Streamlines and a Multiscale Method

Towards Scalable, Robust and Fast Reservoir Simulation

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SINTEF Applied Mathematics

Motivation

- Gap between geological and reservoir simulation models.

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  Modern geostatistical methods can produce models of size $10^7 - 10^9$, which currently is well out of reach for any reservoir simulator.

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Upscaling is often a manual and very time-consuming procedure. Many methods exist, but no universal approach. Aslo, upscaling inherently means loss of fine-scale information that may effect the global flow behaviour.

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Fast simulation is necessary not only for large models, but also for special applications like history-matching, uncertainty assessment, and process optimization.

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- Well-known, fast simulation method which allows million grid-block simulations on single workstations.
- Works by convecting the phase saturations along streamlines given a mass conservative velocity field.

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- Recent approach for solving elliptic equations with strongly heterogeneous coefficients.
- Capable of producing conservative velocity fields at multiple scales.

Talk Outline

- Mathematical model
- The streamline method
- The multiscale method
- Example
- Concluding remarks

Assumptions

The simple model considered here covers two-phase flow including gravity, but disregards the following effects:

- Compressibility
- Dispersivity
- Miscibility
- Thermal effects
- Reactive terms

Mathematical Model

Mass balance and Darcy's law yields:

$$-\nabla \cdot \vec{u} = q,$$

and,

$$\phi \frac{\partial S_w}{\partial t} + \vec{u} \cdot \nabla f_w + \nabla \cdot \vec{G}_w = q f_w,$$

where $\vec{u}$ is the total Darcy velocity,

$$\vec{u} = \vec{\vec{K}} \cdot (\lambda_t \nabla P + \lambda_g \nabla D).$$

The Streamline Method

- Based on an IMPES strategy for solving the flow equations.

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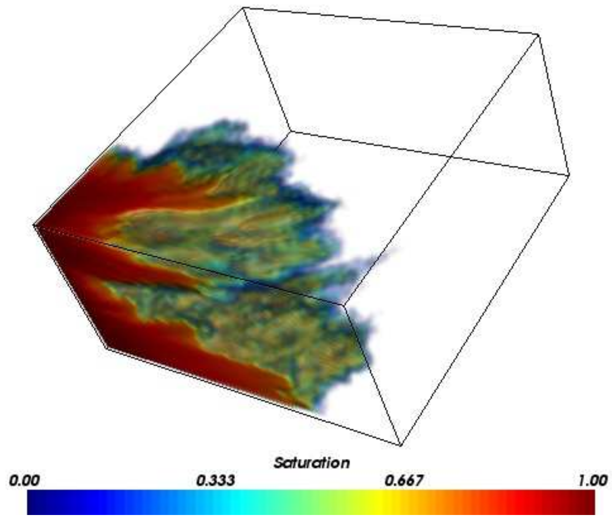
  - IMplicit Pressure, Explicit Saturation.
  - Decouples the flow equations by simply evaluating initial phase mobilities and solving the pressure equation separately.
  - Assumes constant pressure during a time step to be able to move the phase saturations forward in time.

The Streamline Method

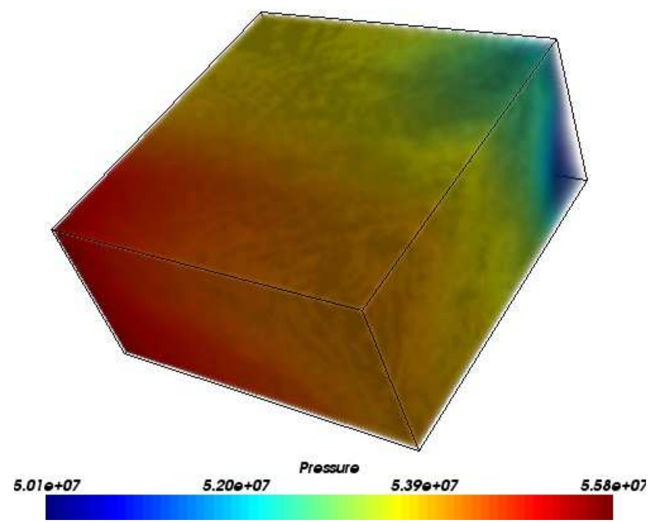
- Based on an IMPES strategy for solving the flow equations.
- The full 3D saturation equation is decoupled into multiple 1D equations to be solved along streamlines.

