Coarsening of three-dimensional structured and unstructured grids for subsurface flow

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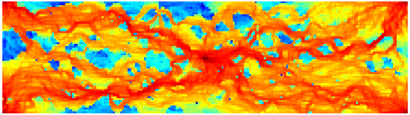
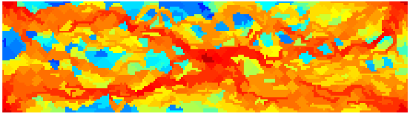
Logarithm of velocity on geomodel



Logarithm of velocity on coarse grid



Task: Given ability to model velocity on geomodels, and transport on coarse grids:

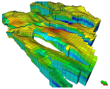
Find a suitable coarse grid that resolves flow patterns and minimize accuracy loss.

Today:

Geomodels too large and complex for flow simulation:
Upscaling performed to obtain

- Simulation grid(s).
- Effective parameters and pseudofunctions.

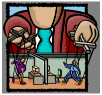
Reservoir simulation workflow



Geomodel →

Upscaling →

Flow simulation →



Management

Tomorrow:

Earth Model shared between geologists and reservoir engineers —
Simulators take Earth Model as input, users specify grid-resolution to fit available computer resources and project requirements.

Objectives

Main objective:

Develop a generic grid coarsening algorithm for reservoir simulation that resolves dominating flow patterns.

- *generic:* one implementation applicable to all types of grids.
- *resolve flow patterns:* separate high flow and low flow regions.

Secondary objective:

Reduce the need for pseudofunctions.

Simulation model and solution strategy

Simulation model

Pressure equation and component mass-balance equations

Primary variables:

- Darcy velocity v,
- Liquid pressure p_o,
- Saturations s_j, j=aqueous, liquid, vapor.

Iterative sequential solution strategy:

$$\begin{aligned} v_{\nu+1} &= v(s_{j,\nu}), \\ p_{o,\nu+1} &= p_o(s_{j,\nu}), \end{aligned} \qquad s_{j,\nu+1} = s_j(p_{o,\nu+1}, v_{\nu+1}).$$

(Fully implicit with fixed point rather than Newton iteration).

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(Fully implicit with fixed point rather than Newton iteration).

Advantages with sequential solution strategy:

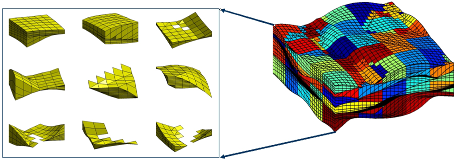
- Grid for pressure and mass balance equations may be different.
- Multiscale methods may be used to solve pressure equation.
- Pressure eq. allows larger time-steps than mass balance eqs.

Discretization

Pressure equation:

- **Solution grid:** Geomodel — no effective parameters.
- **Discretization:** Multiscale mixed / mimetic method

Coarse grid: obtained by up-gridding in index space

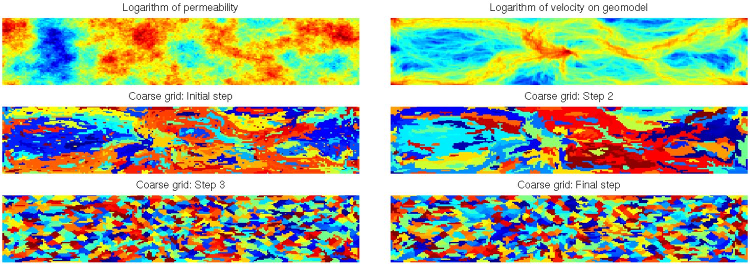


Mass balance equations:

- **Solution grid:** Non-uniform coarse grid.
- **Discretization:** Two-scale upstream weighted FV method — integrals evaluated on geomodel.
- **Pseudofunctions:** No.

Coarsening algorithm

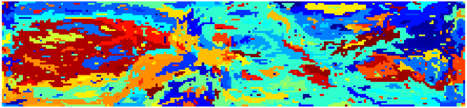
1. Separate regions with different magnitude of flow.
2. Combine small blocks with a neighboring block.
3. Refine blocks with too much flow.
4. Repeat step 2.



Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern.

