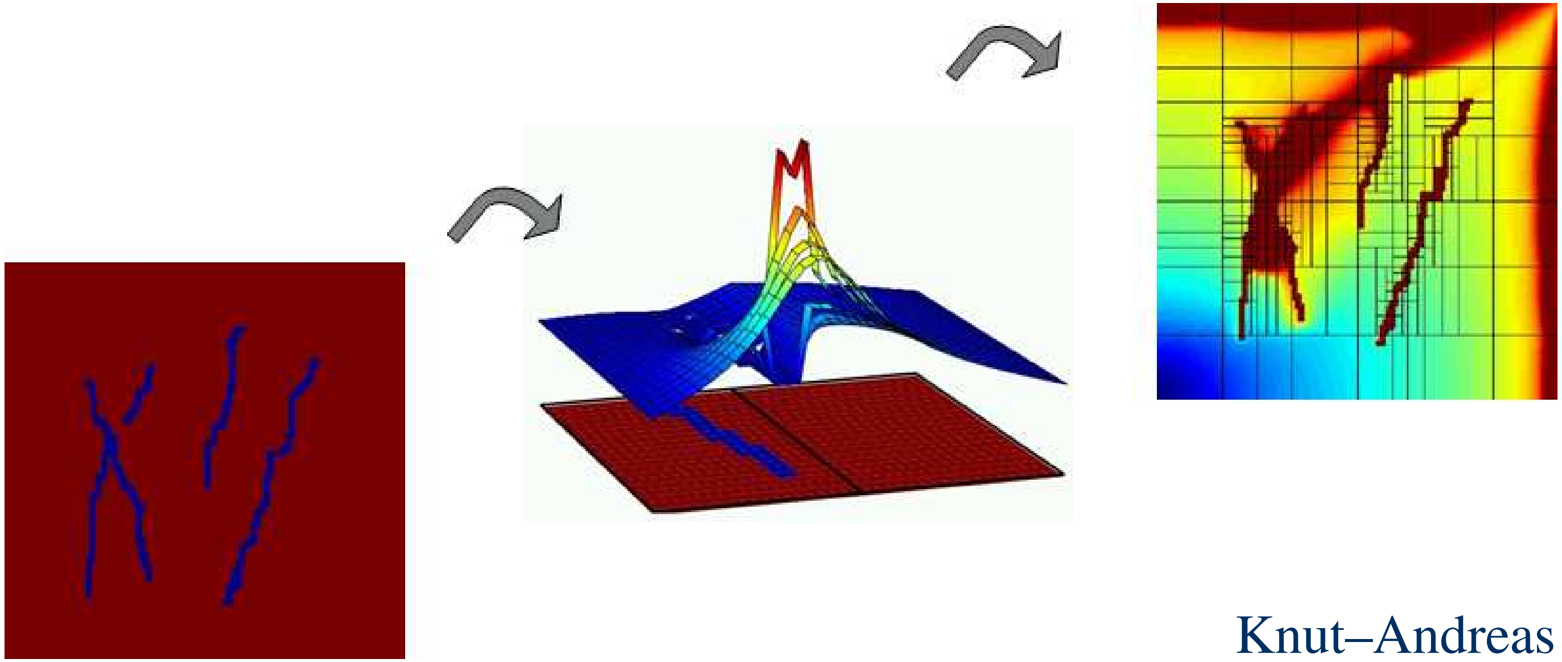
Multiscale Methods for Flow in Porous Media



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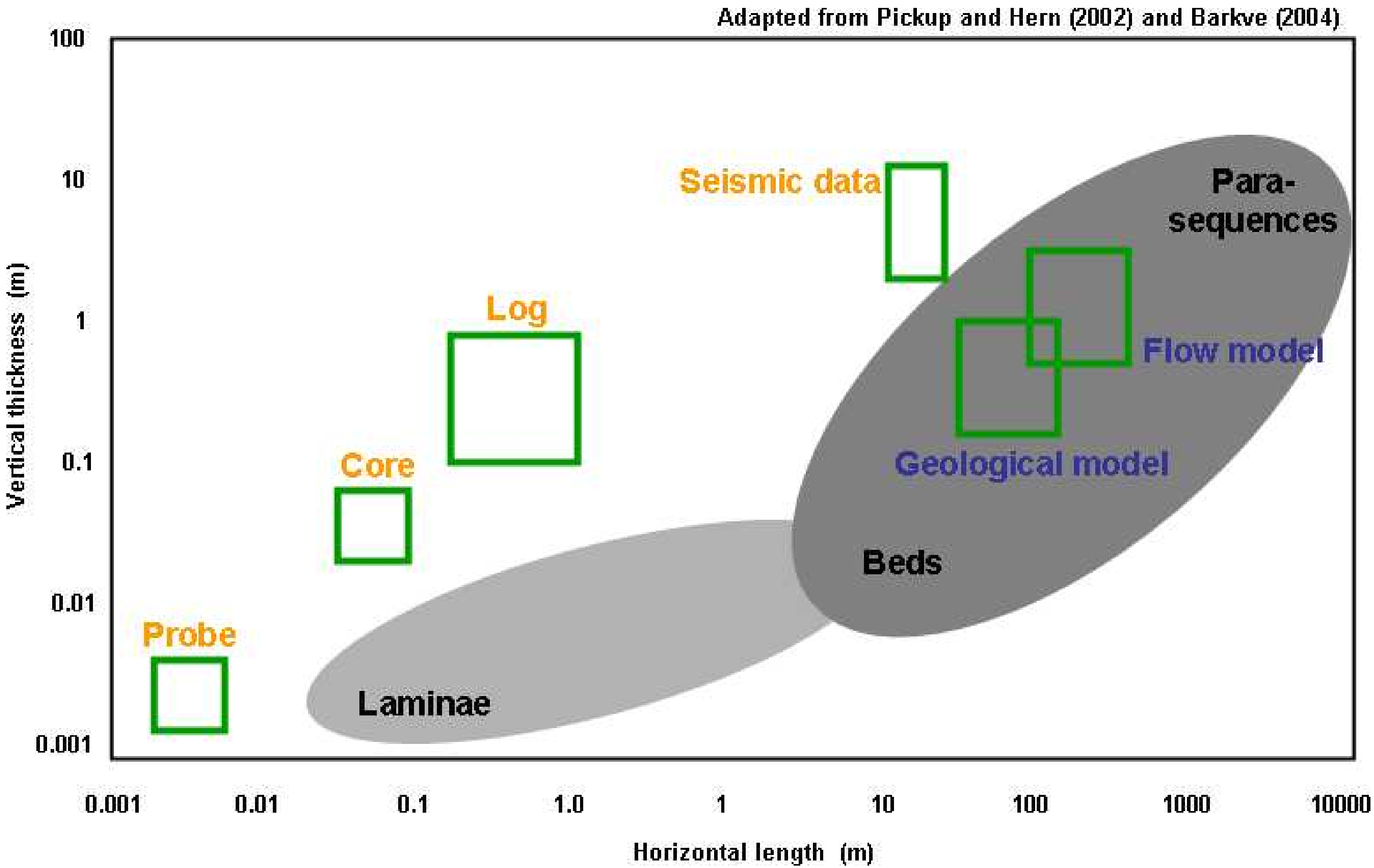
Scales in porous media

Porous media often have repetitive layered structures, but faults and fractures caused by stresses in the rock disrupt flow patterns.

