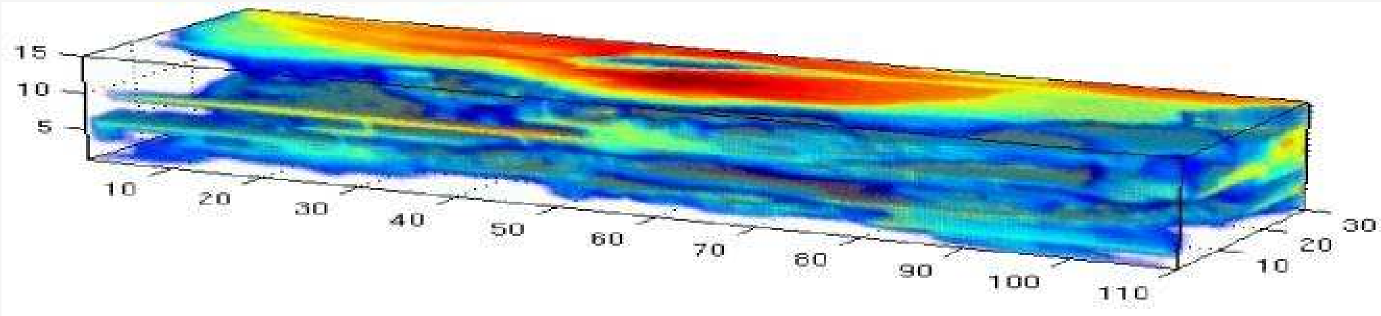
A multiscale framework for modeling flow in porous media with multiscale structures



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8th European Multigrid Conference
Scheveningen The Hague, The Netherlands

This work is part of the **GeoScale Research Project**:

> Develop a numerical methodology that facilitates reservoir simulation studies on multi-million cell geological models.

Simulations should run within a few hours on desktop computers.

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A cornerstone in the project is a multiscale mixed finite element method (MsMFEM) that models pressure and filtration velocity.

To model the transport we explore two different strategies:
- streamline methods for convection dominated flow.
- an adaptive multiscale finite volume method.

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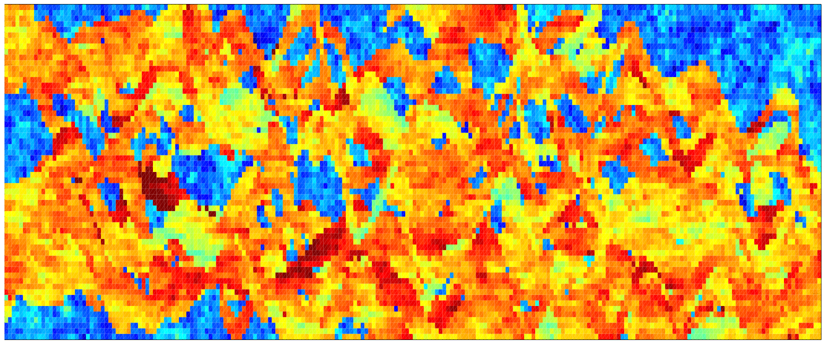
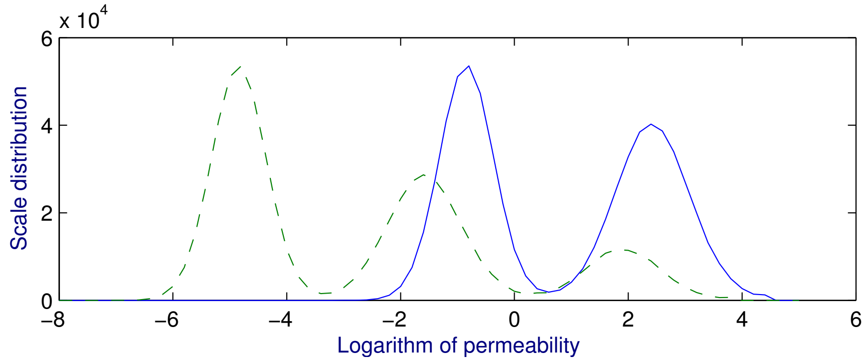
Geological models

Geological reservoir models give a geometric reservoir description and a plausible distribution of rock permeability - the rocks ability to transmit fluid - and porosity - the volume fraction open to flow.





Geological models may contain 10^6–10^9 grid cells and are often characterized by large contrasts in the permeability field.

Simulation models

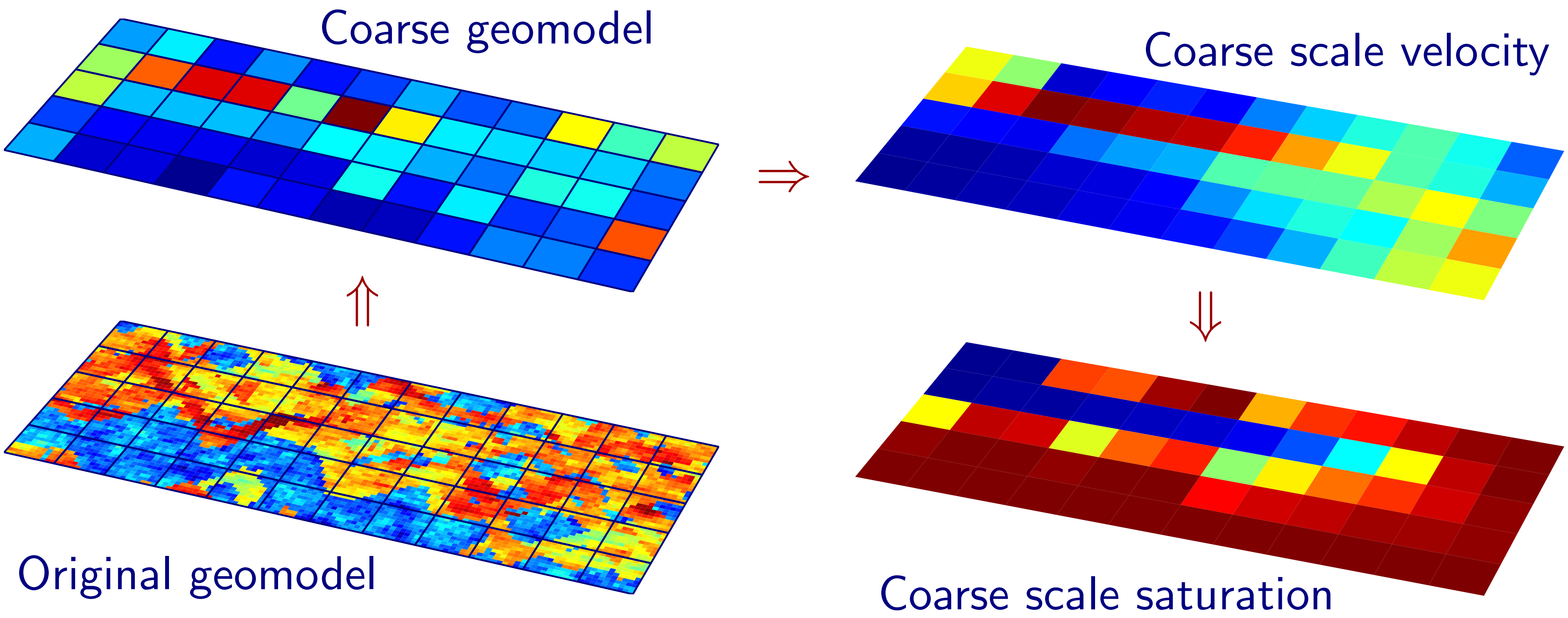
For presentational simplicity we consider a model for incompressible and immiscible two-phase flow without gravity and capillary forces:

