Direct Flow Simulation of High-Resolution Geo-Cellular Models

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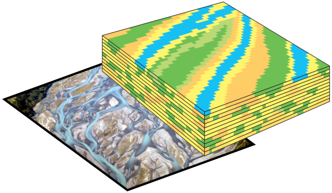
PGP Wine Seminar

Research group

- 5 researchers
- 3–4 postdocs
- 2–4 PhD students
- 1–2 programmers



Collaboration with national and international partners in industry and academia

Research vision:

Direct simulation of complex grid models of highly heterogeneous and fractured porous media - a technology that bypasses the need for upscaling.

http://www.math.sintef.no/GeoScale/

Applications:

- Validation during development of geomodels
- Fast simulations of multiple realizations
- Optimization of production, well placement, etc
- History matching
- Geological storage of CO_2

Funding:

- Strategic research grant and PhD/postdoc grants
- Research grants with end-user involvement (KMB, BIP, SFI)
- Industry projects

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Geological models as direct input to ....

.... efficient multiscale simulation techniques

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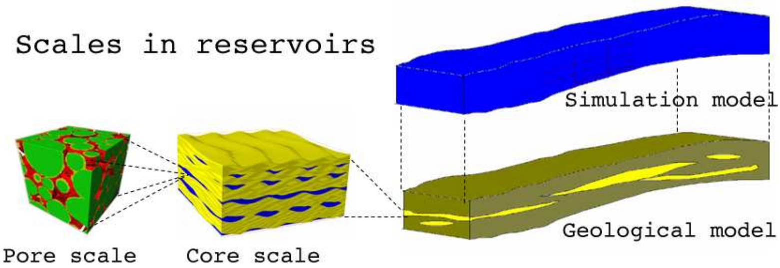
Physical Scales in Porous Media Flow
... one cannot resolve them all at once

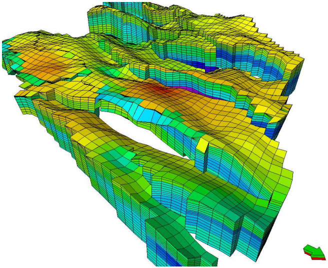
The scales that impact fluid flow in oil reservoirs range from

- the micrometer scale of pores and pore channels
- via dm–m scale of well bores and laminae sediments
- to sedimentary structures that stretch across entire reservoirs.



Geomodels:

- are articulations of the experts perception of the reservoir
- describe the reservoir geometry (horizons, faults, etc)
- give rock parameters (e.g., permeability $\mathbf{K}$ and porosity ϕ) that determine flow

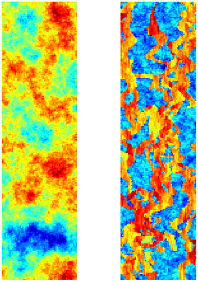


In the following: the term “geomodel” will designate a grid model where rock properties have been assigned to each cell

Geological Models as Direct Input to Simulation

The impact of rock properties

Ex: Brent sequence



Tarbert   Upper Ness

Rock properties are used as parameters in flow models

- Permeability $\mathbf{K}$ spans many length scales and have multiscale structure

$$\max \mathbf{K} / \min \mathbf{K} \sim 10^3\text{–}10^{10}$$

- Details on all scales impact flow

Challenges:

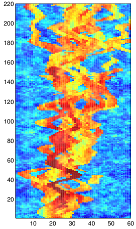
- How much details should one use?
- Need for good linear solvers, preconditioners, etc.

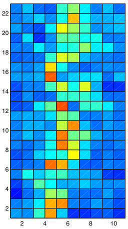
Gap in resolution:

- High-resolution geomodels may have $10^7 - 10^9$ cells
- Conventional simulators are capable of about $10^5 - 10^6$ cells

Traditional solution: upscaling of parameters

- Upscaling the geomodel is not always the answer
  - Loss of details and lack of robustness
  - Bottleneck in the workflow
- Need for fine-scale computations?
- In the future: need for multiphysics on multiple scales?

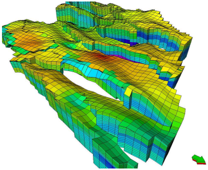




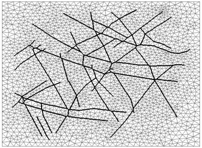
Challenge:

- Industry-standard grids are often nonconforming and contain skewed and degenerate cells
- There is a trend towards unstructured grids
- Standard discretization methods produce wrong results on skewed and rough cells

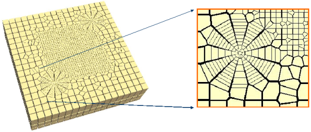
Corner point:



Tetrahedral:



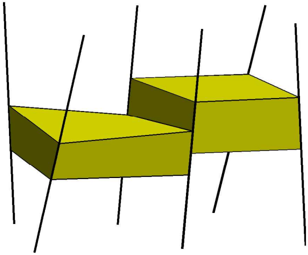
PEBI:

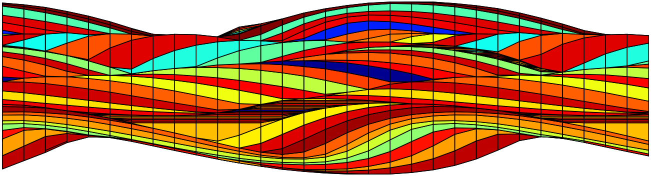


Corner-Point Grids
Industry standard for modelling complex reservoir geology

Specified in terms of:

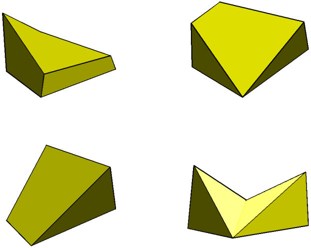
- areal 2D mesh of vertical or inclined pillars
- each volumetric cell is restriced by four pillars
- each cell is defined by eight corner points, two on each pillar



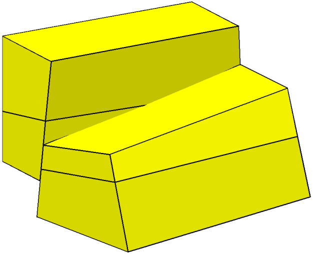


Accurate simulation of industry-standard grid models is challenging!

