MMsFEM and Streamlines

Combining a mixed multiscale FEM with streamline simulation for enhanced reservoir performance prediction on large grid models.

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SINTEF ICT/Applied Mathematics

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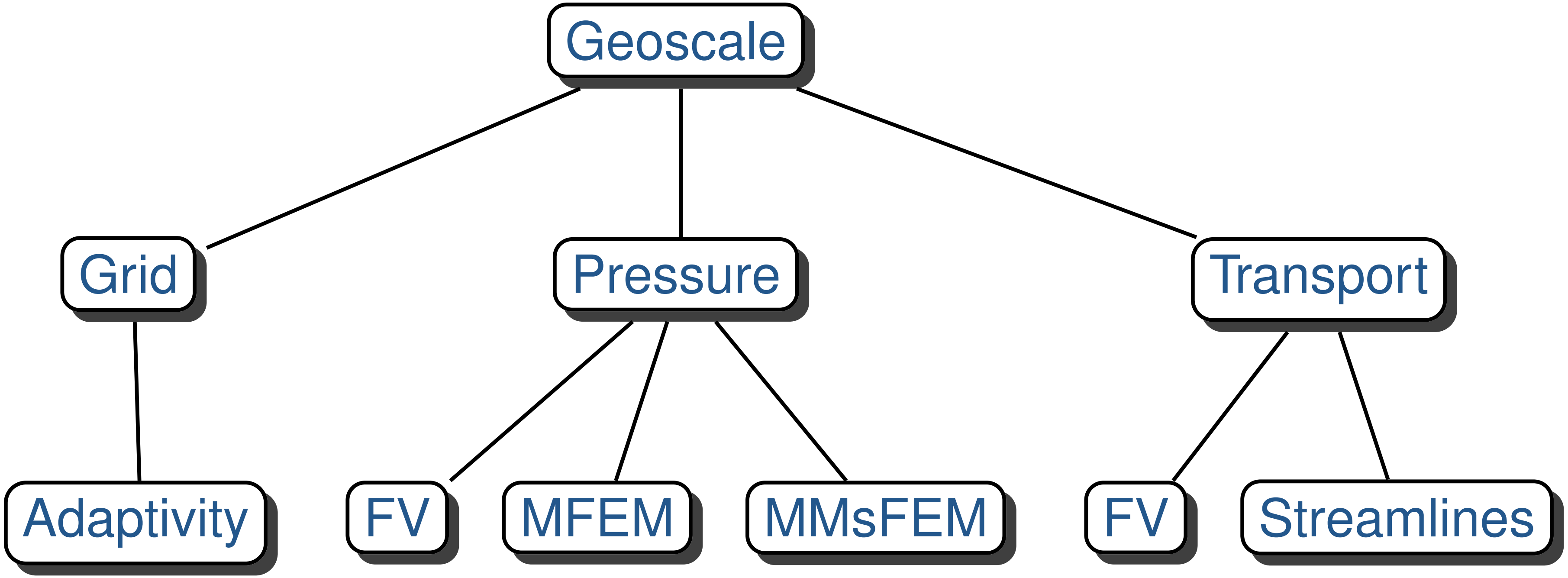
- ✘ Upscaling is not always very robust (no universal appoach).
- ✘ Upscaling may be a time-consuming process.

Geoscale Overview



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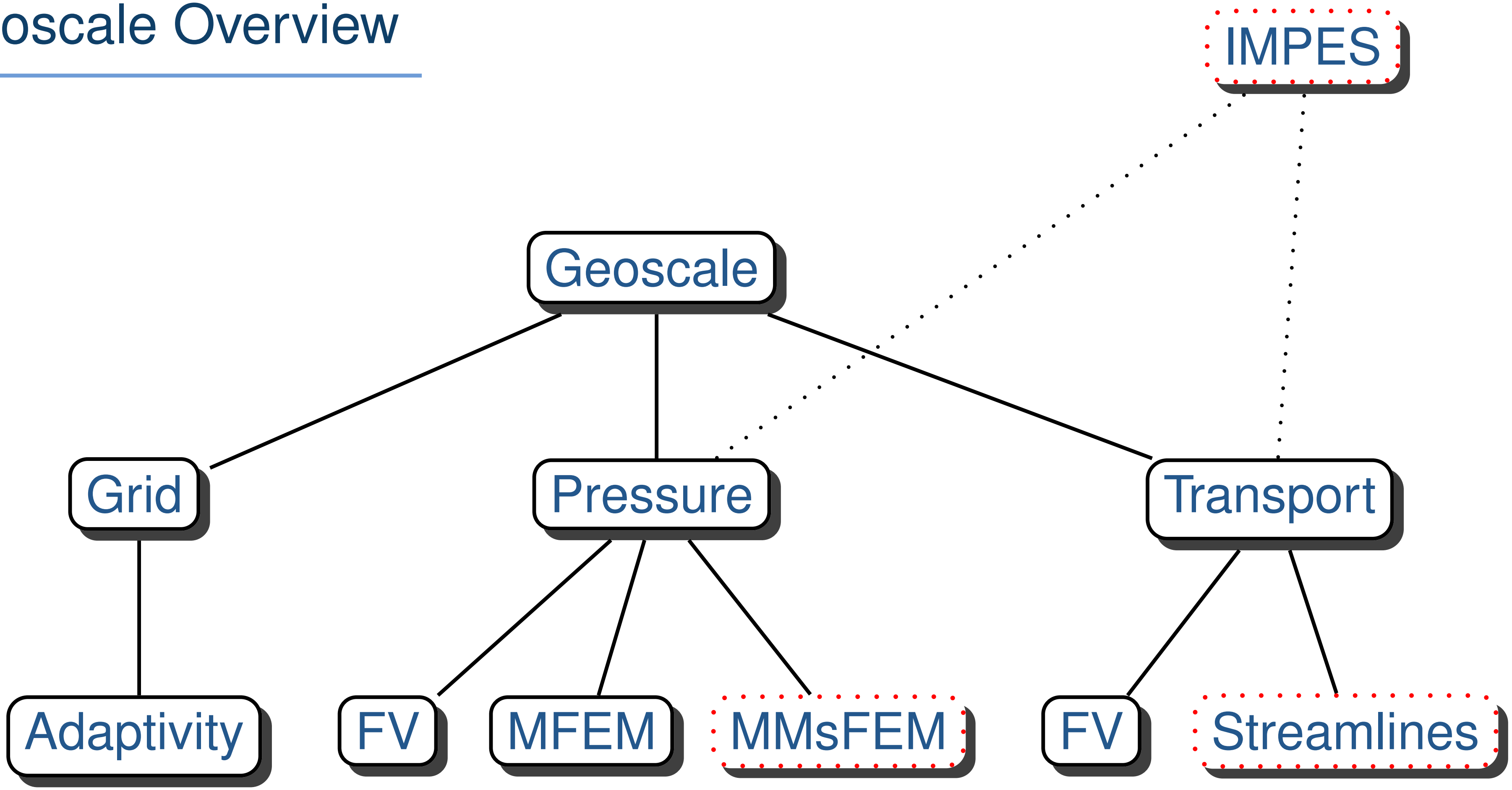

