Vehicle routing problems with alternative paths

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Outline

- Data modeling in vehicle routing problems
- Methodological impact of the introduction of alternative paths
- Practical impact on computing times and quality of results

Data collection from a Geographical Information System

- Too many pieces of information delivered by the GIS
  - every consistent portion of road is described
- Usual approach for vehicle routing:
  - introduce a vertex for every important location (depot, customer location…)
  - consider the best route between every pair of vertices

Limit

- How to compute the best route between two vertices when arcs are described with more than one attribute?
  - multicriteria shortest path problem
  - the solution is a set of Pareto efficient solutions

Illustration: VRPTW

From St Olavs Gate
To Grefsen-Kjelsas

Route A:
7 kilometers (cost)
15 minutes

Route B:
5 kilometers (cost)
20 minutes

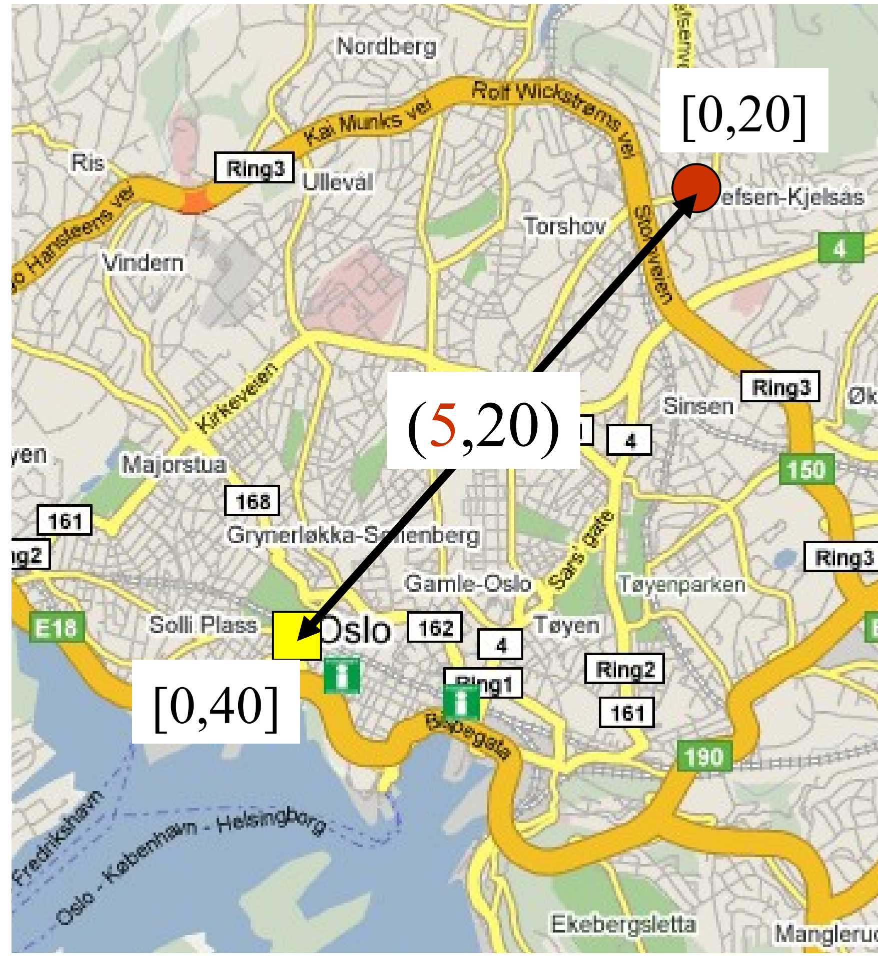
Illustration: VRPTW

Route Olavs – Grefsen - Olavs

Feasible, cost 14 kms

Unfeasible

Illustration: VRPTW

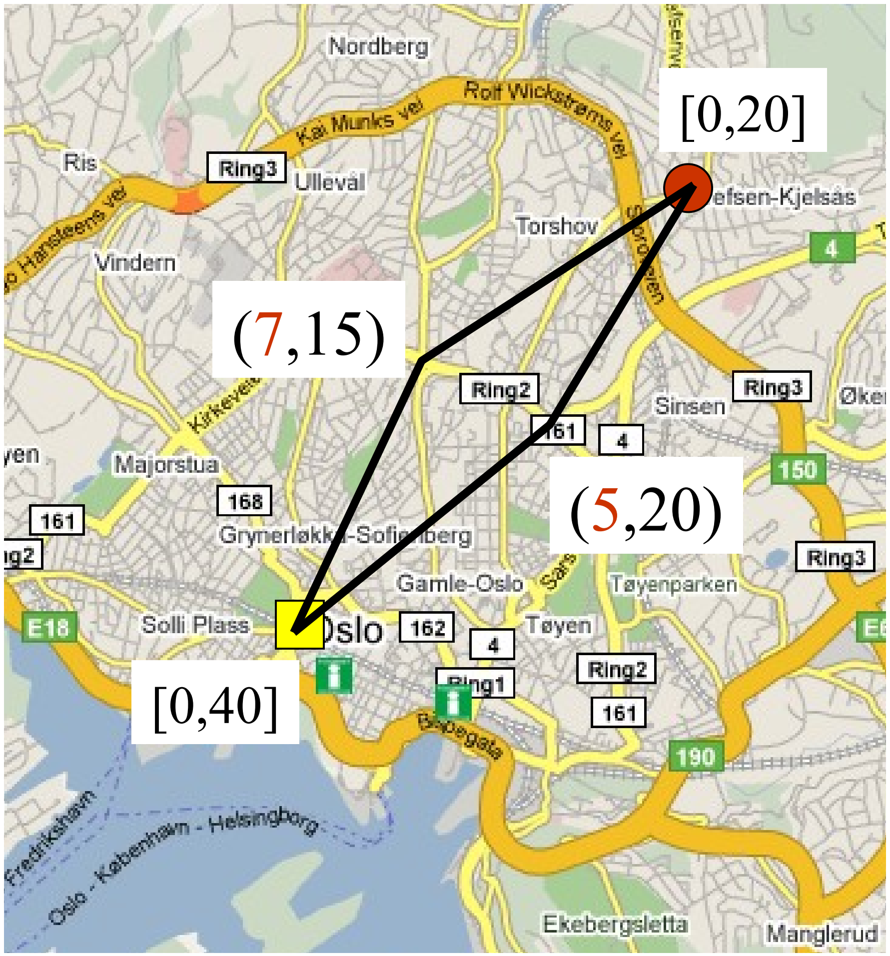


Route Olavs – Grefsen - Olavs

Feasible, cost 14 kms

Feasible, cost 10 kms

Multigraph modeling



Multigraph G=(V,A)

V = set of important locations

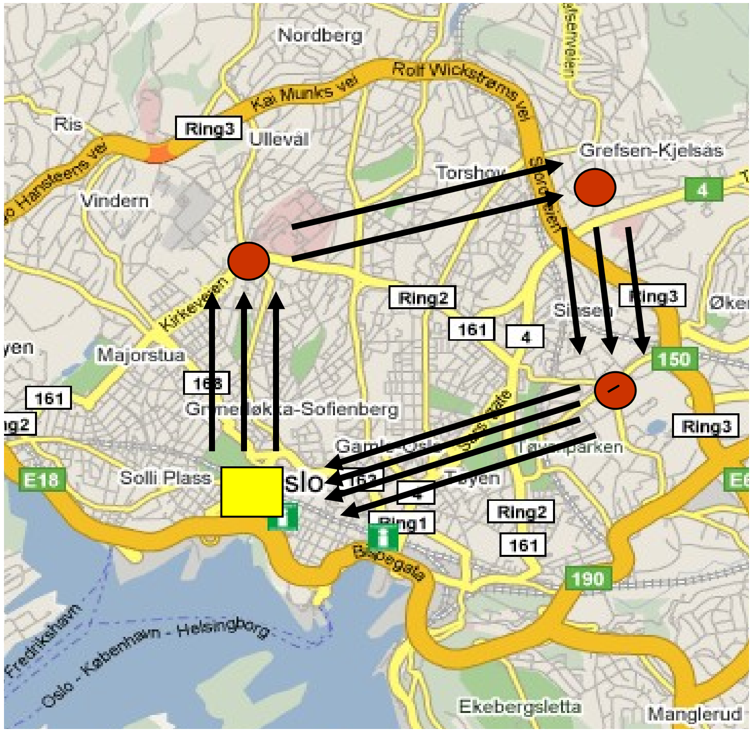
An arc $(i,j)^k$ exists in A for every pair of vertices (i,j) and every Pareto efficient path P^k between i and j

Methodological impact

- Construction of the graph
  - difficult to find the set of Pareto efficient paths
    - multicriteria shortest path problem
    - the set might be of very large size
  - in practice
    - one can expect a limited number of attributes on arcs
    - one can expect a set of limited size due to correlations
    - one can consider a subset of the efficient set (possibly user-defined)