Streamline Steps



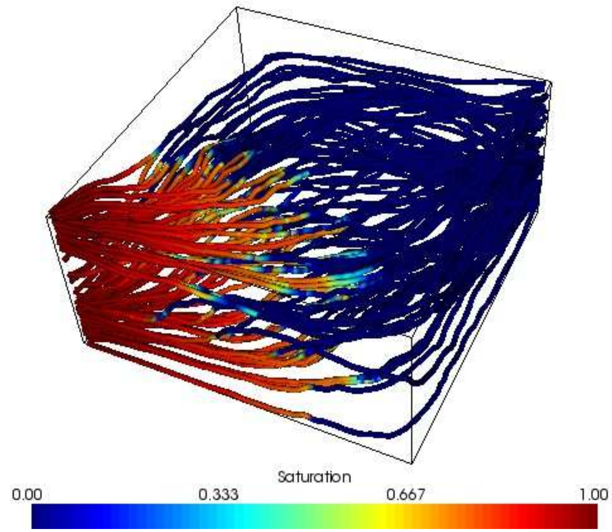
The starting point is an initial saturation field.

Streamline Steps



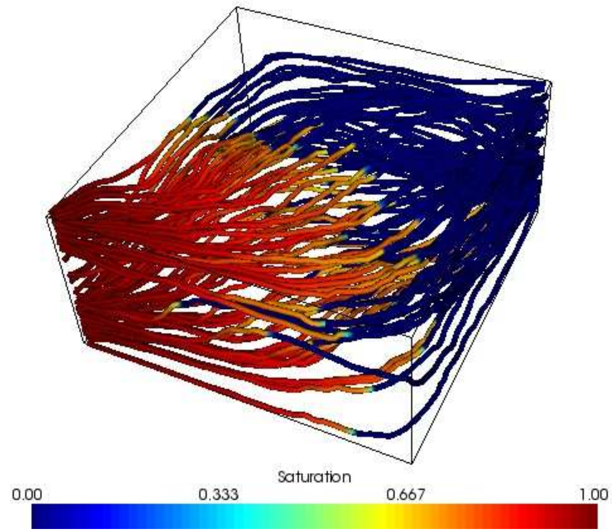
The pressure is computed using the initial saturations to evaluate the mobility terms.

Streamline Steps



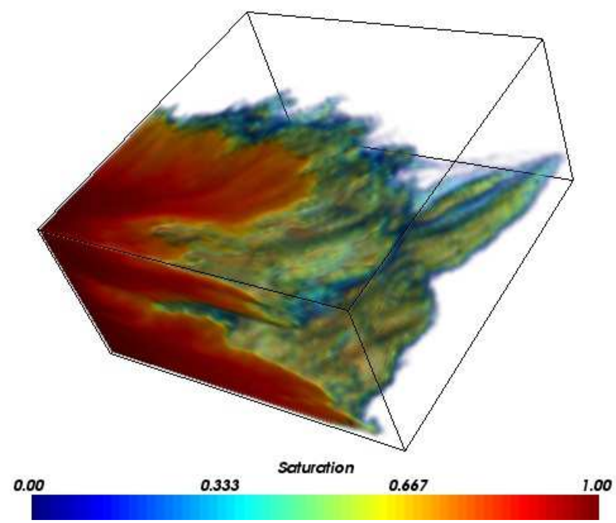
The pressure defines a velocity field and the streamline are traced from injectors to producers while picking up the grid block saturations.