$$\begin{aligned} \nabla \cdot k[\lambda_w(S) + \lambda_o(S)]\nabla p &= q \\ \phi \partial_t S + \nabla \cdot (f_w v) &= q_w. \end{aligned}$$

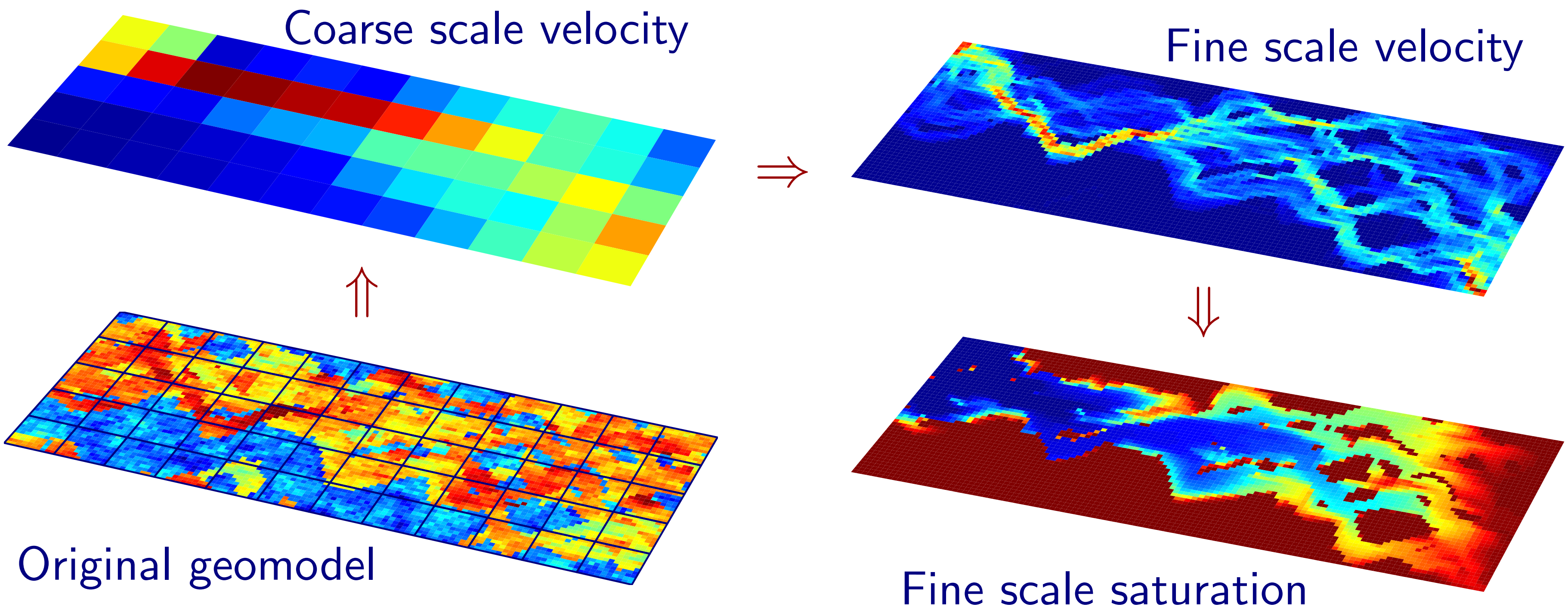
Here k denotes permeability, λ_i the mobility of phase i, ϕ porosity, p pressure, S water saturation, $f_w = \lambda_w/(\lambda_w + \lambda_o)$ the fraction of water in the flowing fluid, and $v = v_w + v_o$ the total Darcy velocity.

Reservoir simulation models usually consist of 10^4–10^6 grid blocks.