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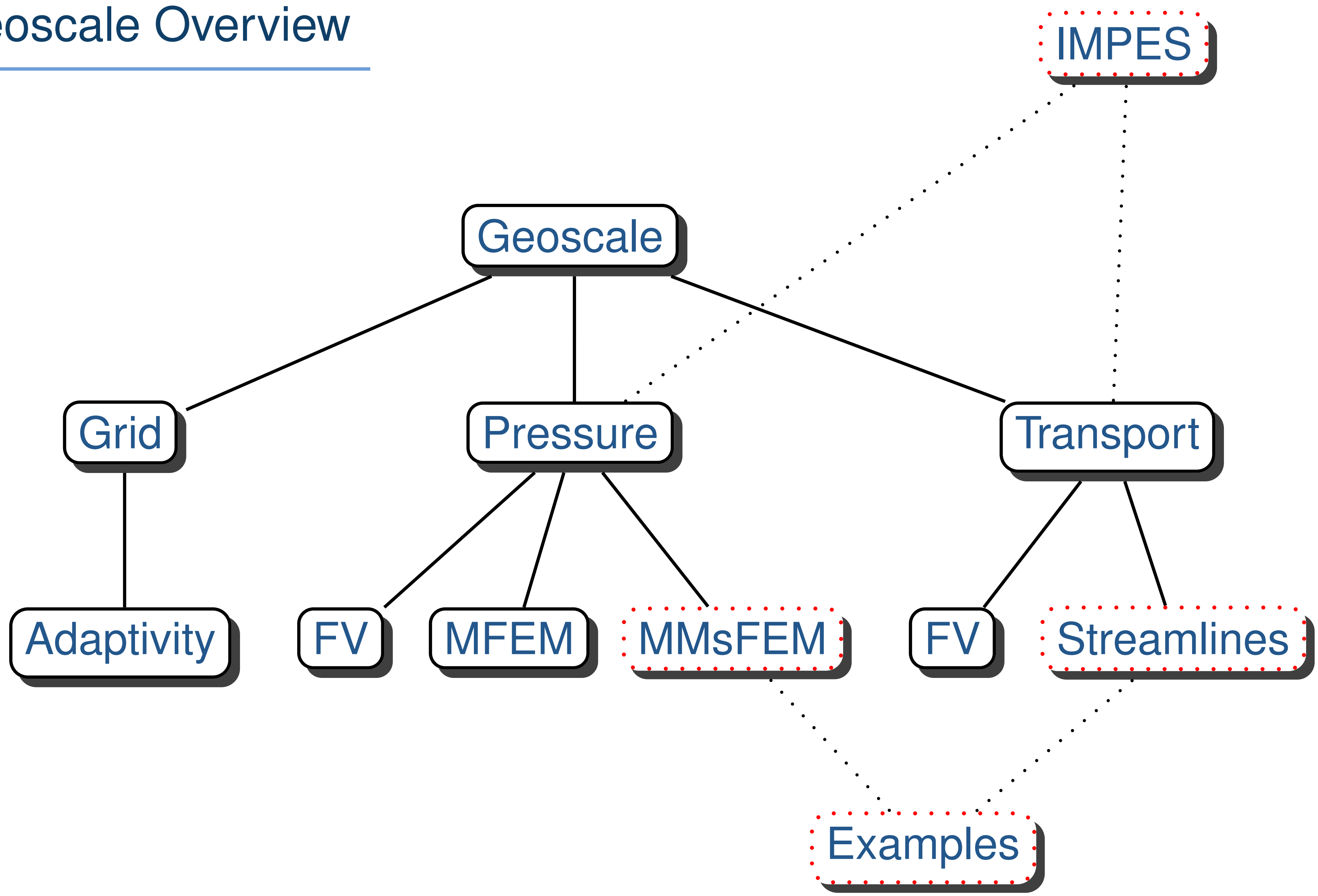

IMPES Equations (2-phase)

IMPES = IMplicit Pressure, Explicit Saturation.

IMPES Equations (2-phase)

Elliptic pressure equation:

$$v = -\lambda(S)K\nabla p$$

$$\nabla \cdot v = q$$

Hyperbolic saturation equation:

$$\phi\frac{\partial S}{\partial t} + \nabla \cdot (vf(S)) = q_w$$

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Hyperbolic saturation equation:

$$\phi\frac{\partial S}{\partial t} + \nabla \cdot (vf(S)) = q_w$$

- Total velocity:

$$v = v_o + v_w$$

- Total mobility:

$$\lambda = \lambda_w(S) + \lambda_o(S) = k_{rw}(S)/\mu_w + k_{ro}(S)/\mu_o$$

- Saturation water: S
- Fractional flow water:

$$f(S) = \lambda_w(S)/\lambda(S)$$

Back...

