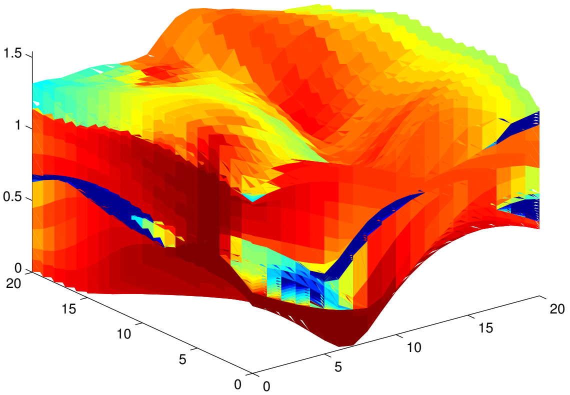
Multiscale methods for modeling flow in porous media : approaching industrial applications

Jørg E. Aarnes, Stein Krogstad and Knut–Andreas Lie

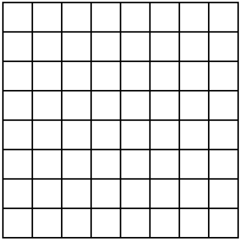


Outline

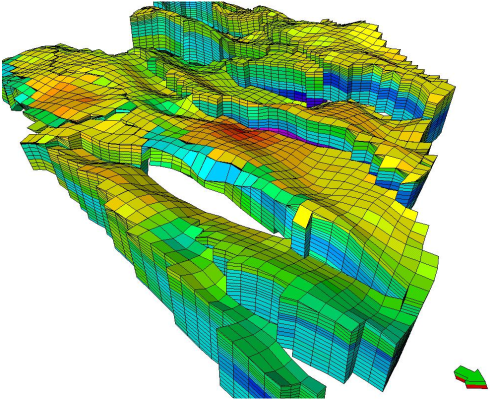
1. How can we promote multiscale methods to the oil-industry
   - Gap between academic research and industry needs
2. Industry-standard geological models
   - The corner-point format
   - Coarse grid-generation
3. Multiscale mixed finite element method
   - Coarse grid formulation
   - Subgrid discretization
4. Why consider multiscale methods

Academic models:

- Simple domains
- Structured grids
- Conforming grids



Industry models:



The gap between academia and industry

Academia:

- Incompressible
- Immiscible
- Gravity?
- Capillary forces?
- Pseudofunctions
- etc.

Industry:

- Compressible
- Miscible
- Yes!
- Yes.
- Relative permeability???
- etc., etc., etc., ...

The gap between academia and industry
What is important

Academia:

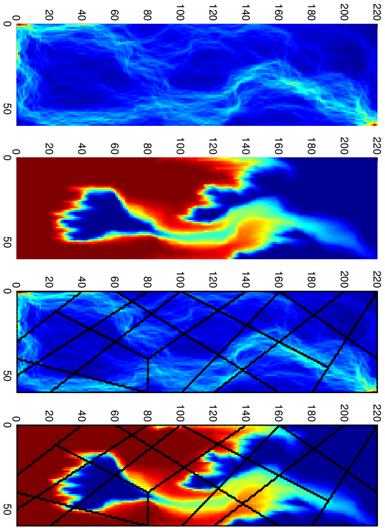
- Can it be published?
- Nice plots!
- Accuracy
- Efficiency
- Practical importance

Industry:

- Money
- Risk
- Can it handle our models
- Efficiency
- Robustness

Conjecture: MsMFEM has the following key features

Based on experience with synthetic Cartesian petroleum reservoir models



Accurate: flow scenarios match closely fine grid simulations.

Mass conservative: conserves mass on coarse and fine grids.

Efficient: basis functions can be computed in parallel and need not be recomputed.

Flexible: unstructured and irregular coarse grids are handled easily.

Robust: suitable for models with highly oscillatory coefficients and large grid-cell aspect ratios.

Do you want an amazing multiscale method?

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A1: Multiscale method???

Do you want an amazing multiscale method?

A2: But, multiscale methods are new and very complex, right?

Do you want an amazing multiscale method?

A3: Have they been tested on realistic models?

Do you want an amazing multiscale method?

A4: Yes, when it is implemented in my favourite software!