Separate: Define $g = \ln|v|$ and $D = (\max(g) - \min(g))/10$.

$$\text{Region } i = \{c : \min(g) + (i-1)D < g(c) < \min(g) + iD\}.$$



Initial grid: connected subregions — 733 blocks

Grid generation procedure

Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

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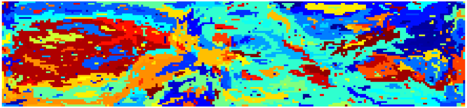
Coarse grid: Initial step

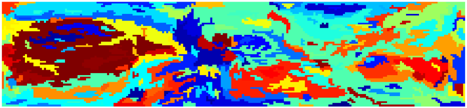


Initial grid: connected subregions — 733 blocks

Merge: If $|B| < c$, merge B with a neighboring block B' with

$$\frac{1}{|B|}\int_B \ln|v|dx \approx \frac{1}{|B'|}\int_{B'} \ln|v|\,dx$$

Coarse grid: Step 2



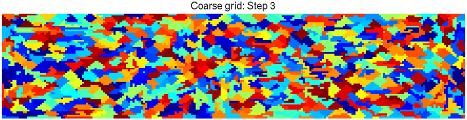
Step 2: 203 blocks

Grid generation procedure

Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

Refine: If criteria — $\int_B \ln|v|dx < C$ — is violated, do

- Start at ∂B and build new blocks B' that meet criteria.
- Define $B = B \backslash B'$ and progress inwards until B meets criteria.



Step3: 914 blocks

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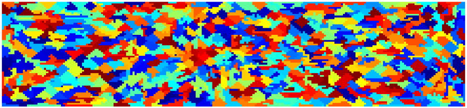
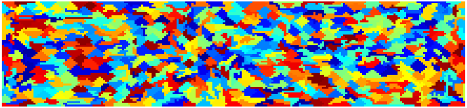
Coarse grid: Step 3



Step3: 914 blocks

Cleanup: Merge small blocks with adjacent block.

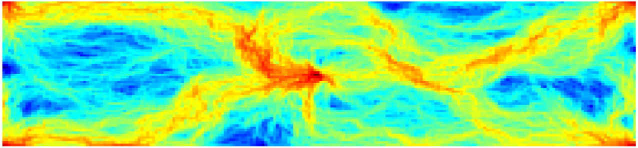
Coarse grid: Final step



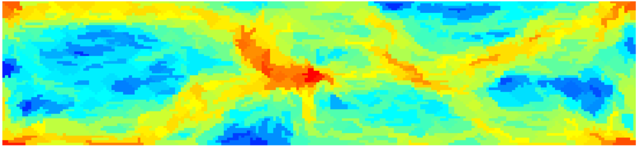
Final grid: 690 blocks

Example: Log of velocity magnitude on different grids

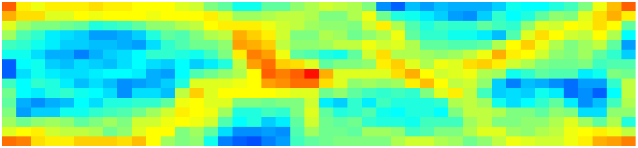
Logarithm of velocity on geomodel



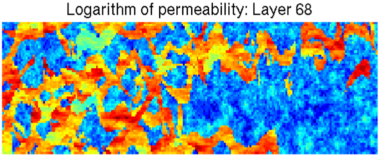
Logarithm of velocity on coarse grid



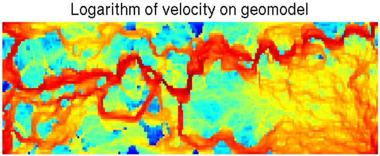
Logarithm of velocity on Cartesian coarse grid

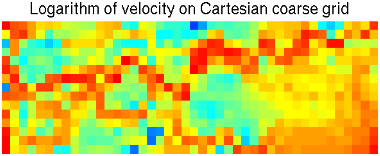


Layer 68 SPE10, 5 spot well pattern

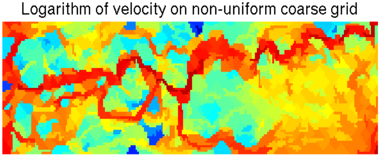


Geomodel: 13200 cells

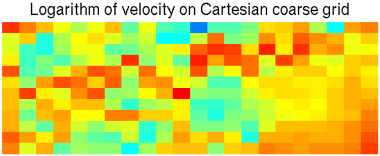




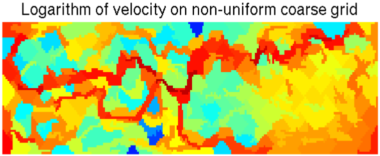
Coarse grid: 660 cells



Coarse grid: 649 cells



Coarse grid: 264 cells



Coarse grid: 257 cells

Experimental setup:

Model: Incompressible two-phase flow (oil and water).

Initial state: Completely oil-saturated.