Mixed formulation of the pressure equation:

Find $(v, p) \in H_0^{1,\text{div}} \times L^2$ such that

$$\int (\lambda K)^{-1} u \cdot v dx - \int p \nabla \cdot v dx = 0, \qquad \forall u \in H_0^{1,\text{div}},$$
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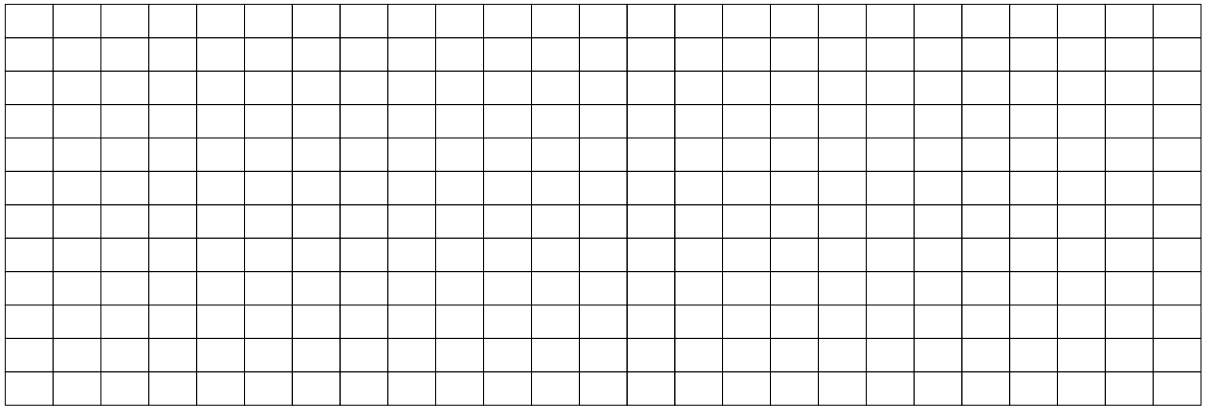
Multiscale discretisation: Seek solutions in low-dimensional subspaces

$$U^{ms} \subset H_0^{1,\text{div}} \text{ and } V \in L^2,$$

where local fine scale properties are incorporated into the basis functions.

Grids and basis functions

We assume we are given a *fine* grid with permeability and porosity attached to each fine grid block.



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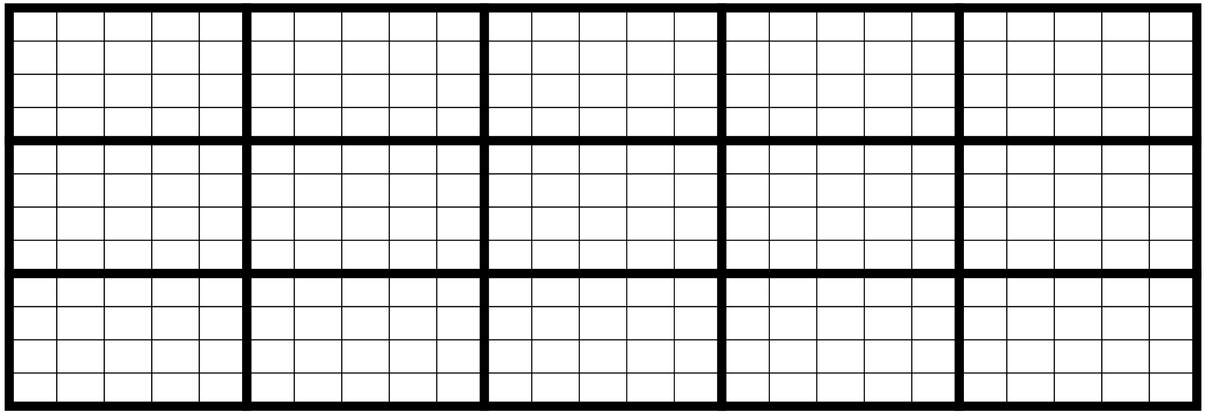

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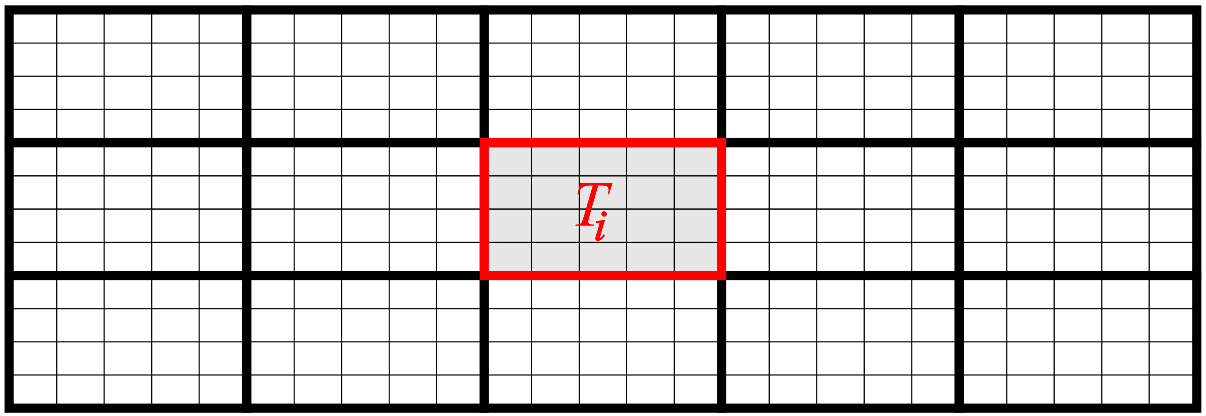

