” to calculate!

$$\boldsymbol{n} = \frac{\nabla \varphi}{|\nabla \varphi|}$$

$$\kappa = \nabla \cdot \left(\frac{\nabla \varphi}{|\nabla \varphi|} \right)$$

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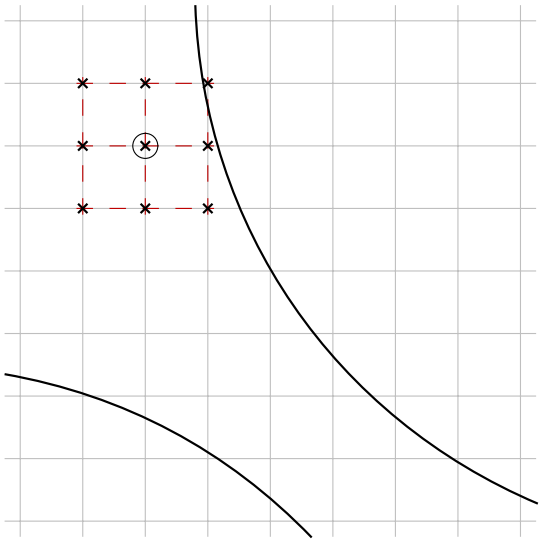
A closer look at the curvature

The discretization of κ requires a nine-point stencil.

$$\kappa = \frac{\varphi_x^2 \varphi_{yy} - 2\varphi_x \varphi_y \varphi_{xy} + \varphi_y^2 \varphi_{xx}}{\left(\varphi_x^2 + \varphi_y^2\right)^{3/2}}$$

$$\varphi_x = \frac{\partial \varphi}{\partial x} \simeq \frac{\varphi_{i+1,j} - \varphi_{i-1,j}}{2\Delta x}$$



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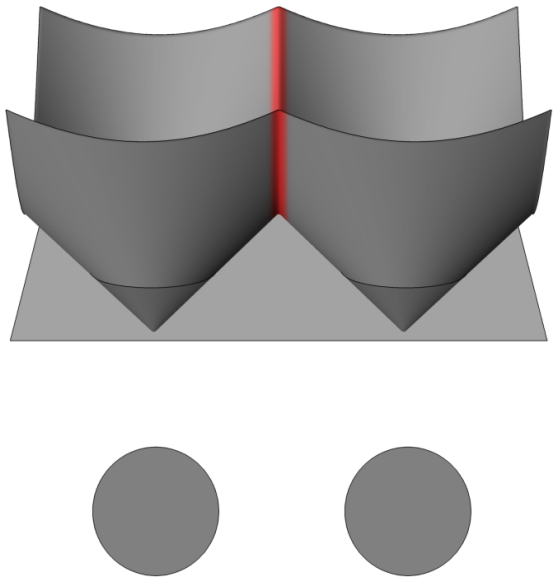
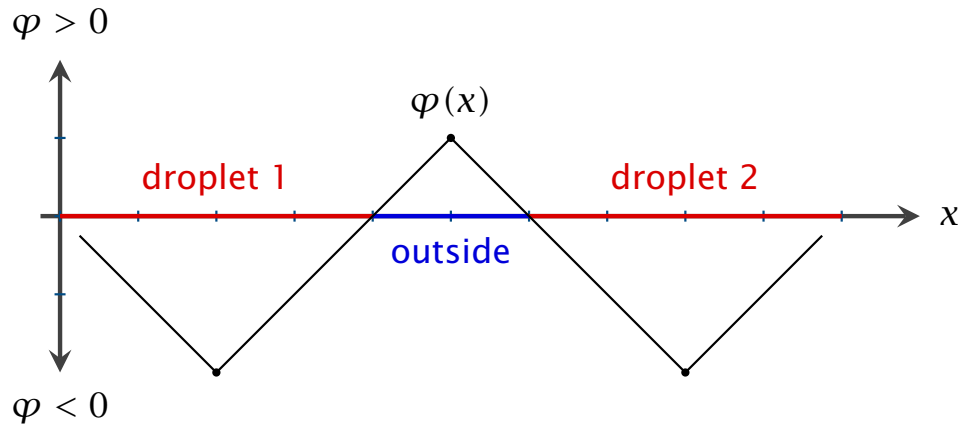
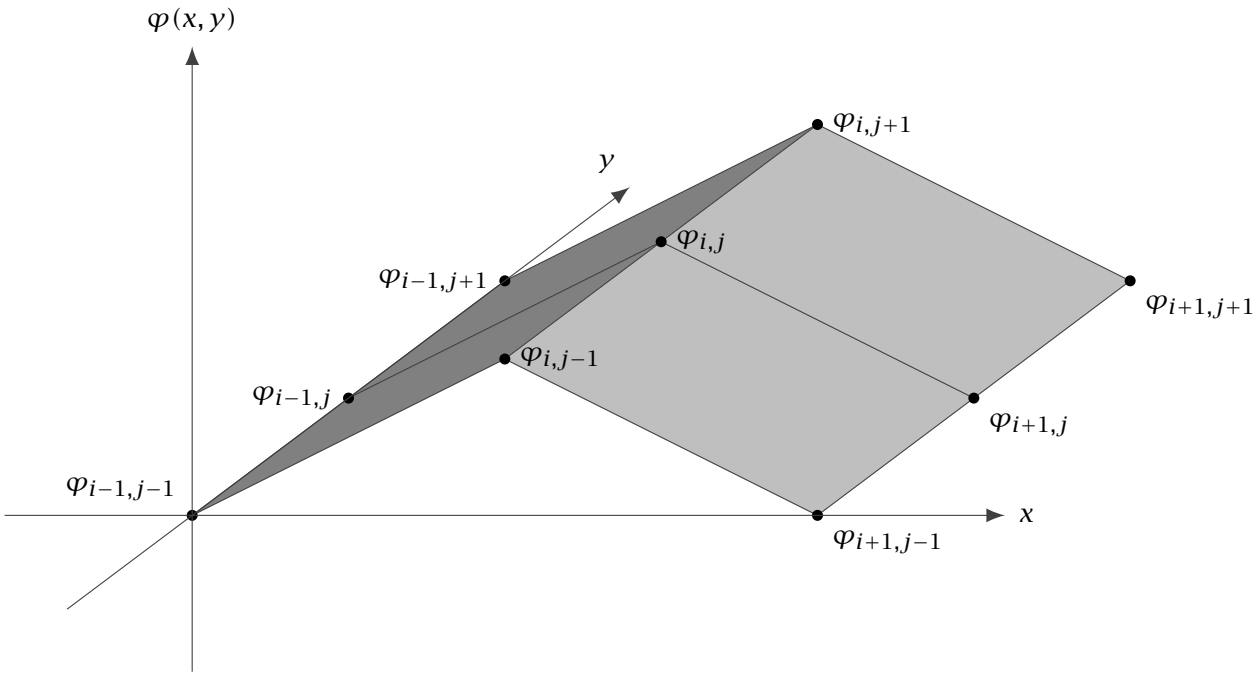
The derivative of the level-set function is not continuous!