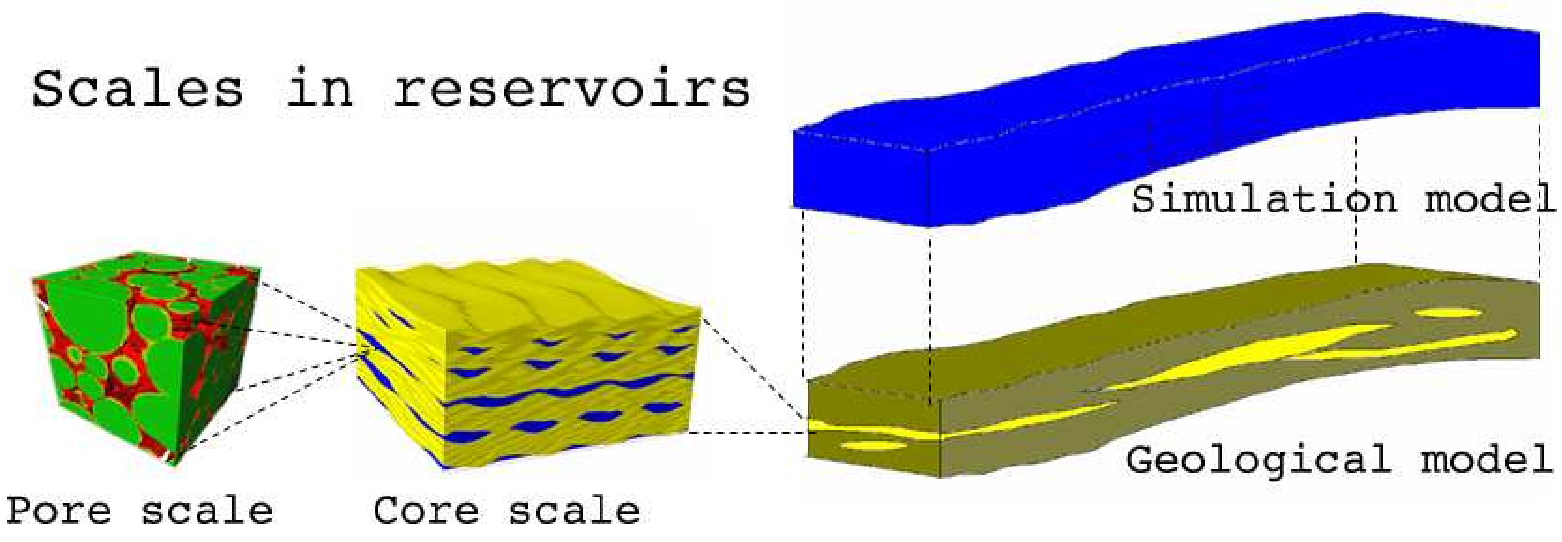
Scales in porous media, cont'd



Scales in porous media, cont'd

The scales that impact fluid flow in oil reservoirs range from

- the micrometer scale of pores and pore channels
- via dm–m scale of well bores and laminae sediments
- to sedimentary structures that stretch across entire reservoirs.



Reservoir simulation

Two-phase flow, modelled by continuity equation for each phase and Darcy's law

$$\phi \partial_t S_i + \nabla \cdot v_i = q_i, \qquad v_i = -k\lambda_i \nabla p_i$$

Model reformulation: pressure and saturation equation

$$-\nabla\big(k\lambda(S)\nabla p\big) = q, \qquad v = -k\lambda(S)\nabla p,$$

$$\phi \partial_t S + \nabla \cdot (v f(S)) = 0$$

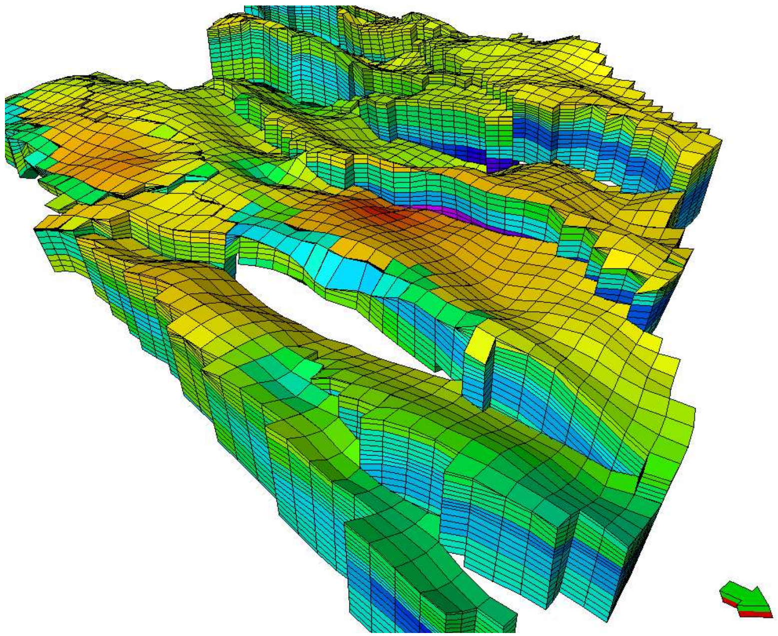
Need for fast (desktop) simulations for decision support:

predictions of production, history matching, ranking, uncertainty, process optimisation,...

Geo(logical) model

Geomodels consist of rock parameters k and ϕ.

- k spans many length scales and has multiscale structure,
- details on all scales impact flow



Gap between simulation and geomodels:

- High-resolution geomodels may have $10^7 - 10^9$ cells
- Conventional (FV/FD) simulators are capable of about $10^5 - 10^6$ cells

Traditional solution: upscaling

Upscaling the pressure equation

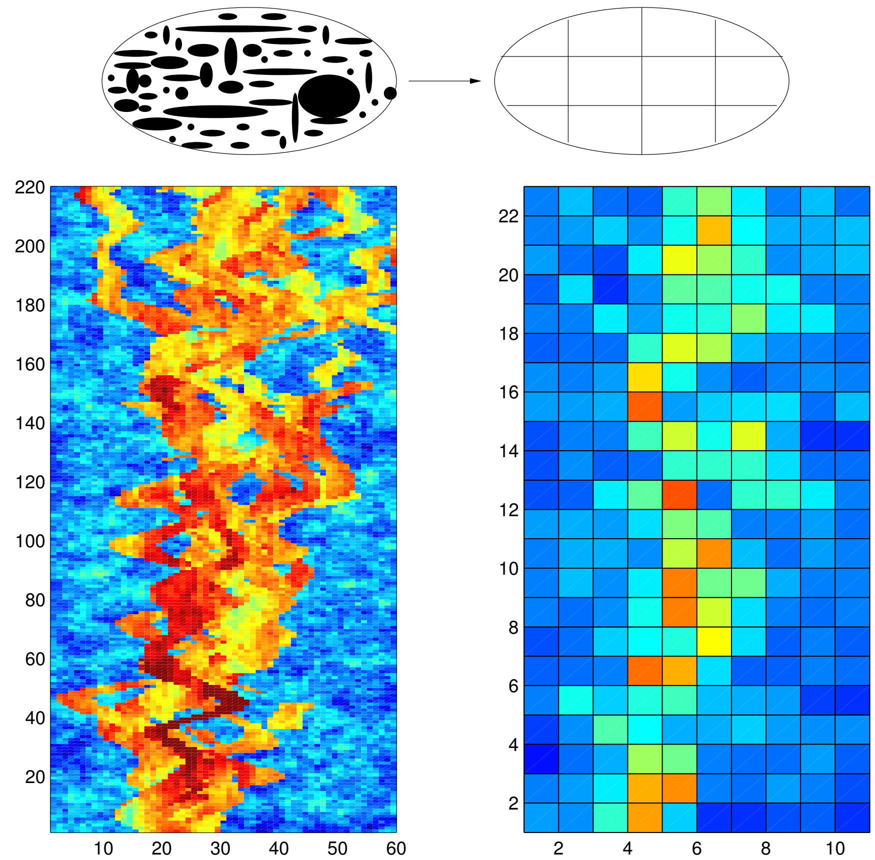
Assume that u satisfies the elliptic PDE:

$$-\nabla\big(a(x)\nabla u\big) = f.$$

Upscaling amounts to finding a new field $a^*(\bar{x})$ on a coarser grid such that

