SPE 106179

Multiscale Mixed Finite Element Modeling of Coupled Wellbore / Near-Well Flow

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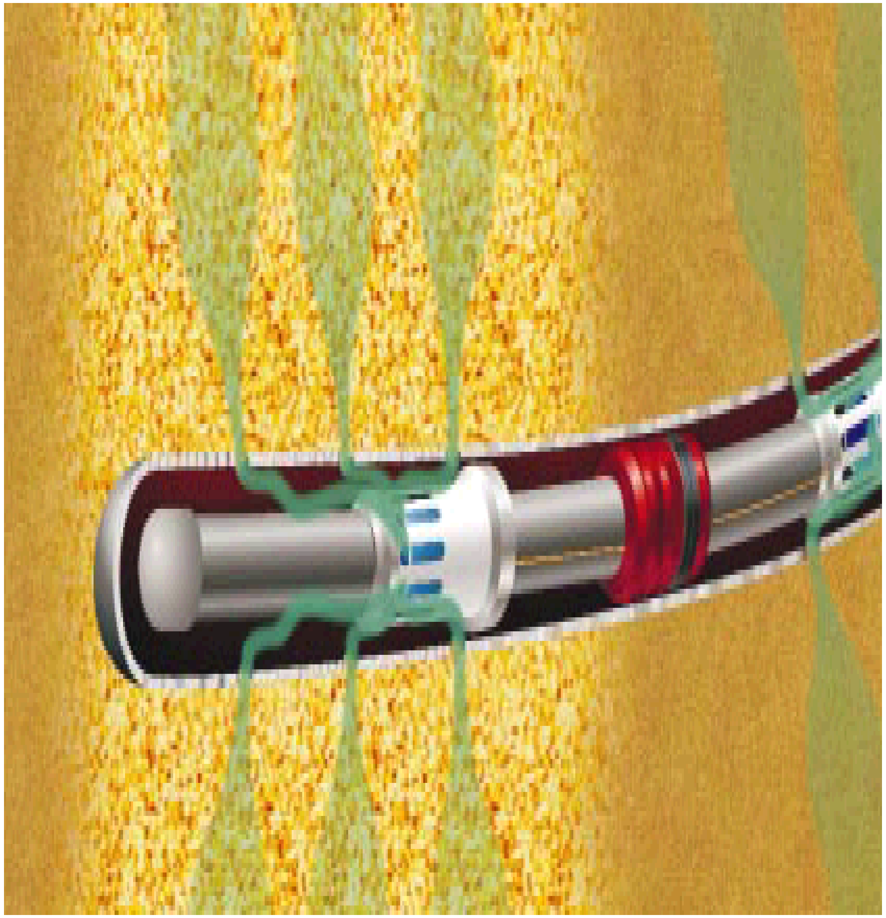
Motivation:



- Near-well region extremely important
- Cannot fully resolve all scales in typical simulation
- Multiscale methods incorporate fine scales in coarse scale equations

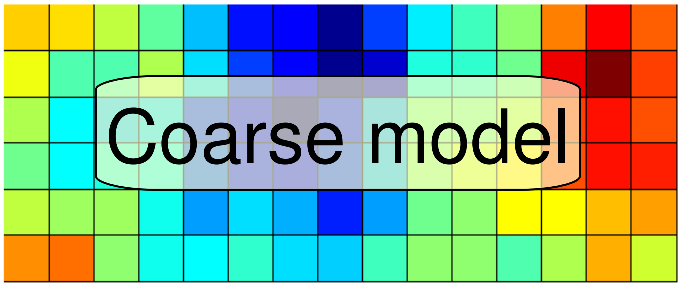
Outline

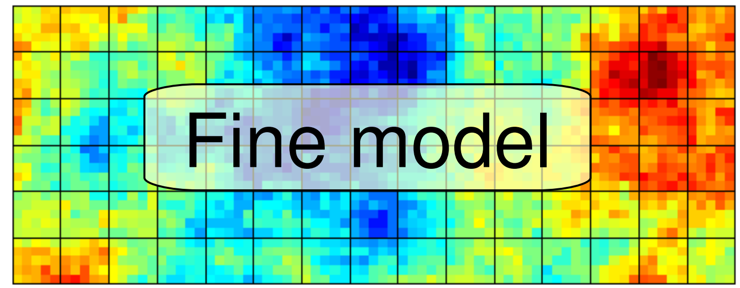
- Motivation
- Multiscale Mixed Methods
- Drift-Flux Wellbore Flow Modeling
- Multiscale – Drift-Flux Coupling
- Numerical Experiments
- Conclusions / Further Work

Multiscale Methods for Reservoir Simulation

- Multiscale Finite Element Method
  - T. Hou, X. H. Wu, 1997
- Multiscale Mixed FEM
  - Z. Chen, T. Hou, 2003
  - T. Arbogast et al., 2000
  - J. Aarnes et al., 2004 (group at SINTEF)
- Multiscale Finite Volume Method
  - P. Jenny, S. H. Lee, H.A. Tchelepi, 2003+

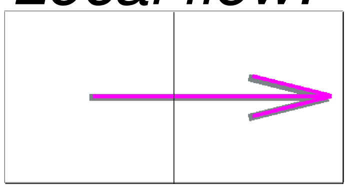
Standard / Multiscale Discretization



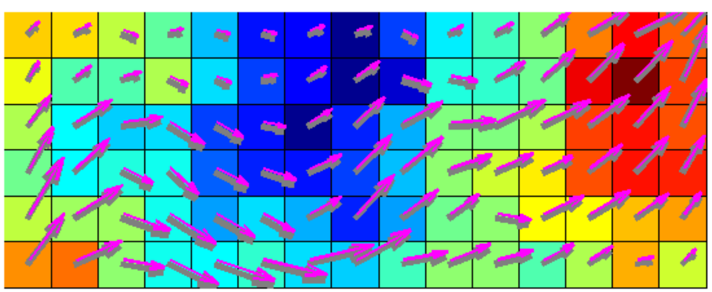


Standard

Local flow:

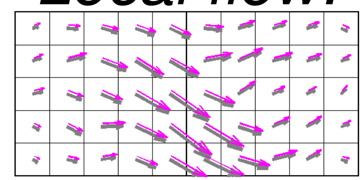


Solve **coarse** equations

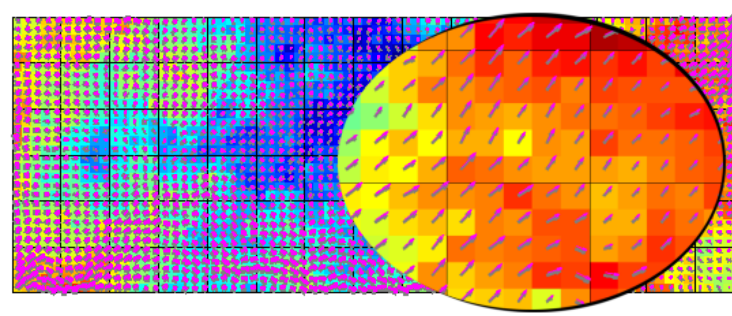


Multiscale

Local flow:



Solve **coarse** equations / form fine scale flow



Mixed Finite Elements (MFEM)

Model equations:

$$\mathbf{u} = -\lambda \mathbf{k} \nabla p$$

$$\nabla \cdot \mathbf{u} = q$$

Choose basis:

$$\mathbf{u} \approx \sum v_i \boldsymbol{\Psi}_i$$

$$p \approx \sum p_j \varphi_j$$

Weak formulation:

$$\int \mathbf{u} \cdot (\lambda \mathbf{k})^{-1} \hat{\mathbf{u}} - \int p \nabla \cdot \hat{\mathbf{u}} = 0$$

$$\int \hat{p} \nabla \cdot \mathbf{u} = \int \hat{p} q$$

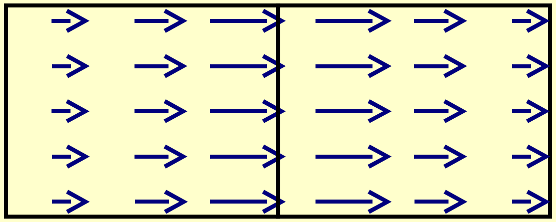
for all $(\hat{\mathbf{u}}, \hat{p}) \in U \times V$.

Mixed discretization:

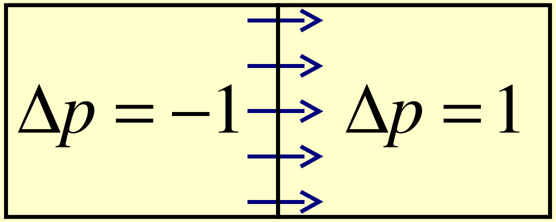
$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{v} \\ -\mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{q} \end{pmatrix}$$

Mixed / Mimetic / Multiscale

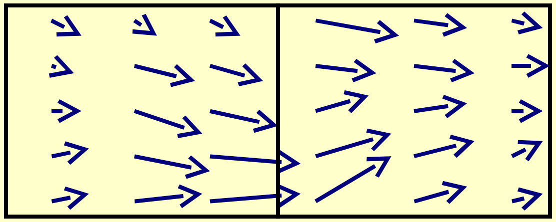
- Raviart-Thomas



- Mimetic Finite Differences



- Multiscale Mixed FEM



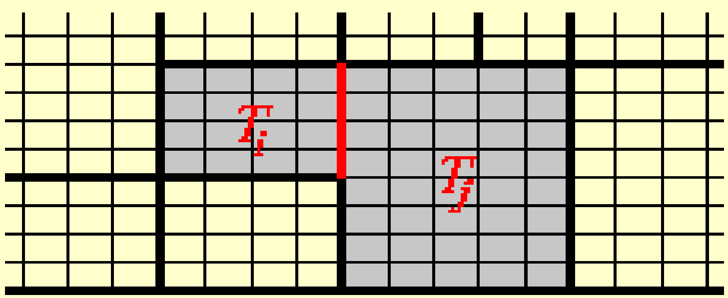
Multiscale MFEM (MsMFEM)

Flux basis function, $\boldsymbol{\psi}$:

$$\boldsymbol{\psi} = -\mathbf{k}\nabla p$$

$$\nabla \cdot \boldsymbol{\psi} = \begin{cases} w_i & \text{for } x \in T_i \\ -w_j & \text{for } x \in T_j \end{cases}$$

$$\boldsymbol{\psi} \cdot \mathbf{n} = 0 \text{ on } \partial(T_i \quad T_j)$$



- Initially compute basis functions

for n=1 **to** N

- Solve coarse system based on current saturation
- Form fine scale fluxes
- Advance fine scale saturation by Δt