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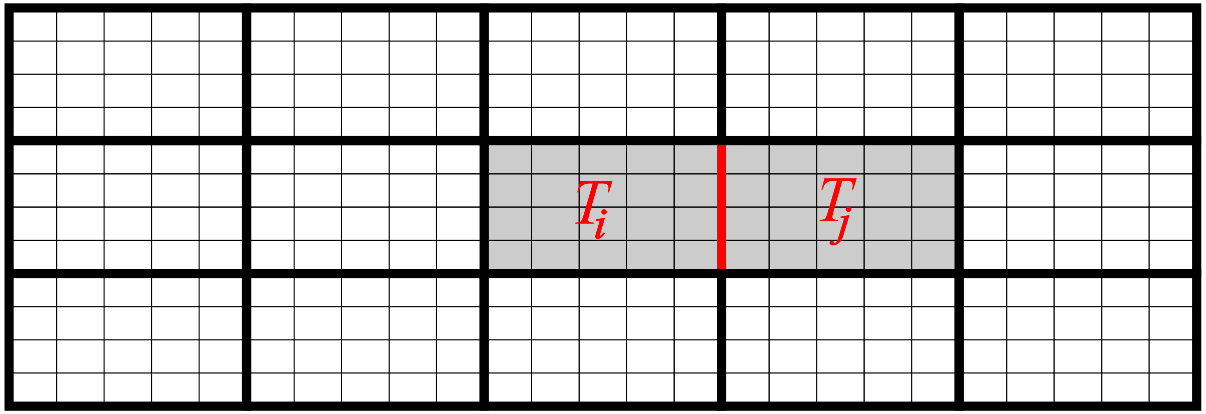

We construct a *coarse* grid, and choose the discretisation spaces V and U^{ms} such that:

- For each coarse block T_i,there is a basis function $\phi_i \in V$.
- For each coarse edge Γ_{ij}, there is a basis function $\psi_{ij} \in U^{ms}$.

Basis functions for the velocity field

For each coarse edge Γ_{ij} define a basis function

$$\psi_{ij} : T_i \cup T_j \to R^2$$

with unit flux through Γ_{ij}, and no flow across $\partial(T_i \cup T_j)$.

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We use $\psi_{ij} = -\lambda K \nabla \phi_{ij}$ with

$$\nabla \cdot \psi_{ij} = \begin{cases} f_i(x) / \int_{T_i} f_i(x) dx & \text{for } x \in T_i, \\ -f_j(x) / \int_{T_j} f_j(x) dx & \text{for } x \in T_j, \\ 0 & \text{otherwise,} \end{cases}$$

with BCs $\psi_{ij} \cdot n = 0$ on $\partial(T_i \cup T_j)$.

Basis functions for the velocity field cont.

If $\int_{T_i} q dx \neq 0$ (T_i contains a source), then

$$f_i(x) = q(x).$$

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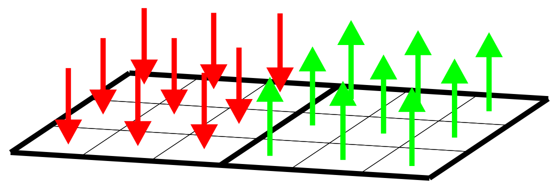
Otherwise we may choose

$$f_i(x) = 1,$$



MMsFEM Velocity Basis - Comments

For homogeneous coefficients the MMsFEM with the basis defined above reduces to the lowest order Raviart-Thomas mixed FEM (for quadrilateral elements).

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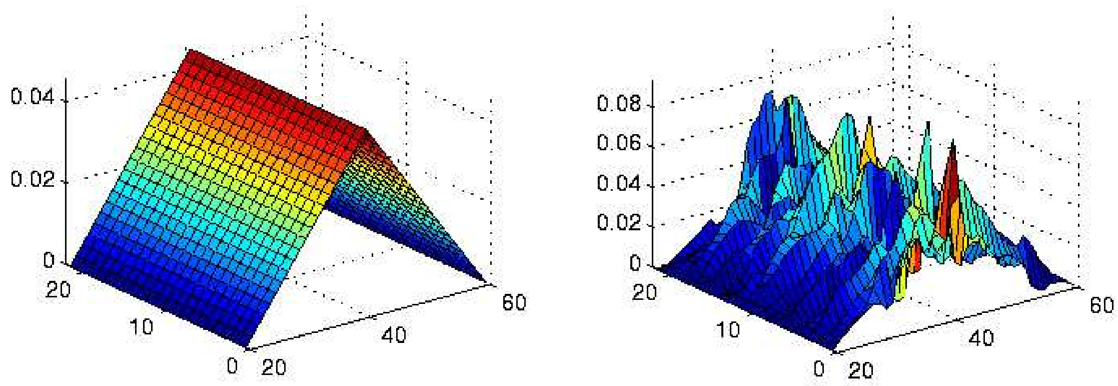
MMsFEM can therefore be viewed as an extension to the case where the coefficients can vary within each element.



x-component of the 2D basis function for homogeneous and heterogeneous coefficients.

Basis functions for the pressure

The pressure basis functions ϕ_i are piecewise constants, i.e., $V = \mathcal{P}_0(\mathcal{T})$.

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However, for consistency with the conservative velocity field and better accuracy in the near-well region we add more resolution in the well blocks by writing $p = \bar{p} + \hat{p}$, with $\bar{p} \in V$ and $\hat{p}$ a well-block correction term having (blockwise) zero average.

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One possible definition for $\hat{p}$:

$$\hat{p}|_{T_i} = \rho_i - \bar{\rho}_i \quad \text{where} \quad \rho_i = \sum_j v_{ij}\phi_{ij}|_{T_i} \quad \text{and} \quad \bar{\rho}_i = \int_{T_i} \rho_i \mathrm{d}x.$$

Here ϕ_{ij} is the pressure solution assiciated with basis function ψ_{ij}, while v_{ij} is the corresponding basis coefficient (i.e., the coefficient in $v = \sum_{ij} v_{ij}\psi_{ij}$).

Basis functions for the pressure

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Using the relation $p = \bar{p} + \hat{p}$ for the pressure a standard Peaceman-type well model can be used on the coarse scale. (The fine-scale well transmissibilities enter the global system in a natural way when writing out the well equations.)