Traditional reservoir simulation



Reservoir simulation using multiscale methods



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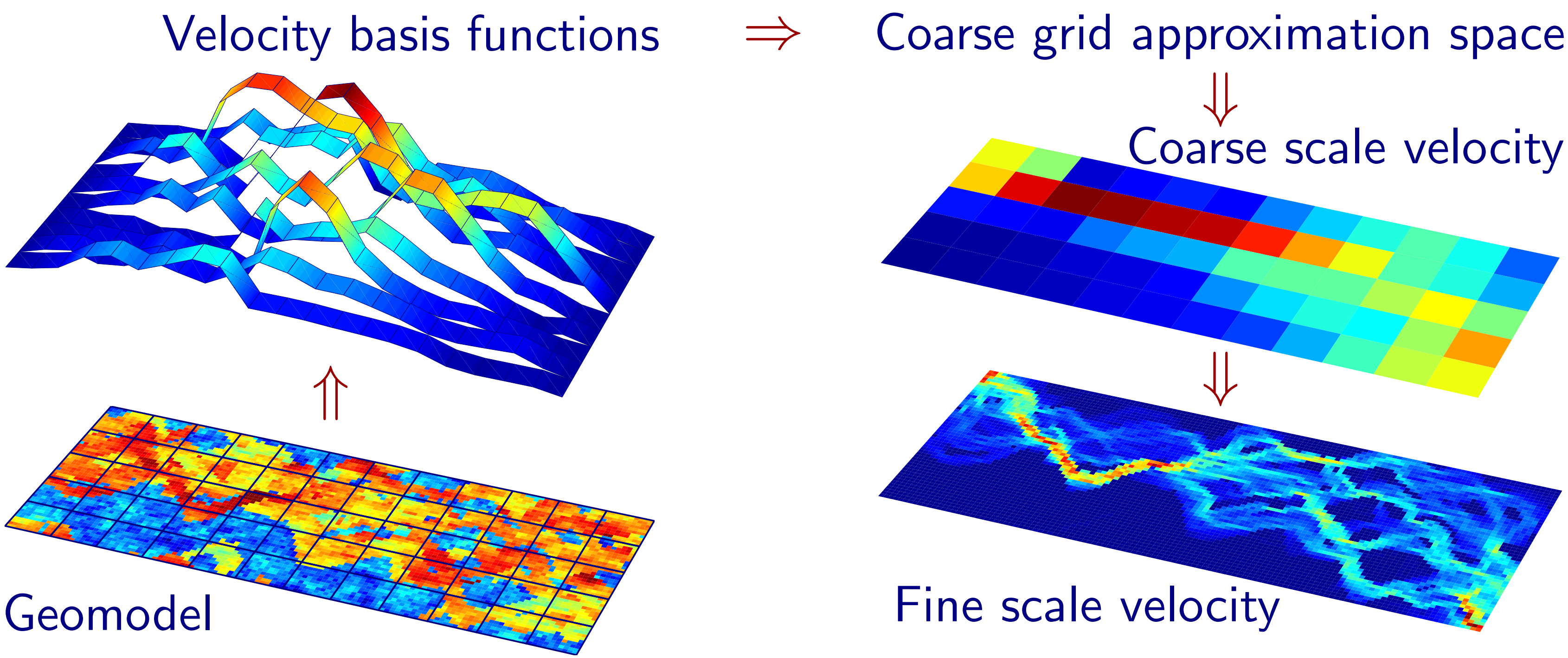
Multiscale mixed finite element methods

In a mixed FEM formulation one seeks $v \in V$ and $p \in U$ such that

$$\begin{aligned} \int_\Omega k^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= 0 && \forall u \in V, \\ \int_\Omega l \, \nabla \cdot v \, dx &= \textstyle\int_\Omega q l \, dx && \forall l \in U. \end{aligned}$$

Here $V \subset \{v \in (L^2)^d : \nabla \cdot v \in L^2, \; v \cdot n = 0 \text{ on } \partial\Omega\}$ and $U \subset L^2$.

In MsMFEMs the approximation space for velocity $V = \text{span}\{\psi_{ij}\}$ is designed so that it embodies the impact of fine scale structures.



The fine scale velocity field is expressed as a linear superposition of the basis functions: $v = \sum_{ij} v_{ij}\psi_{ij}$ where the coefficients v_{ij} are obtained from the solution of the coarse scale system.

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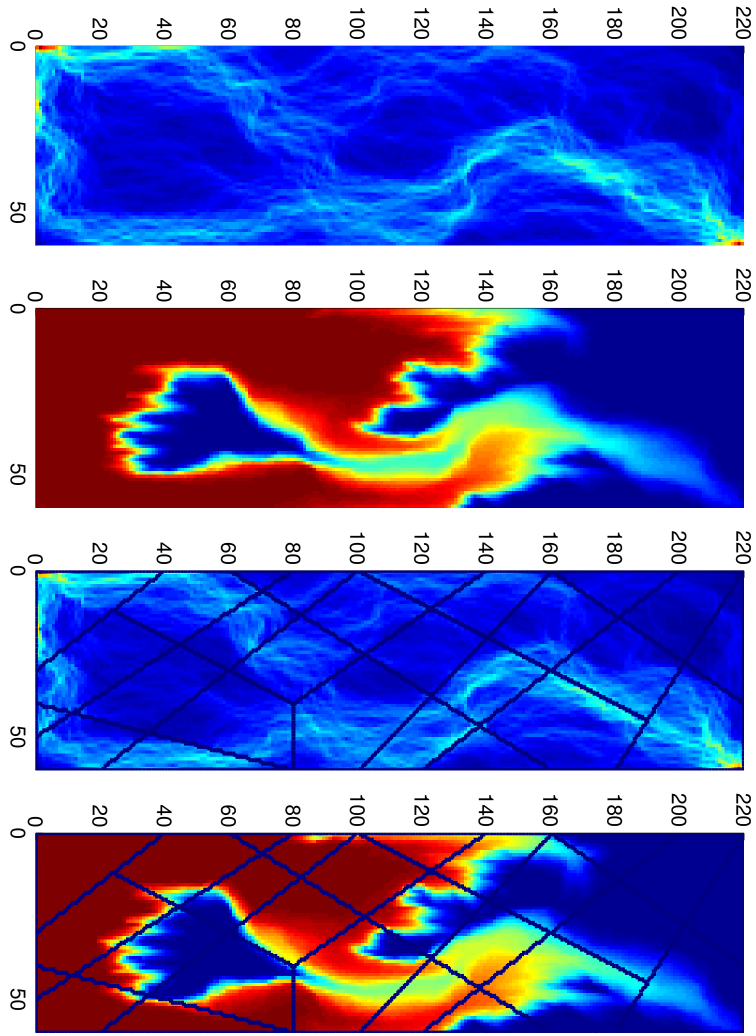


MsMFEMs enjoy the following prop.:

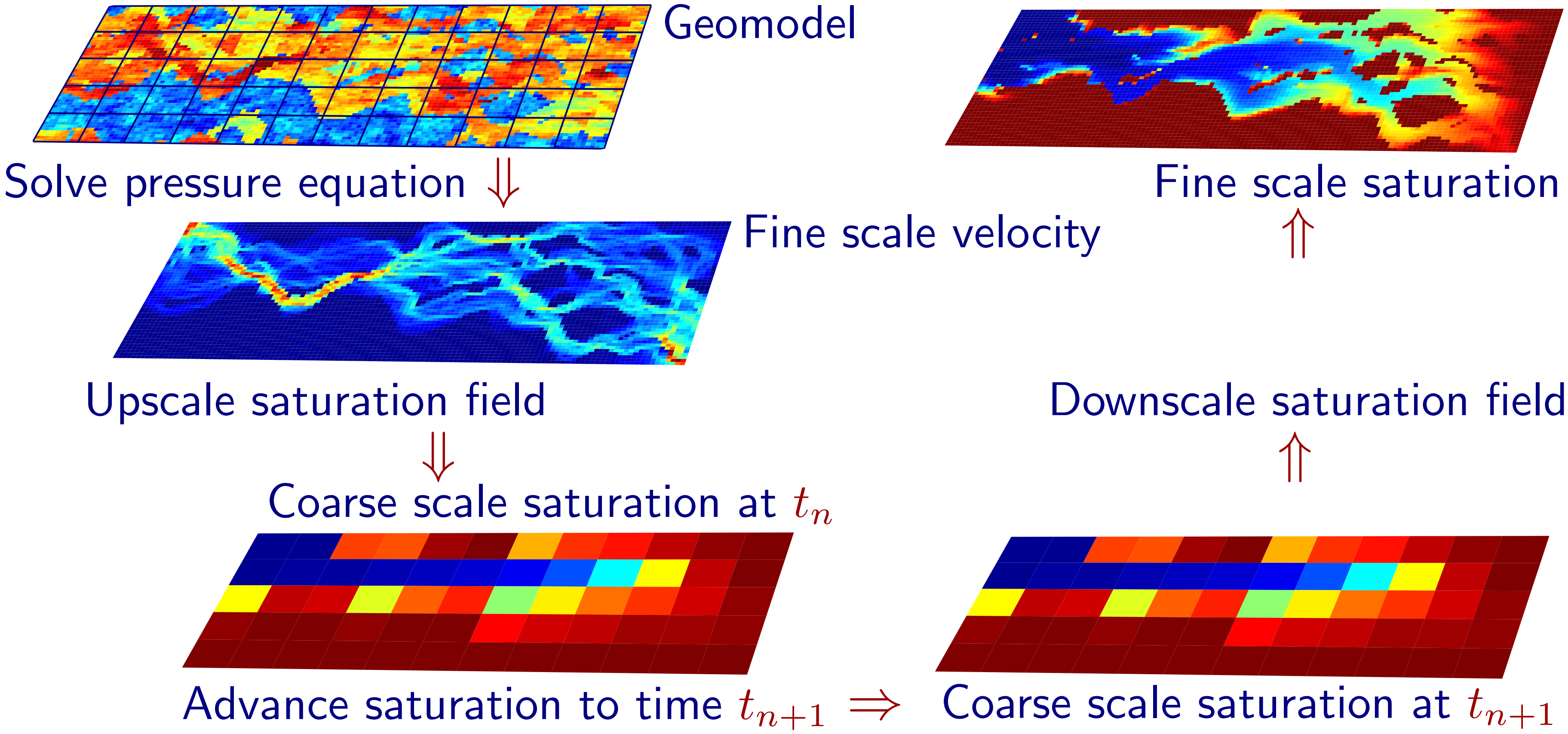
- They are accurate: flow scenarios match closely fine grid simulations.
- They are efficient: basis functions need to be computed only once.
- They are flexible: unstructured and irregular grids are handled easily.
- They are robust: suitable for modeling flow in porous media with very strong heterogeneous structures.

Conclusion I: Multiscale methods for elliptic equations provide a robust and efficient tool to get accurate velocity fields on fine grids,

... but solving the saturation equation on multi-million cell geomodels becomes a bottle-neck in large flow simulations.

Is it possible to develop a similar multiscale methodology for solving the saturation equation more efficiently?

A multiscale framework for the saturation equation



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A multiscale method for the saturation equation:

Assume that S^n is a saturation field on the fine grid $\{T\}$ at $t = t_n$, and denote non-degenerate fine grid interfaces by $\gamma_{ij} = \partial T_i \cap \partial T_j$.

1: For each K in the coarse grid, do

$$\bar{S}^{n+1}|_K = \bar{S}^n|_K + \frac{\triangle t}{\int_K \phi \, dx} \left[\int_K q_w \, dx - \sum_{\gamma_{ij} \subset \partial K} F_{ij}(S^n) \right],$$

where $F_{ij}(S) = \max\{f_w(S_i)v_{ij}, -f_w(S_j)v_{ij}\}$.

2: Map $\bar{S}^{n+1}|_K$ onto the fine grid: $S^{n+1}|_K = I_K(\bar{S}^{n+1})$.

The interpolation operators are defined by $I_K(\bar{S}) = \chi_K(x, t(\bar{S}))$, where χ_K is determined by

$$\phi \frac{\partial \chi_K}{\partial t} + \nabla \cdot [f_w v^0] = q_w \qquad \text{in } K^E = K \cup \{T : \partial K \cap \partial T \neq \emptyset\},$$

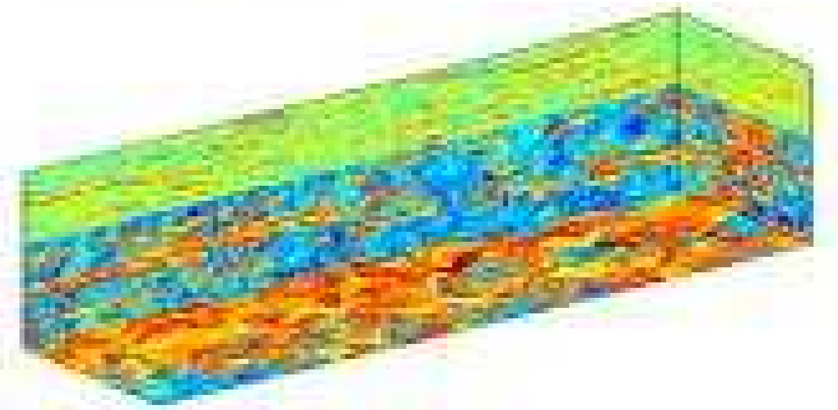
with $v^0 = v(S^0)$, $\chi_K^0 = S^0$, and $f_w = 1$ on the inflow boundary

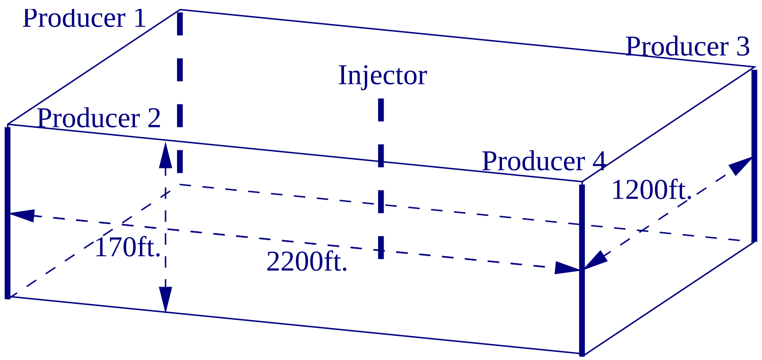
$$\Gamma_{\text{in}}^E = \{\gamma_{ij} \subset \partial K^E : K_i \subset K^E,\ v_{ij} < 0\}.$$

The time $t(\bar{S})$ is determined by requiring mass conservation:

$$\int_K I_K(\bar{S})\phi \, dx = \bar{S} \int_K \phi \, dx.$$

Test case: 10th SPE comparative solution project (model 2).





