Generic framework for taking geological models as input for reservoir simulation

Collaborators:

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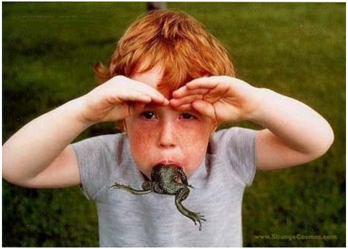
NTNU:	Vegard Stenerud
Stanford	Lou Durlofsky



Nature's input

$\Longrightarrow$

Plausible flow scenario

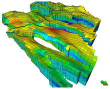
Motivation

Today:

Geomodels too large and complex for flow simulation:
Upscaling performed to obtain

- Simulation grid(s).
- Effective parameters and pseudofunctions.

Reservoir simulation workflow



Geomodel

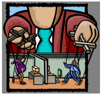
$\longrightarrow$

Upscaling

$\longrightarrow$

Flow simulation

$\longrightarrow$



Management

Tomorrow:

Earth Model shared between geologists and reservoir engineers —
Simulators take Earth Model as input, users specify grid-resolution to fit available computer resources and project requirements.

Objective and implication

Main objective:

Build a generic framework for reservoir modeling and simulation capable of taking geomodels as input.

– *generic:* one implementation applicable to all types of models.

Value: Improved modeling and simulation workflows.

- Geologists may perform simulations to validate geomodel.
- Reservoir engineers gain understanding of geomodeling.
- Facilitate use of geomodels in reservoir management.

Equations:

- Pressure equation

  $c_t \frac{\partial p_o}{dt} + \nabla \cdot v + \sum_j c_j v_j \cdot \nabla p_o = q$

- Mass balance equation for each component

Primary variables:

- Darcy velocity v
- Liquid pressure p_o
- Phase saturations s_j, aqueous, liquid, vapor.

Solution strategy: Iterative sequential

$$\begin{aligned} v_{\nu+1} &= v(s_{j,\nu}), \\ p_{o,\nu+1} &= p_o(s_{j,\nu}), \end{aligned} \qquad s_{j,\nu+1} = s_j(p_{o,\nu+1}, v_{\nu+1}).$$

(Fully implicit with fixed point rather than Newton iteration).

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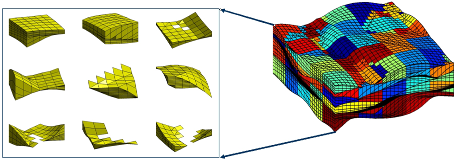
Advantages with sequential solution strategy:

- Grid for pressure and mass balance equations may be different.
- Multiscale methods may be used to solve pressure equation.
- Pressure eq. allows larger time-steps than mass balance eqs.

Pressure equation:

- **Solution grid:** Geomodel — no effective parameters.
- **Discretization:** Multiscale mixed / mimetic method

Coarse grid:
obtained by up-gridding in index space



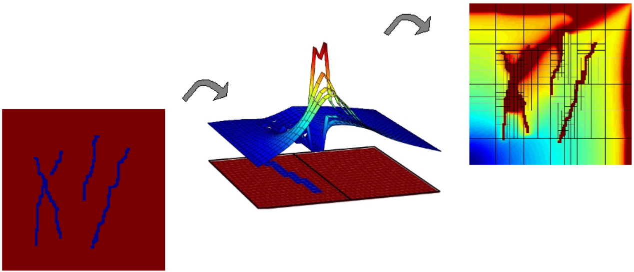
Mass balance equations:

- **Solution grid:** Non-uniform coarse grid.
- **Discretization:** Two-scale upstream weighted FV method — integrals evaluated on geomodel.
- **Pseudofunctions:** No.

Multiscale mixed/mimetic method (4M)

Generic two-scale approach to discretizing the pressure equation:

- Mixed FEM formulation on coarse grid.
- Flow patterns resolved on geomodel with mimetic FDM.