$$-\nabla\big(a^*(\bar{x})\nabla u^*\big) = \bar{f},$$

$$u^* \sim \bar{u}, \qquad q^* \sim \bar{q}\ .$$

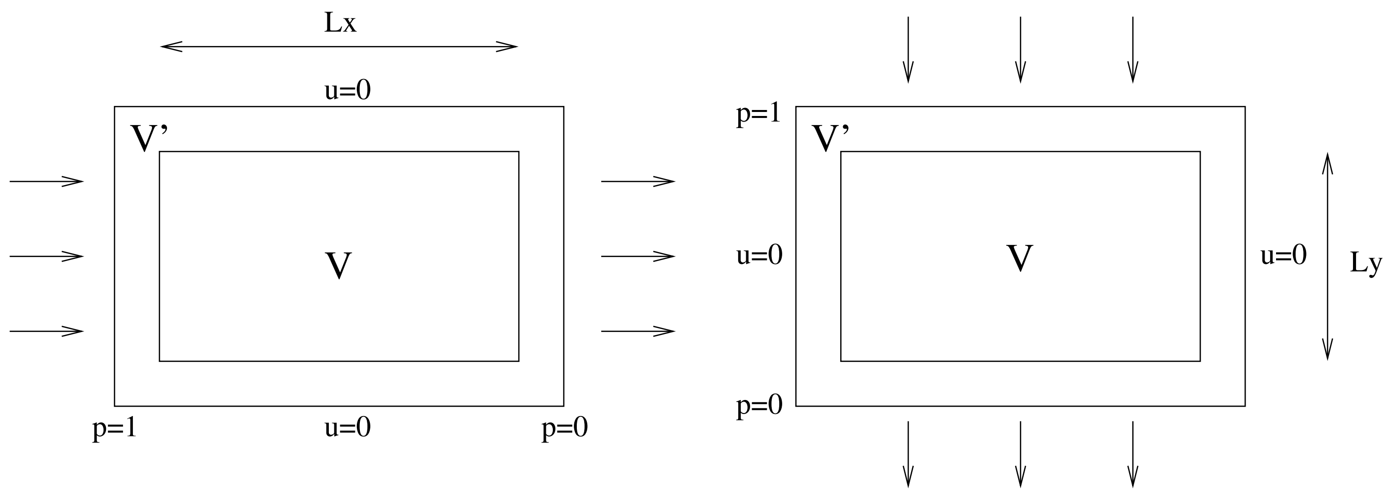


Here the overbar denotes averaged quantities on a coarse grid.

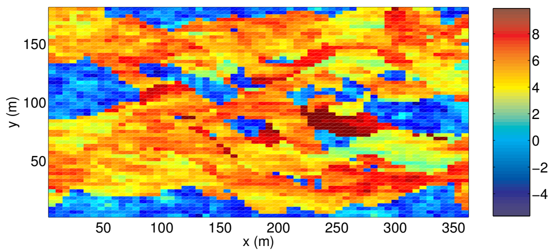
Upscaling permeability

How do we represent fine-scale heterogeneities on a coarse scale?

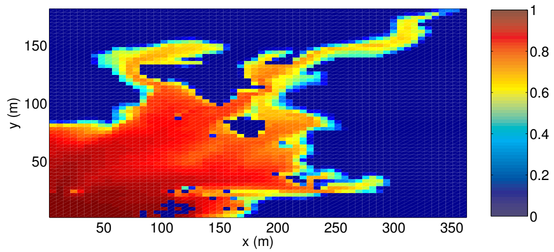
- Arithmetic, geometric, harmonic, or power averaging $\left(\frac{1}{|V|}\int_V a(x)^p\, dx\right)^{1/p}$
- Equivalent permeabilities ($a^*_{xx} = -Q_x L_x / \Delta P_x$)



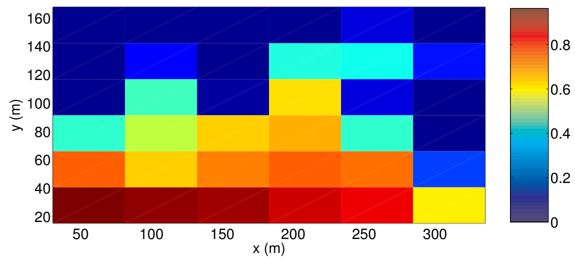
Multiscale simulation rather than upscaling?



permeability



reference solution



upscaled solution

- Upscaling the geomodel is not always the answer
  - Loss of details and lack of robustness
  - Bottleneck in the workflow
- Need for fine-scale computations?
- In the future: need for multiphysics on multiple scales?

Mixed formulation of the pressure equation:

Find $(v, p) \in H_0^{1,\text{div}} \times L^2$ such that

$$\int (\lambda K)^{-1} u \cdot v dx - \int p \nabla \cdot u dx = 0, \qquad \forall u \in H_0^{1,\text{div}},$$

$$\int l \nabla \cdot v dx = \int q l dx, \quad \forall l \in L^2.$$

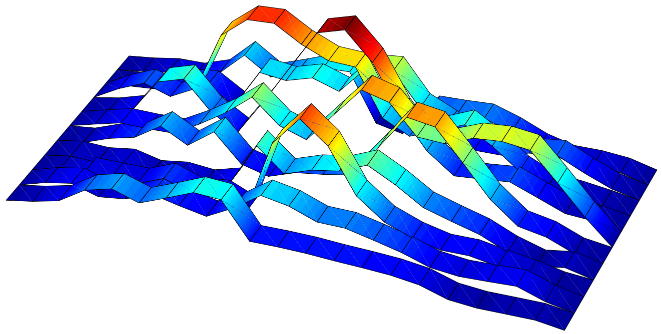
Multiscale discretisation: Seek solutions in low-dimensional subspaces

$$U^{ms} \subset H_0^{1,\text{div}} \text{ and } V \in L^2,$$

where local fine scale properties are incorporated into the basis functions.

Multiscale mixed finite element method

Velocity basis functions $\Longrightarrow$ Coarse grid approximation space



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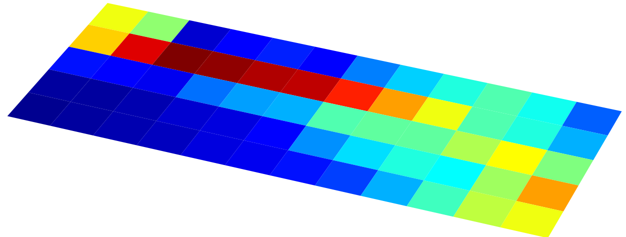
Coarse scale velocity



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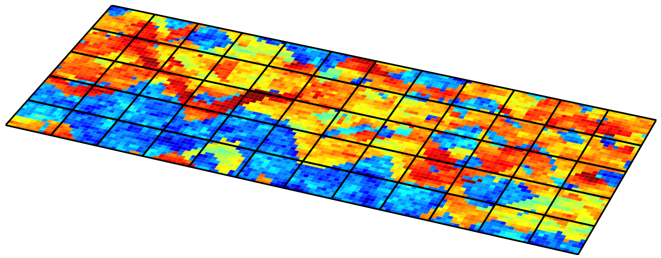
Geomodel



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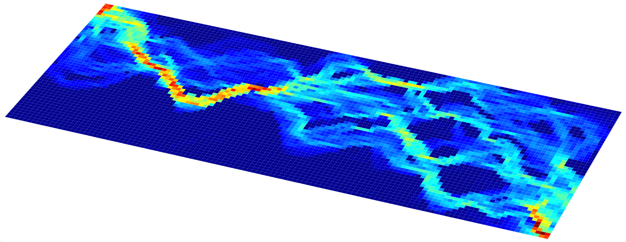
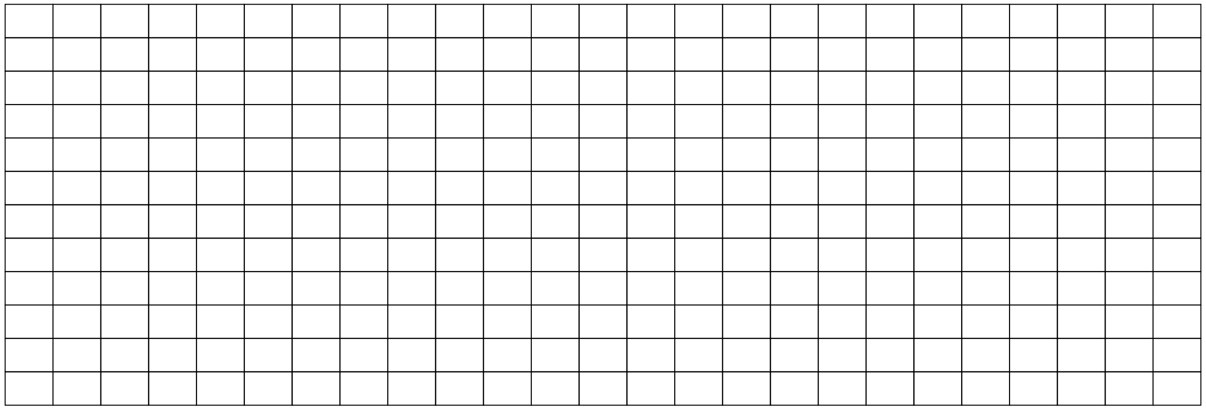
Fine scale velocity



For the MsMFEM the fine scale velocity field is a linear superposition of the base functions: $v = \sum_{ij} v_{ij}^{*} \psi_{ij}$.