MMsFEM Properties

Multiscale:
Incorporates small-scale effects into coarse-scale solution

Conservative:
Mass conservative on a subgrid scale

Scalable:
Well suited for parallel implementation since basis functions are processed independently

Flexible:
No restrictions on subgrids and subgrid numerical method. Few restrictions on the shape of the coarse blocks

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Fast:
The method is fast when avoiding regeneration of (most of) the basis functions at every time step

Back...

Streamlines

The saturation equation

$$\phi \frac{\partial S}{\partial t} + v \cdot \nabla f(S) = 0$$

is transformed to an equation along streamlines using

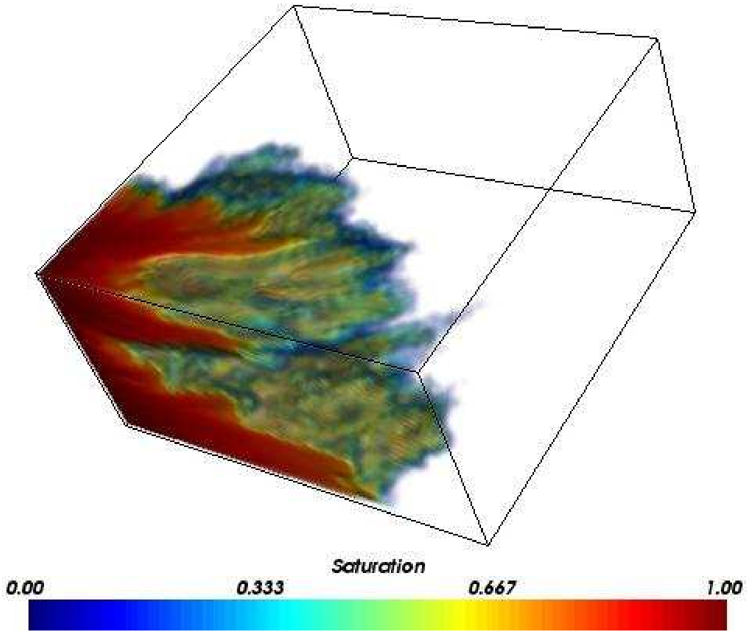
$$v \cdot \nabla = |v| \frac{\partial}{\partial s} = \phi \frac{\partial}{\partial \tau}$$

to give a 1D equation along streamlines:

$$\phi \frac{\partial S}{\partial t} + \frac{\partial f(S)}{\partial \tau} = 0.$$

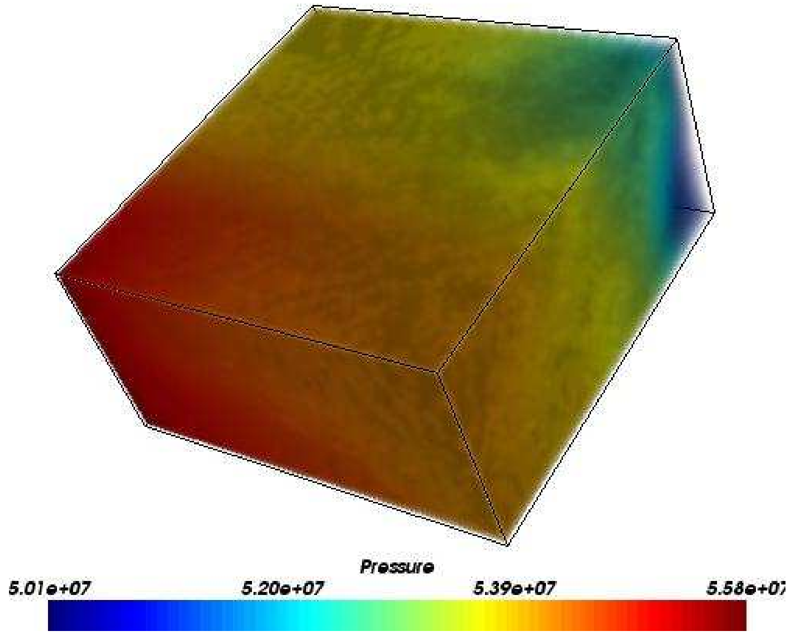
Here τ is the **time-of-flight** along the streamline.

IMPES Streamline Method



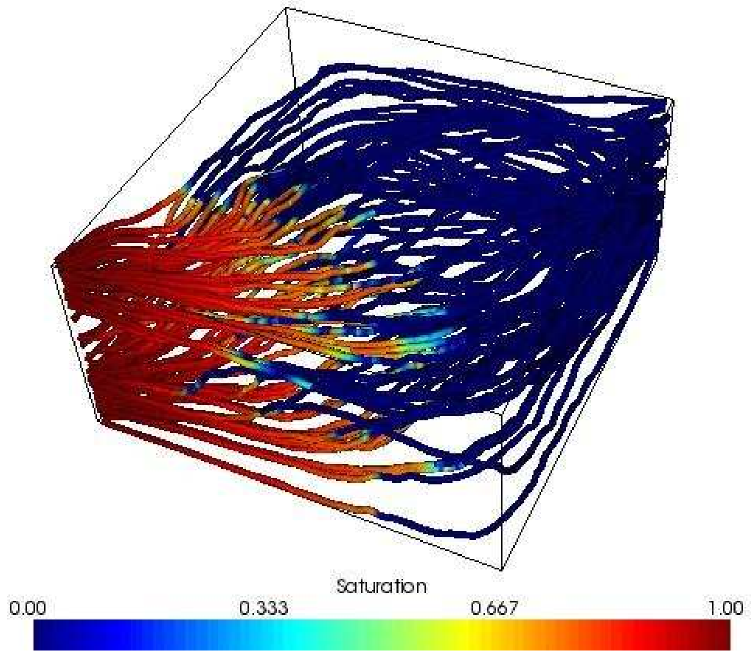
The starting point is an initial saturation field.