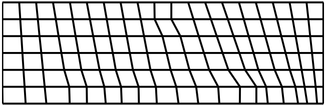
Skew and deformed grid blocks:



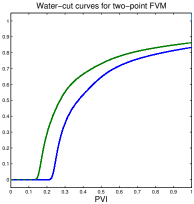
Non-matching cells:

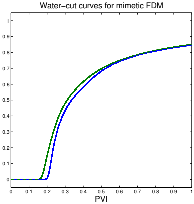


Standard method + skew grids = grid-orientation effects

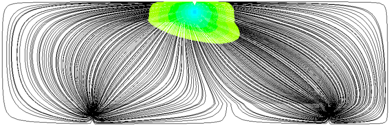


$\mathbf{K}$: homogeneous and isotropic, symmetric well pattern
$\longrightarrow$ symmteric flow

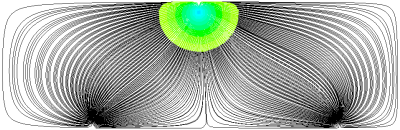




Streamlines with standard method



Streamlines with mimetic method



Direct Simulation on Geomodels

How to approach this vision ...

Research vision:

Direct simulation of complex grid models of highly heterogeneous and fractured porous media - a technology that bypasses the need for upscaling.

Geological models as direct input to ....

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Direct Simulation on Geomodels
How to approach this vision ...

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multiscale pressure solver

fast transport solvers

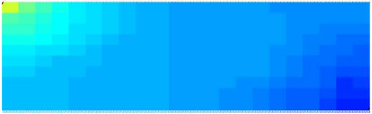
Basic idea:

- Upscaling and downscaling in one step
- Pressure on coarse grid (subresolution near wells)
- Velocity with subgrid resolution everywhere

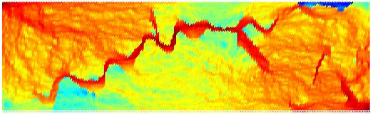
Example: Layer 36 from SPE 10

Pressure field computed with mimetic FDM

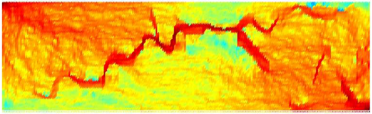
Pressure field computed with 4M



Velocity field computed with mimetic FDM

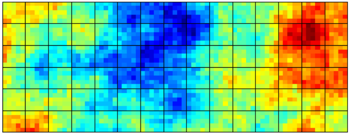


Velocity field computed with 4M



From Upscaling to Multiscale Methods

Standard upscaling:



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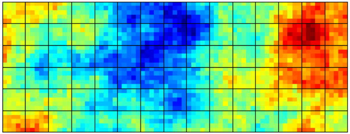

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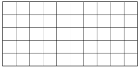
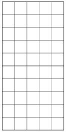
Coarse grid blocks:





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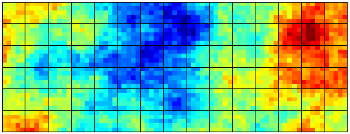

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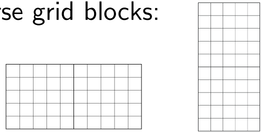

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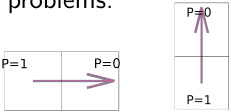
Flow problems:



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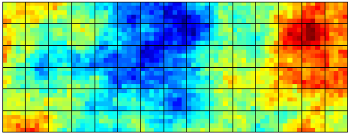

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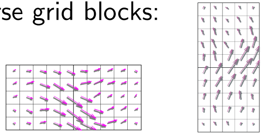

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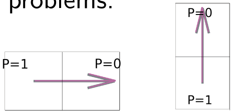

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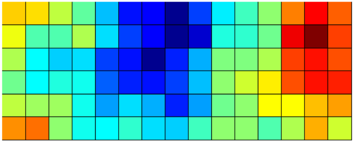

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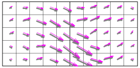

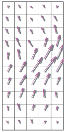

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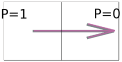

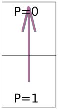

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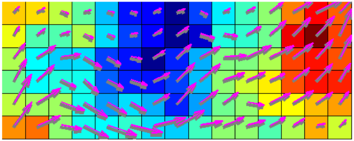

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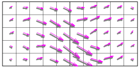

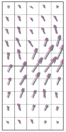

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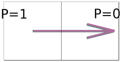

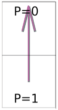

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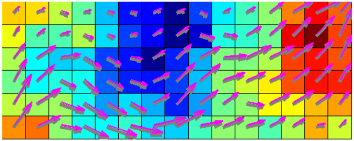

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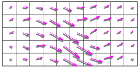

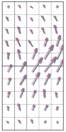

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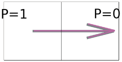
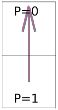
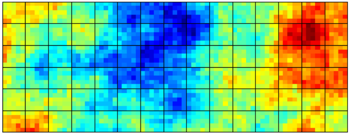




Multiscale method:



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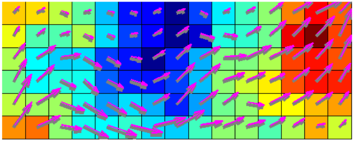

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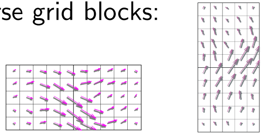