What does this mean?



$$\varphi_{i+1,j} = \varphi_{i-1,j}$$
$$\varphi_{i,j+1} = \varphi_{i,j-1}$$
$$\Downarrow$$
$$\varphi_x = \varphi_y = 0$$

$$\Rightarrow \kappa = \frac{\varphi_x^2\varphi_{yy} - 2\varphi_x\varphi_y\varphi_{xy} + \varphi_y^2\varphi_{xx}}{\left(\varphi_x^2 + \varphi_y^2\right)^{3/2}} \rightarrow \pm\infty$$

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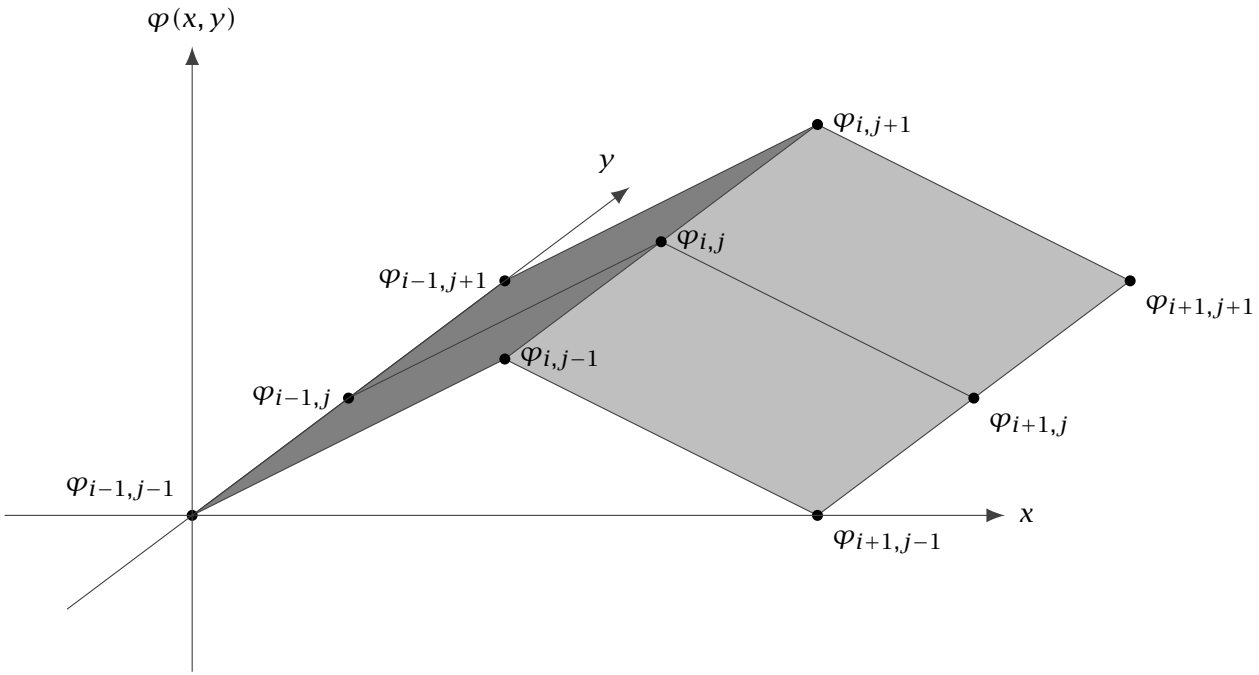

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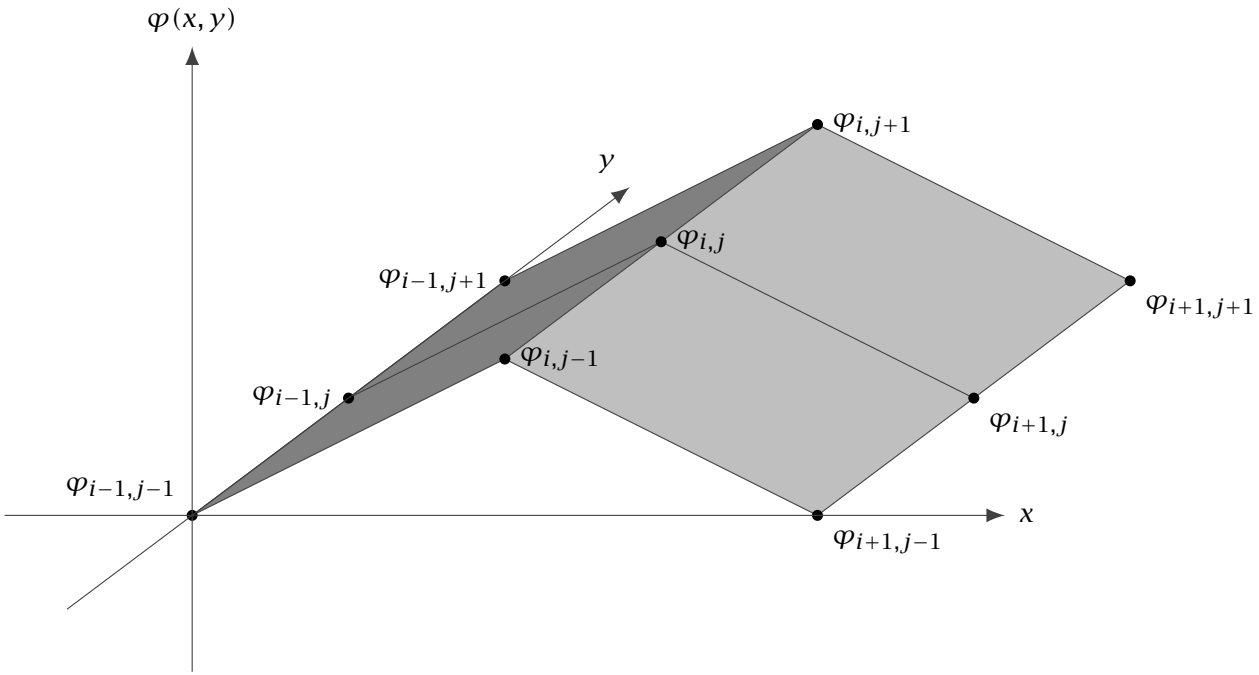