- Fine grid: $60 \times 220 \times 85$ ($1.122 \cdot 10^6$ fine grid cells.)
- Coarse grid: $6 \times 22 \times 17$ (2244 coarse grid blocks).
- Mobilities: $\lambda_w = S^2/\mu_w$ and $\lambda_o = (1-S)^2/\mu_o$.
- Viscosities: $\mu_w = 3.0 \cdot 10^{-4}$ and $\mu_o = 3.0 \cdot 10^{-3}$.

To assess the accuracy of a solution S we compute the discrepancy between S and a reference solution S_{ref} using the following norms:

$$e_F(S(\cdot,t)) = \frac{\|S_{\text{ref}}(\cdot,t) - S(\cdot,t)\|_{L^2_\phi}}{\|S_{\text{ref}}(\cdot,t) - S_{\text{ref}}(\cdot,0)\|_{L^2_\phi}},$$

$$e_C(S(\cdot,t)) = \frac{\|\bar{S}_{\text{ref}}(\cdot,t) - \bar{S}(\cdot,t)\|_{L^2_\phi}}{\|\bar{S}_{\text{ref}}(\cdot,t) - \bar{S}_{\text{ref}}(\cdot,0)\|_{L^2_\phi}}.$$

Here $\bar{S}$ denotes the coarse grid saturations corresponding to S, and

$$\|S\|^2_{L^2_\phi} = \int_\Omega (S\phi)^2\,dx.$$

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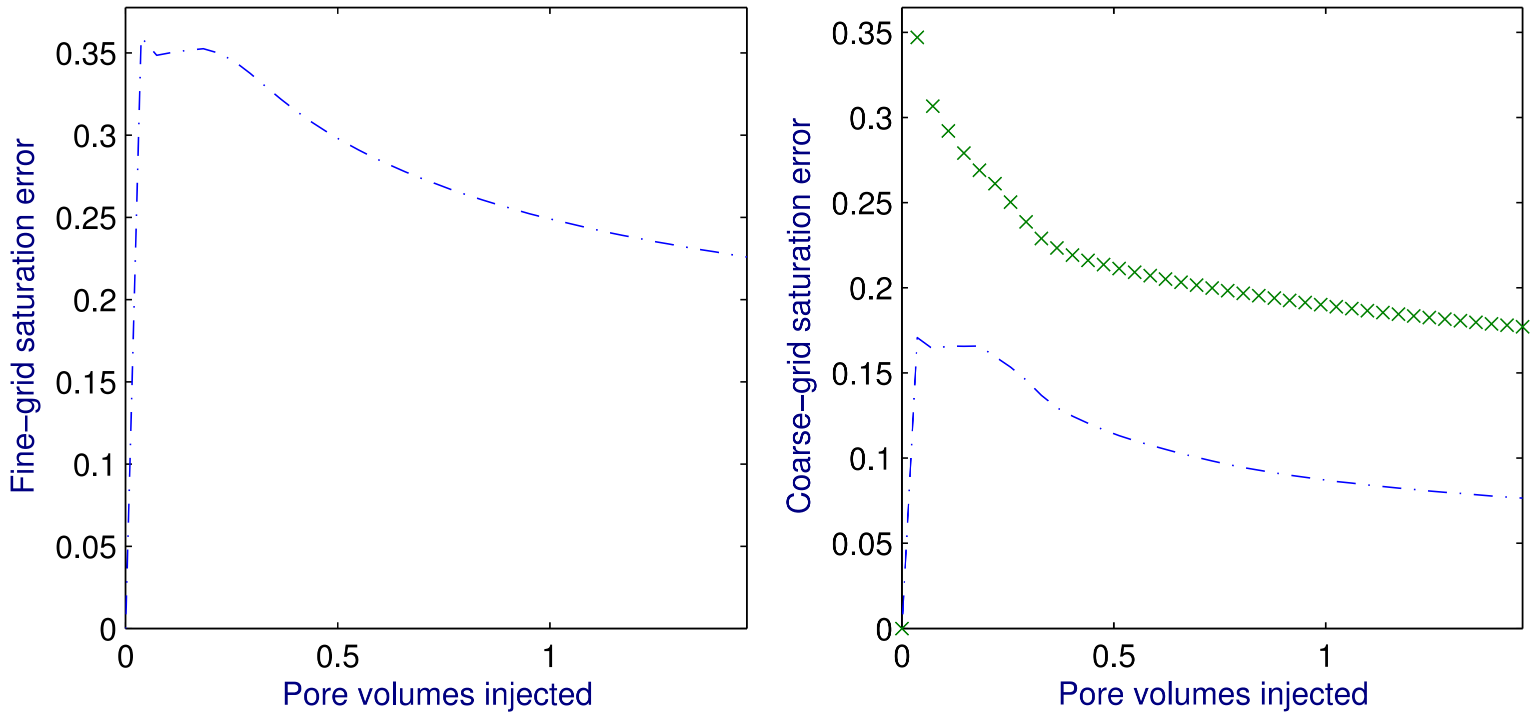
Results for pure multiscale algorithm:



The x–marks correspond to the standard upstream scheme on $\{K\}$.