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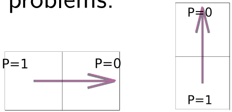

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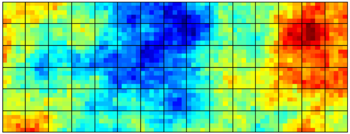

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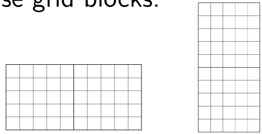

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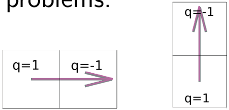
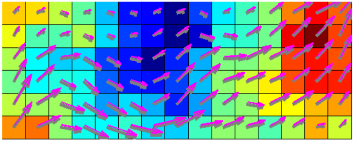


From Upscaling to Multiscale Methods

Standard upscaling:



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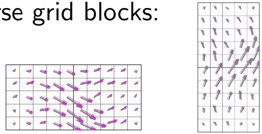

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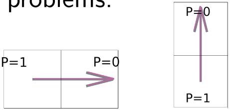

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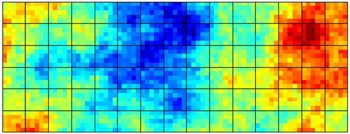

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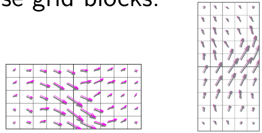

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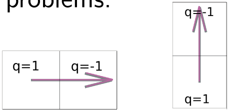

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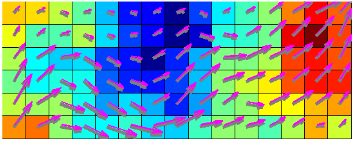

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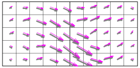

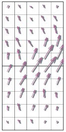

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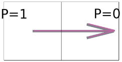
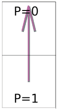
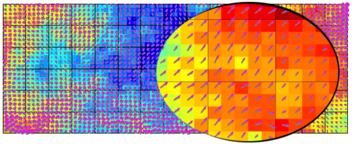
Flow problems:



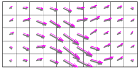


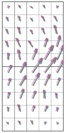
Multiscale method:



⇓ ⇑

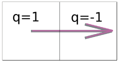
Coarse grid blocks:

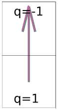




⇓ ⇑

Flow problems:





Ability to handle industry-standard grids

- highly skewed and degenerate cells
- non-matching cells and unstructured connectivities

Compatible with current solvers

- can be built on top of commercial/inhouse solvers
- can utilize existing linear solvers

More efficient than standard solvers

- automated generation of coarse simulation grids
- easy to parallelize
- less memory requirements than fine-grid solvers

Standard finite-element method (FEM):

Piecewise polynomial approximation to pressure

Mixed finite-element methods (MFEM):

Piecewise polynomial approximations simultaneously to pressure and velocity

Standard finite-element method (FEM):

Piecewise polynomial approximation to pressure

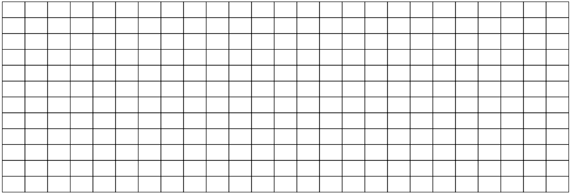
Mixed finite-element methods (MFEM):

Piecewise polynomial approximations simultaneously to pressure and velocity

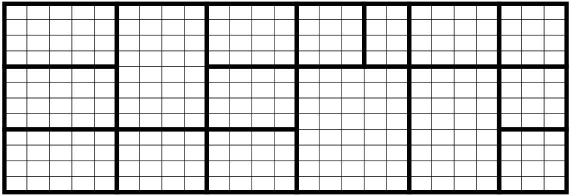
Multiscale mixed finite-element method (MsMFEM):

Velocity approximated in a (low-dimensional) space designed to embody the impact of fine-scale structures.

Assume we are given a *fine* grid with permeability and porosity attached to each fine-grid block:

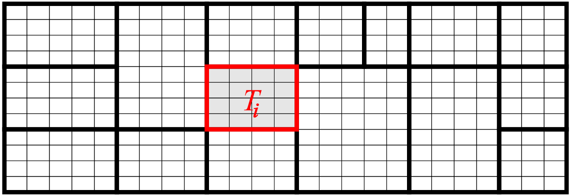


Assume we are given a *fine* grid with permeability and porosity attached to each fine-grid block:



We construct a *coarse* grid, and choose the discretisation spaces U and V^{ms} such that:

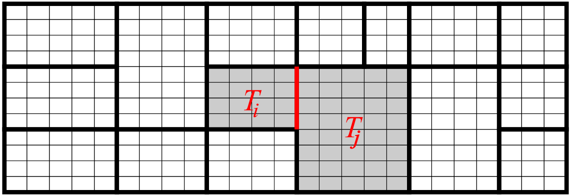
Assume we are given a *fine* grid with permeability and porosity attached to each fine-grid block:



We construct a *coarse* grid, and choose the discretisation spaces U and V^{ms} such that:

- For each coarse block T_i, there is a basis function $\phi_i \in U$.

Assume we are given a *fine* grid with permeability and porosity attached to each fine-grid block:



We construct a *coarse* grid, and choose the discretisation spaces U and V^{ms} such that:

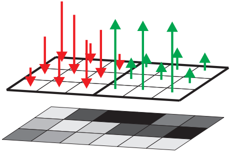
- For each coarse block T_i, there is a basis function $\phi_i \in U$.
- For each coarse edge Γ_{ij}, there is a basis function $\psi_{ij} \in V^{ms}$.

