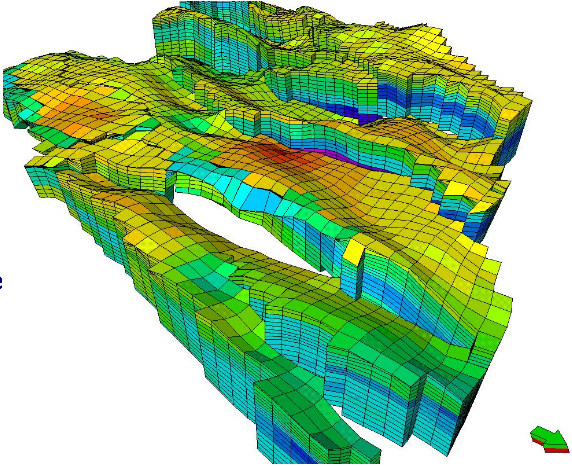
Multiscale Mixed/Mimetic FEM on Complex Geometries



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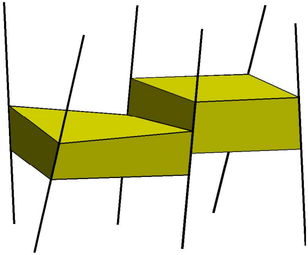
Outline

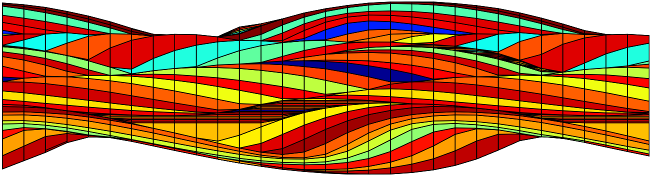
Corner-Point Grids

Industry standard for modelling complex reservoir geology

Specified in terms of:

- areal 2D mesh of vertical or inclined pillars
- each volumetric cell is restriced by four pillars
- each cell is defined by eight corner points, two on each pillar





Motivation

Often too much details in *geomodels* to run reservoir simulations directly $\Rightarrow$ model coarsening is necessary.

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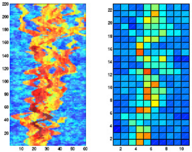
- Standard upscaling
  - Difficult to obtain coarse scale parameters consistently.
  - Need to *resample*: coarse grid does not match fine grid.



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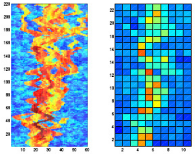
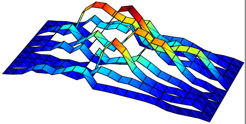


- Multiscale Mixed FEM (MsMFEM);
  - Incorporates fine scale features in coarse model basis functions.
  - Coarse grid can (in principle) be any partition of the fine grid.



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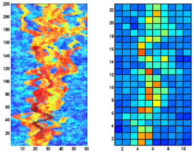

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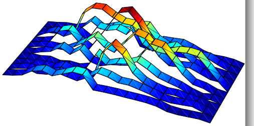

Goal: Automated accurate upgridding

Model Equations

Elliptic pressure equation:

$$v = -\lambda(S)\mathbf{K}\nabla p$$
$$\nabla \cdot v = q$$

Hyperbolic saturation equation:

$$\phi\frac{\partial S}{\partial t} + \nabla \cdot (vf(S)) = q_w$$

- Total velocity:

$$v = v_o + v_w$$

- Total mobility:

$$\begin{aligned}\lambda &= \lambda_w(S) + \lambda_o(S)\\ &= k_{rw}(S)/\mu_w + k_{ro}(S)/\mu_o\end{aligned}$$

- Saturation water: S
- Fractional flow water:

$$f(S) = \lambda_w(S)/\lambda(S)$$

Mixed Methods

Weak formulation:

Find $(v, p) \in H_0^{1,\text{div}} \times L^2$ such that

$$\begin{aligned}
\int (\lambda \mathbf{K})^{-1} \hat{v} \cdot v \, dx - \int p \nabla \cdot \hat{v} \, dx &= 0, && \forall \hat{v} \in H_0^{1,\text{div}}, \\
\int \hat{p} \nabla \cdot v \, dx &= \int q \hat{p} \, dx, && \forall \hat{p} \in L^2.
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Multiscale discretization:

Seek solutions in low-dimensional subspaces

$$U^{ms} \subset H_0^{1,\text{div}} \text{ and } V \subset L^2,$$

where local fine-scale properties are incorporated into the basis functions.

Mixed Methods – Lowest Order Discretization

Given finite bases $\{\phi_i\} = V \subset L^2$ and $\{\psi_k\} = U \subset H_0^{1,\mathrm{div}}$, the resulting linear system reads

$$\begin{pmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{C}^T & \mathbf{O} \end{pmatrix} \begin{pmatrix} \mathbf{v} \\ -\mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{q} \end{pmatrix},$$

where

$$\mathbf{B}_{kl} = \int \psi_k^T (\lambda \mathbf{K})^{-1} \psi_l \, dx \quad \text{and} \quad \mathbf{C}_{ki} = \int \phi_i \nabla \cdot \psi_k \, dx.$$

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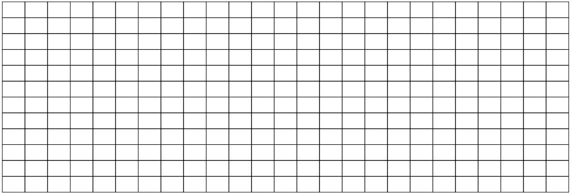
- Use hybridization to obtain SPD system

Multiscale Mixed FEM (MsMFEM)

Grids and Basis Functions

Assume we are given a *fine* grid with permeability and porosity attached to each fine-grid cell:



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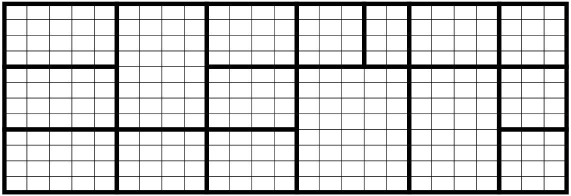

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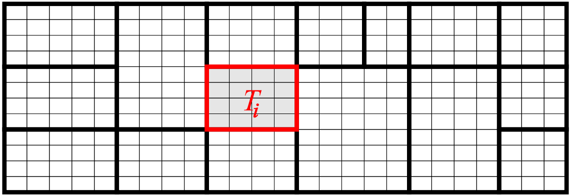

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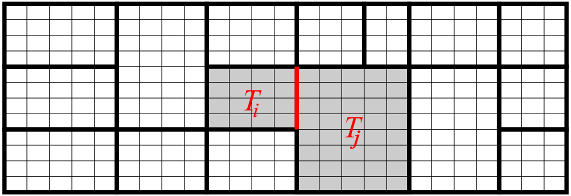

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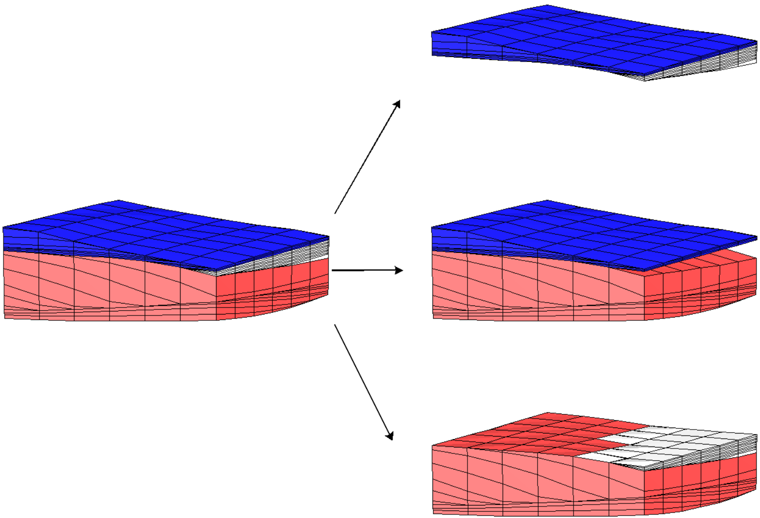
- For each coarse block T_i, there is a basis function $\phi_i \in V$.
- For each coarse edge Γ_{ij}, there is a basis function $\psi_{ij} \in U^{ms}$.