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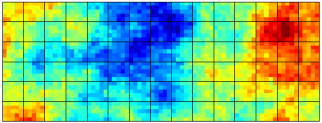

Multiscale mixed/mimetic method

Flow based upscaling versus multiscale method

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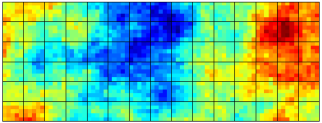

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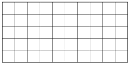
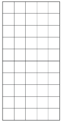
Coarse grid blocks:





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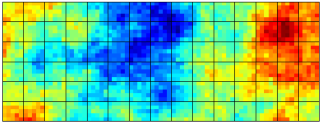

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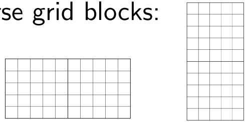

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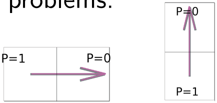

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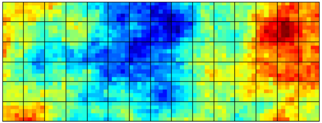

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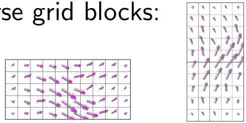

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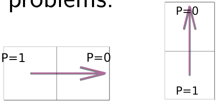

Multiscale mixed/mimetic method

Flow based upscaling versus multiscale method

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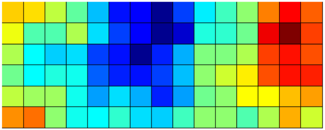

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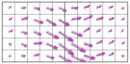

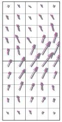

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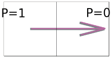

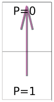

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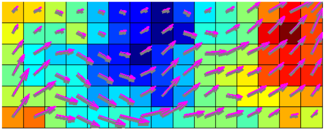

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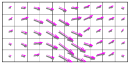

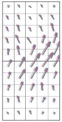

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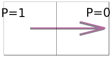

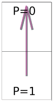

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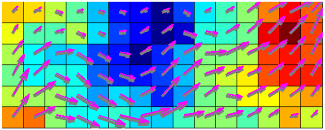

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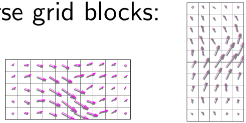

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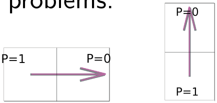
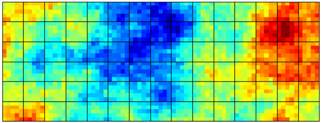


Multiscale method (4M):



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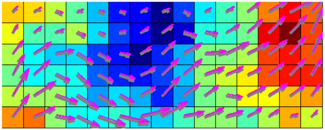

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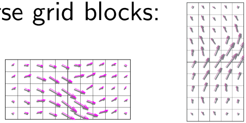

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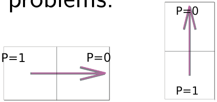

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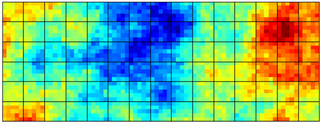

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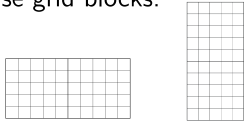

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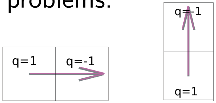

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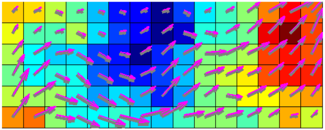

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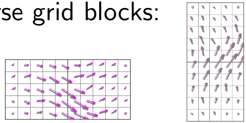

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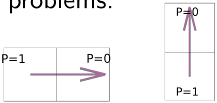

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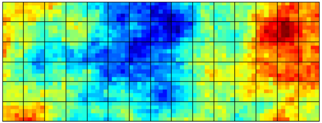

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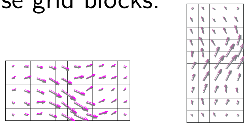

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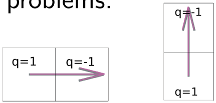

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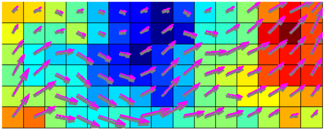

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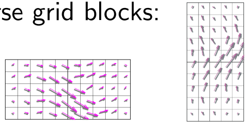

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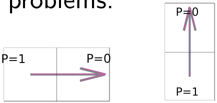

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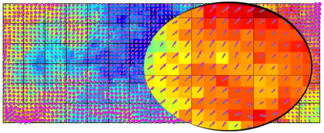

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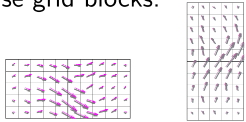

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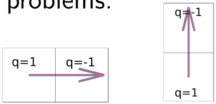