Grids and basis functions

We assume we are given a *fine* grid with permeability and porosity attached to each fine grid block.



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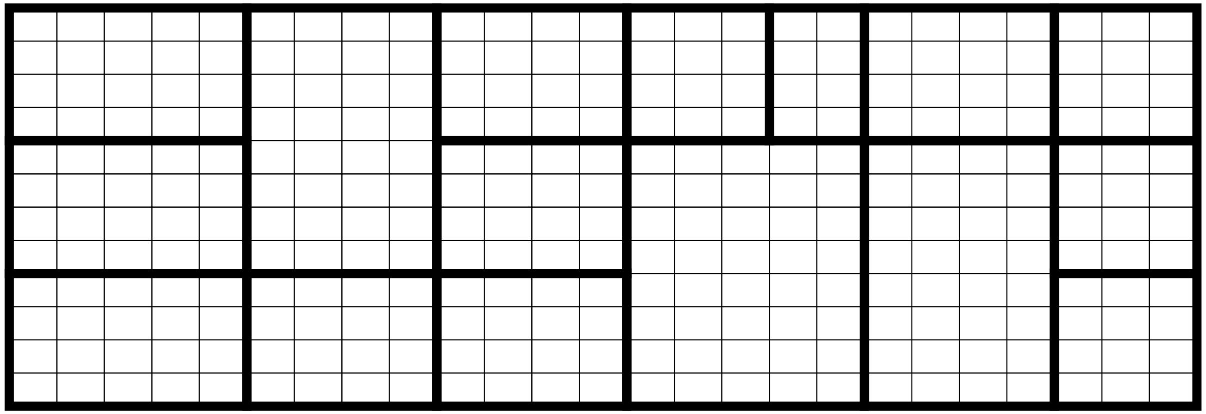

We construct a *coarse* grid, and choose the discretisation spaces V and U^{ms} such that:

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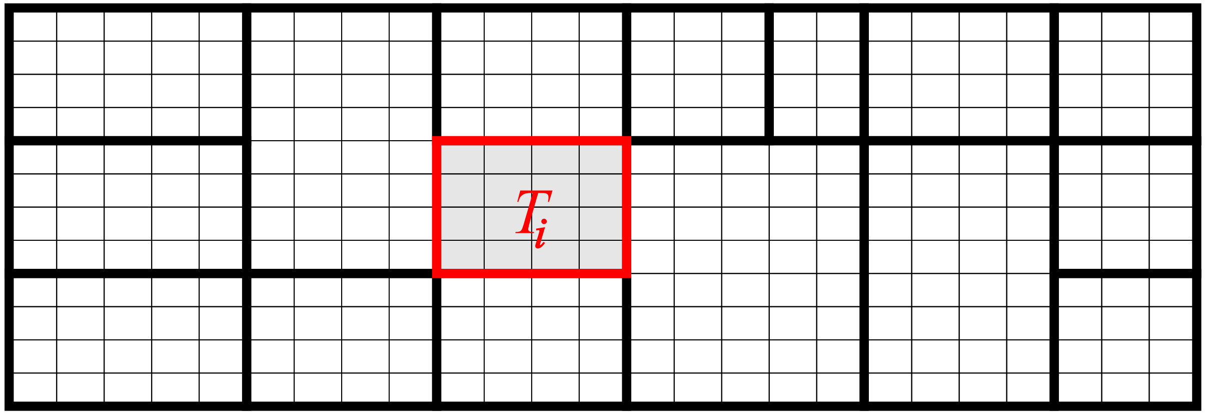

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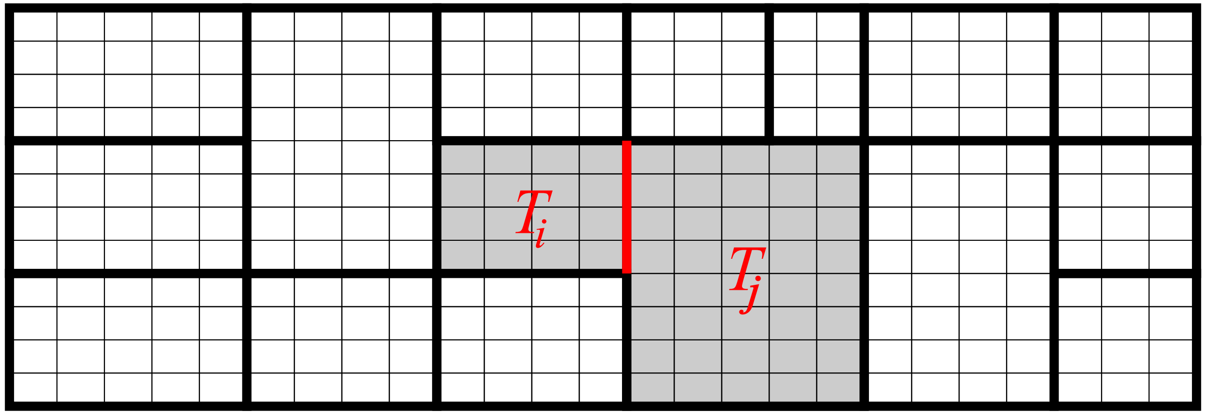
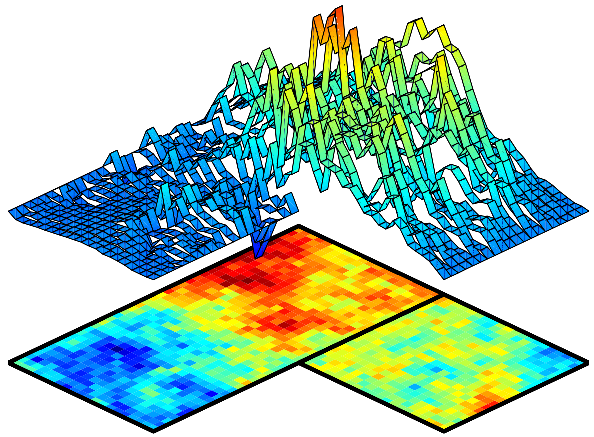


We construct a *coarse* grid, and choose the discretisation spaces V and U^{ms} such that:

- For each coarse block T_i, there is a basis function $\phi_i \in V$.
- For each coarse edge Γ_{ij}, there is a basis function $\psi_{ij} \in U^{ms}$.

Basis functions for the velocity field

For each coarse edge Γ_{ij} define a basis function



$$\psi_{ij} : T_i \cup T_j \to R^2$$

with unit flux through Γ_{ij}, and no flow across $\partial(T_i \cup T_j)$.