Domain decomposition method for the saturation equation:

For all grid blocks K, let $K^E = K \cup \{T : \partial K \cap \partial T \neq \emptyset\}$, and do

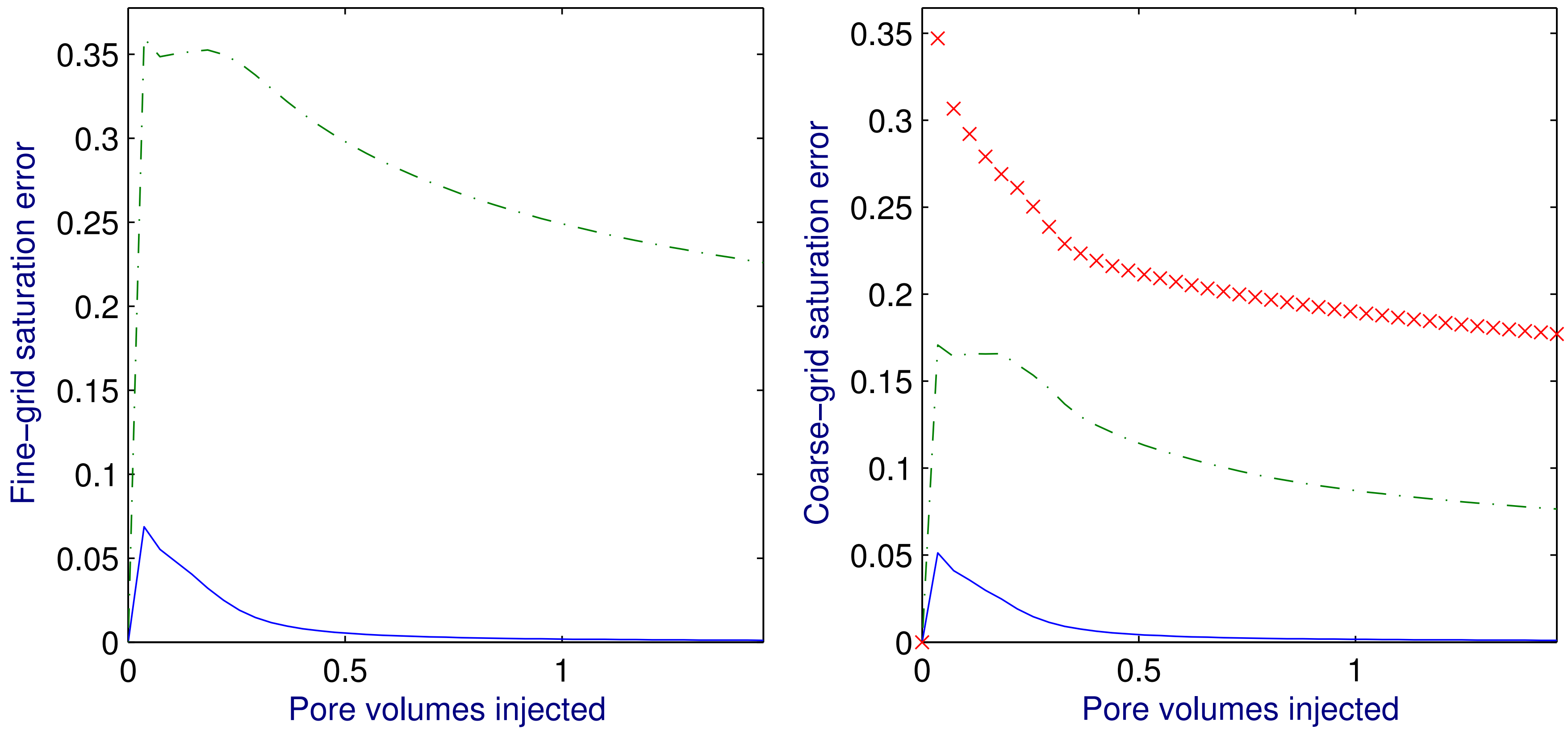
1: For $T_i \in K^E$, compute:

$$S_i^{n+1/2} = S_i^n + \frac{\triangle t}{\phi_i |T_i|} \left(\int_{T_i} q_w(S^{n+1/2})\, dx - \sum_{j \neq i} F_{ij}^* \right),$$

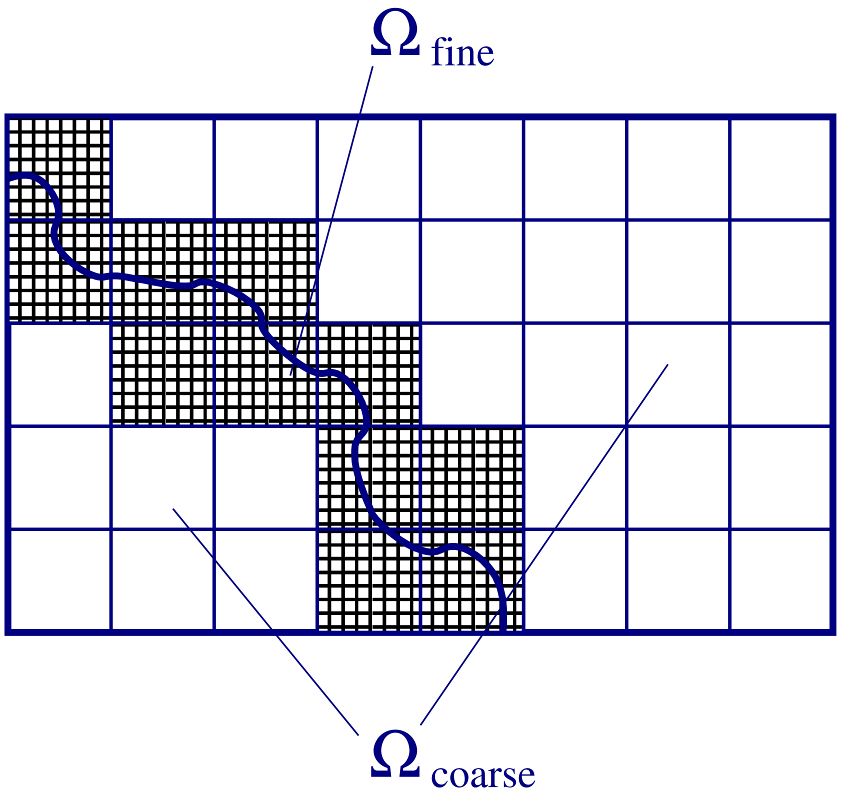
where $F_{ij}^* = \begin{cases} F_{ij}(S^n) & \text{if } \gamma_{ij} \subset \partial K \text{ and } v_{ij} < 0. \\ F_{ij}(S^{n+1/2}) & \text{otherwise.} \end{cases}$

2: For $T_i \in K^E$, set $S_i^{n+1} = S_i^{n+1/2}$.

Results for multiscale and domain decomposition algorithm:



Domain decomposition type localization procedures provide a natural environment for the development of adaptive schemes.



Adaptive algorithm:

- Use DD method in transient flow regions (Ω_{fine}).
- Update coarse grid saturation in regions with slow transients.
- Map saturation in Ω_{coarse} onto fine grid using $\{I_K\}$.