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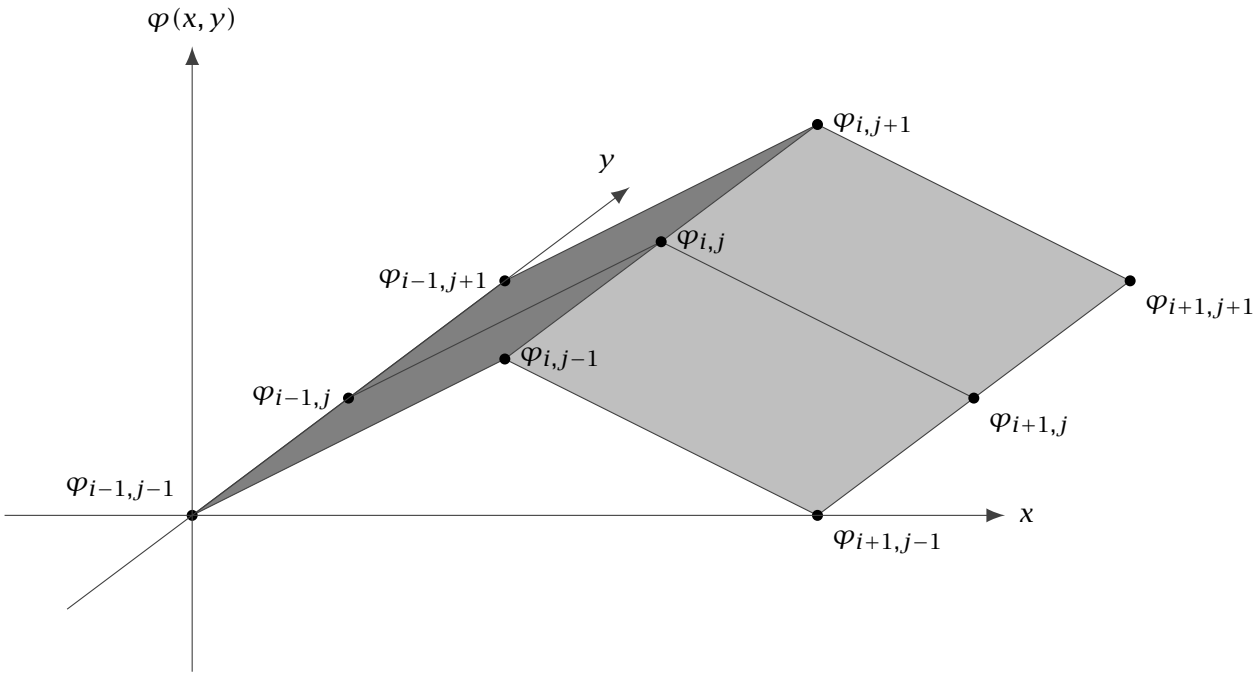

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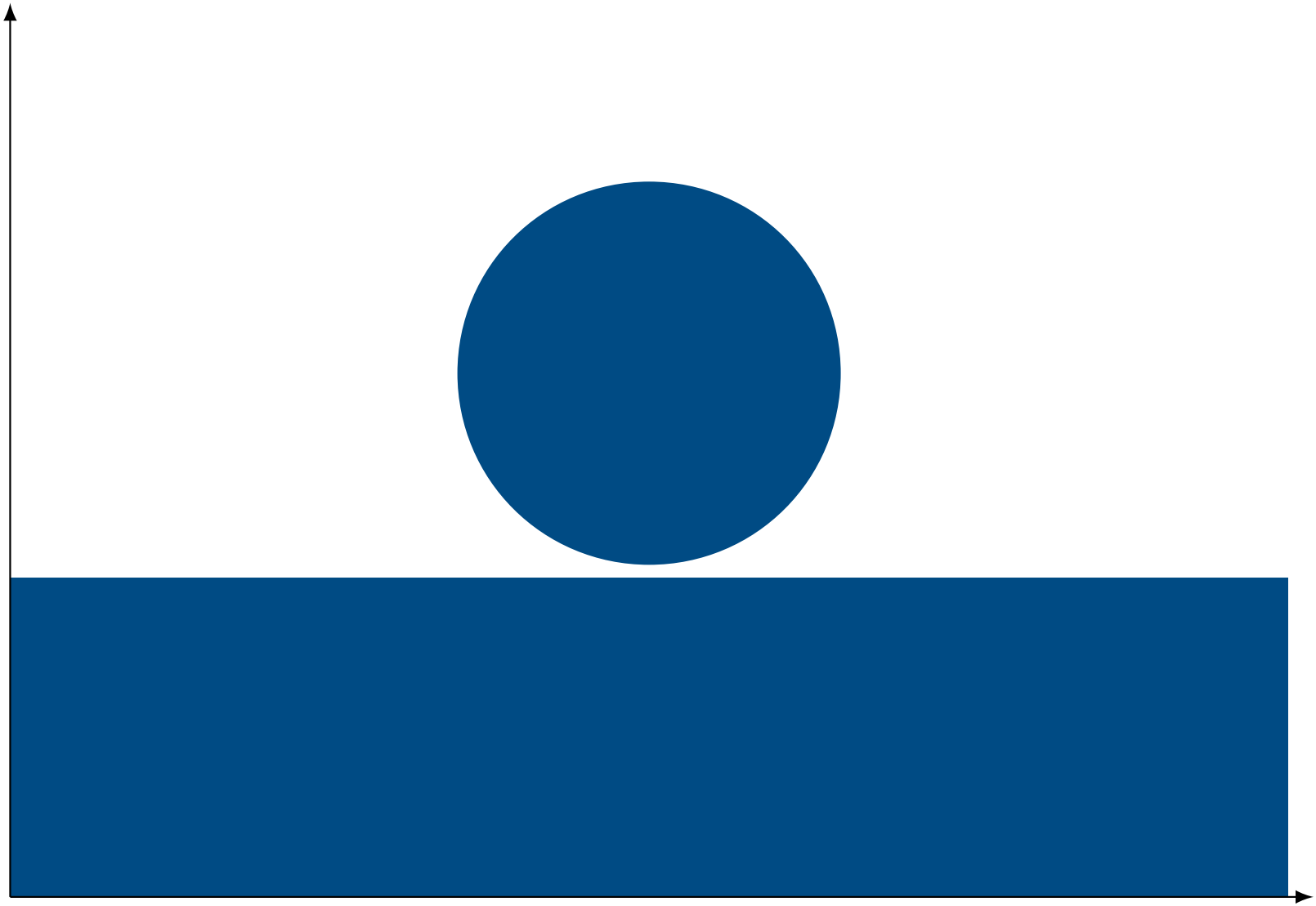
An example of a collision