Discrete hybrid formulation: $(u, v)_m = \int_{T_m} u \cdot v \, dx$

Find $v \in V$, $p \in U$, $\pi \in \Pi$ such that for all blocks T_m we have

$$\begin{aligned} (\lambda^{-1} v, u)_m - (p, \nabla \cdot u)_m + \int_{\partial T_m} \pi u \cdot n \, ds &= (\omega g \nabla D, u)_m \\ (c_t \tfrac{\partial p_o}{dt}, l)_m + (\nabla \cdot v, l)_m + (\textstyle\sum_j c_j v_j \cdot \nabla p_o, l)_m &= (q, l)_m \\ \int_{\partial T_m} \mu v \cdot n \, ds &= 0. \end{aligned}$$

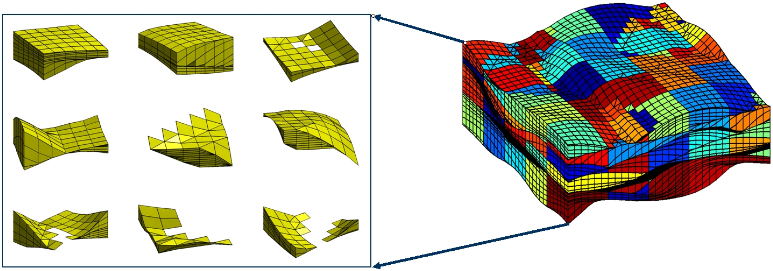
for all $u \in V$, $l \in U$ and $\mu \in \Pi$.

Solution spaces and variables: $\mathcal{T} = \{T_m\}$

$V \subset H^{\text{div}}(\mathcal{T})$,	$U = \mathcal{P}_0(\mathcal{T})$,	$\Pi = \mathcal{P}_0(\{\partial T_m \cap \partial T_n\})$.
v = velocity,	p = block pressures,	π = interface pressures.

Each coarse grid block is a connected set of cells from geomodel.

Example: Coarse grid obtained with uniform coarsening in index space.



Grid adaptivity at well locations:

One block assigned to each cell in geomodel with well perforation.

Definition of approximation space for velocity:

The approximation space V is spanned by basis functions ψ_m^i that are designed to embody the impact of fine-scale structures.

Definition of basis functions:

For each pair of adjacent blocks T_m and T_n, define ψ by

$$\psi = -K\nabla u \text{ in } T_m \cup T_n, \qquad \nabla \cdot \psi = \begin{cases} w_m & \text{in } T_m, \\ -w_n & \text{in } T_n, \end{cases}$$
$$\psi \cdot n = 0 \text{ on } \partial(T_m \cup T_n),$$

Split ψ: $\quad \psi_m^i = \psi|_{T_m}, \quad \psi_n^j = -\psi|_{T_n}.$

Basis functions time-independent if w_m is time-independent.

Role of weight functions

Let $(w_m, 1)_m = 1$ and let v_m^i be coarse-scale coefficients.

$$v = \sum_{m,i} v_m^i \psi_m^i \quad \Rightarrow \quad (\nabla \cdot v)|_{T_m} = w_m \sum_i v_m^i.$$

$\longrightarrow$ w_m gives distribution of $\nabla \cdot v$ among cells in geomodel.

Choice of weight functions

$$\nabla \cdot v \sim c_t \frac{\partial p_o}{dt} + \sum_j c_j v_j \cdot \nabla p_o$$

- Use adaptive criteria to decide when to redefine w_m.
- Use $w_m = \phi$ ($c_t \sim \phi$ when saturation is smooth).

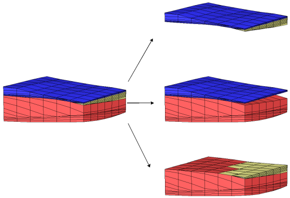
$\longrightarrow$ **Basis functions computed once, or updated infrequently.**

Multiscale mixed/mimetic method

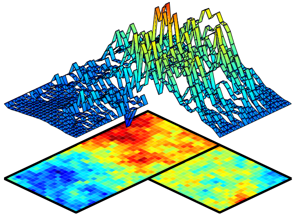
Workflow

At initial time

Detect all adjacent blocks



Compute ψ for each domain



For each time-step:

- Assemble and solve coarse grid system.
- Recover fine grid velocity.
- Solve mass balance equations.

Velocity basis functions computed using mimetic FDM

Mixed FEM for which the inner product $(u, \sigma v)$ is replaced with an approximate explicit form ($u, v \in H^{\text{div}}$ and σ SPD),
— no integration, no reference elements, no Piola mappings.

May also be interpreted as a multipoint finite volume method.