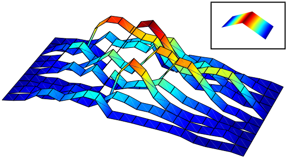
Velocity basis function ψ_{ij}: unit flow through Γ_{ij} defined as

$$\nabla \cdot \psi_{ij} = \begin{cases} w_i(x), & \text{for } x \in T_i, \\ -w_j(x), & \text{for } x \in T_j, \end{cases}$$

and no flow $\psi_{ij} \cdot n = 0$ on $\partial(T_i \cup T_j)$.

Multiscale space:
$V^{ms} = \text{span}\{\psi_{ij} = -\lambda K \nabla \phi_{ij}\}$





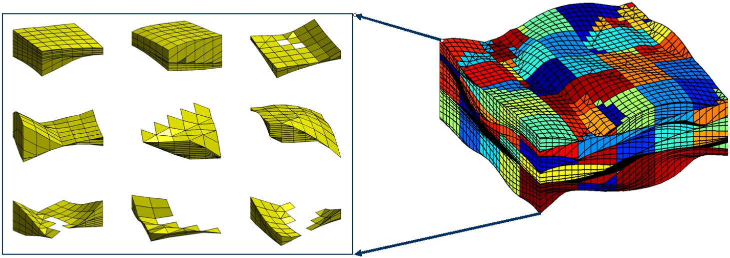
Global velocity:

$v = \sum_{ij} v_{ij}\psi_{ij}$, where v_{ij} are (coarse-scale) coefficients.

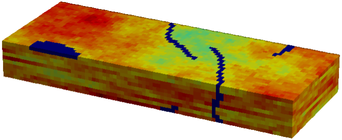
Blocks in coarse grid: connected sets of cells from geomodel

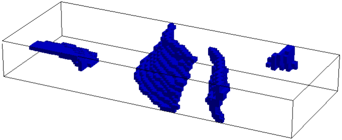
Example: Depositional bed model

Coarse grid obtained with uniform coarsening in index space

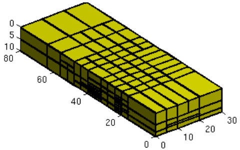


Examples of exotic grids – an indication of 4M's grid flexibility

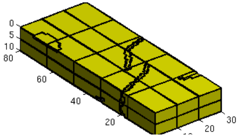




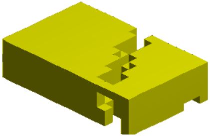
Non-uniform grid, hexahedral cells

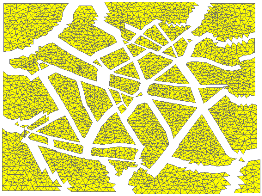


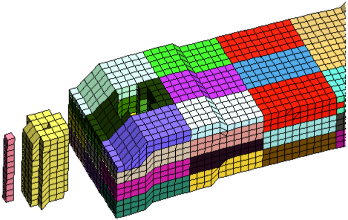
Non-uniform grid, general cells



General grid-cell

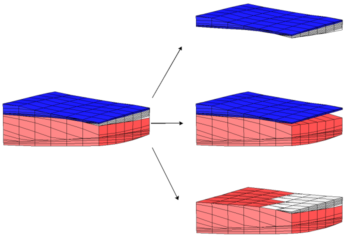




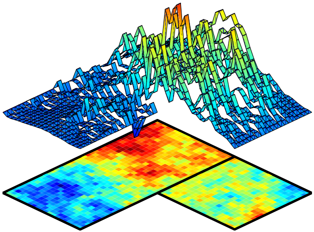


At initial time:

Detect all adjacent blocks



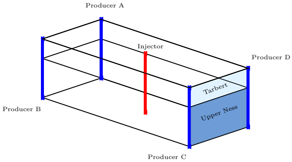
Compute ψ for each domain



For each time step:

- Assemble and solve coarse-grid system
- Recover fine-grid velocity
- Solve fluid-transport equations

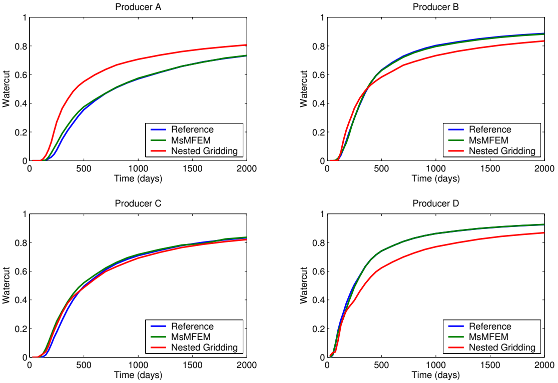
SPE 10, Model 2:



Fine grid: $60 \times 220 \times 85$
Coarse grid: $5 \times 11 \times 17$
2000 days production

4M + streamlines:
2 min 22 sec on 2.4 GHz desktop PC

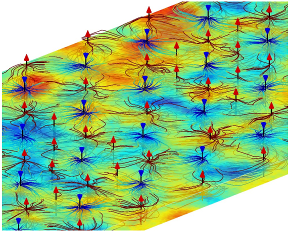
Water-cut curves at the four producers

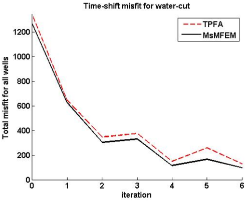


upscaling/downscaling, 4M/streamlines, fine grid

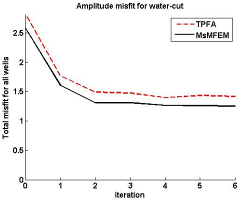
Assimilation of production data to calibrate model

- 1 million cells, 32 injectors, and 69 producers
- 2475 days $\approx$ 7 years of water-cut data





Time-residual



Amplitude-residual

Computation time: $\sim$ 17 min on desktop PC (6 iterations).