- **Promoting multiscale methods to the industry is a challenge, but academia must make the first move!**

Bridging the gap between academia and industry

A prerequisite for conducting simulation studies on full-scale real-field petroleum reservoir models is the ability to handle grids on a corner-point format.

- **Model:** corner-point grid without fractures and faults.
- **Physics:** incompressible and immiscible two-phase flow, neglecting effects from (gravity and) capillary forces.

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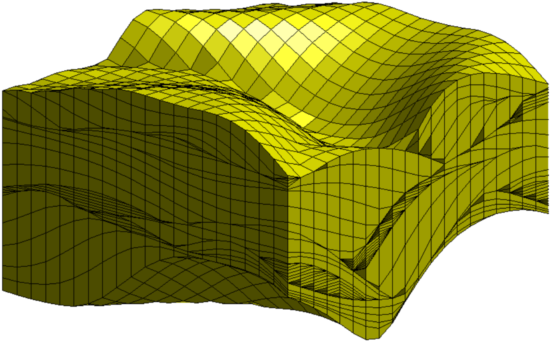
Next: MsMFEM on corner-point grid geological models

The corner-point, or pillar grid format, has become the industry standard for reservoir modeling and simulation.



In a corner-point grid the grid-cell corner-points lie on pillars (lines) that extend from the top to the bottom of the reservoir.

Corner-point grids, cont.

The data structure for corner-point grids is logically Cartesian, i.e.,

1. the pillars are ordered in a logical Cartesian manner, and
2. each layer extends throughout the entire reservoir.

Layers may collapse to a hyperplane in certain regions.

Collapsed cells are labeled non-active. Active cells have polyhedral shape with 5 – 8 corners.

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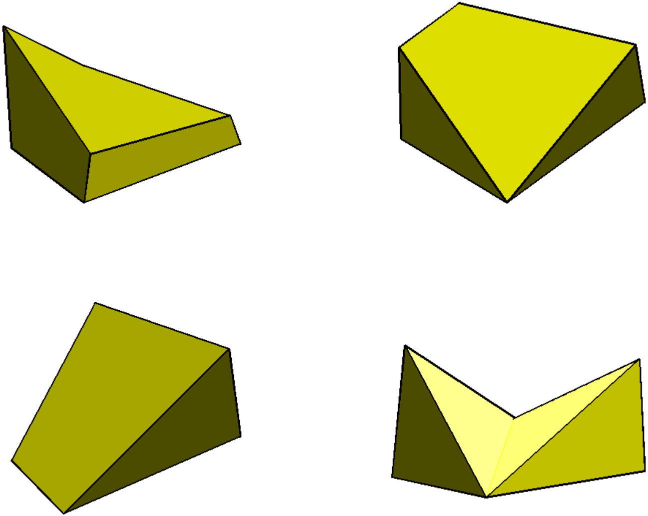
In physical space, corner-point grids are unstructured!

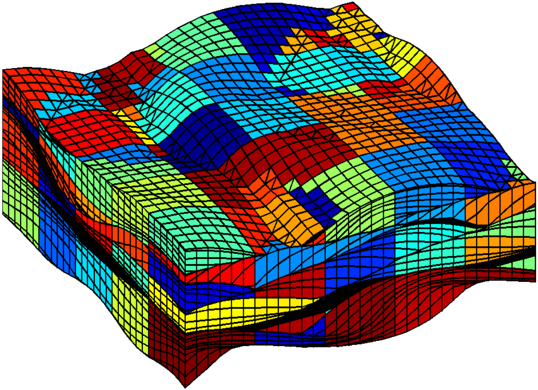
Corner-point grids, cont.

Examples of degenerate hexahedral cells in corner-point grids



Generating a coarse grid for MsMFEM

Let $\mathcal{K} = \{K\}$ be a coarse grid with blocks of "arbitrary" shape, and denote by $\mathcal{T} = \{T\}$ a fine subgrid of $\mathcal{K}$