Properties:

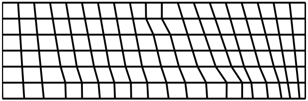
- Exact for linear pressure.
- Same implementation applies to all grids.
- Mimetic inner product *needed* to evaluate terms in multiscale formulation, e.g., $(\psi_m^i, \lambda^{-1}\psi_m^j)$ and $(\omega g \nabla D, \psi_{m,j})$.

Multiscale mixed/mimetic method

Mimetic finite difference method vs. Two-point finite volume method

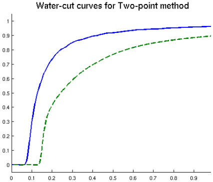
Two-point FD method is "generic", but ...

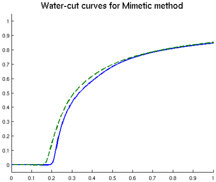
Example:



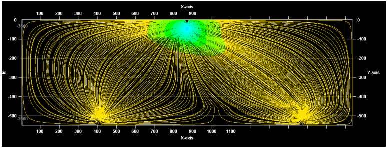
Homogeneous+isotropic, symmetric well pattern
$\longrightarrow$ equal water-cut.

Two-point method + skewed grids
= grid orientation effects.

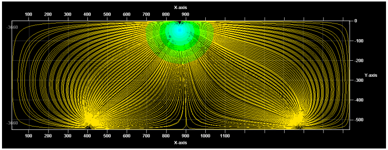




Two-point FV method

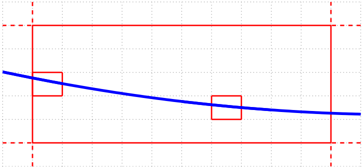


Mimetic FD method



Grid block for cells with a well

- correct well-block pressure
- no near well upscaling
- free choice of well model.



Alternative well models

1. Peaceman model:

   $$q_{\text{perforation}} = -W_{\text{block}}(p_{\text{block}} - p_{\text{perforation}}).$$

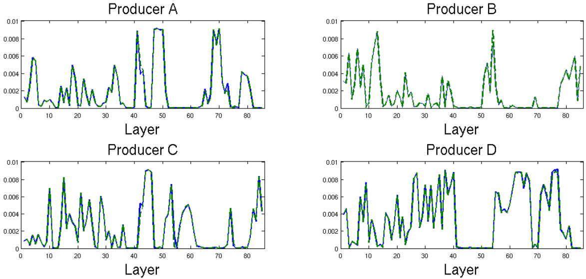
   Calculation of well-index grid dependent.

2. Exploit pressures on grid interfaces:

   $$q_{\text{perforation}} = -\sum_i W_{\text{face}i}(p_{\text{face}i} - p_{\text{perforation}}).$$

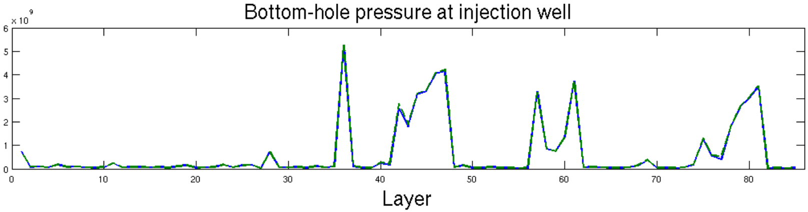
   Generic calculation of $W_{\text{face}i}$.

5-spot: 1 rate constr. injector, 4 pressure constr. producers
Well model: Interface pressures employed.

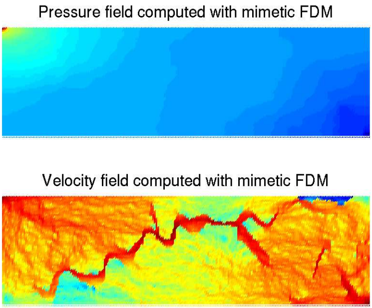


Distribution of production rates

- Reference (60×220)
- Multiscale (10×22)



Example: Layer 36 from SPE10 (Christie and Blunt, 2001).



Primary features

- Coarse pressure solution, subgrid resolution at well locations.
- Coarse velocity solution with subgrid resolution everywhere.