Research vision:

Direct simulation of complex grid models of highly heterogeneous and fractured porous media - a technology that bypasses the need for upscaling.

Geological models as direct input to ....

.... efficient multiscale simulation techniques

multiscale pressure solver

fast transport solvers

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Direct simulation of complex grid models of highly heterogeneous and fractured porous media - a technology that bypasses the need for upscaling.

Geological models as direct input to ....

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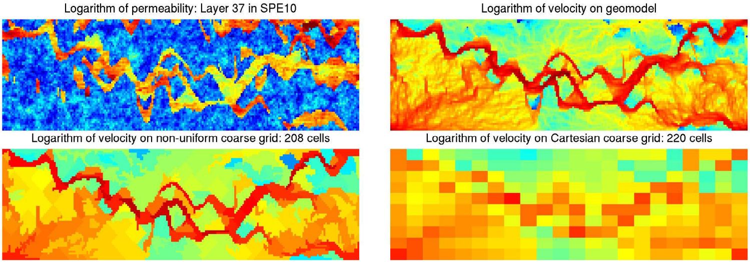
fast transport solvers

Adaptive Model Reduction of Transport Grids
Flow-based nonuniform coarsening

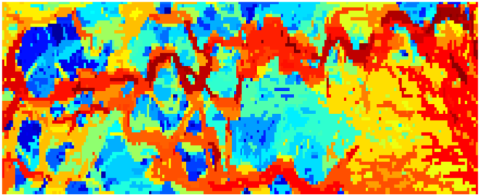
Task

Given the ability to model velocity on geomodels and transport on coarse grids:
Find a suitable coarse grid that best resolves fluid transport and minimizes accuracy loss.

SPE 10, Layer 37



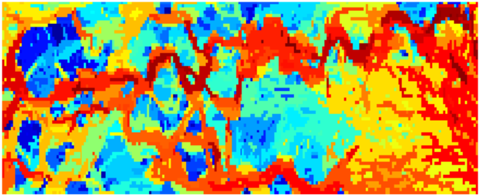
Step 1: Segment ln $|v|$ into N level sets



Robust choice: $N = 10$

Step 1: 1411 cells

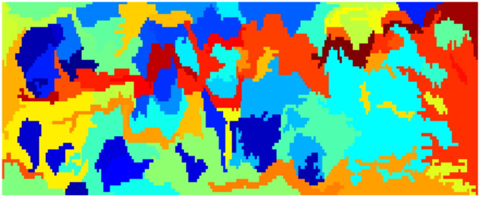
Step 1: Segment ln $|v|$ into N level sets



Robust choice: $N = 10$

Step 1: 1411 cells

Step 2: Combine small blocks ($|B| < c$) with a neighbour

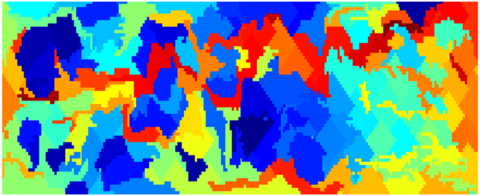


Merge B and B' if

$$\frac{1}{|B|}\int_B \ln|v| \approx \frac{1}{|B'|}\int_{B'} \ln|v|$$

Step 2: 94 cells

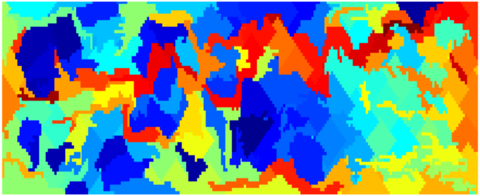
Step 3: Refine blocks with too much flow ($\int_B \ln |v| dx > C$)



Build B' inwards from ∂B
Restart with $B = B \setminus B'$

Step 3: 249 cells

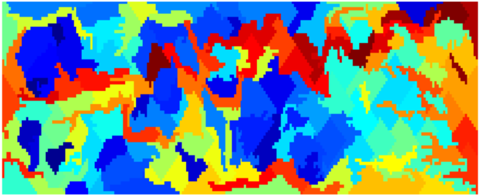
Step 3: Refine blocks with too much flow ($\int_B \ln |v| dx > C$)



Build B' inwards from ∂B
Restart with $B = B \setminus B'$

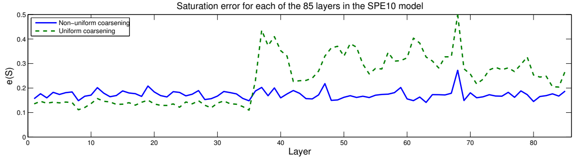
Step 3: 249 cells

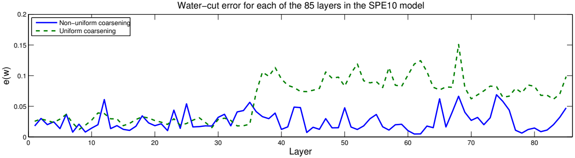
Step 4: Combine small blocks with a neighbouring block



Step 2 repeated

Step 4: 160 cells



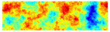
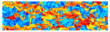


Geomodel:
$60 \times 220 = 13\,200$

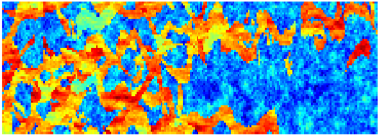
Uniform:
$15 \times 44 = 660$

Non-uniform:
619–734 blocks

Observations:

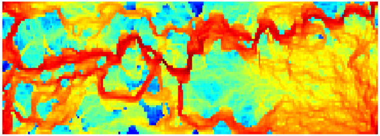
- First 35 layers:  $\Rightarrow$ uniform grid adequate.
- Last 50 layers:  $\Rightarrow$ uniform grid inadequate.
- Non-uniform grid gives consistent results for all layers.