Streamline Steps



Saturations are moved forward along the streamlines under the assumption that the streamlines remain fixed during the time step.

Streamline Steps



Finally the streamline saturations are mapped back onto the grid to yield a new saturation field, and the process may now be repeated.

Properties

- Speed.

Properties

- Speed.

  The method is very fast and allows simulation of million grid block models on single workstations.

Properties

- Speed.
- Scalability.

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  Low memory requirements and completely independent processing of streamlines makes the streamline method scalable both on serial and parallel computer architectures.

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- Also worth noting...

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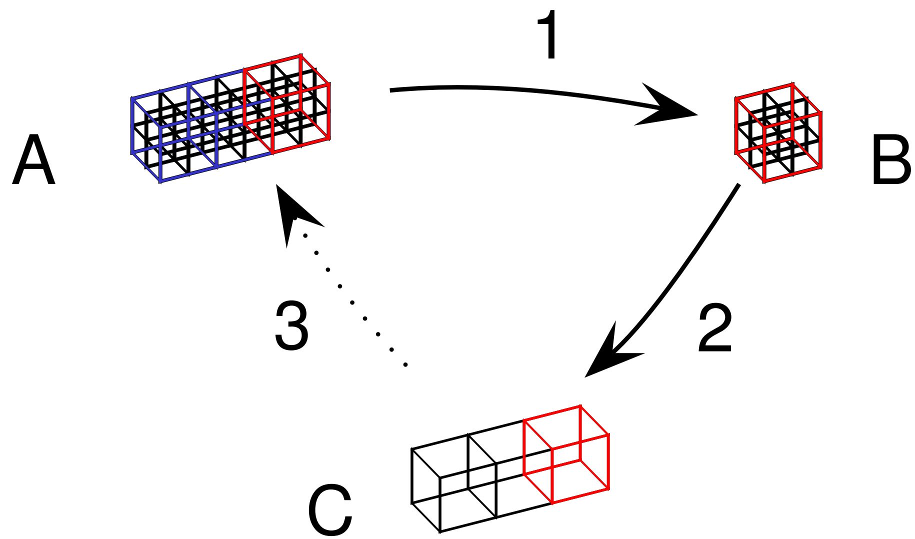
The method is not restricted to the simple model used here, and has been successfully applied to multiphase, dispersive, compositional displacement. Has also been used on unstructured grids.

Multiscale



Schematic view of a multiscale method.

MMsFEM

A Mixed Multiscale Finite Element Method

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$$-\nabla \cdot \vec{u} = -\nabla \cdot \vec{\vec{K}} \cdot (\lambda_t \nabla P + \lambda_g \nabla D) = q$$

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A Mixed Multiscale Finite Element Method

$$-\nabla \cdot \vec{u} = -\nabla \cdot \vec{\vec{K}} \cdot (\lambda_t \nabla P + \lambda_g \nabla D) = q$$

Find $(\vec{u}_h, P_h) \in \mathcal{V}_h \times \Pi_h$ *such that,*

$$\int_\Omega (\vec{\vec{K}}\lambda_t)^{-1}\, \vec{u}_h \cdot \vec{v}_h \,\mathrm{d}\vec{x} = \int_\Omega P_h \,\nabla \cdot \vec{v}_h \,\mathrm{d}\vec{x} - \int_\Omega \frac{\lambda_g}{\lambda_t} D\, \nabla \cdot \vec{v}_h \,\mathrm{d}\vec{x} \quad \forall \vec{v}_h \in \mathcal{V}_h,$$

$$\int_\Omega Q_h \,\nabla \cdot \vec{u}_h \,\mathrm{d}\vec{x} = \int_\Omega q Q_h \,\mathrm{d}\vec{x}, \qquad \forall Q_h \in \Pi_h.$$

MMsFEM

A Mixed Multiscale Finite Element Method

$\mathcal{V}_h =$ span $\{\vec{\psi}\}$, where $\vec{\psi}$ captures the local behaviour of the differential operator $L = -\nabla \cdot \vec{\vec{K}} \lambda_t \nabla$.