Grid coarsening procedures

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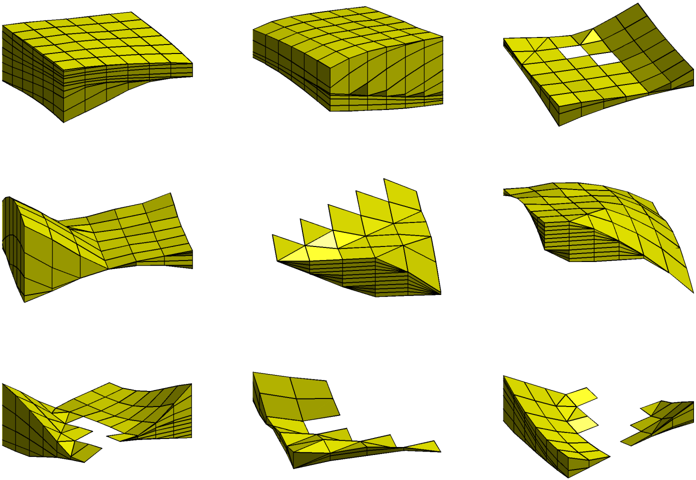
- Volume constrained partitioning in index space:
  - Blocks are irregular and number of neighbors increases :=(
  - Blocks have smooth faces, and approximately equal volume :=)

Disconnected blocks are split into a family of connected subblocks.



Let Ω denote a computational domain, and consider the following model problem

$$
\begin{aligned}
v &= -k\nabla p, \\
\nabla \cdot v &= q \quad \text{in } \Omega, \\
v \cdot n &= 0 \quad \text{on } \partial\Omega.
\end{aligned}
$$

Here k is a symmetric and positive definite tensor with uniform upper and lower bounds in Ω.

We will refer to p as pressure and v as velocity.

In mixed FEMs one seeks $v \in V$ and $p \in U$ such that

$$\begin{aligned} \int_\Omega k^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= 0 && \forall u \in V, \\ \int_\Omega l \, \nabla \cdot v \, dx &= \int_\Omega q l \, dx && \forall l \in U. \end{aligned}$$

Here $V \subset \{v \in (L^2)^d : \nabla \cdot v \in L^2\}$ and $U \subset L^2$.

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In multiscale mixed FEMs the approximation space for velocity is designed so that it embodies the impact of fine scale structures.

Associate a basis function χ_m for pressure with each grid block:

$$U = \text{span}\{\chi_m : K_m \in \mathcal{K}\} \quad \text{where} \quad \chi_m = \begin{cases} 1 & \text{if } x \in K_m, \\ 0 & \text{else.} \end{cases}$$

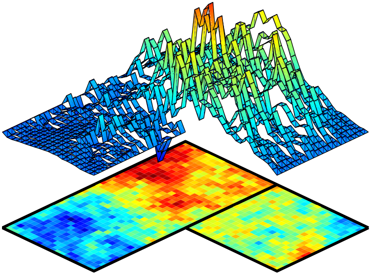
Construct a velocity basis function for each interface $\partial K_i \cap \partial K_j$:

$V = \text{span}\{\psi_{ij}\}$ where $\psi_{ij} = -k\nabla\phi_{ij}$ and ϕ_{ij} is determined by no-flow boundary conditions on $(\partial K_i \cup \partial K_j)\backslash(\partial K_i \cap \partial K_j)$, and

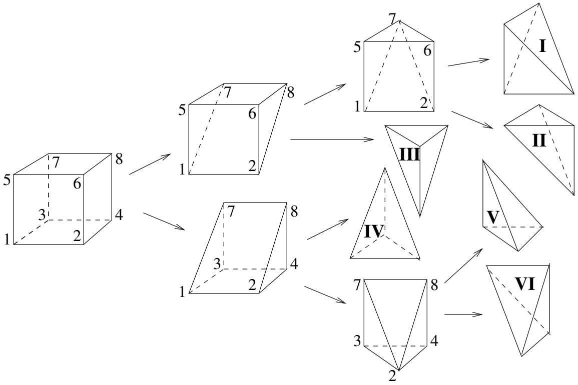
$$\nabla \cdot \psi_{ij} = \begin{cases} q(K_i) & \text{in } K_i, \\ -q(K_j) & \text{in } K_j, \end{cases}$$

where

$$q(K) = \begin{cases} \frac{|k|}{\int_K |k|} & \text{if } \int_K f\, dx = 0, \\ \frac{f}{\int_K f} & \text{if } \int_K f\, dx \neq 0. \end{cases}$$



To implement a mixed FEM on a CPG is a bit cumbersome because degenerate cells have less than eight corners. Conforming CPGs can, however, be subdivided into tetrahedra in such a way that the non-degenerated tetrahedra form a conforming grid.