Logarithm of permeability: Layer 68

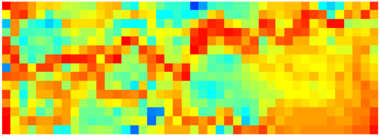


Geomodel: 13200 cells

Logarithm of velocity on geomodel

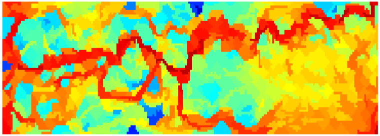


Logarithm of velocity on Cartesian coarse grid



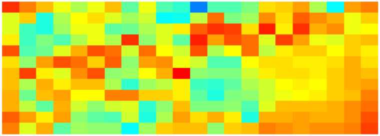
Coarse grid: 660 cells

Logarithm of velocity on non-uniform coarse grid



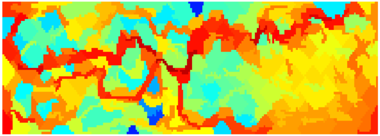
Coarse grid: 649 cells

Logarithm of velocity on Cartesian coarse grid



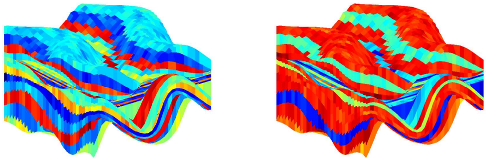
Coarse grid: 264 cells

Logarithm of velocity on non-uniform coarse grid

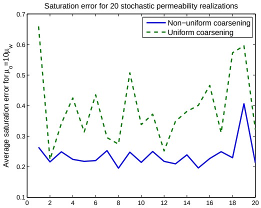


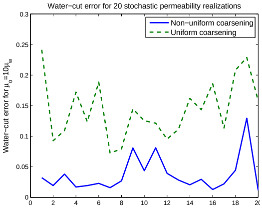
Coarse grid: 257 cells

Example 2: Depositional bed, 20 lognormal realizations, q5-spot



⇐ 2 realizations
15206 cells





Uniform:
838 blocks

Non-uniform:
647–704 blocks

Observations:

- Coarsening algorithm applicable to unstructured grids — accuracy consistent with observations for SPE10 models.
- Results obtained with uniform grid (in index space) inaccurate.

Fast Methods Based on Topological Sorting

Flow models are typically on the form

$$au + \mathbf{v} \cdot \nabla f(u) = b(u), \qquad u \text{ given on } \partial\Omega^-$$

Examples:

- Steady-state tracer: $\mathbf{v} \cdot \nabla c = 0$
- Time-of-flight: $\mathbf{v} \cdot \nabla \tau = \phi$
- Implicit schemes for multiphase/multicomponent transport:

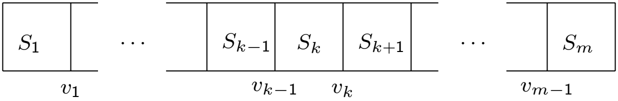
$$S^{n+1} + \Delta t \mathbf{v} \cdot \nabla f(S^{n+1}) = \Delta t q(S^{n+1}) + S^n$$

Basic idea

1. Utilize the unidirectional flow property to solve cell by cell
2. High order: discontinous Galerkin + upwind flux to preserve unidirectional flow property

Fast Methods Based on Topological Sorting

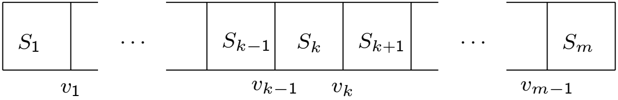
Motivation: Implicit scheme in 1D



First-order upwind scheme ($v_k > 0$, $\forall k$):

$$\frac{\phi}{\Delta t}(S_k^{n+1} - S_k^n) + \frac{1}{\Delta x}\Big(v_{k-1} f(S_{k-1}^{n+1}) - v_k f(S_k^{n+1})\Big) = Q_k(S_k^{n+1})$$

Lower triangular matrix $\Longrightarrow$ equations can be solved in sequence



First-order upwind scheme ($v_k > 0$, $\forall k$):

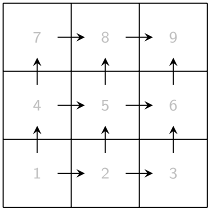
$$\frac{\phi}{\Delta t}(S_k^{n+1} - S_k^n) + \frac{1}{\Delta x}\Big(v_{k-1} f(S_{k-1}^{n+1}) - v_k f(S_k^{n+1})\Big) = Q_k(S_k^{n+1})$$

Lower triangular matrix $\Longrightarrow$ equations can be solved in sequence

Multidimensions

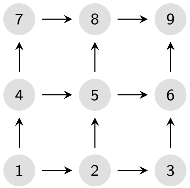
Same idea applies by using *a topological sort* of the directed graph of fluxes

Grid and flux matrix

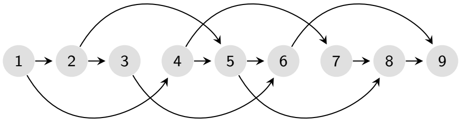


$$V = \begin{bmatrix} * & & & & & & & & \\ * & * & & & & & & & \\ & * & * & & & & & & \\ * & & & * & & & & & \\ & * & & * & * & & & & \\ & & * & & * & * & & & \\ & & & * & & & * & & \\ & & & & * & & * & * & \\ & & & & & * & & * & * \end{bmatrix}$$

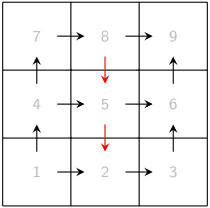
Graph interpretation



Flattened graph (unidirectional)