An adaptive multiscale method for the saturation equation

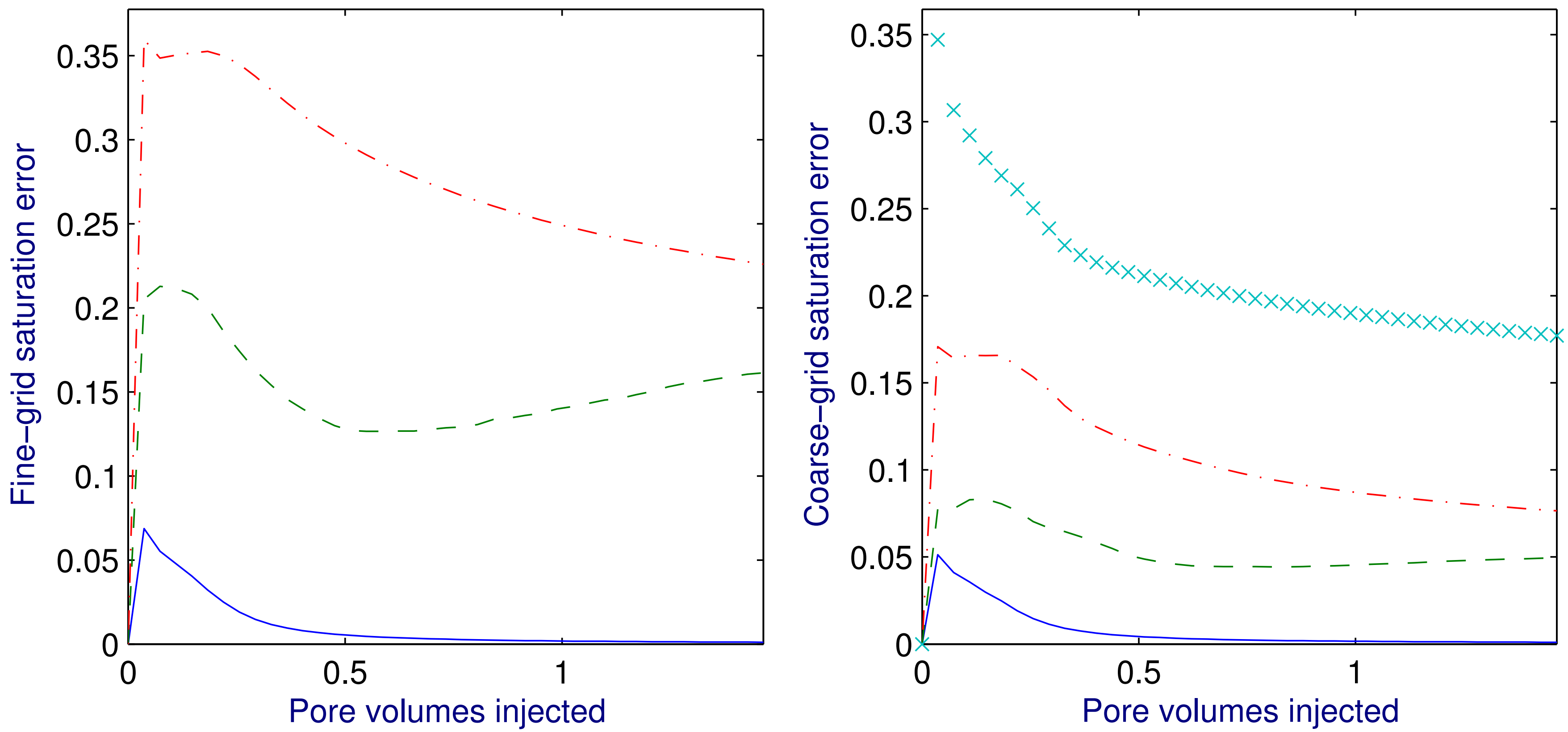
1: Compute S^{n+1} in Ω_{fine} using the DD method.

2: Set $S^{n+1} = S^n$ in Ω_{coarse} and compute

$$\bar{S}^{n+1}|_K = \bar{S}^n|_K + \frac{\triangle t}{\int_K \phi \, dx} \left[\int_K q_w \, dx - \sum_{\gamma_{ij} \subset \partial K} F_{ij}(S^{n+1}) \right].$$

3: Map $\bar{S}^{n+1}|_K$ onto the fine grid: $S^{n+1}|_K = I_K(\bar{S}^{n+1})$.

Results for the adaptive multiscale algorithm:



Water-cut curves: fraction of water in produced fluid.