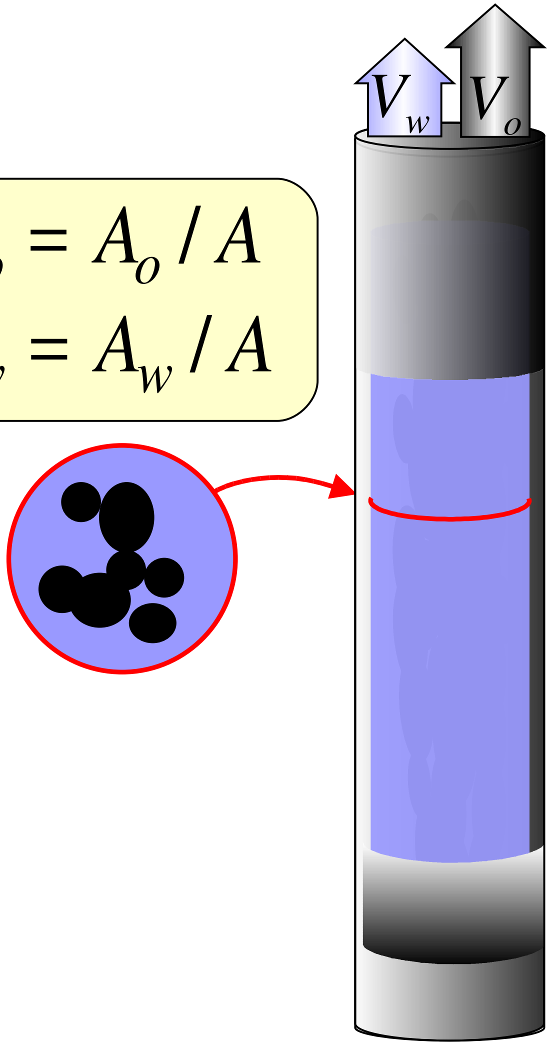
end

Drift-Flux Wellbore Flow Model

- Mixture velocity (oil/water):

$$V_m = \underbrace{\alpha_o V_o}_{V_{so}} + \underbrace{\alpha_w V_w}_{V_{sw}}$$

$$\alpha_o = A_o / A$$
$$\alpha_w = A_w / A$$



- Oil velocity:

$$V_o = C_0 V_m + m(\theta) V_d$$

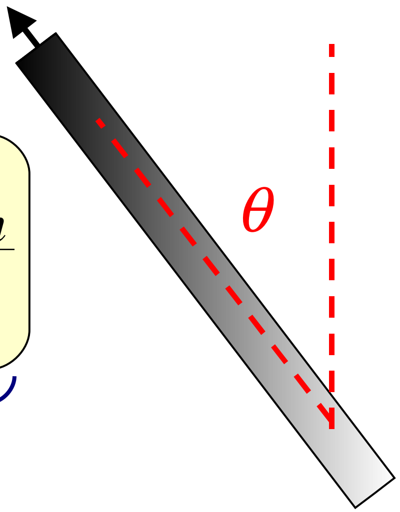
Shi *et al.* (2005):

$$C_0 = 1$$
$$V_d = 1.53 V_c (1 - \alpha_o)$$
$$m(0) = 1.07$$

Governing Equations for Wellbore Flow

- Wellbore pressure:

$$\frac{\partial p}{\partial x} = \underbrace{\rho_m g \cos(\theta)}_{\text{hydrostatic}} + \underbrace{\frac{2 f_{tp} \rho_m V_m^2}{D}}_{\text{friction}} + \underbrace{\frac{2 q_m \rho_m V_m}{A}}_{\text{acceleration}}$$

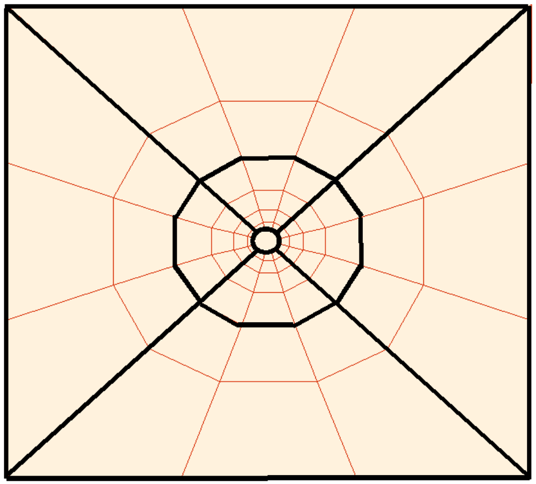


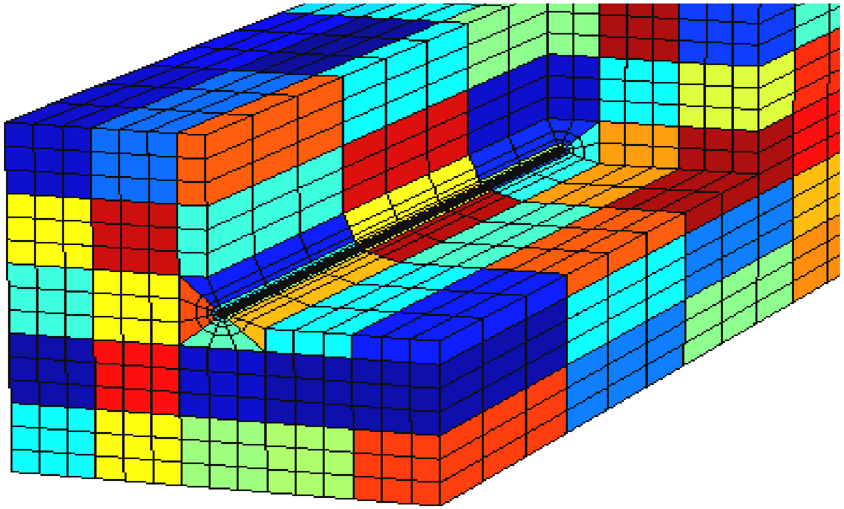
- In-situ volume fraction:

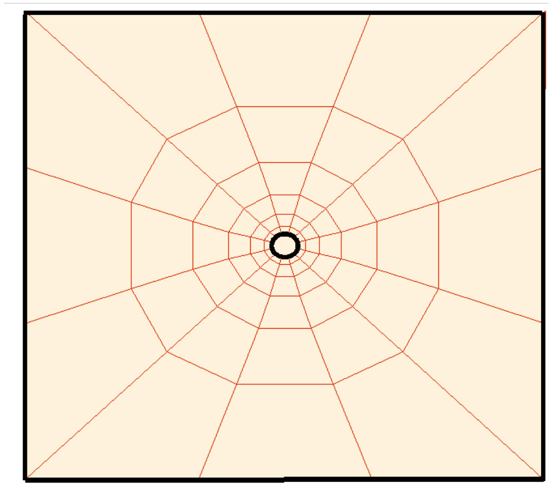
$$\frac{\partial \alpha_o}{\partial t} - \frac{\partial V_{so}}{\partial x} = q_o$$

Well – Reservoir Linkage

- Fine grid to the annulus, well segments included in the grid
- Well segments treated as coarse blocks - no well model is used







Well – Reservoir Coupling

Pressure:

- Linearize wellbore pressure equation
- Couple to MsMFEM equations
- Fixed-point iteration for initial pressure

Saturation / Holdup:

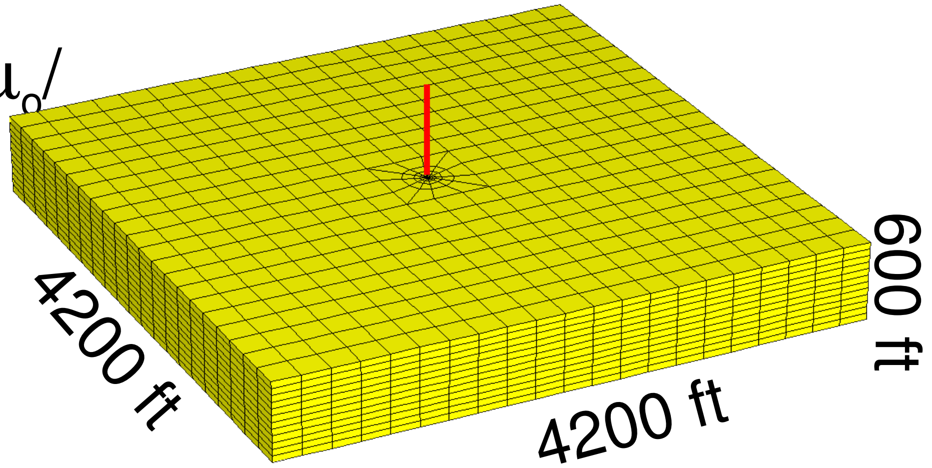
- Implicit finite volume
- Optimal ordering of cells
  - Newton iteration in sequence for each cell / small cluster

Prototype implementation:

Enhancements required for full generality

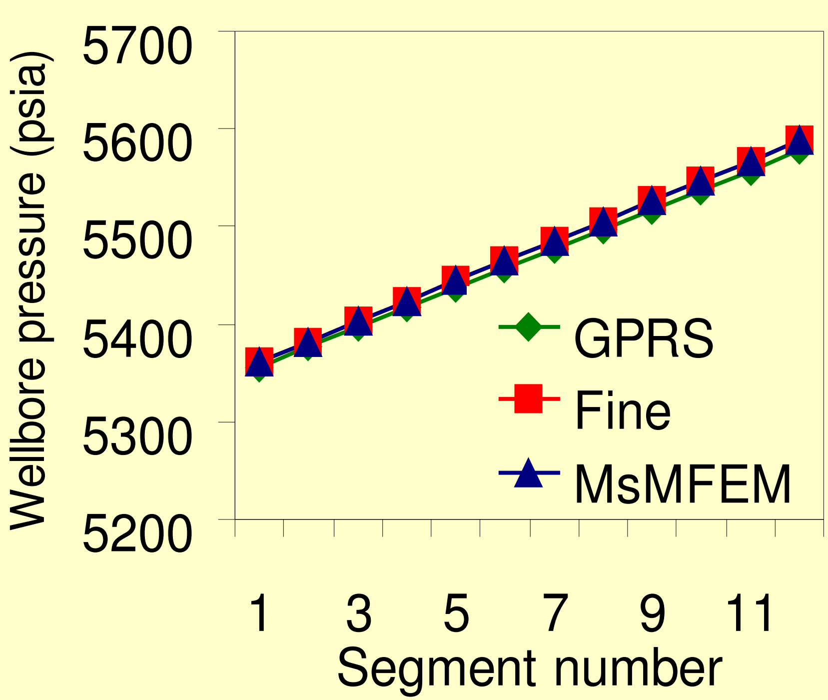
Numerical Example 1: Validation

- Homogeneous permeability
- Compressibility (*psi* $^{-1}$), 3×10^{-4}
- Wellbore radius *(inch),* 2.0
- Pipe roughness (*inch*), 0.001
- Initial saturation, 0.5
- Quadratic relative perms, $\mu_o/\mu_w = 1$
- 7056 fine cells
- 284 coarse blocks
- 12 well segments