Example: A Three-Block Domain $\longrightarrow$ Three Basis Functions



Multiscale Basis Functions for Velocity

Each basis function ψ is the (numerical) solution of a one-phase local flow-problem over two neighboring blocks T_i, T_j:
$\psi = -\mathbf{K}\nabla\phi$ with

$$\nabla \cdot \psi = \begin{cases} w_i(x), & \text{for } x \in T_i \\ -w_j(x), & \text{for } x \in T_j, \end{cases}$$

with BCs $\psi \cdot n = 0$ on $\partial(T_i \cup \Gamma_{ij} \cup T_j)$.

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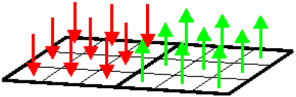
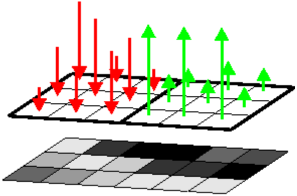
Weights w_i, w_j:





Subgrid Solvers

MsMFEM requires that a conservative numerical method is used to compute velocity basis functions.

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Alternatives for corner-point grids:

- Mixed FEM on tetrahedral subgrid of corner-point grid
- TPFA or MPFA finite-volume methods
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All of the above can be recast in mixed form as a *discrete* approximation of the bilinear form:

$$\int_\Omega u^T(\lambda \mathbf{K})^{-1}v \approx \sum_{E_i} \mathbf{u}_i \mathbf{M}_i \mathbf{v}_i,$$

where $\mathbf{u}_i$ and $\mathbf{v}_i$ contain the fluxes of u and v over the cell-faces of E_i.

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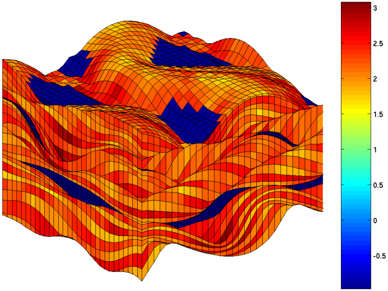
where $\mathbf{u}_i$ and $\mathbf{v}_i$ contain the fluxes of u and v over the cell-faces of E_i. $\Rightarrow$ All applicable in the MsMFEM framework.

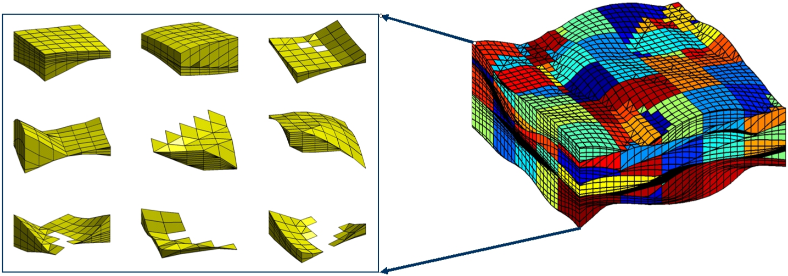
Numerical Example: A Wavy Depositional Bed (1)

- $30 \times 30 \times 100$ logically Cartesian.
- Corner to corner flow
- Three different perm fields
- Varying levels of coarsening