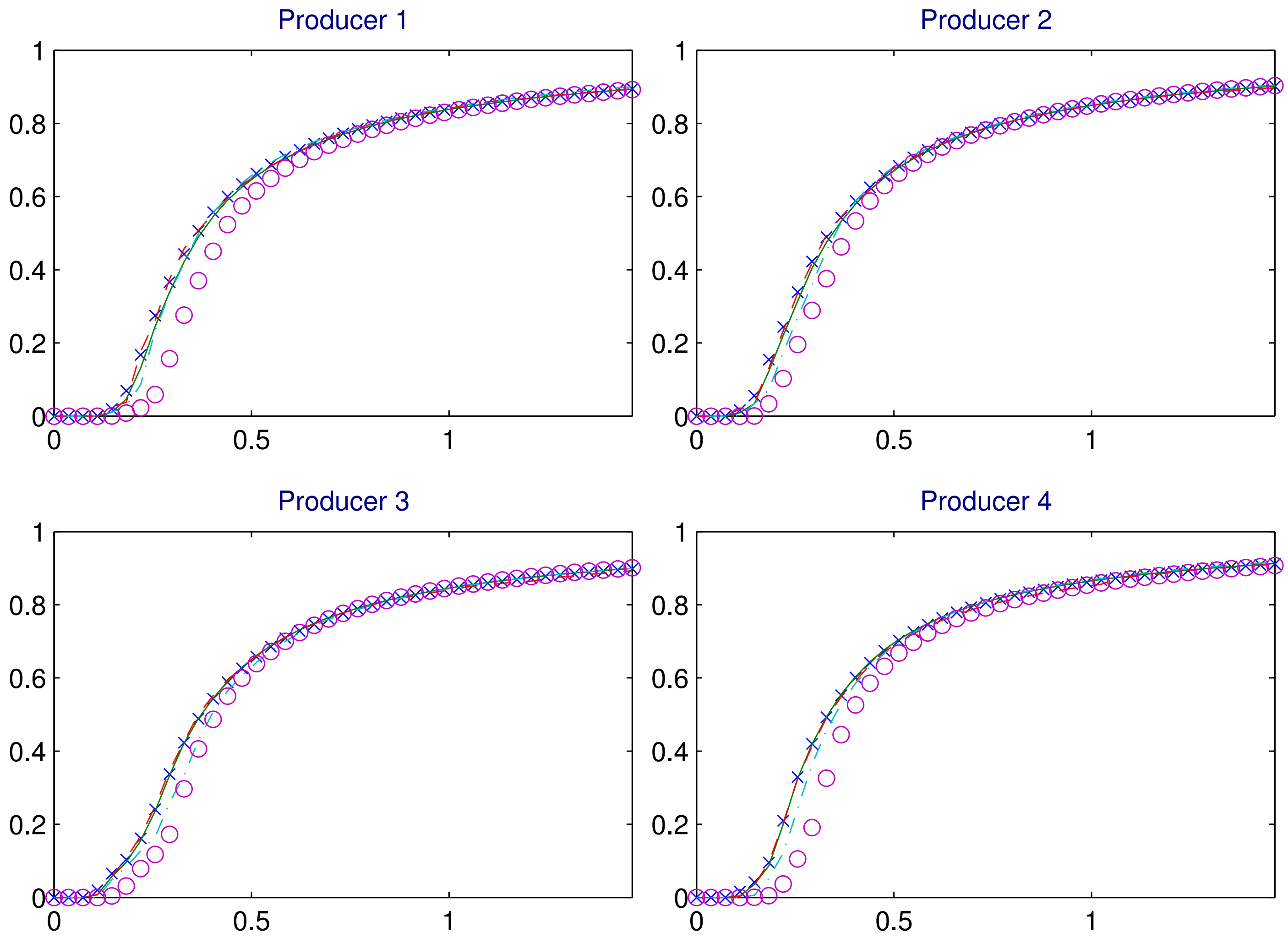
x: Ref.

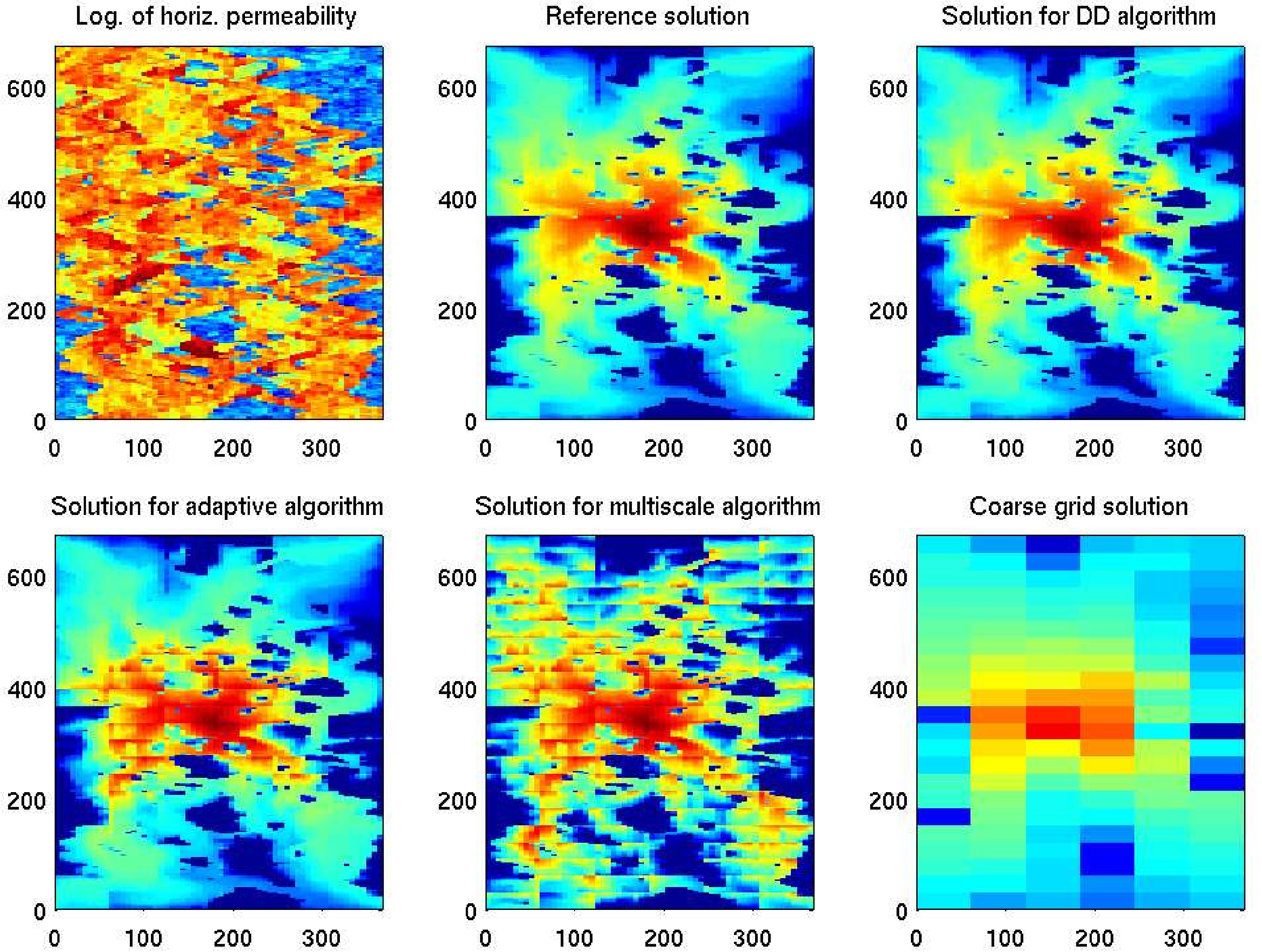
–: DD.

--: AdMs.

-·: Ms.

o: Coarse.




Log. of horiz. permeability
Reference solution
Solution for DD algorithm
Solution for adaptive algorithm
Solution for multiscale algorithm
Coarse grid solution
600
400
200
0
0
100
200
300

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Conclusions

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Acknowledgments: This work is done in collaboration with Stein Krogstad, Vegard Kippe and Knut-Andreas Lie at **SINTEF**.