IMPES Streamline Method



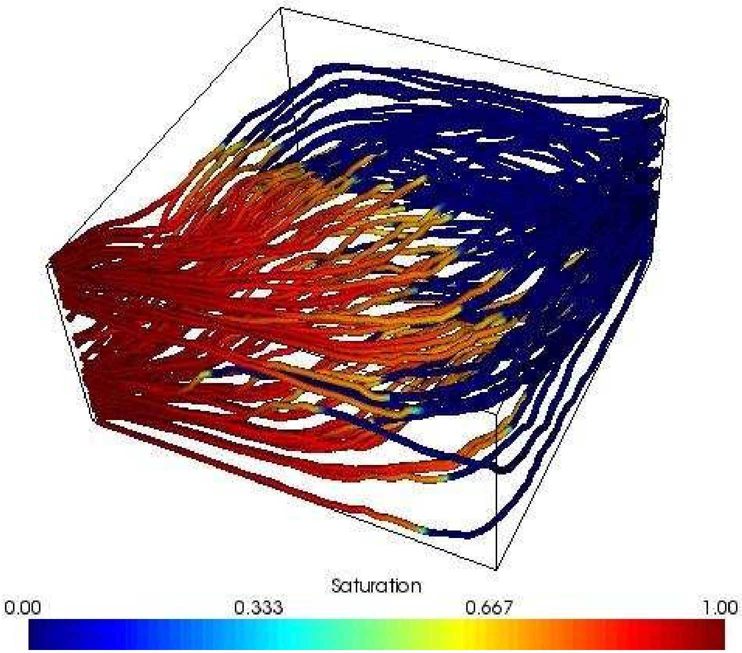
The pressure is computed using the initial saturations to evaluate the mobility terms.

IMPES Streamline Method



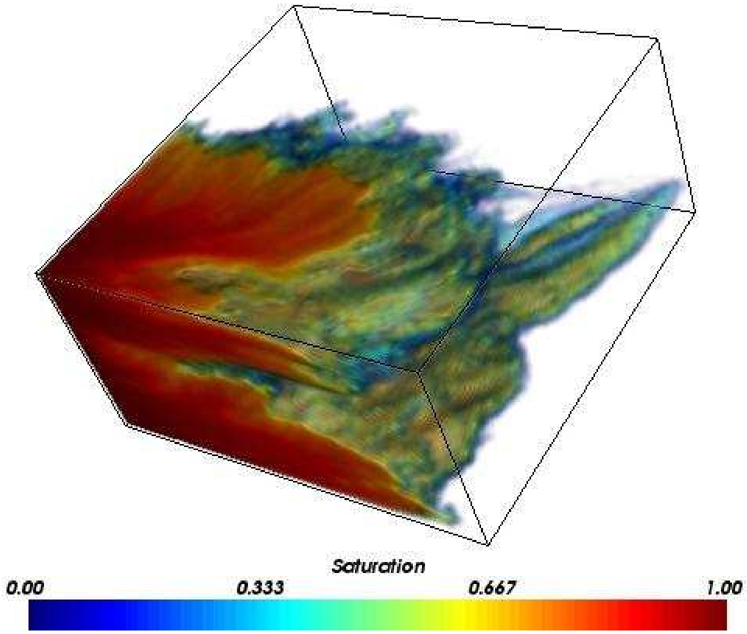
The pressure defines a velocity field and the streamline are traced from injectors to producers while picking up the grid block saturations.

IMPES Streamline Method



Saturations are moved forward along the streamlines under the assumption that the streamlines remain fixed during the time step.

IMPES Streamline Method



Finally the streamline saturations are mapped back onto the grid to yield a new saturation field, and the process may now be repeated.

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Streamline Method Properties

Speed:
Solving a series of 1D problems along streamlines is considerably faster than solving the full 3D equation.

Scalability:
The method scales well with increasing model size. Also, streamlines are processed independently, thus making the method well-suited for parallel implementation.

Limitations:

- ✘ Hard to account for capillary pressure.
- ✘ For highly compressible flow the efficiency advantage over conventional FV-methods may vanish.

Back...

2D Example

A simple quarter five-spot of size 60 x 60.

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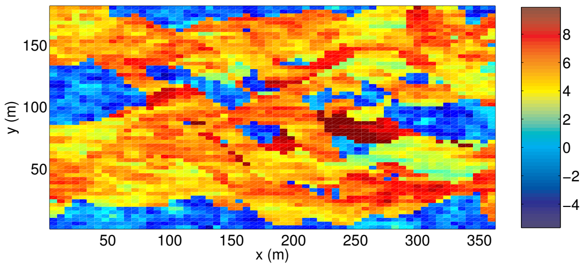
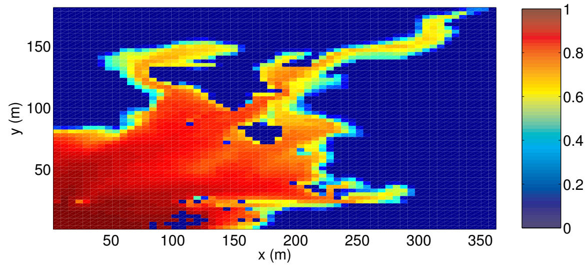
Fluvial reservoir, Upper Ness formation. Strongly varying permeability and porosity. (Excerpted from the SPE case to be described later.)