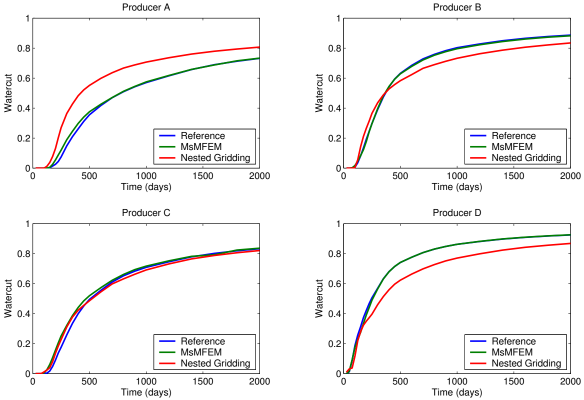
Model: SPE10 model 2, 1.1 M cells, 1 injector, 4 producers.

Coarse grid:
$5 \times 11 \times 17$

- Reference
- 4M
- Upscaling + downscaling

4M+streamlines:
$\sim$ 2 minutes on desktop PC.

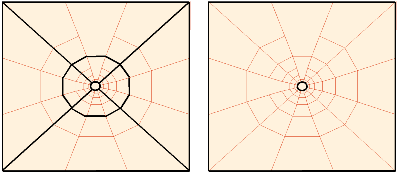
Water-cut curves at producers A–D

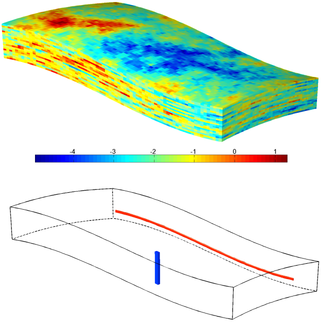


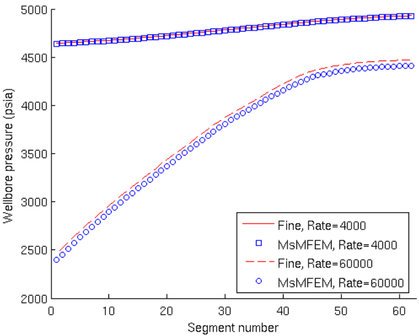
Krogstad and Durlofsky, 2007:

Fine grid to annulus,
block for each well segment

- No well model needed.
- Drift-flux wellbore flow.

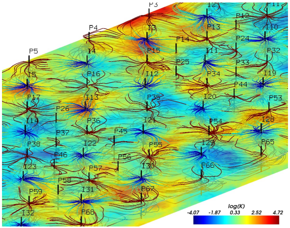


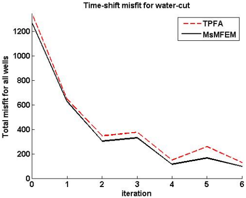




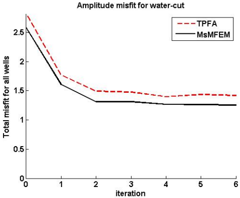
Stenerud, Kippe, Datta-Gupta, and Lie, RSS 2007:

- 1 million cells, 32 injectors, and 69 producers
- Matching travel-time and water-cut amplitude at producers
- Permeability updated in blocks with high average sensitivity
  $\longrightarrow$ Only few multiscale basis functions updated.





Time-residual



Amplitude-residual

Computation time: $\sim$ 17 min. on desktop PC. (6 iterations).

Conclusions

Multiscale mixed/mimetic method:

- Reservoir simulation tool that can take geomodels as input.
- Solutions in close correspondence with solutions obtained by solving the pressure equation directly.
- Computational cost comparable to flow based upscaling.

Applications:

- Reservoir simulation on geomodels
- Near-well modeling / Improved well models
- History matching on geomodels

Coarsening of three-dimensional structured and unstructured grids for subsurface flow

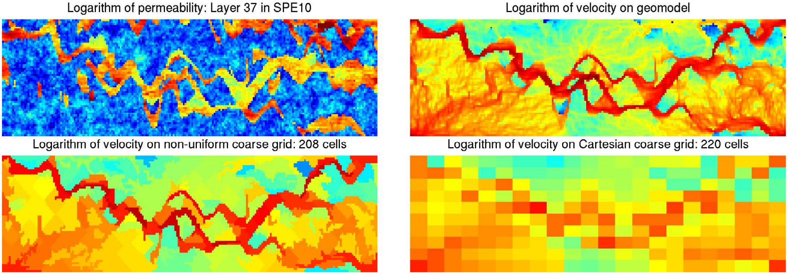
Collaborators:

Vera Louise Hauge, SINTEF ICT

Yalchin Efendiev, Texas A&M

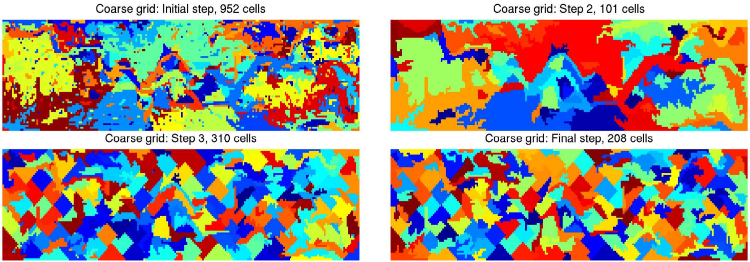
Task: Given ability to model velocity on geomodels, and transport on coarse grids:

Find a suitable coarse grid that resolves flow patterns and minimize accuracy loss.



Coarsening algorithm

1. Separate regions with different magnitude of flow.
2. Combine small blocks with a neighboring block.
3. Refine blocks with too much flow.
4. Repeat step 2.



Example: Layer 37 SPE10 (Christie and Blunt), 5 spot well pattern.

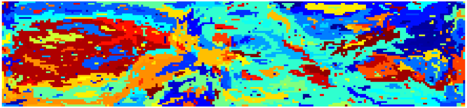
Grid generation procedure

Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

Separate: Define $g = \ln|v|$ and $D = (\max(g) - \min(g))/10$.

$$\text{Region } i = \{c : \min(g) + (i-1)D < g(c) < \min(g) + iD\}.$$

Coarse grid: Initial step



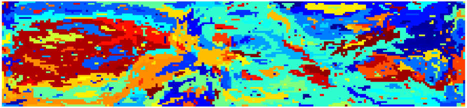
Initial grid: connected subregions — 733 blocks

Grid generation procedure

Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

Separate: Define $g = \ln|v|$ and $D = (\max(g) - \min(g))/10$.

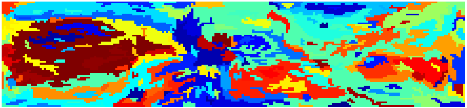
$$\text{Region } i = \{c : \min(g) + (i-1)D < g(c) < \min(g) + iD\}.$$



Initial grid: connected subregions — 733 blocks

Merge: If $|B| < c$, merge B with a neighboring block B' with

$$\frac{1}{|B|}\int_B \ln|v|dx \approx \frac{1}{|B'|}\int_{B'} \ln|v|\,dx$$



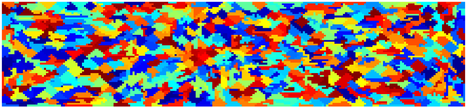
Step 2: 203 blocks

Grid generation procedure

Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

Refine: If criteria — $\int_B \ln|v| dx < C$ — is violated, do

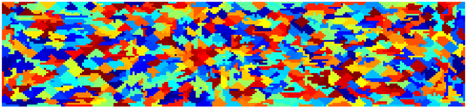
- Start at ∂B and build new blocks B' that meet criteria.
- Define $B = B \backslash B'$ and progress inwards until B meets criteria.



Step3: 914 blocks

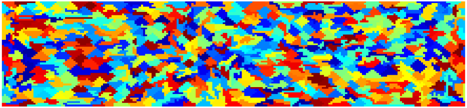
Refine: If criteria — $\int_B \ln|v|dx < C$ — is violated, do

- Start at ∂B and build new blocks B' that meet criteria.
- Define $B = B \backslash B'$ and progress inwards until B meets criteria.



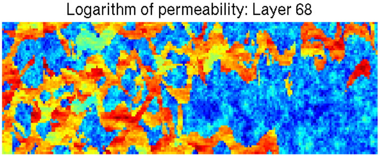
Step3: 914 blocks

Cleanup: Merge small blocks with adjacent block.

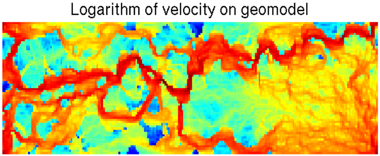


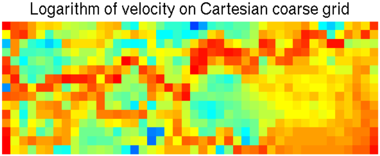
Final grid: 690 blocks

Layer 68 SPE10, 5 spot well pattern

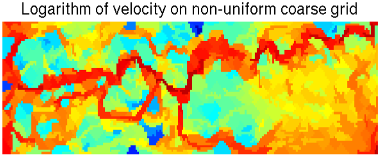


Geomodel: 13200 cells

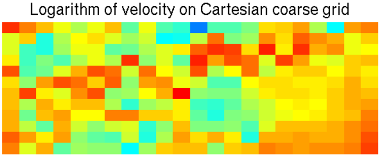




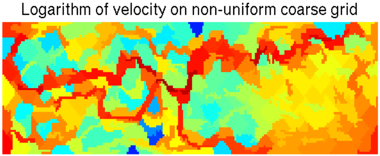
Coarse grid: 660 cells



Coarse grid: 649 cells



Coarse grid: 264 cells



Coarse grid: 257 cells

Experimental setup:

Model: Incompressible two-phase flow (oil and water).