MMsFEM Basis

One possibility:

Let $\mathcal{K} = \{K\}$ be a partition (grid) of Ω and define the basis functions $\vec{\psi}_{ij}$ by,

$$(\nabla \cdot \vec{\psi}_{ij})_{|K} = -\nabla \cdot \vec{\vec{K}} \lambda_t \nabla \phi_{ij} = \begin{cases} \frac{1}{|K|}, & \text{in non-well blocks,} \\ \frac{q}{\int_K q \, \mathrm{d}\vec{x}} & \text{in well blocks,} \end{cases}$$

where each $\vec{\psi}_{ij}$ is associated with $\Gamma_{ij} = \partial K_i \cap \partial K_j$ and $\vec{\psi}_{ij} \cdot \vec{n} = \nu_{ij}$ on Γ_{ij} and zero elsewhere on the boundary. The boundary conditions ν_{ij} should reflect the heterogenities at the boundaries and the radial flow pattern near wells. Also they must be scaled to ensure compatibility.

MMsFEM Basis

For homogeneous coefficients the MMsFEM with the basis defined above reduces to the lowest order Raviart-Thomas mixed FEM (for quadrilateral elements).

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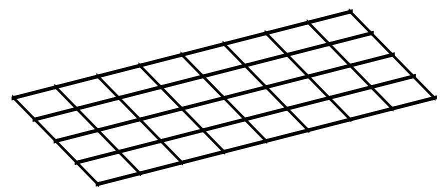
MMsFEM can therefore be viewed as an extension to the case where the coefficients can vary within each element.

MMsFEM Basis

2D Example:



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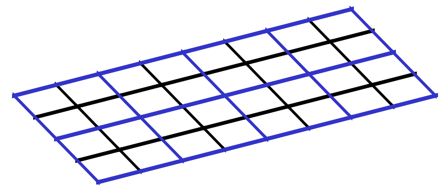

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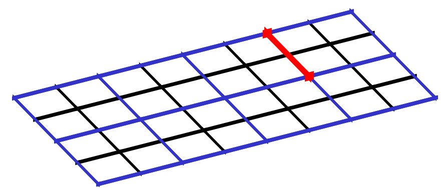

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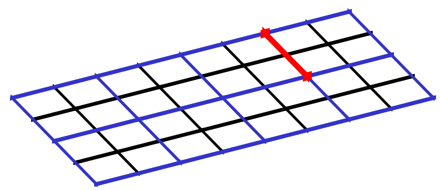

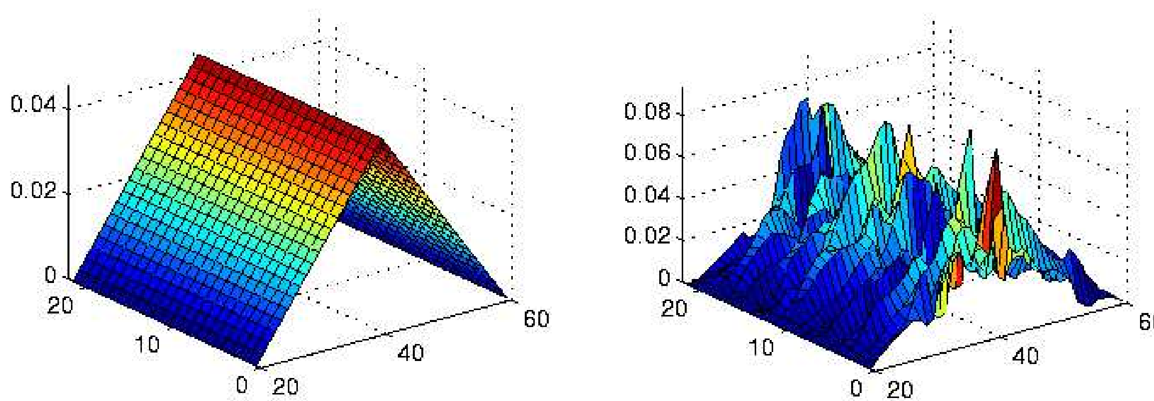

x-component of the 2D basis function for homogeneous and heterogeneous coefficients.