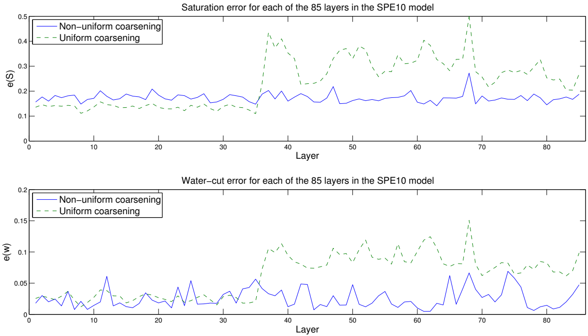
Relative permeability: $k_{rj} = s_j^2, \quad 0 \le s_j \le 1.$

Viscosity ratio: $\mu_o/\mu_w = 10.$

Error measures: (Time measured in PVI)

Saturation error: $e(S) = \int_0^1 \frac{\|S(\cdot,t) - S_{\text{ref}}(\cdot,t)\|_{L^1(\Omega)}}{\|S_{\text{ref}}(\cdot,t)\|_{L^1(\Omega)}}\, dt.$

Water-cut error: $e(w) = \|w - w_{\text{ref}}\|_{L^2([0,1])}/\|w_{\text{ref}}\|_{L^2([0,1])}.$



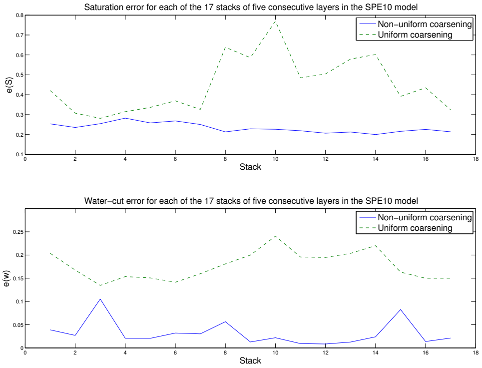
Geomodel: $60 \times 220 \times 1$

Uniform grid: $15 \times 44 \times 1$

Non-uni. grid: 619–734 blocks

Observations:

- First 35 layers smooth $\Rightarrow$ Uniform grid adequate.
- Last 50 layers fluvial $\Rightarrow$ Uniform grid inadequate.
- Non-uniform grid gives consistent results for all layers.



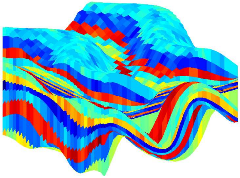
Geomodel:
$60 \times 220 \times 5$

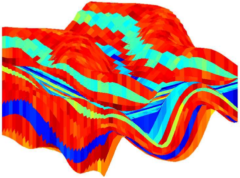
Uniform grid:
$15 \times 44 \times 1$

Non-uniform grid:
655–714 blocks

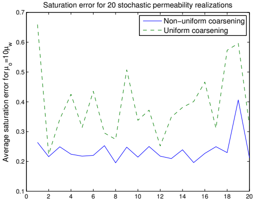
Observations:

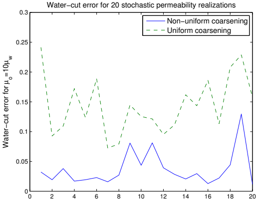
- Uniform grid inadequate, also for stacks from layers 1–35 — lognormal mean of permeability in layers varies significantly.
- Non-uniform grid gives consistent results for all stacks.





⇐ 2 realizations.
Geomodel:
15206 cells





Uniform grid:
838 blocks

Non-uni. grid:
647–704 blocks

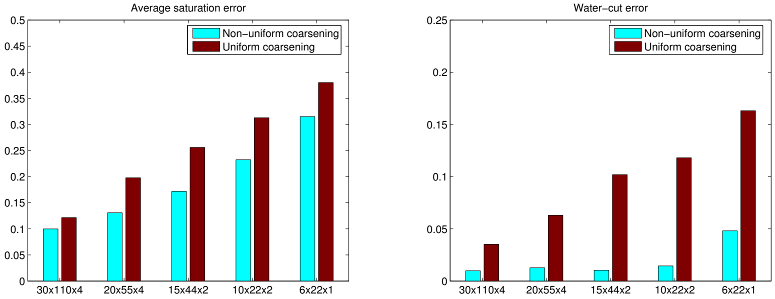
Observations:

- Coarsening algorithm applicable to unstructured grids
  — accuracy consistent with observations for SPE10 models.
- Results obtained with uniform grid (in index space) inaccurate.