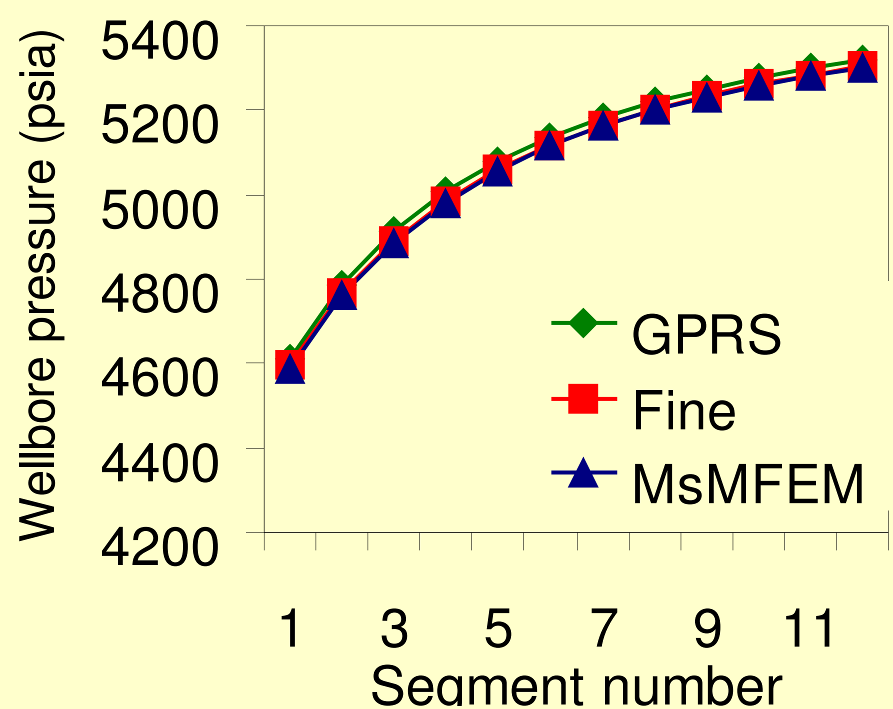


Numerical Example 1:
Pressure profiles:

Total rate: 1,600 STB/d

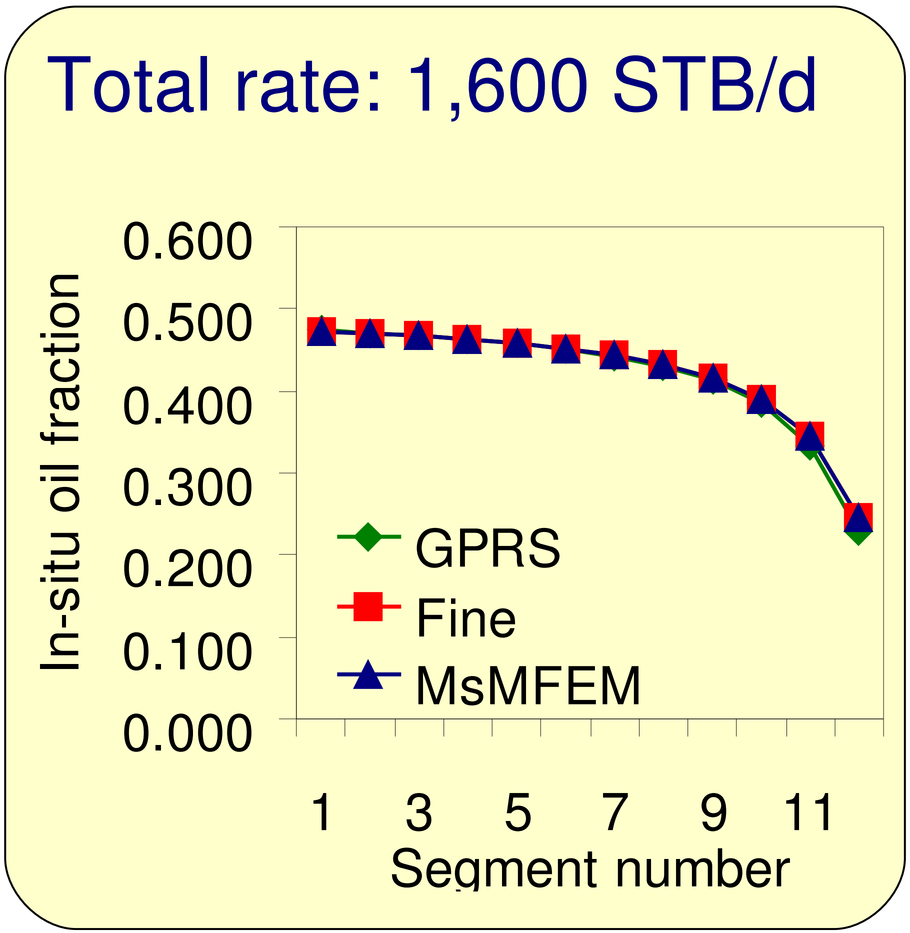


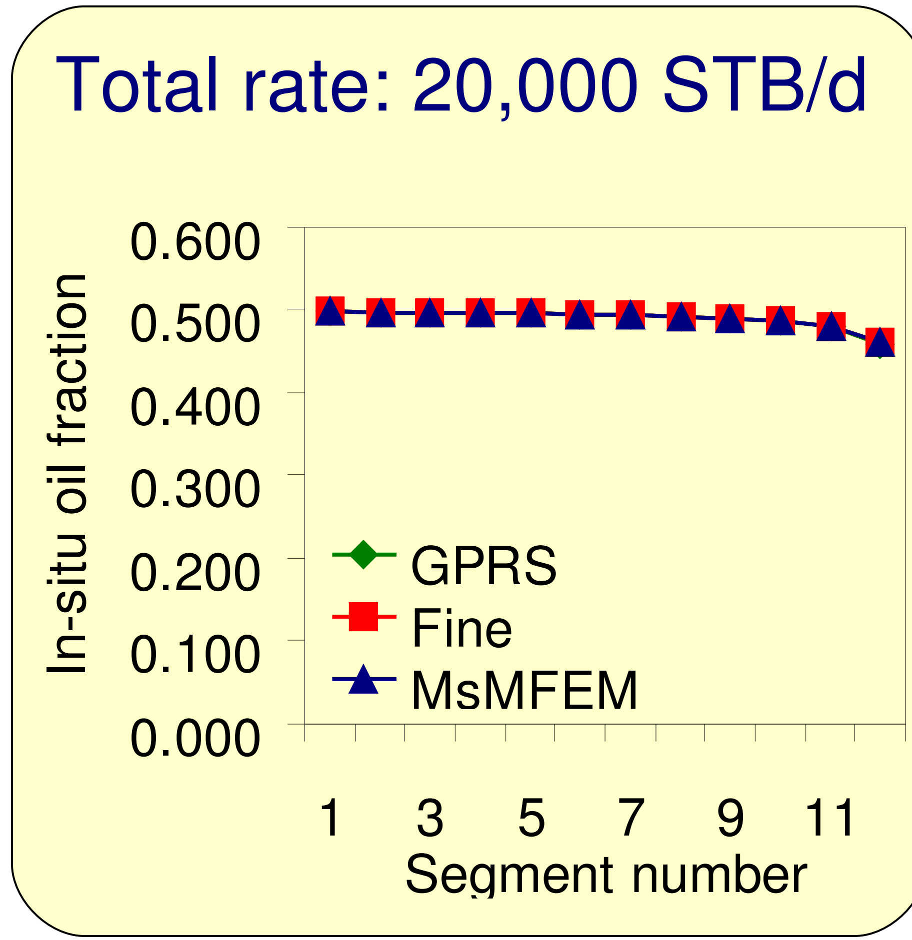
Total rate: 20,000 STB/d



Numerical Example 1:

In-situ oil fraction:



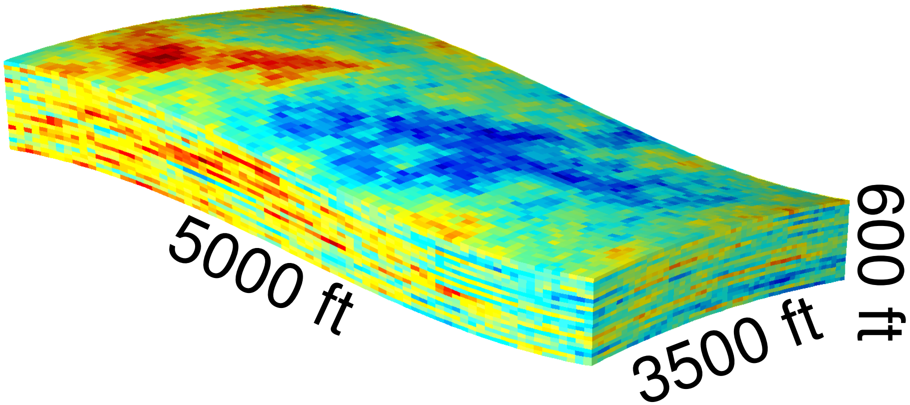


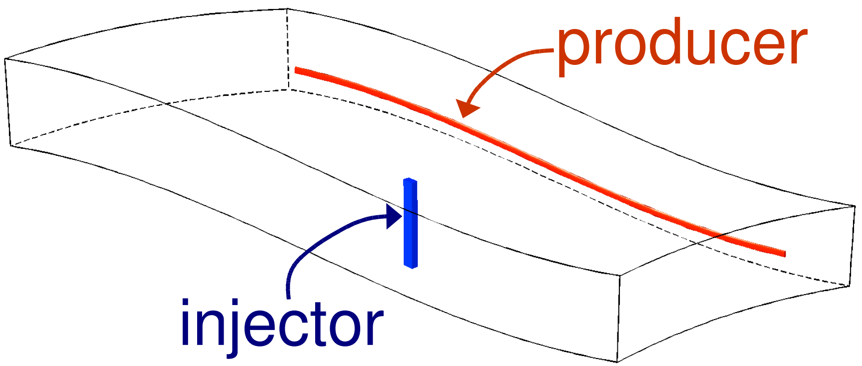
Numerical Example 2:

Long inclined producer in heterogeneous reservoir

- Two-phase oil/water incompressible flow
- Prescribed total flowrate
- Quadratic relative perms, $\mu_o/\mu_w = 10$
- Initially saturated with oil