Properties

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The particular choice of basis described yields a mass-conservative fine grid velocity field which can be used for streamline tracing.

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  Thus the method can be viewed as a robust alternative to upscaling if computations are continued on the coarse grid.

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  The method puts no restrictions on the subgrids, and any numerical method may be used for the subgrid problems.

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  The basis functions are processed individually, thus the method is well suited for parallel implementation.

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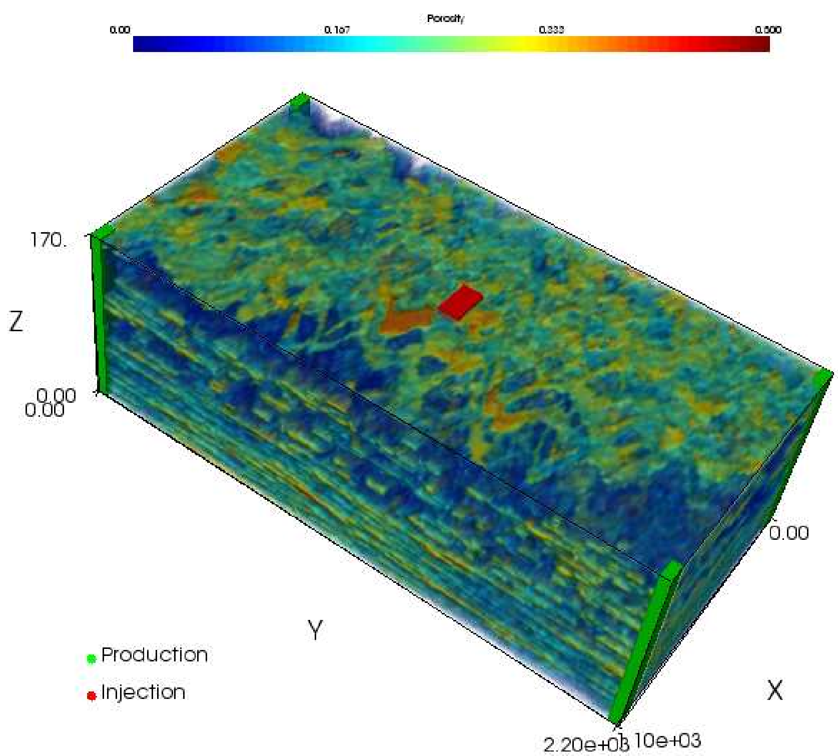
  Can be computationally efficient if recomuptation of the basis functions at every time step is avoided.