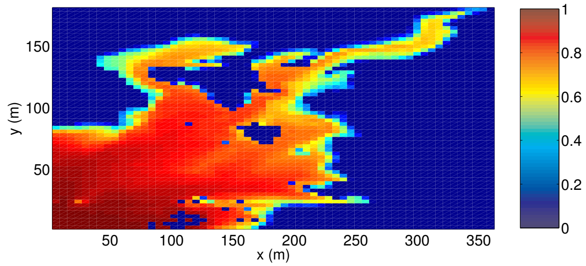
2D Example



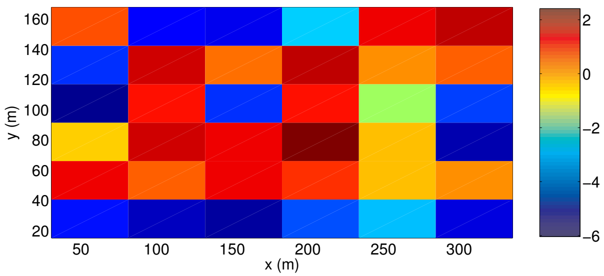
Permeability



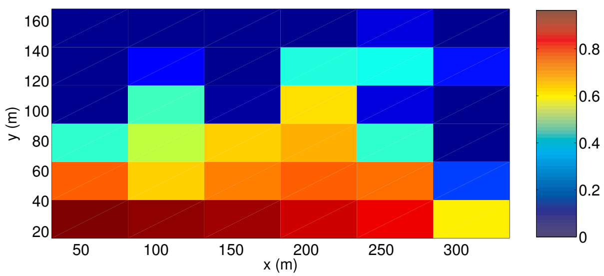
Reference



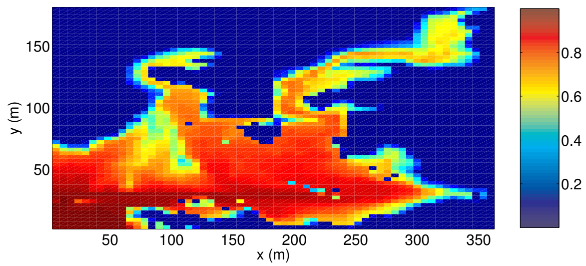
MMsFEM/SL



Upscaled Perm.



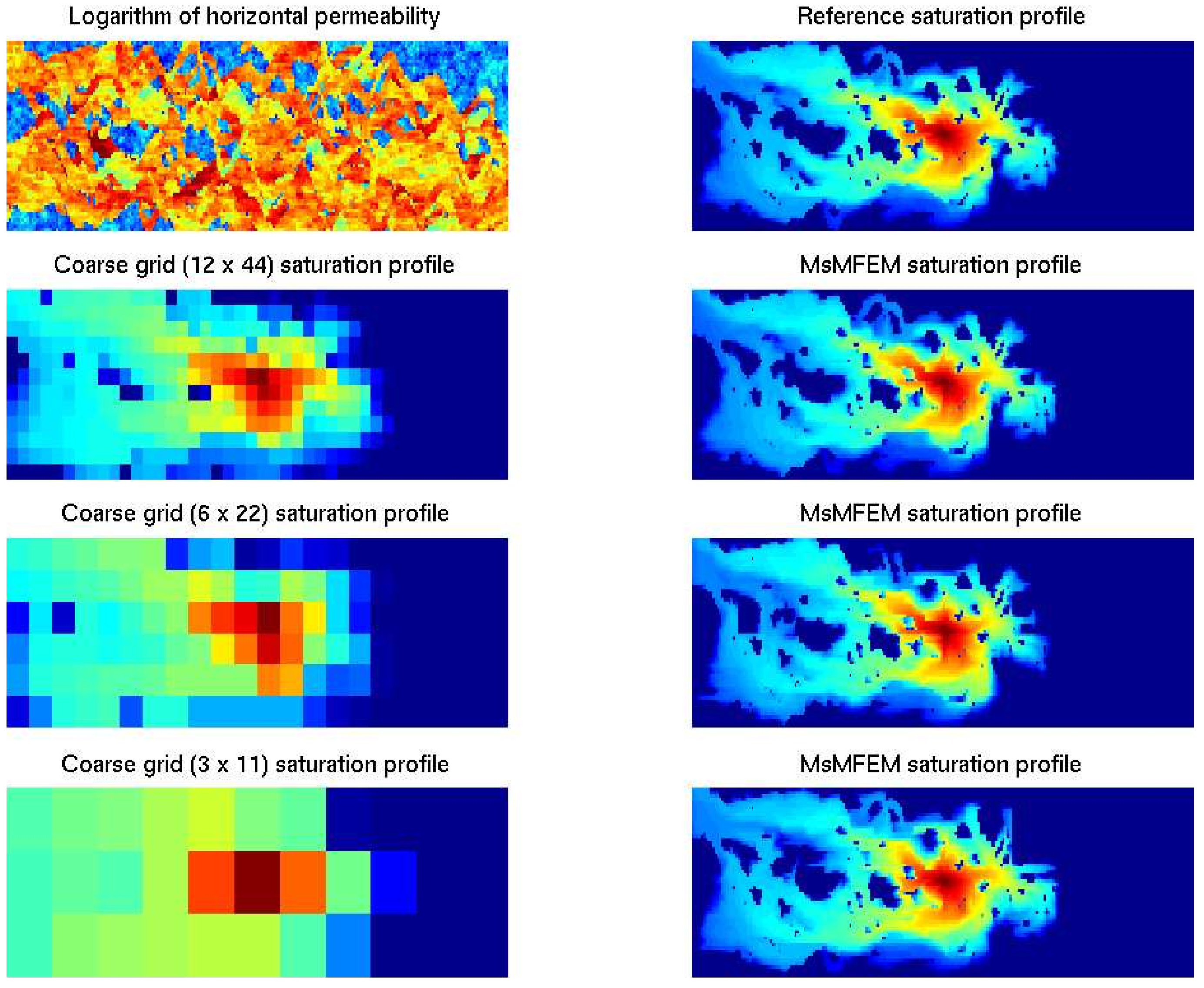
Upscaled Sol.