We use $\psi_{ij} = -\lambda K \nabla \phi_{ij}$ with

$$\nabla \cdot \psi_{ij} = \begin{cases} w_{ij}(x), & \text{for } x \in T_i, \\ -w_{ij}(x), & \text{for } x \in T_j, \\ 0, & \text{otherwise,} \end{cases}$$

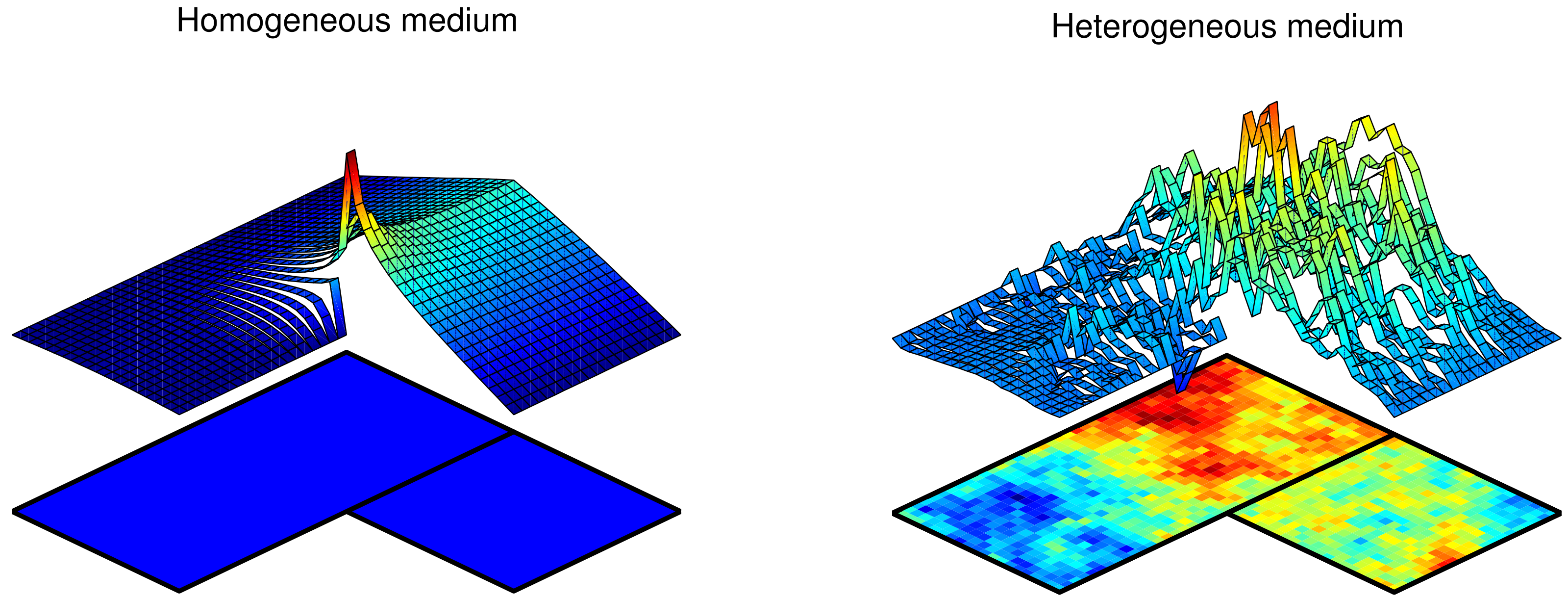
with boundary conditions $\psi_{ij} \cdot n = 0$ on $\partial(T_i \cup T_j)$.

Global boundary conditions: specify $v|_{\Gamma_{ij}}$ if known initially

MsMFEM velocity basis

Homogeneous coefficients and rectangular support domain:
basis function = lowest order Raviart-Thomas basis

MsMFEM = extension to cases with subscale variation in coefficients and non-rectangular support domain



x-component of the 2D basis function

MsMFEM properties

Multiscale:
Incorporates small-scale effects into coarse-scale solution

Conservative:
Mass conservative on a subgrid scale

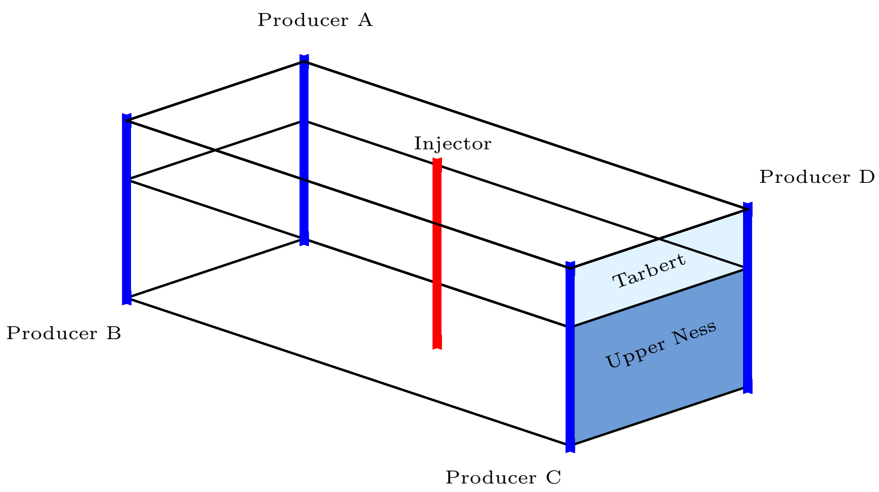
Scalable:
Well suited for parallel implementation since basis functions are processed independently

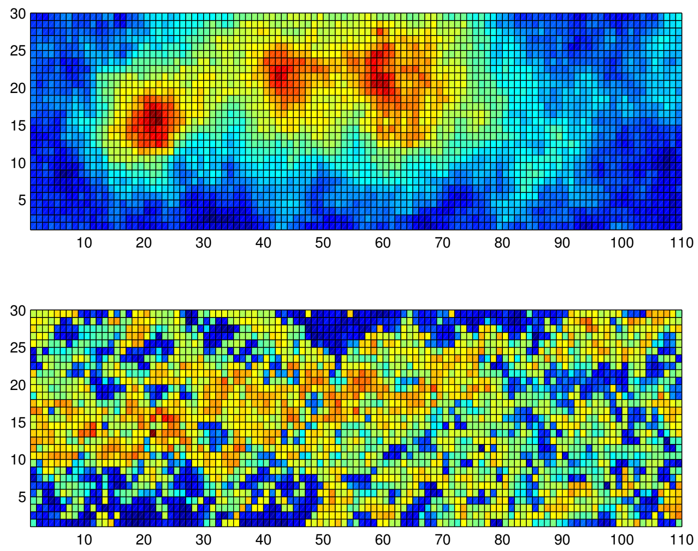
Flexible:
No restrictions on subgrids and subgrid numerical method. Few restrictions on the shape of the coarse blocks

Fast:
The method is fast when avoiding regeneration of (most of) the basis functions at every time step

Numerical examples: SPE 10th CSP

Industrial benchmark for upscaling:



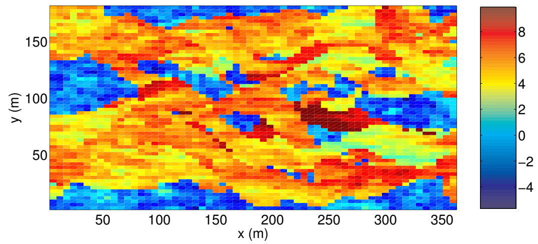


$60 \times 220 \times 85$ grid, $\lambda_w \propto S^2$, $\lambda_o \propto (1-S)^2$, $\mu_o = 3.0$ cP, $\mu_w = 0.3$ cP

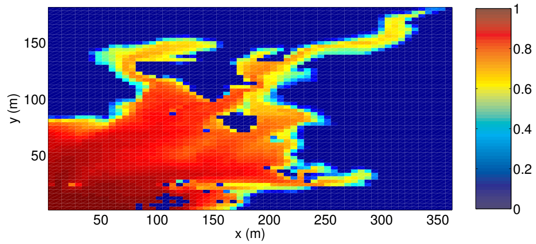
2000 days of production at bhp 4000 psi. Injection: 5000 bbl/day.

In the following we consider both 2D subsets and the full 3D case.