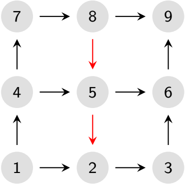


Grid and flux matrix

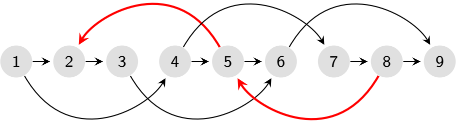


$$V = \begin{bmatrix} \ast & & & & & & & & \\ \ast & \ast & & & \ast & & & & \\ & \ast & \ast & & & & & & \\ \ast & & & \ast & & & & & \\ & & & \ast & \ast & & & \ast & \\ & & \ast & & \ast & \ast & & & \\ & & & \ast & & & \ast & & \\ & & & & & & \ast & \ast & \\ & & & & & \ast & & \ast & \ast \end{bmatrix}$$

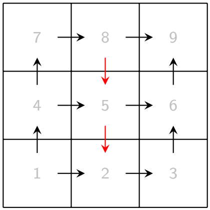
Graph interpretation



Flattened graph

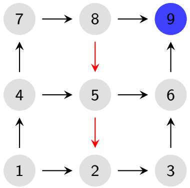


Grid and flux matrix



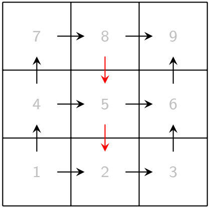
$$V = \begin{bmatrix} \ast & & & & & & & & \\ \ast & \ast & & & {\color{red}\ast} & & & & \\ & \ast & \ast & & & & & & \\ \ast & & & \ast & & & & & \\ & & & \ast & \ast & & & {\color{red}\ast} & \\ & & \ast & & \ast & \ast & & & \\ & & & \ast & & & \ast & & \\ & & & & & & \ast & \ast & \\ & & & & & \ast & & \ast & \ast \end{bmatrix}$$

Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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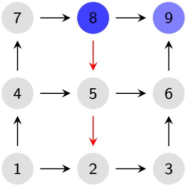
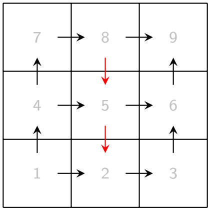
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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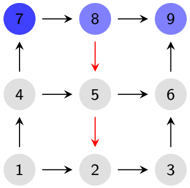
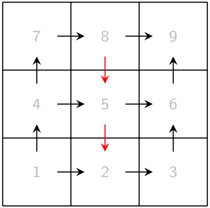
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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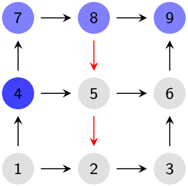
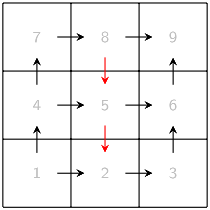
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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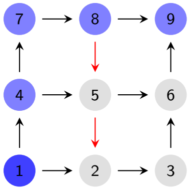
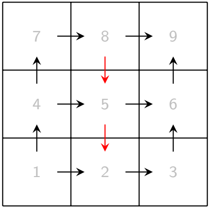
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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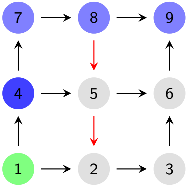
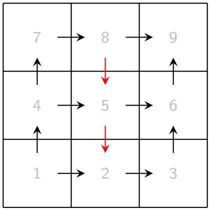
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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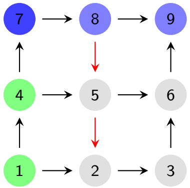
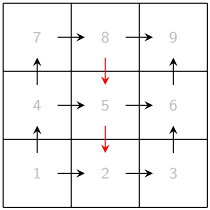
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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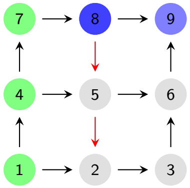
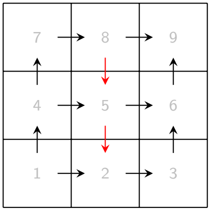
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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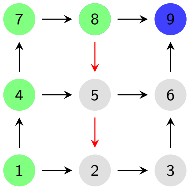
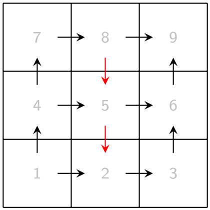
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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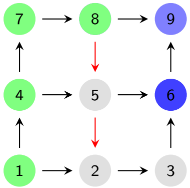
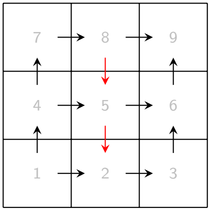
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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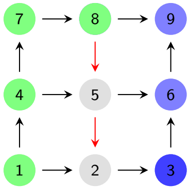
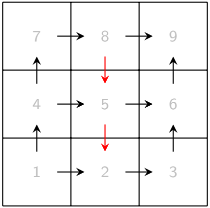
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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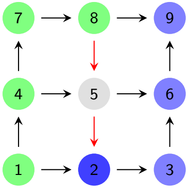
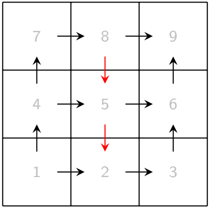
Graph interpretation and topological sorting



Flattened graph (unidirectional)

Grid and flux matrix



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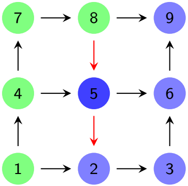
Graph interpretation and topological sorting

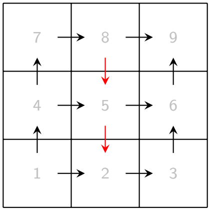


Flattened graph (unidirectional)

Fast Methods Based on Topological Sorting

Motivation: Heterogeneous quarter-five spot in 2D

Grid and flux matrix



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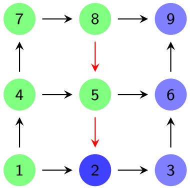
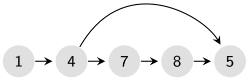
Graph interpretation and topological sorting