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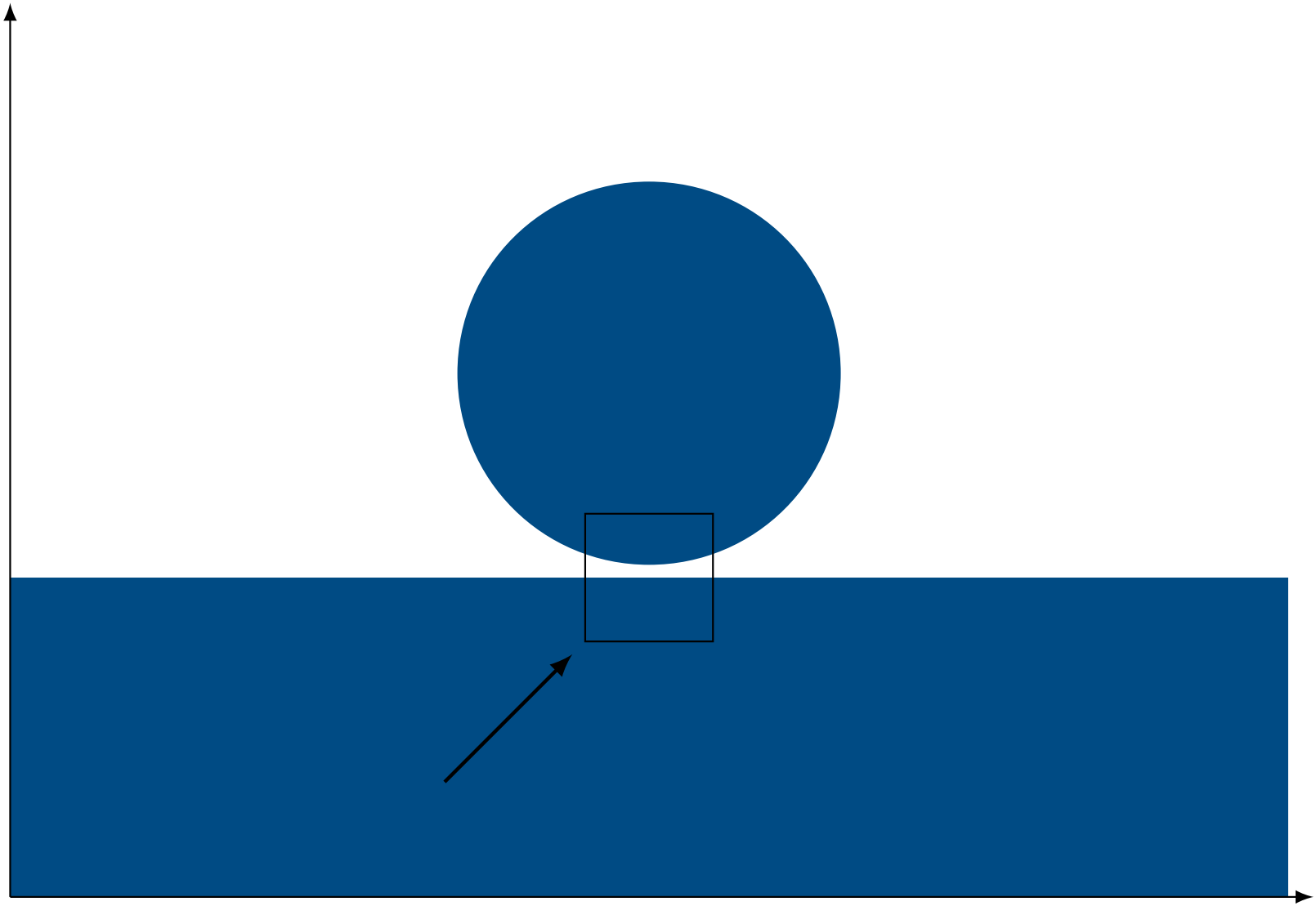