- 85,000 fine cells
- 62 well segments
- θ between 70° and 80°



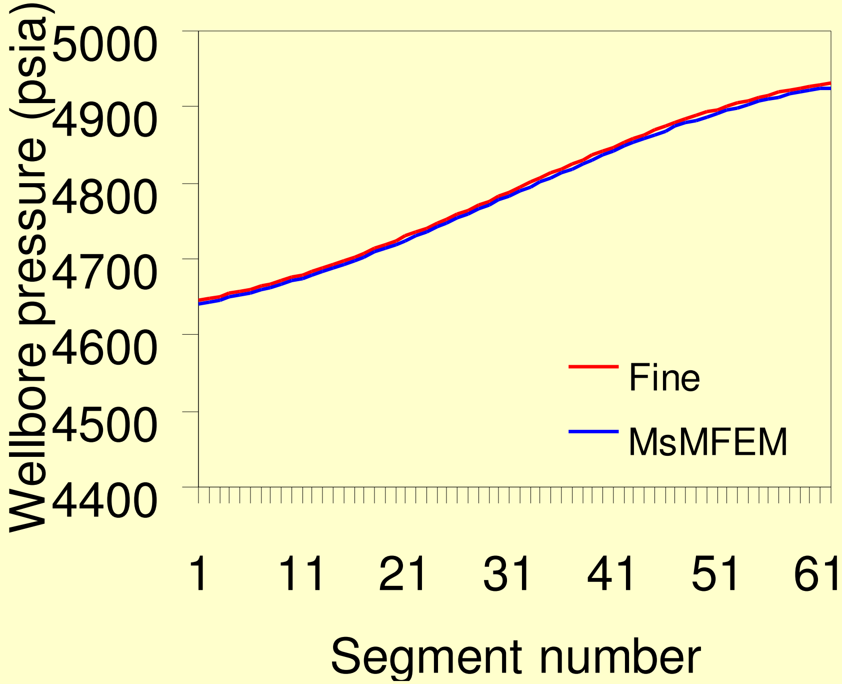


Numerical Example 2:

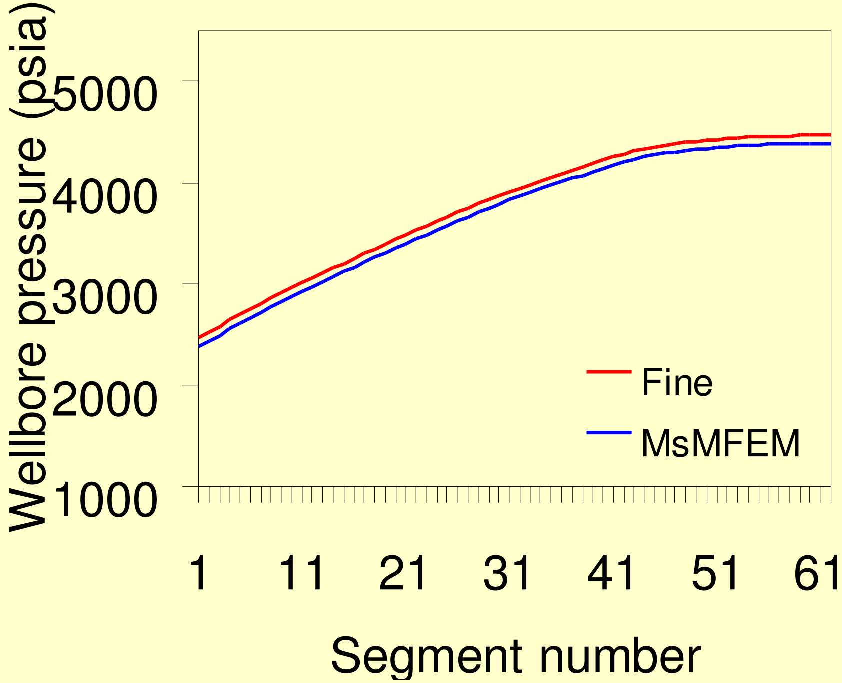
Pressure profiles at 0.12 PVI:

- Coarse grid: 2856 blocks (factor of 30 coarsening)

Flowrate: 4,000 STB/d

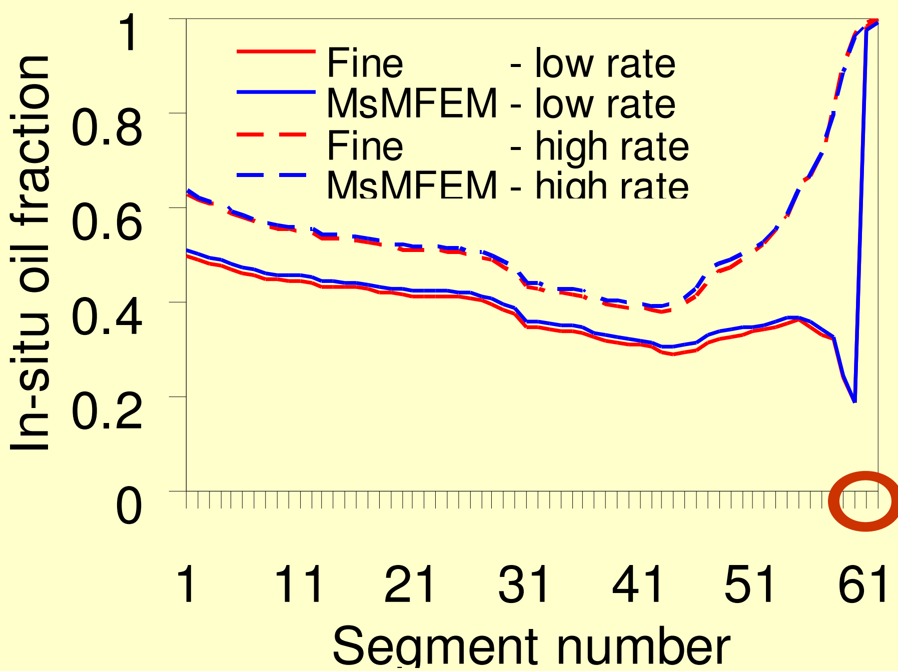


Flowrate: 60,000 STB/d

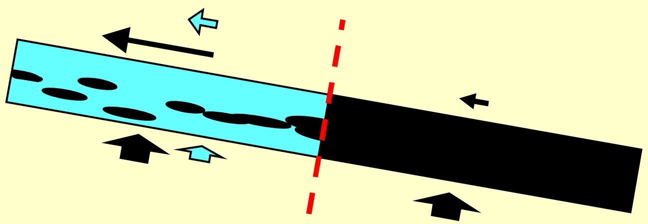


Numerical Example 2:

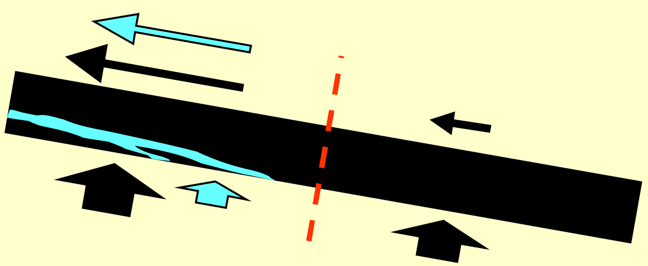
In-situ oil fraction:



Low flowrate:

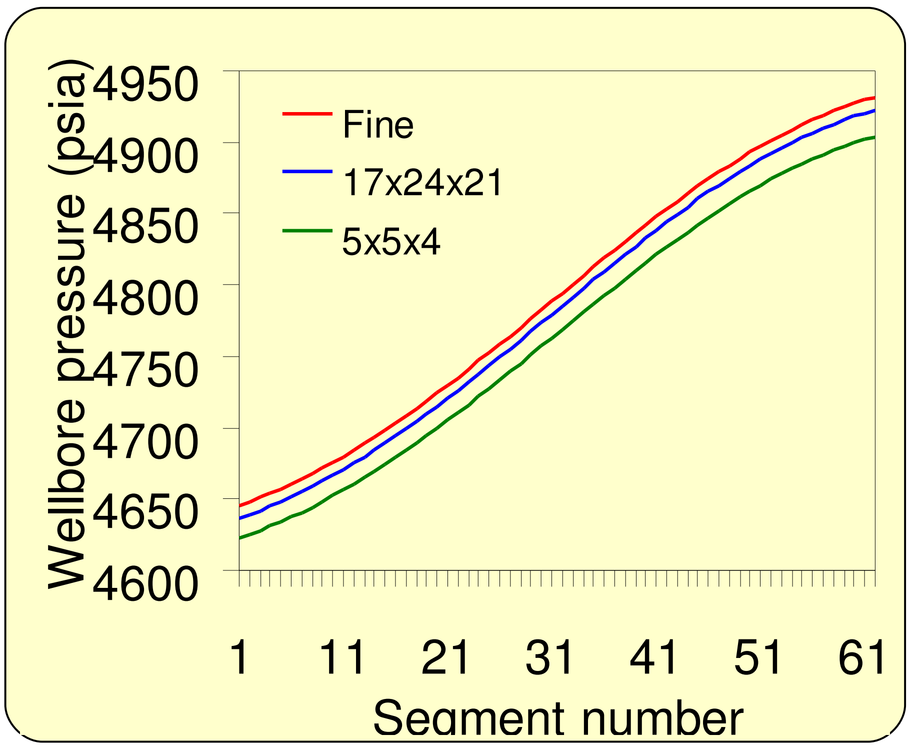


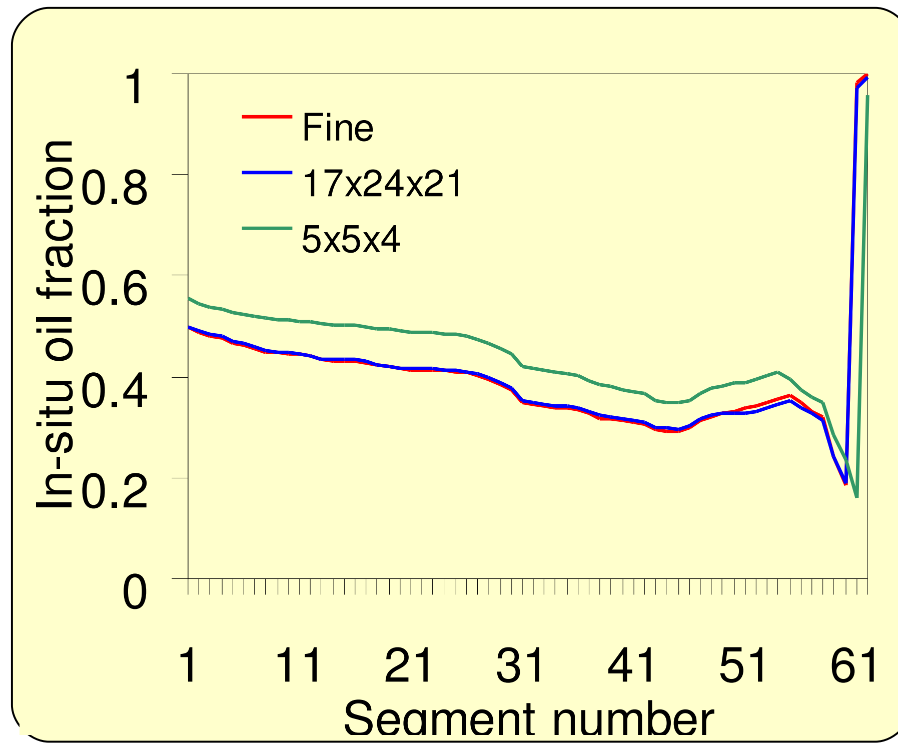
High flowrate:



Numerical Example 2:

- Coarsening factor varied from 10 to 850
- Accuracy degrades with coarsening, but physically reasonable results in all cases





Conclusions / Further Work

- Extended MsMFEM for oil-water systems to include drift-flux wellbore flow model
- Demonstrated and validated through numerical experiments involving vertical and deviated wells
- Achieved accurate results for significantly coarsened models
- Extend to three-phase flow