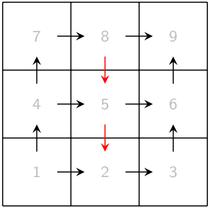
Flattened graph (unidirectional)



Fast Methods Based on Topological Sorting

Motivation: Heterogeneous quarter-five spot in 2D

Grid and flux matrix



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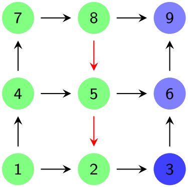
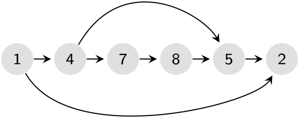
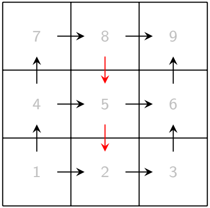
Graph interpretation and topological sorting



Flattened graph (unidirectional)



Grid and flux matrix



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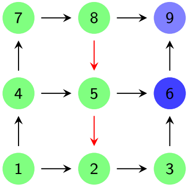
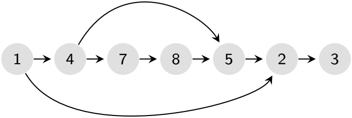
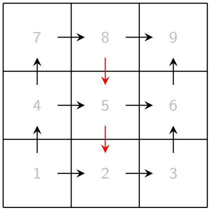
Graph interpretation and topological sorting



Flattened graph (unidirectional)



Grid and flux matrix



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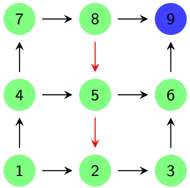
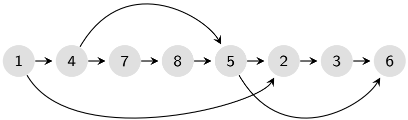
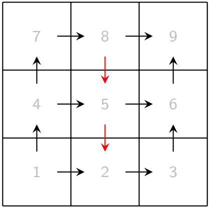
Graph interpretation and topological sorting



Flattened graph (unidirectional)



Grid and flux matrix



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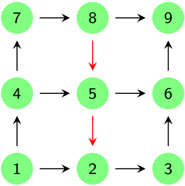
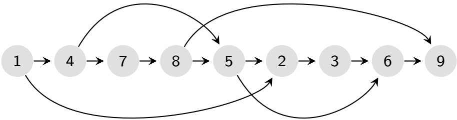
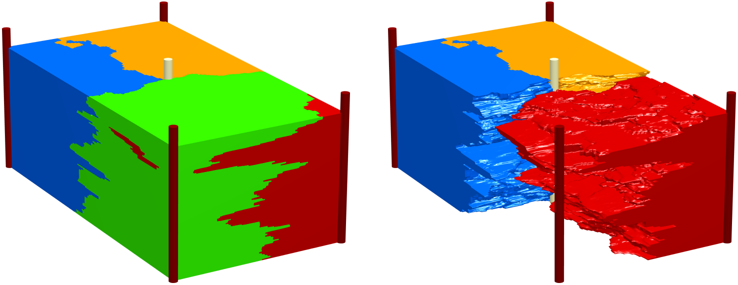
Graph interpretation and topological sorting



Flattened graph (unidirectional)





SPE 10, Model 2, 60 × 220 × 85 (1.122 million grid blocks)

Stationary tracer:

Solve $\mathbf{v} \cdot \nabla c = q_i$ for i wells
Contour $c = 0.5$

Scheme: dG(n)+upwinding

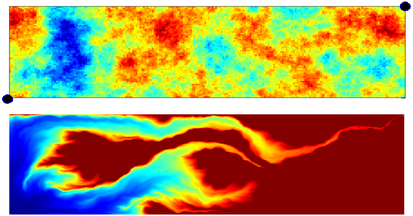
Timings:

order	dof's	time
0	1	3.1 sec
1	4	9.9 sec
2	10	86.8 sec

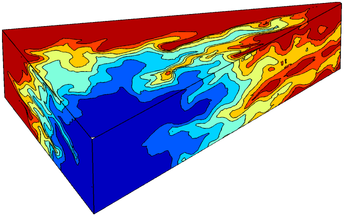
CPU: AMD Athlon X2 4400+

Time-of-flight

- Travel time for a neutral particle injected at boundary/well
- Timelines for single phase flow



Layer 1 of SPE 10



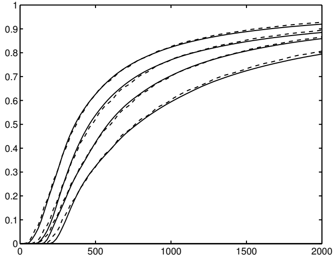
$64 \times 64 \times 16$ grid, vertical q5-spot

Key idea:

Implicit time discretization: element-wise Newton-Raphson solution gives high efficiency.

- $\mathcal{O}(n)$ operations for n unknowns
- Local control over Newton iteration.
- Small memory requirements.
- Small, simple code.
- Well-known *conservative* discretisation.
- Valid for general polyhedral grids.

Water-cut, SPE 10, Model 2



Δt=20 days, 2 minutes

Summary

The research will enable simulator technology to better aid more work processes – by striking balances between reduced computational time, geological representation, and complexity of flow physics

A key to efficient simulation methods – operator splitting:

- Multiscale pressure solvers:
  - Upscaling and downscaling in one step
  - Robust and efficient alternative to upscaling
  - Flow field on coarse, intermediate, and fine grid
- Fast transport solvers, (coarse–intermediate–fine grids):
  - Adaptive nonuniform coarsening
  - Discontinuous Galerkin with topological sorting
  - Streamlines