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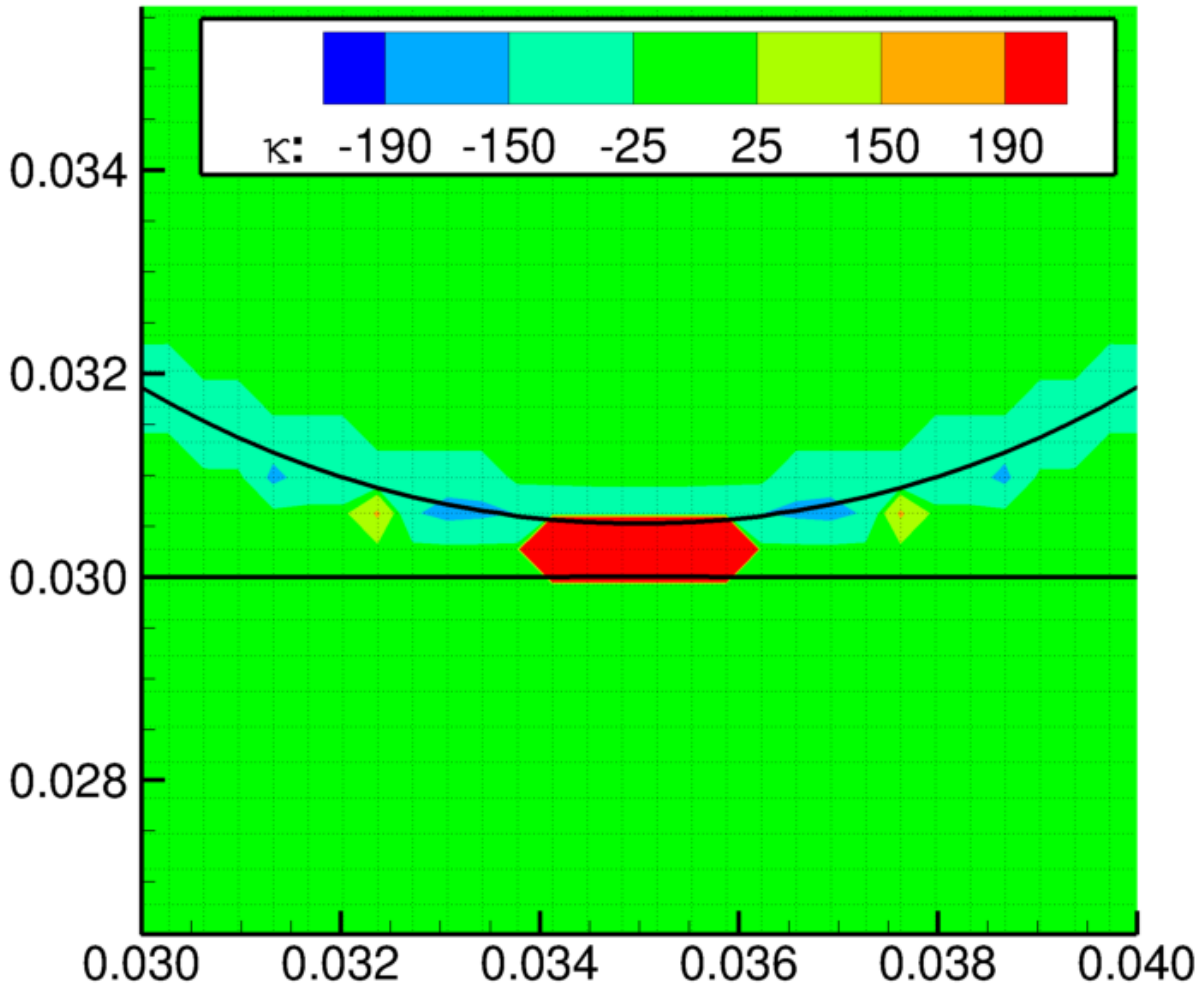

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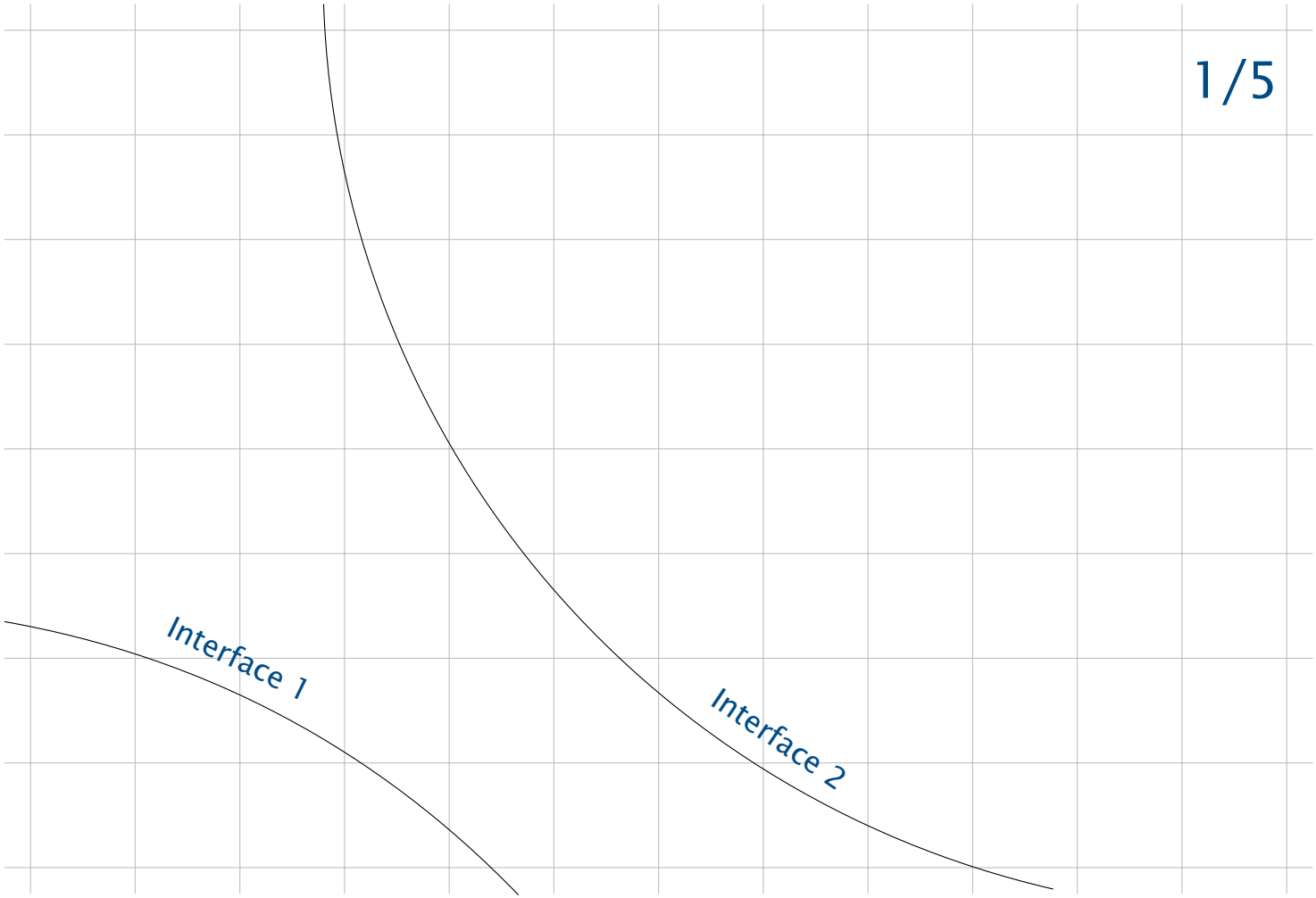