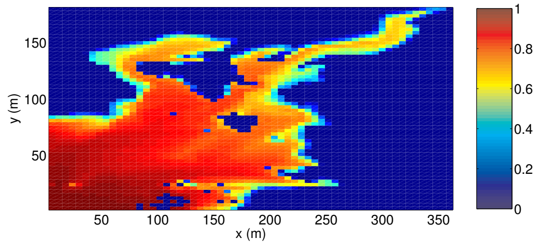
2D section from Upper Ness



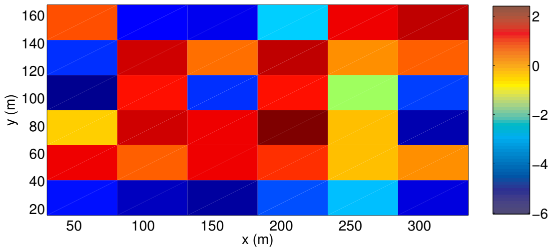
permeability



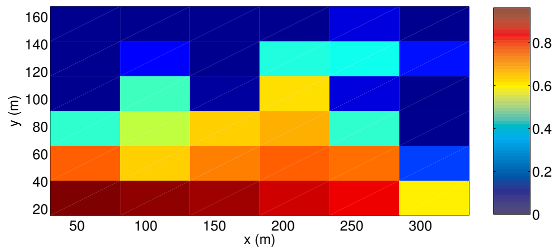
reference solution



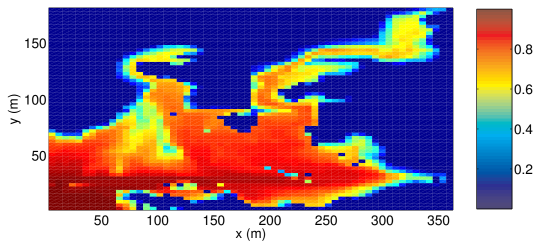
MsMFEM + SL



upscaled perm.



upscaled solution

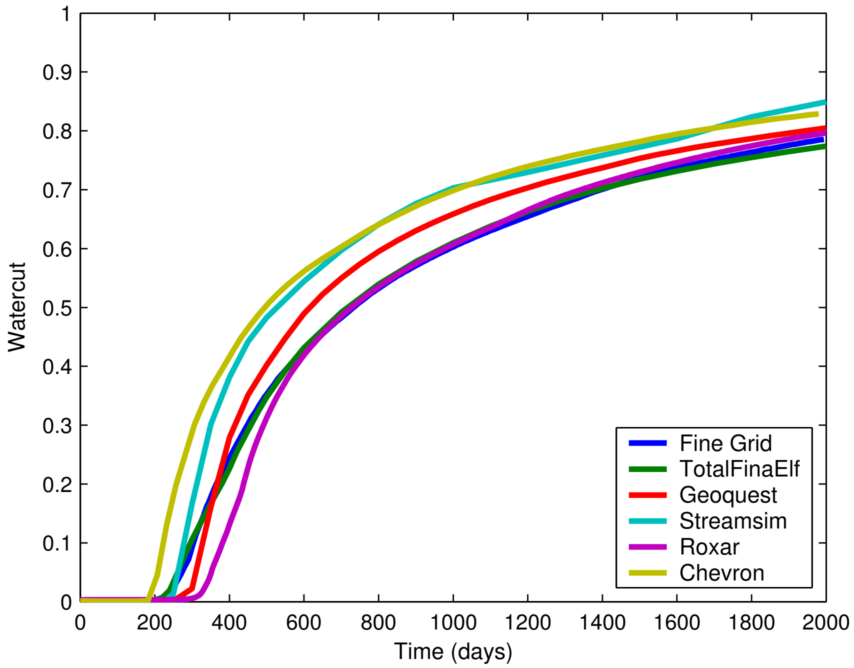


nested gridding

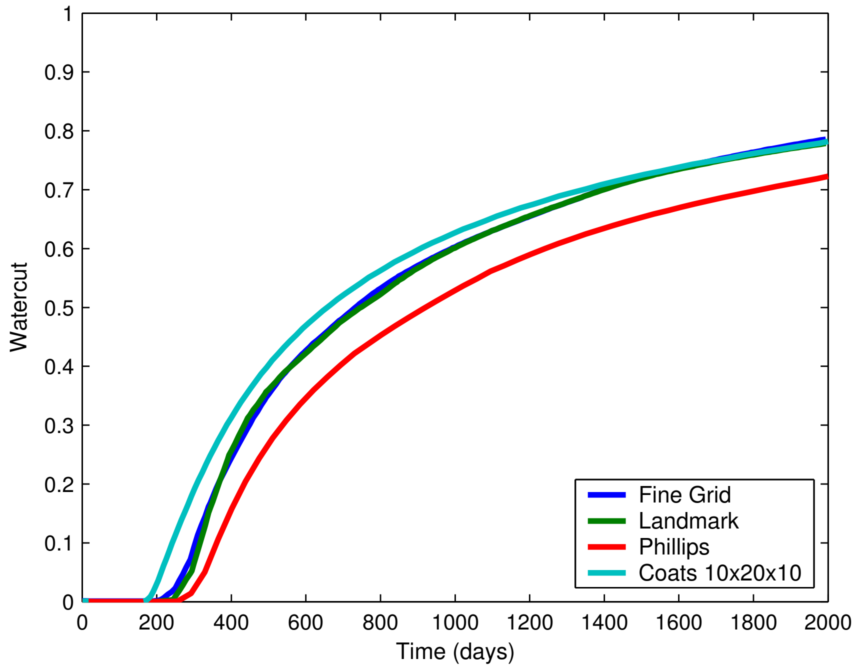
Nested gridding: upscale $(k\lambda)$, solve for pressure and then subgrid problem for velocities, i.e., a method with subscale resolution but *without coupling* between the fine and coarse scale

The SPE benchmark results

Producer A:

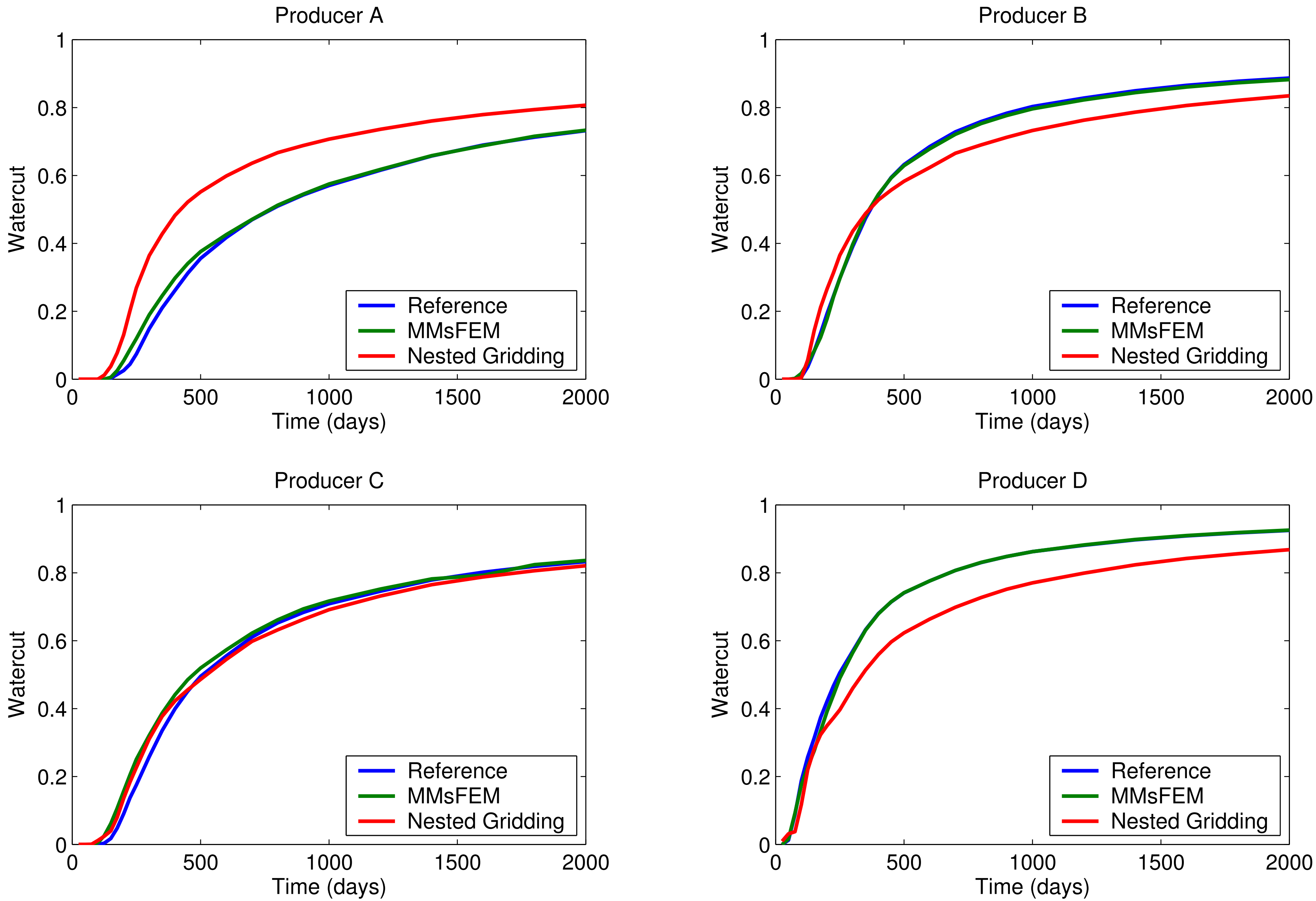


nonpseudo upscaling

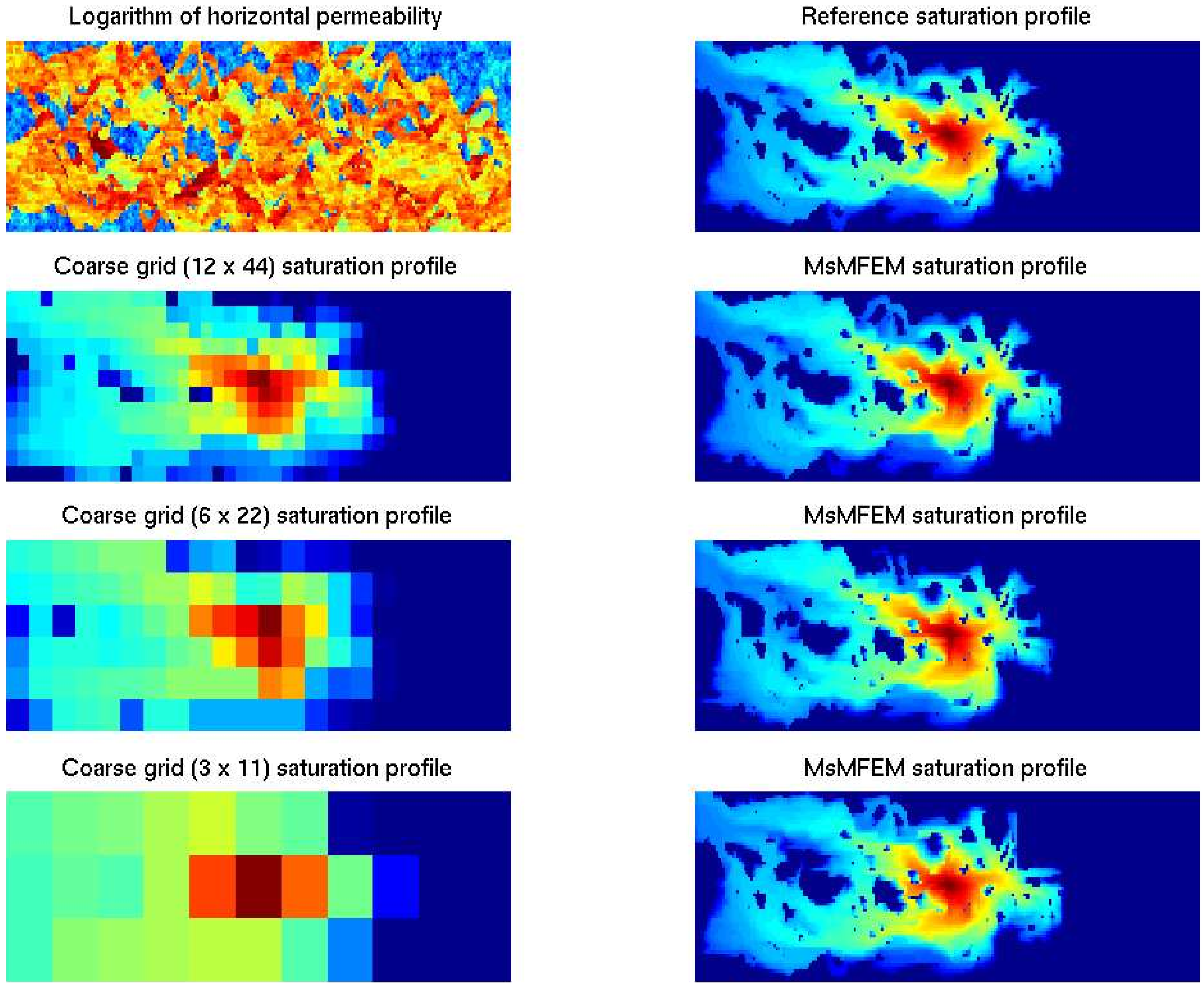


pseudo upscaling

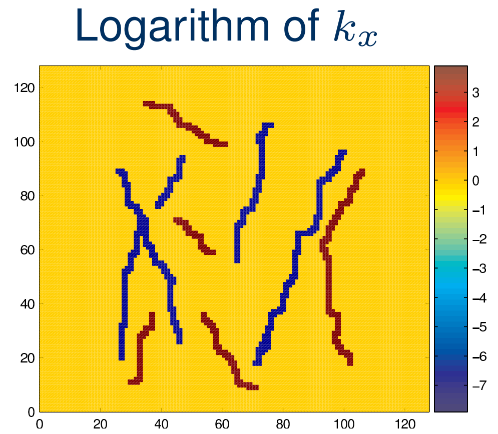
MsMFEM results (coarse grid: 5 x 11 x 17)