Initial state: Completely oil-saturated.

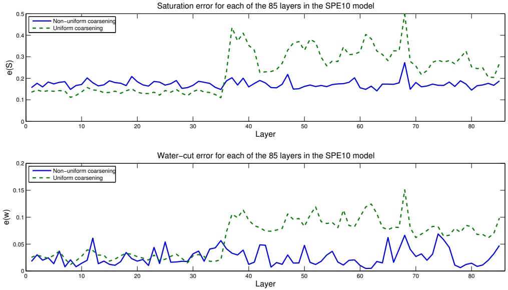
Relative permeability: $k_{rj} = s_j^2, \quad 0 \leq s_j \leq 1.$

Viscosity ratio: $\mu_o/\mu_w = 10.$

Error measures: (Time measured in PVI)

Saturation error: $e(S) = \int_0^1 \frac{\|S(\cdot,t) - S_{\text{ref}}(\cdot,t)\|_{L^1(\Omega)}}{\|S_{\text{ref}}(\cdot,t)\|_{L^1(\Omega)}}\, dt.$

Water-cut error: $e(w) = \|w - w_{\text{ref}}\|_{L^2([0,1])} / \|w_{\text{ref}}\|_{L^2([0,1])}.$



Geomodel:
$60 \times 220 \times 1$

Uniform grid:
$15 \times 44 \times 1$

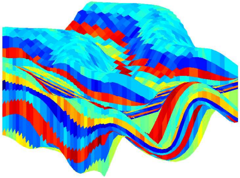
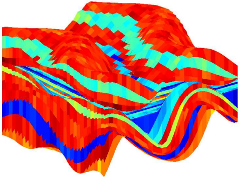
Non-uni. grid:
619–734 blocks

Observations:

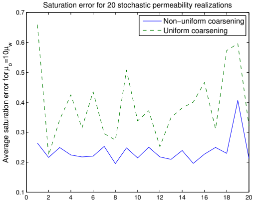
- First 35 layers smooth ⇒ Uniform grid adequate.
- Last 50 layers fluvial ⇒ Uniform grid inadequate.
- Non-uniform grid gives consistent results for all layers.

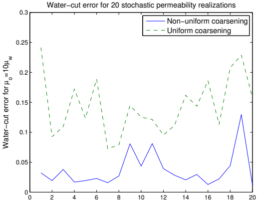
Example 2: Geomodel = unstructured corner-point grid

20 realizations from lognormal distribution, Q-of-5-spot well pattern, upsc. factor ~ 25

$\Leftarrow$ 2 realizations.
Geomodel:
15206 cells





Uniform grid:
838 blocks

Non-uni. grid:
647–704 blocks

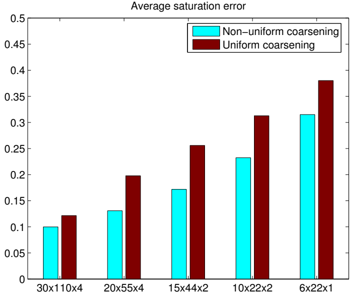
Observations:

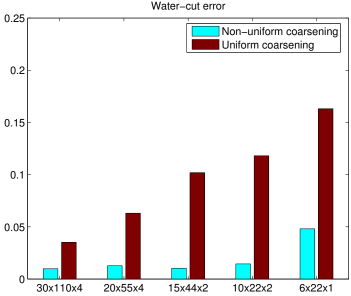
- Coarsening algorithm applicable to unstructured grids — accuracy consistent with observations for SPE10 models.
- Results obtained with uniform grid (in index space) inaccurate.