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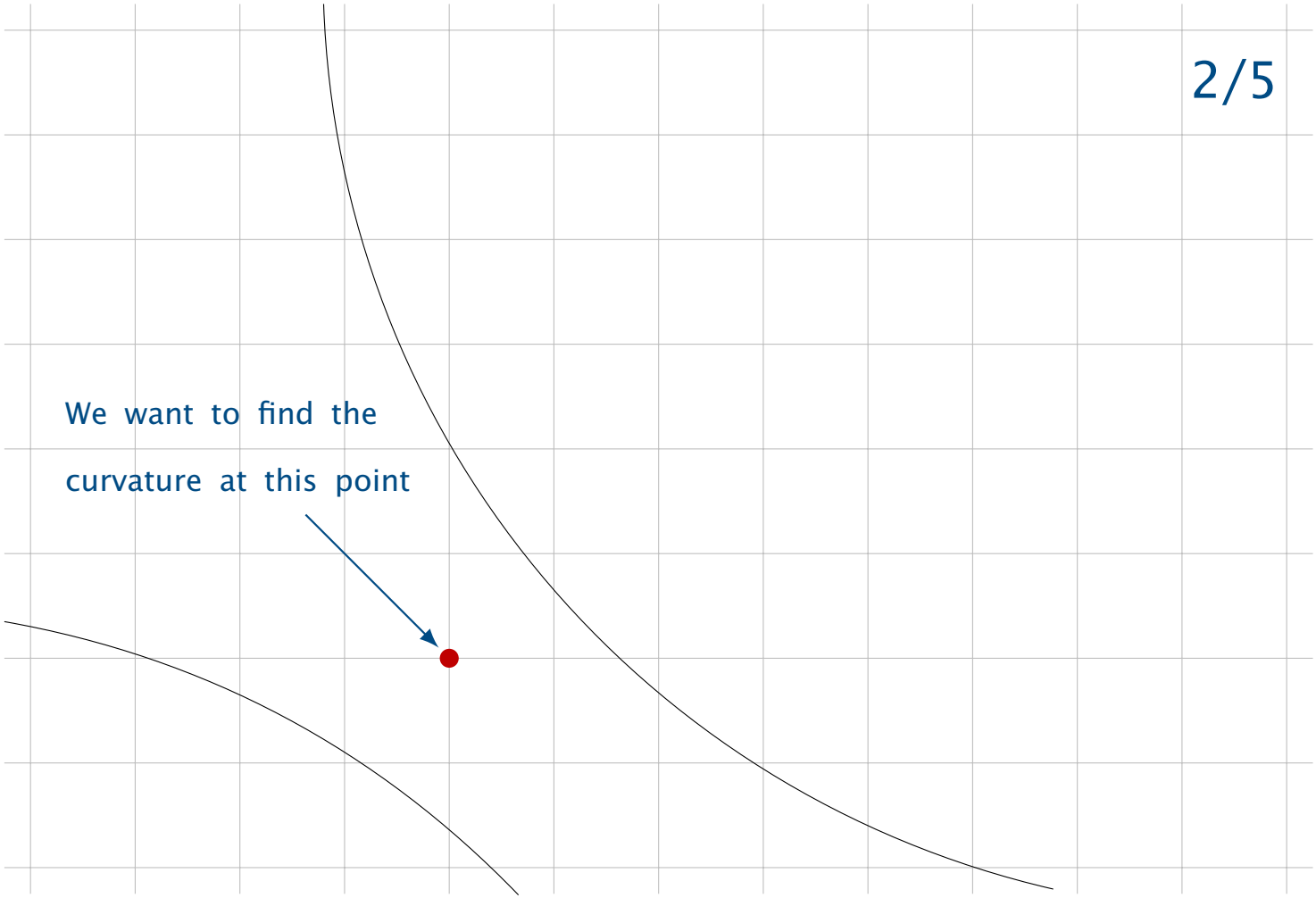

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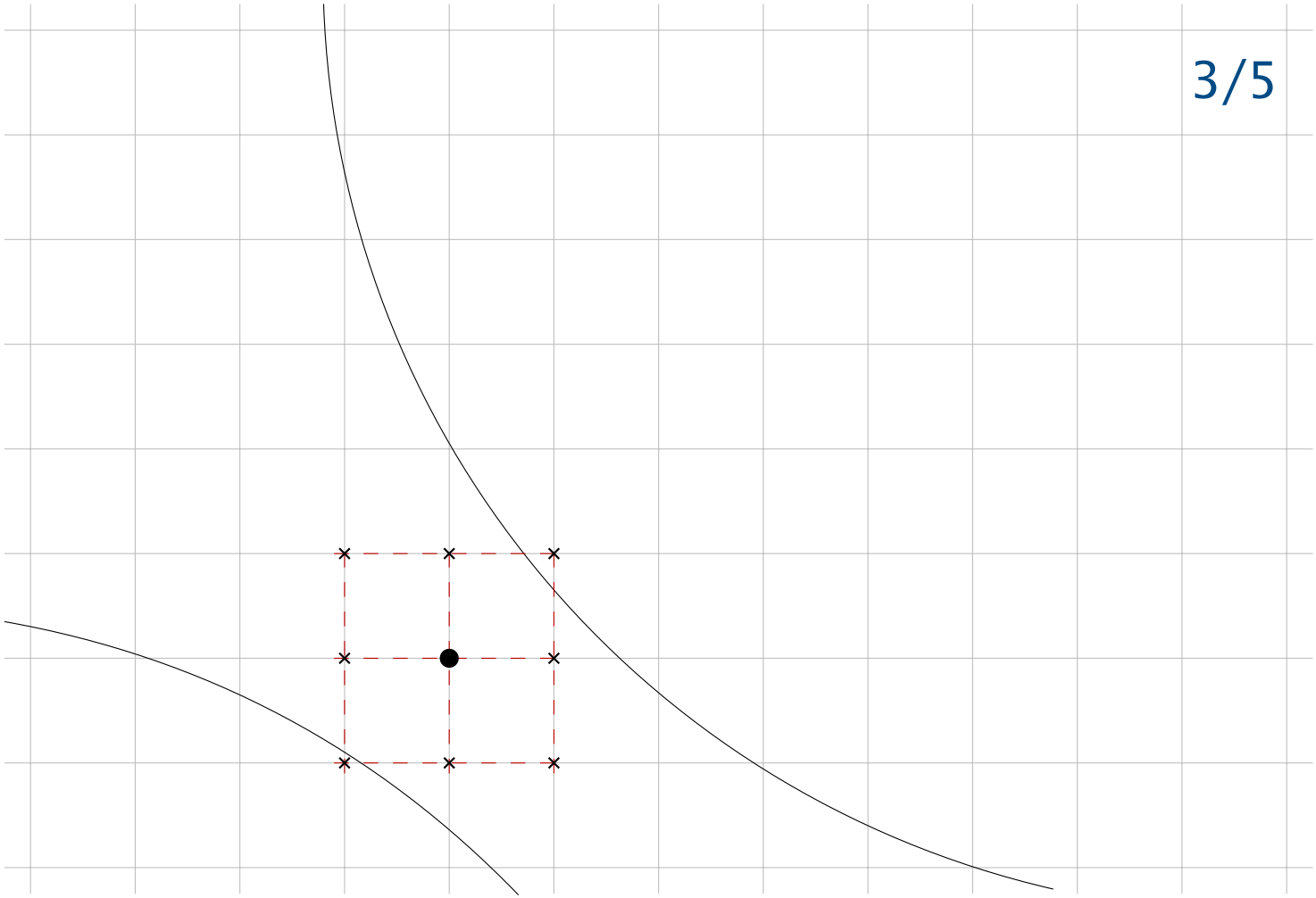