Example 4: Geomodel = four bottom layers from SPE10

Robustness with respect to degree of coarsening, 5-spot well pattern

	Number of cells in grid (upscaling factor 4–400)				
Uniform grid	30x110x4 13200	20x55x4 4400	15x44x2 1320	10x22x2 440	6x22x1 132
Non-U. grid	7516	3251	1333	419	150

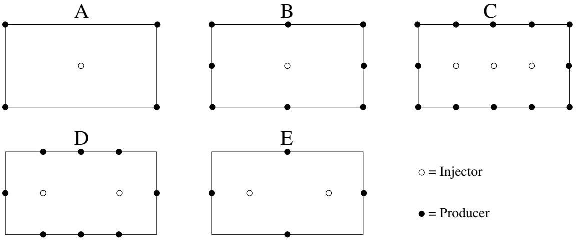


Observations:

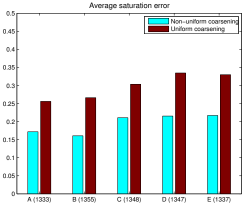
- Non-uniform grid gives better accuracy than uniform grid.
- Water-cut error almost grid-independent for non-uniform grid.

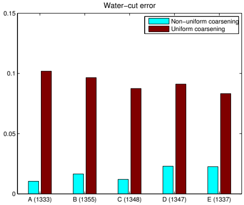
Example 5: Geomodel = four bottom layers from SPE10

Robustness with respect to well configuration, upscaling factor ~ 40



Wellpatterns





Uniform grid:

$15 \times 44 \times 2$

Non-uniform grid

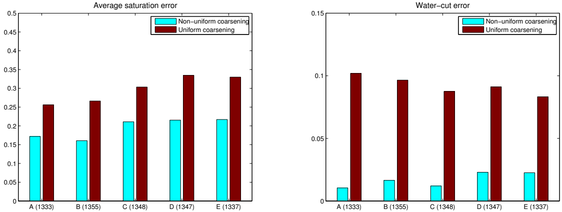
$\sim$ 1320 blocks

- Non-uniform grid gives better accuracy than uniform grid — substantial difference in water-cut error for all cases.

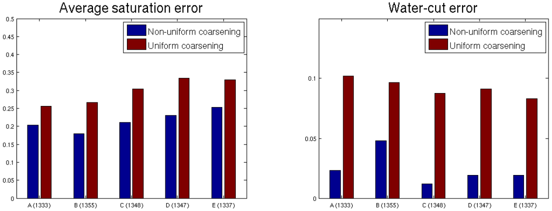
Example 6: Geomodel = four bottom layers from SPE10

Dependency on initial flow conditions, upscaling factor ~ 40

Grid generated with respective well patterns.



Grid generated with pattern C



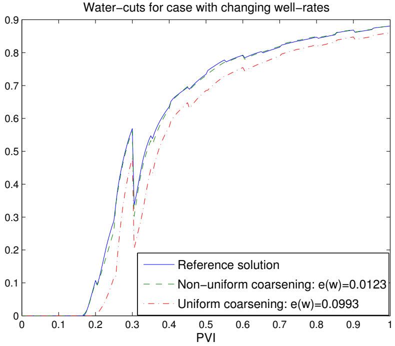
Observation:

Grid resolves high-permeable regions with good connectivity

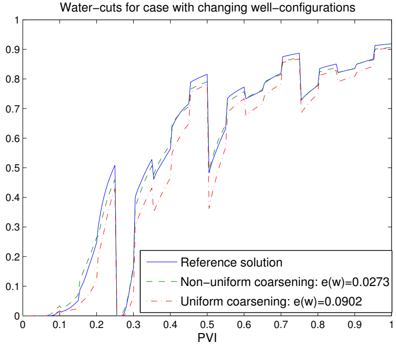
— Grid need *not* be regenerated if well pattern changes.

Example 7: Geomodel = four bottom layers from SPE10

Robustness with respect changing well positions and well rates, upscaling factor ~ 40



5-spot, random prod. rates
grid generated with equal rates



well patterns: 4 cycles A–E
grid generated with pattern C

Observations:

- NU water-cut tracks reference curve closely: 1%–3% error.
- Uniform grid gives $\sim$ 10% water-cut error.

Conclusions

Flashback:

- A generic semi-automated algorithm for generating coarse grids that resolve flow patterns has been presented.
- Solutions are significantly more accurate than solutions obtained on uniform coarse grids with similar number of cells.
- Water-cut error: 1%–3% — pseudofunctions superfluous.
- Grid need **not** be regenerated when flow conditions change!

Potential application:

User-specified grid-resolution to fit available computer resources.

Relation to other methods:

Belongs to family of flow-based grids[a]: designed for flow scenarios where heterogeneity, rather than gravity, dominates flow patterns.

[a] Garcia, Journel, Aziz (1990,1992), Durlofsky, Jones, Milliken (1994,1997)