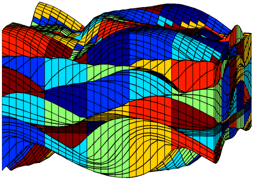
Numerical Example: A Wavy Depositional Bed (2)

Coarse Partitioning in Index space

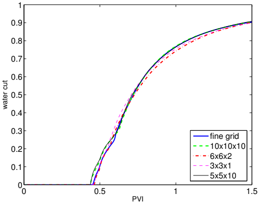


Relative error in saturation at 0.5PVI:

Coarse grid	Isotropic	Anisotropic	Heterogeneous
$10 \times 10 \times 10$	0.026	0.143	0.094
$6 \times 6 \times 2$	0.042	0.169	0.141
$3 \times 3 \times 1$	0.065	0.127	0.106
$5 \times 5 \times 10$	0.060	0.138	0.142



Logically $5 \times 5 \times 10$



Watercut

Can we improve results by altering the coarse grid?

Potential problems for MsMFEM

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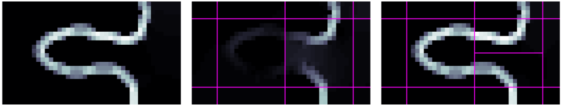
Bidirectional flow over interfaces:



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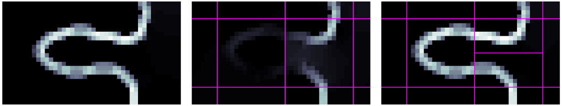
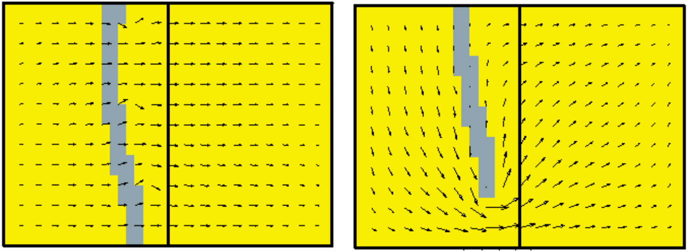
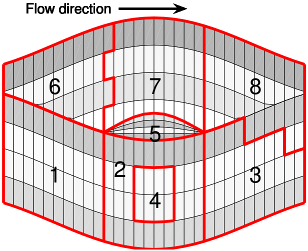
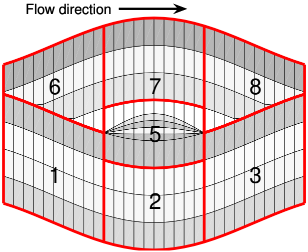


Flow barriers traversing blocks:



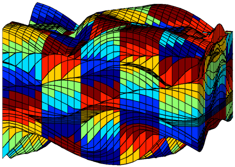
1. Minimize bidirectional flow over interfaces:
   - Avoid unnecessary irregularity (Blocks 6+7 and 3+8)
   - Avoid single neighbors (Block 4)
   - Ensure faces transverse to major flow (Block 5).
2. Blocks and faces should follow geological layers (Block 3+8)
3. Blocks should adapt to flow obstacles whenever possible.
4. For efficiency: reduce number of connections
5. Avoid having too many small blocks



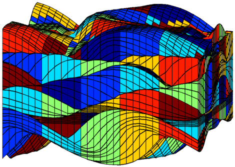


Relative error in saturation at 0.5PVI :

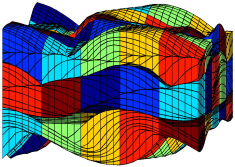
Coarse grid	Isotropic	Anisotropic	Heterogeneous
Physical	0.134	0.274	0.200
Logical	0.060	0.138	0.142
Constrained	0.057	0.148	0.099



Physical



Logical



Constrained

Concluding Remarks and Further Work

- Presented a multiscale mixed FEM that efficiently eliminates the need for upscaled properties and resampling on complex geomodels.
- Suggested guidelines for automated upgridding.
- Further testing on real field models.

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Paper:

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