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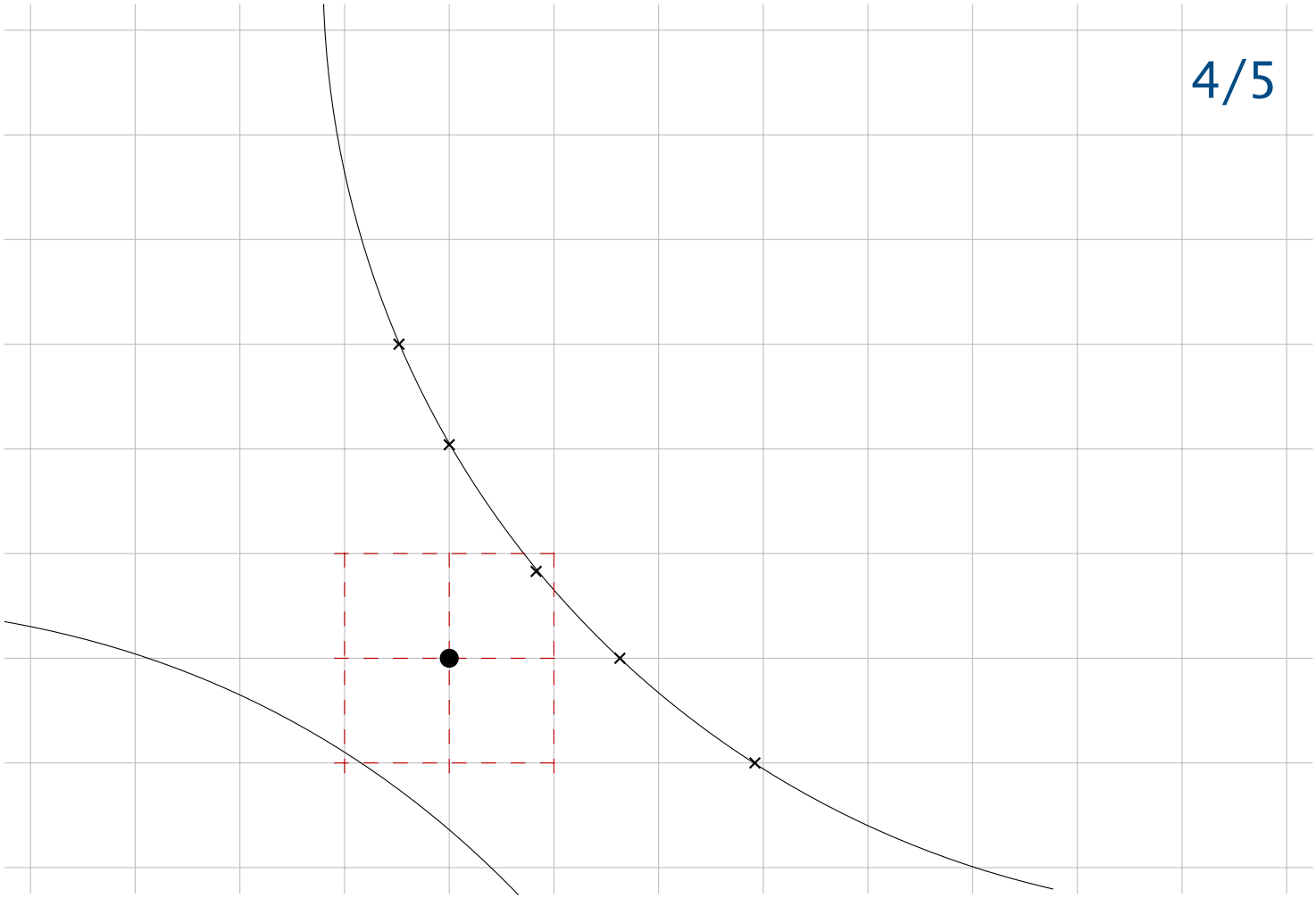