- Most commercial simulators employ a two-point flux approximation scheme to discretize the pressure equation.
    - TPFA schemes are generally not convergent for CPGs.
    - Convergent MPFA schemes exist, but are difficult to implement on CPGs with degenerated cells, and are not capable of handling non-conforming grids.
- Finite volume methods provide fluxes, but not velocity fields.
- Implementation of MsMFEM requires that we can evaluate (approximate) integrals of the following form:

$$\int_{K_i} \psi_{ij} \cdot k^{-1} \psi_{il} \, dx.$$

- Mimetic FDMs allow easy treatment of non-conforming grids with complex grid-cell geometries (including curved faces).
- They employ a mixed formulation, but the local inner-product

$$(u, v)_{T_i} = \int_{T_i} u \cdot k^{-1} v \, dx, \quad u, v \in H^{1,\text{div}}(T_i),$$

is replaced with a matrix-based inner-product

$$(\mathbf{u}, \mathbf{v})_{\mathbf{B}_i} = \mathbf{u}^T \mathbf{B}_i \mathbf{v}.$$

Here $\mathbf{u}, \mathbf{v} \in \mathcal{R}^{n_i}$, where n_i is the number of cell faces, and $\mathbf{B}_i \in \mathcal{R}^{n_i \times n_i}$ is a symmetric and positive definite matrix.

Formula for calculating $\mathbf{B}_i$ in mimetic FDM

- For polygons (with planar faces), a discrete version of the Gauss-Greens formula can be written on the following form:

$$\mathbf{B}_i \mathbf{N} k = \mathbf{C}.$$

  The rows of $\mathbf{N}$ are the unit normals for each face, and the rows of $\mathbf{C}$ are the centroids of each face scaled by the area.

- A class of solutions of this equation has the following form:

$$\mathbf{B}_i = \frac{1}{|T_i|} \mathbf{C} k^{-1} \mathbf{C}^T + \mathbf{Z}\mathbf{U}\mathbf{Z}^T,$$

  where $\mathbf{U}$ is a given symmetric and positive definite matrix and the columns of $\mathbf{Z}$ span the null space of $\mathbf{N}^T$.

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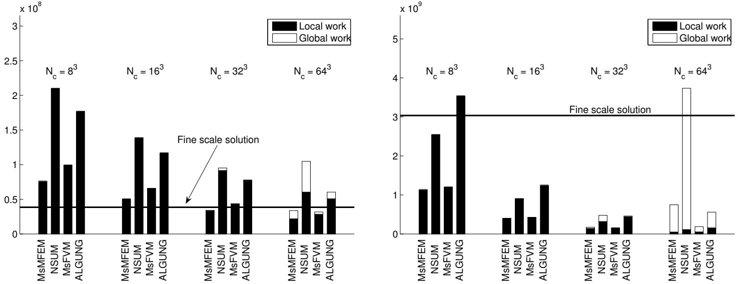
- **Mimetic finite difference methods:**
  - Easy to implement and very flexible wrt. grids :=)
  - New? Less rigorous than mixed FEM? :=|

Why multiscale?

Time $t(n)$ to solve a linear system of dimension n: $t(n) \sim O(n^{\alpha})$.



$\alpha = 1.2$ $\alpha = 1.5$

Computation time comparable to solving global fine-scale system using a (very) efficient linear solver.

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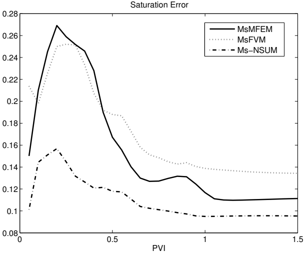
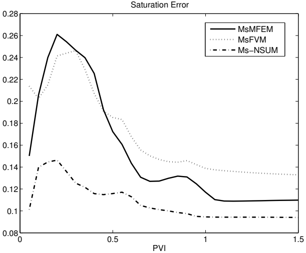
- Multiscale methods have low memory requirements.
- Robust and efficient linear solvers for systems that stem from real-field petroleum reservoir models are (very) hard to find.
- **Computation of basis functions can often be made part of a preprocessing step for multi-phase flow simulations.**



BF computed only once



BF comp. at each timestep

The road ahead

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- **Related activity:** We are trying to develop a parallel technology for the flow transport equations.