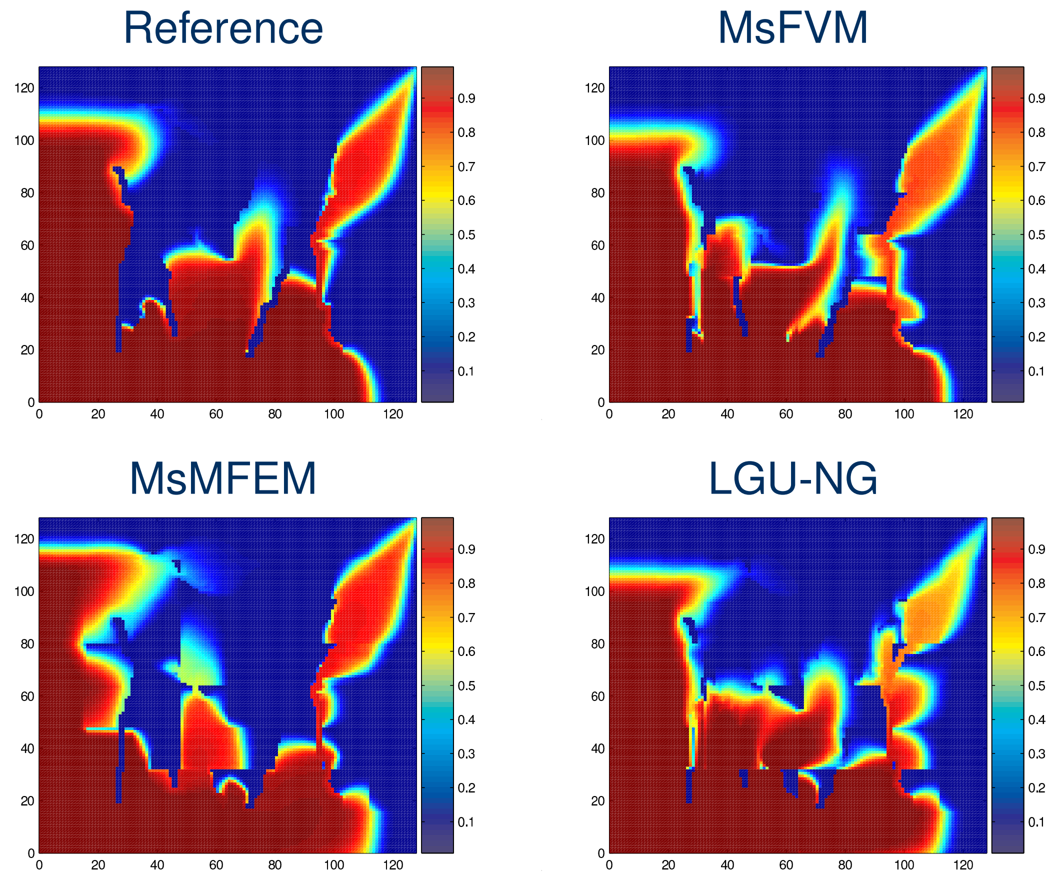


Robust wrt coarse-grid size



Resolves strongly heterogeneous structures

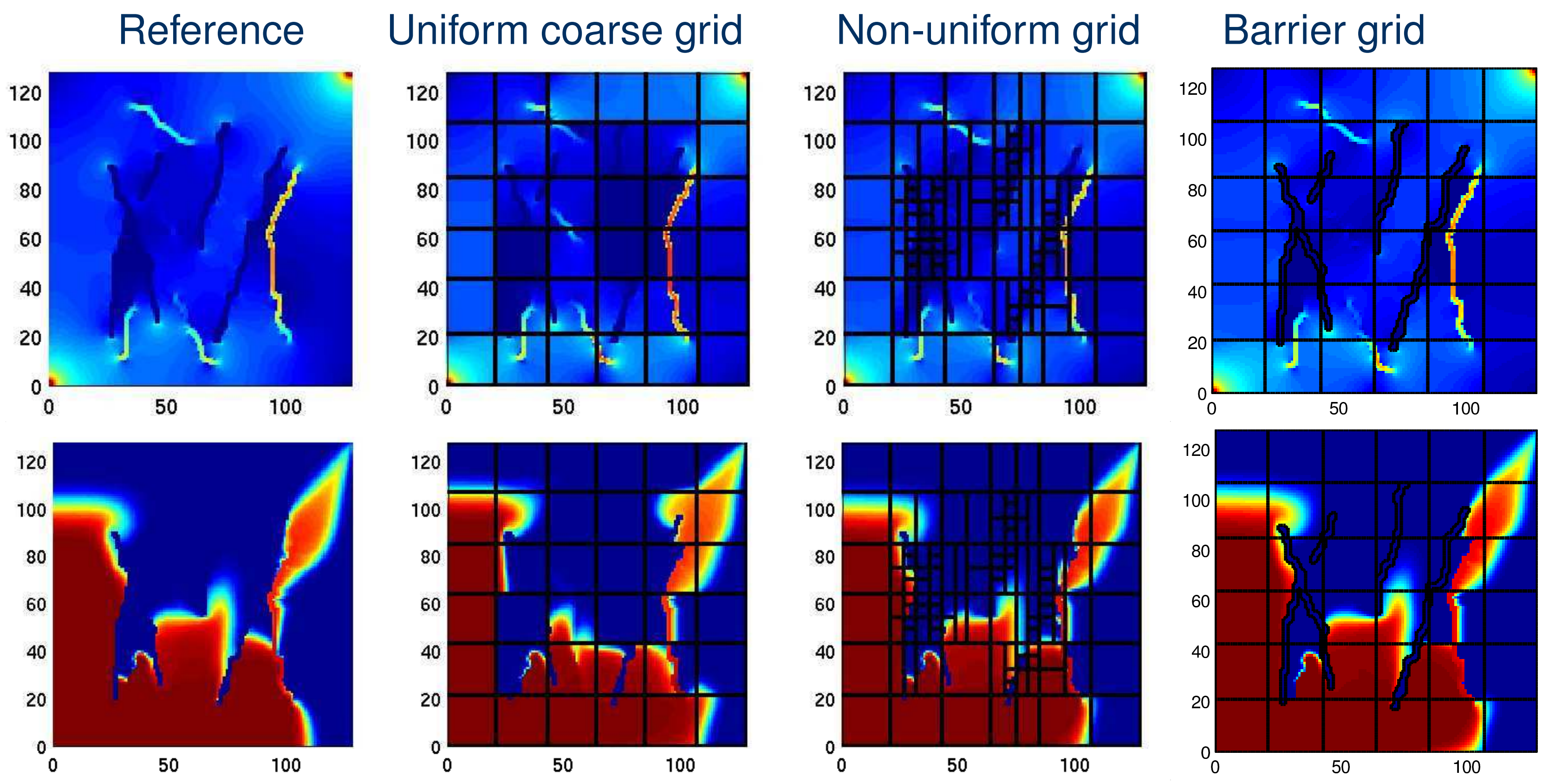




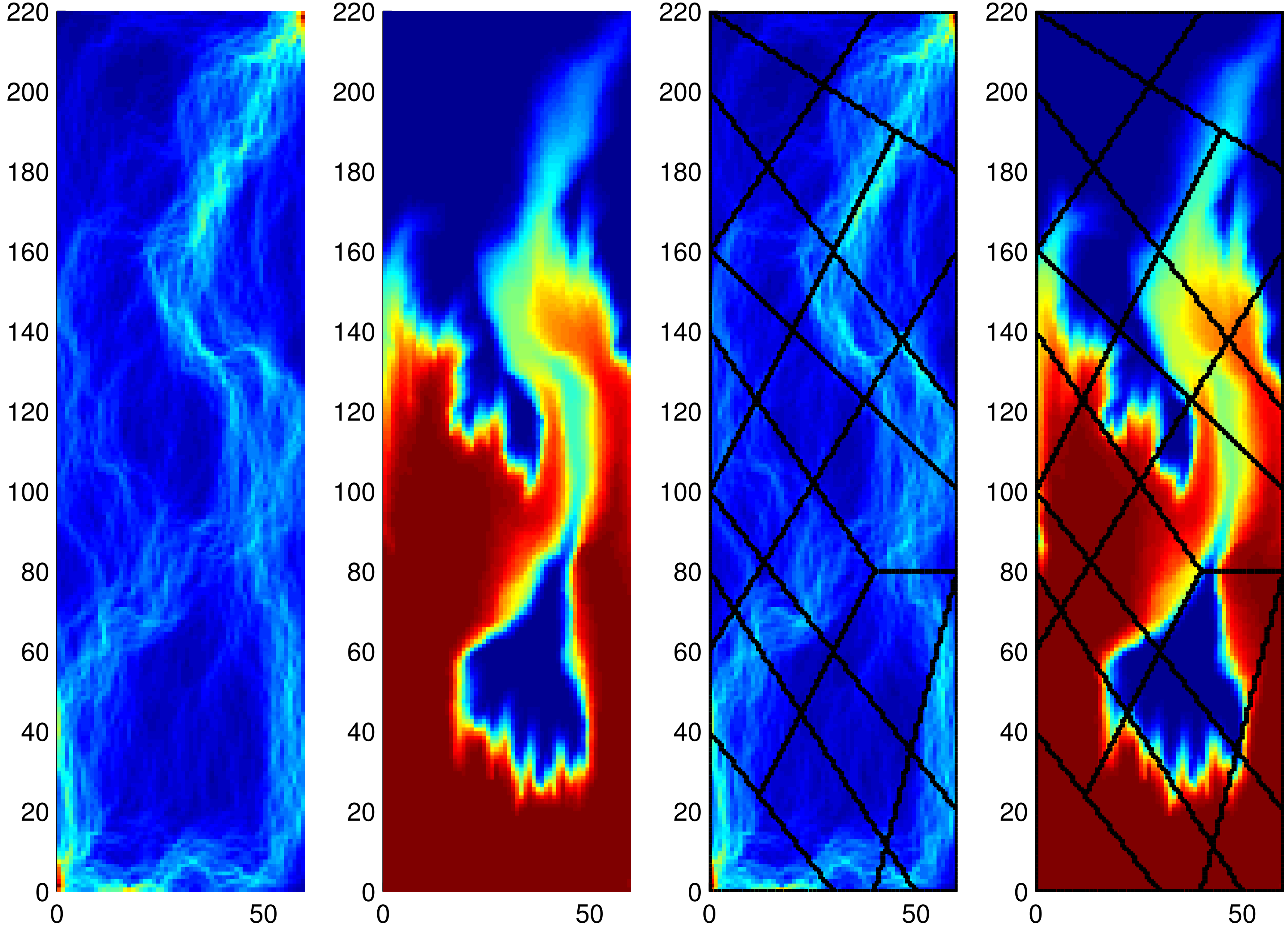
$$k_{\text{red}} = 10^4$$
$$k_{\text{yellow}} = 1$$
$$k_{\text{blue}} = 10^{-8}$$

Coarse grid = 8×8.

Grid refinement is straightforward



Irregular and ustructured grids



Irregular and ustructured grids, cont'd