Example

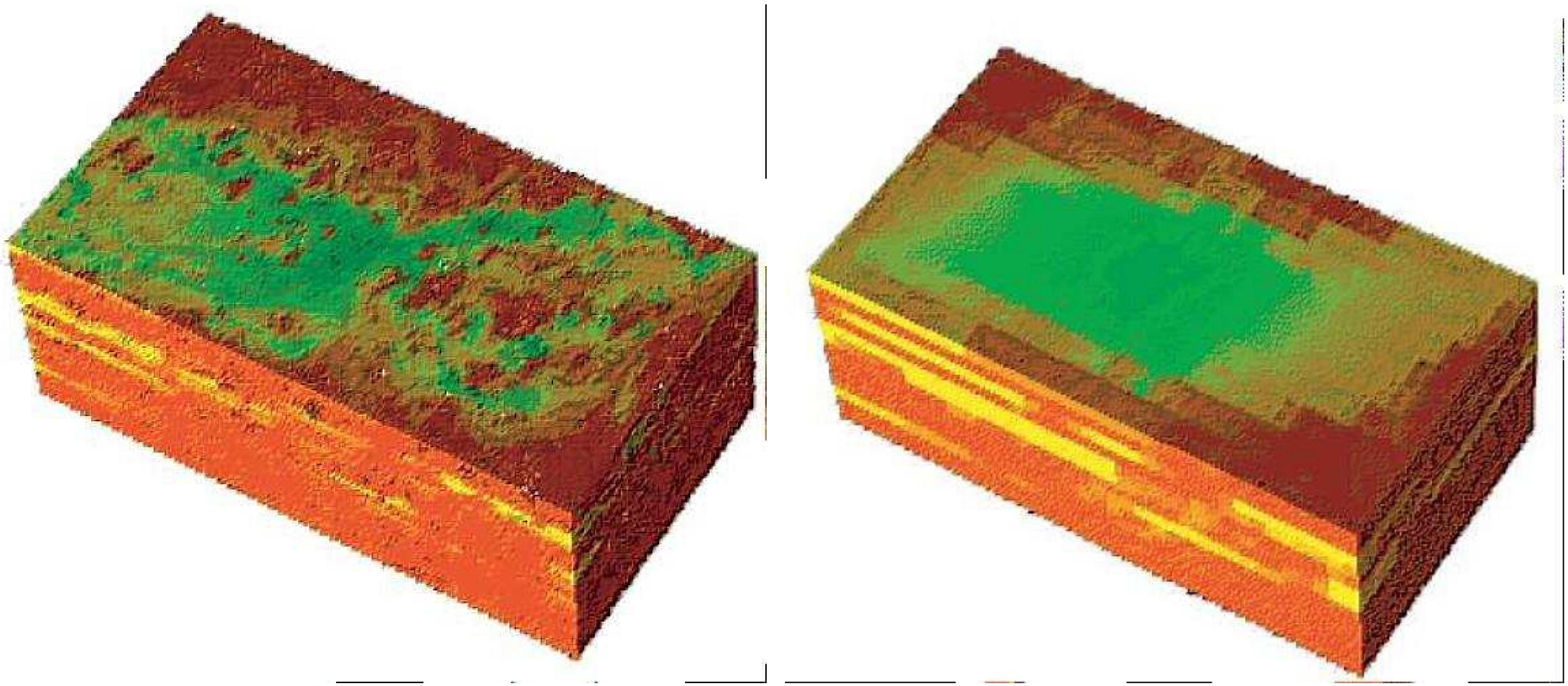
10th SPE Comparative Solution Project, Model II:



- 1200 x 2200 x 170 ft.
- 60 x 110 x 85 blocks.
- 5 x 11 x 17 coarse blocks.