Methodological impact

- Finding the sequence of arcs when the sequence of vertices is known is NP-hard



Called Fixed Sequence Arc Selection Problem (FSASP)

= Multidimensional Multiple Choice Knapsack Problem

Addressed as a shortest path problem with resource constraints (in an acyclic graph)

Methodological impact

- Three complex decisions when solving vehicles routing on multigraphs
  - assign customers to vehicles
  - sequence customers
  - select arcs (FSASP)

Methodological impact

- Heuristic / Metaheuristic
  - Any algorithm can be applied, accepting that evaluating the cost and the feasibility of a solution involves the solution of a FSASP

- Integer programming
  - Classical models can be adapted, with new decision variables (new flow variables for new arcs…)

Computational impact

- Set of random Euclidean instances for a Dial-A-Ride problem

	Simple graph		Multigraph*	
	Best insertion	Exact	Best insertion	Exact
VALUE	12%	0%	-8%	-17%

* : generated from a simple graph with 10% of additional arcs about 30% cheaper and slower ; results in a graph with up to 10 times more arcs

Computational impact

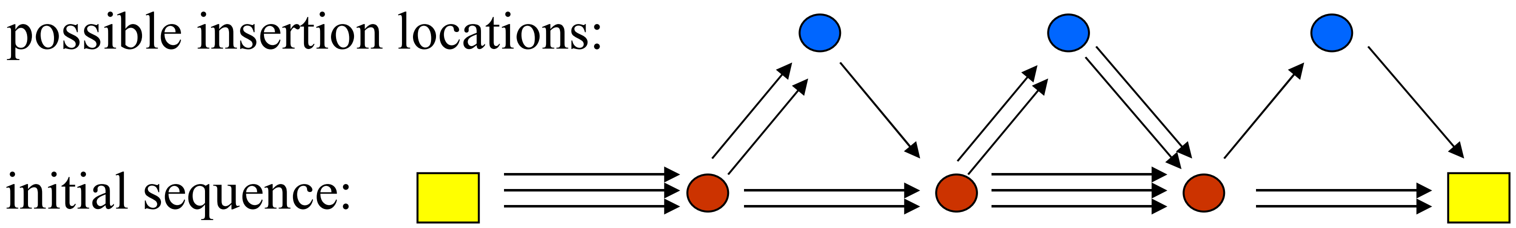
- Set of realistic instances (computed from a GIS) for a Dial-A-Ride problem in a rural zone

	Simple graph		Multigraph*	
	Best insertion	Exact	Best insertion	Exact
VALUE	12%	0%	2%	-8%

* : up to 10 times more arcs, with arcs up to 50% cheaper or slower

Some improvements

- Best insertion
  - Find the best neighbor with the solution of a single Shortest Path Problem with Resource Constraints
    - add a vertex for every possible insertion location
    - add a binary resource that imply to visit exactly one of these vertices



Some improvements

- Exact method
  - Branch and Price
  - The multigraph only impacts:
    - the subproblem: extend labels with every outgoing arcs
    - the branching rule: select or forbid a successor (i.e., a set of parallel arcs)

Computational impact

- Set of random Euclidean instances for a Dial-A-Ride problem

	Simple graph		Multigraph*	
	Best insertion	Exact	Best insertion	Exact
VALUE	12%	0%	-8%	-17%
TIME	0s	100s	10s	1000s

* : generated from a simple graph with 10% of additional arcs about 30% cheaper and slower ; results in a graph with up to 10 times more arcs

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Other improvements

- Exact method
  - When solving the subproblem, replace set of arcs $(i,j)^k$ with a single idealized arc (i,j)
    - $c(i,j) = \min \{c((i,j)^k)$
    - $t(i,j) = \min \{t((i,j)^k)$
  - A set of promising vertex-sequences are obtained
  - Solve the FSASP on these sequences and only keep the feasible routes of negative reduced cost
  - If no route is obtained, solve the original subproblem

Conclusion

- Improving the completeness of the data is a real issue
  - A very simple heuristic can beat an exact method with the multigraph representation
  - An « automatic » adaptation of the algorithms looks simple most of the times (once the FSASP tool is developed)
  - Some possibilities exist to really consider the multigraph issue in the algorithms

Perspectives

- Evaluate this modeling in other contexts
  - Multimodal transportation (time, cost)
  - Transportation with congestion (time, cost)
  - Tourist tours (time, scenic interest)
- Implement more efficient algorithms