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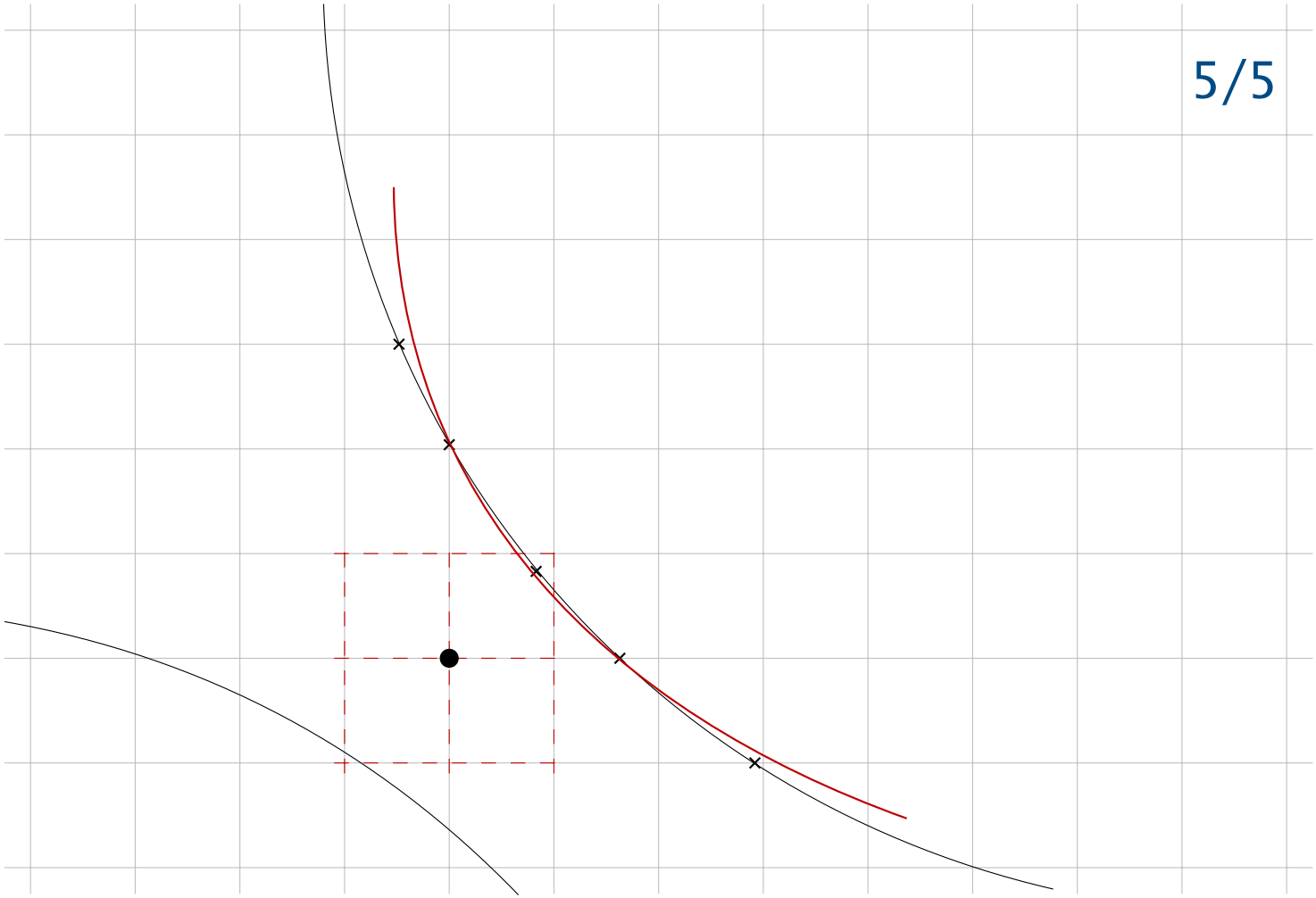
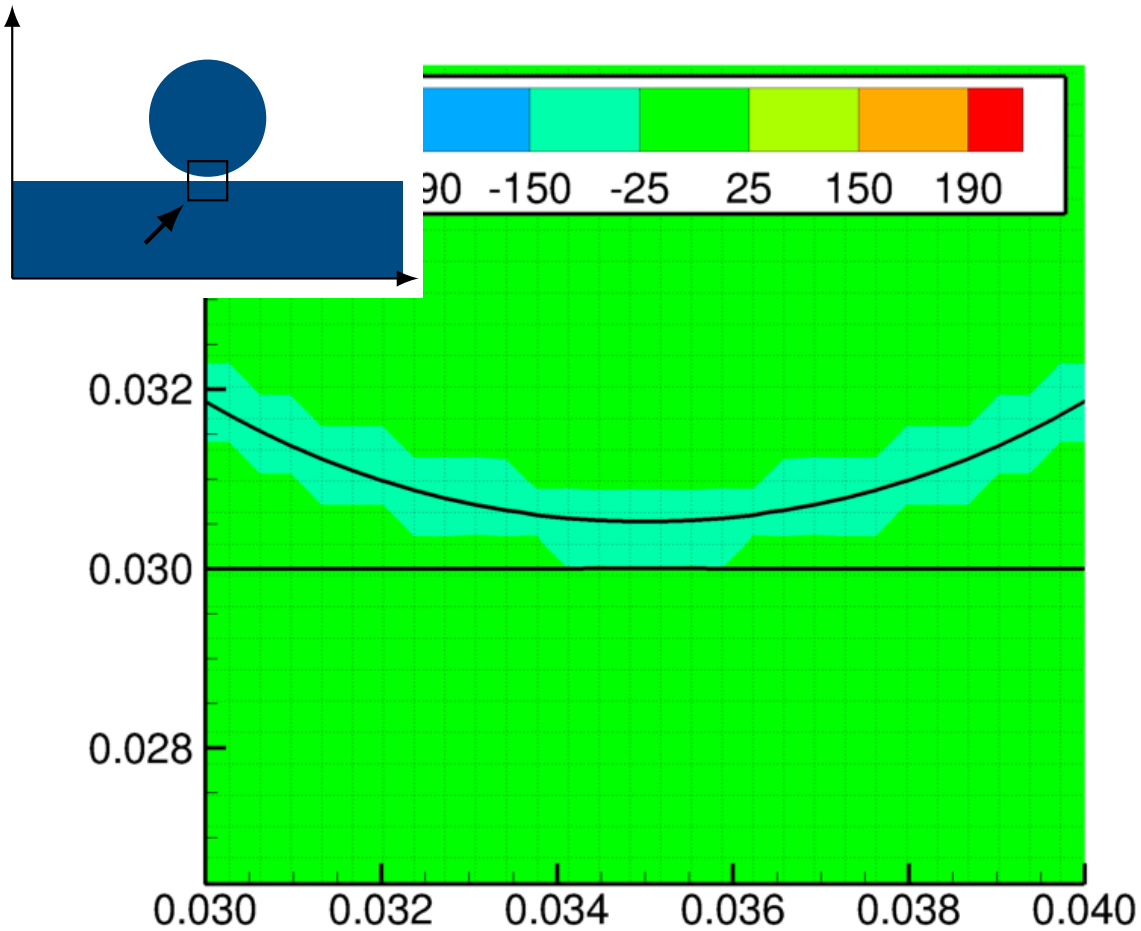


A preliminary and promising result



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- The method involves a lot of details — there are many possibilities for introducing small but dangerous bugs!
- The work is still in progress: The method needs to be refined before it can be used for more general simulations
- Hopefully, this work will lead to better simulations of two-phase flow with topological changes

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Questions?

Acknowledgement

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