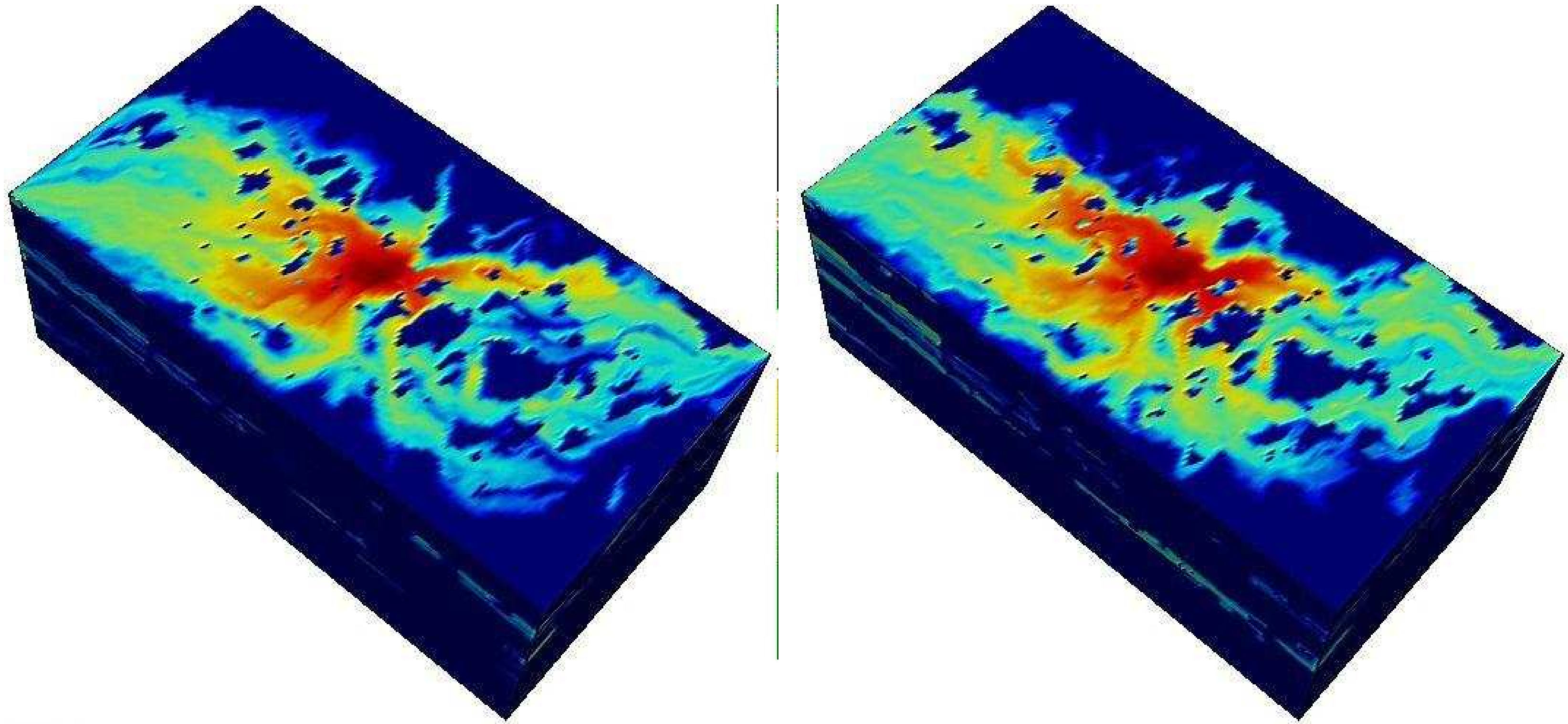
Too large for conventional reservoir simulators!