Example 3: Geomodel = four bottom layers from SPE10

Robustness with respect to degree of coarsening, 5-spot well pattern

	Number of cells in grid (upscaling factor 4–400)				
Uniform grid	30x110x4 13200	20x55x4 4400	15x44x2 1320	10x22x2 440	6x22x1 132
Non-U. grid	7516	3251	1333	419	150



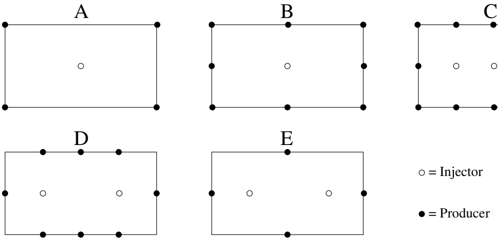


Observations:

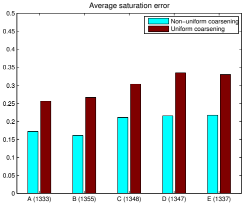
- Non-uniform grid gives better accuracy than uniform grid.
- Water-cut error almost grid-independent for non-uniform grid.

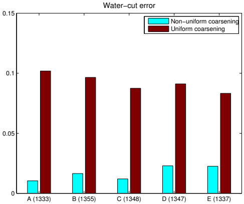
Example 4: Geomodel = four bottom layers from SPE10

Robustness with respect to well configuration, upscaling factor ∼ 40



Wellpatterns



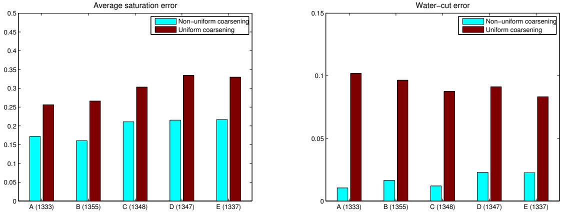


Uniform grid:
$15 \times 44 \times 2$

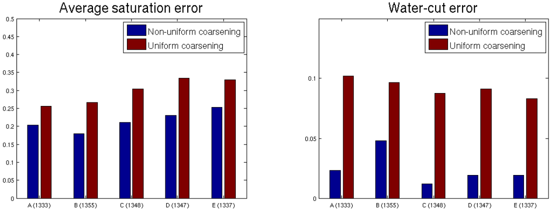
Non-uniform grid
~ 1320 blocks

- Non-uniform grid gives better accuracy than uniform grid — substantial difference in water-cut error for all cases.

Grid generated with respective well patterns.



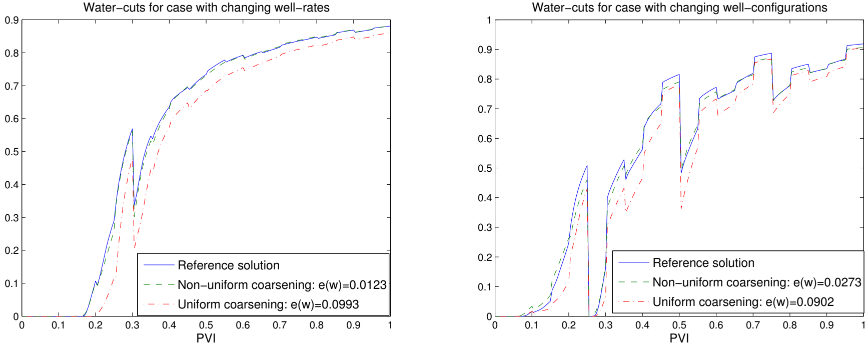
Grid generated with pattern C



Observation:

Grid resolves high-permeable regions with good connectivity — Grid need *not* be regenerated if well pattern changes.

Example 6: Geomodel = four bottom layers from SPE10
Robustness with respect changing well positions and well rates, upscaling factor ~ 40



5-spot, random prod. rates
grid generated with equal rates

well patterns: 4 cycles A–E
grid generated with pattern C

Observations:

- NU water-cut tracks reference curve closely: 1%–3% error.
- Uniform grid gives $\sim$ 10% water-cut error.

Conclusions

Flashback:

- A generic semi-automated algorithm for generating coarse grids that resolve flow patterns has been presented.
- Solutions are significantly more accurate than solutions obtained on uniform coarse grids with similar number of cells.
- Water-cut error: 1%–3% — pseudofunctions superfluous.
- Grid need **not** be regenerated when flow conditions change!

Potential application:

User-specified grid-resolution to fit available computer resources.

Relation to other methods:

Belongs to family of flow-based grids[a]: designed for flow scenarios where heterogeneity, rather than gravity, dominates flow patterns.

[a] Garcia, Journel, Aziz (1990,1992), Durlofsky, Jones, Milliken (1994,1997)

I have a dream ...