Example



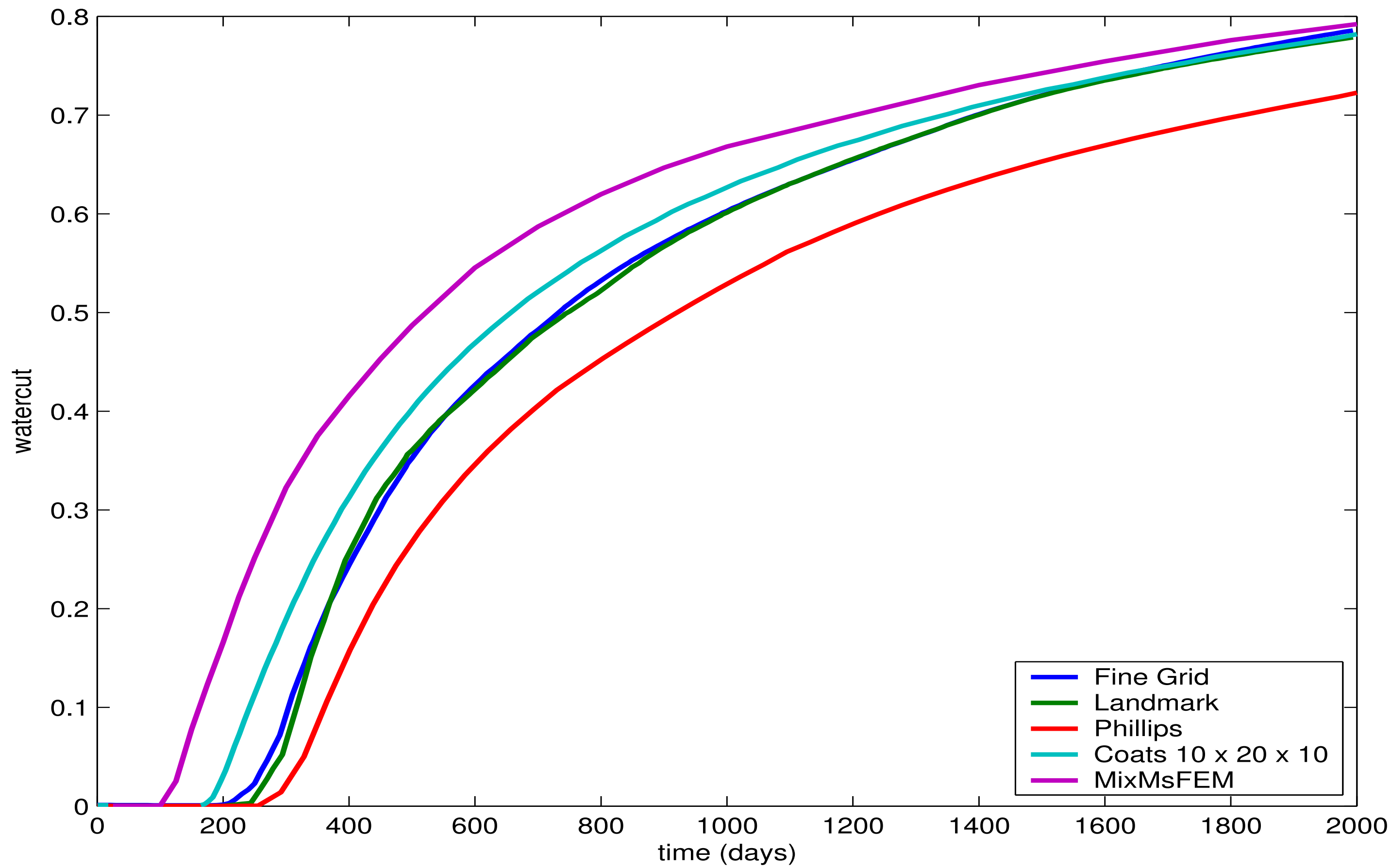
3DSL fine grid and upscaled solution after 800 days of simulation.

Example



Our fine grid solution (left) and MMsFEM/Streamline solution (right) after 800 days of simulation.

Example



Water-cut curves comparing MMsFEM with various upscaled solutions.

Concluding Remarks

- By combining the streamline method with MMsFEM we obtain an overall strategy which is very scalable and may help bridge the gap between geological and reservoir simulation models.

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- By combining the streamline method with MMsFEM we obtain an overall strategy which is very scalable and may help bridge the gap between geological and reservoir simulation models.
- In particular true for a parallel implementation, but experiments with adaptive basis recomputation have shown that typically less than 10% of the MMsFEM basis functions need to be recomputed at every time step. Thus, there is a great potential for accelerating also serial computations.

Concluding Remarks

- Even our plain implementation of the MMsFEM provides alternative to upscaling for large reservoir models. We believe the MMsFEM will prove to be more robust because of its inherent flexibility and firm mathematical foundation.

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- Even this plain implementation of the MMsFEM provides alternative to upscaling for large reservoir models. We believe the MMsFEM will prove to be more robust because of its inherent flexibility and firm mathematical foundation.
- Moreover, with more information available, for instance knowledge of the initial flow pattern, better boundary conditions can be developed, which improves the accuracy of the MMsFEM considerably.