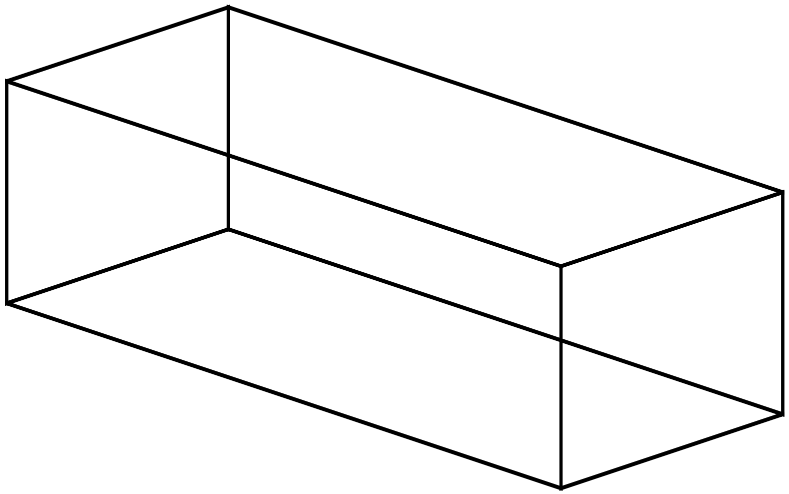
Nested Gridding

Nested Gridding: Upscale $\lambda(S)K$ in each step and solve subgrid problems to obtain velocities on the fine scale.

Different Upscaling Factors

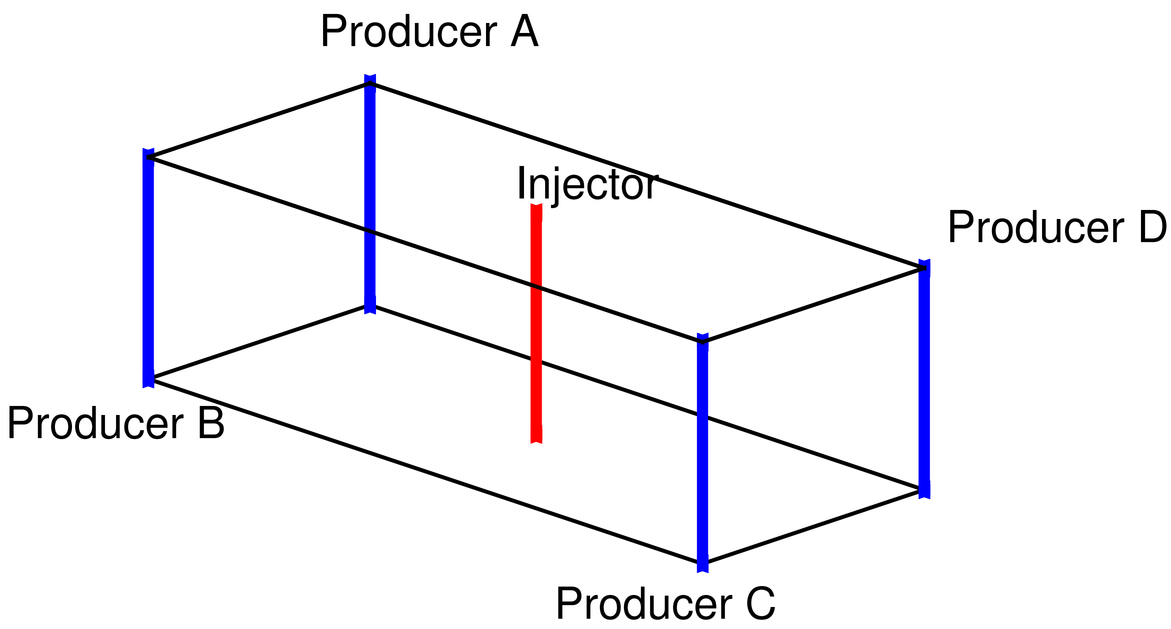


10th SPE Comparative Solution Project - Model 2



Model size is 1200 x 2200 x 170 ft.

10th SPE Comparative Solution Project - Model 2



Injection rate is 5000 bbl/day. The producers are specified with a bottom hole pressure of 5000 psi.

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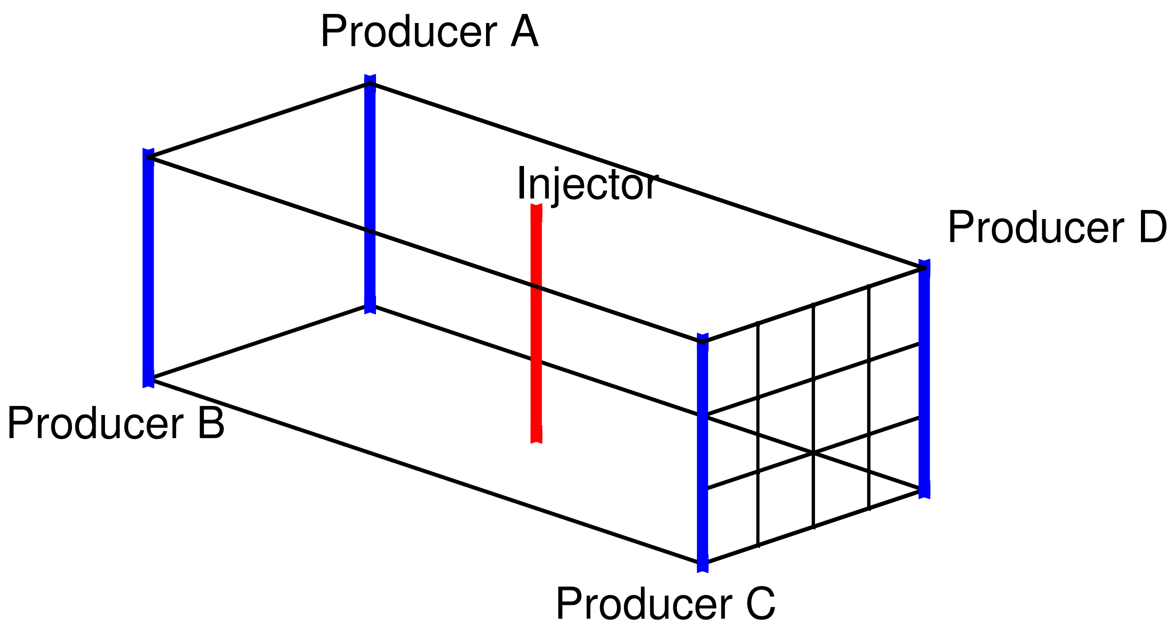
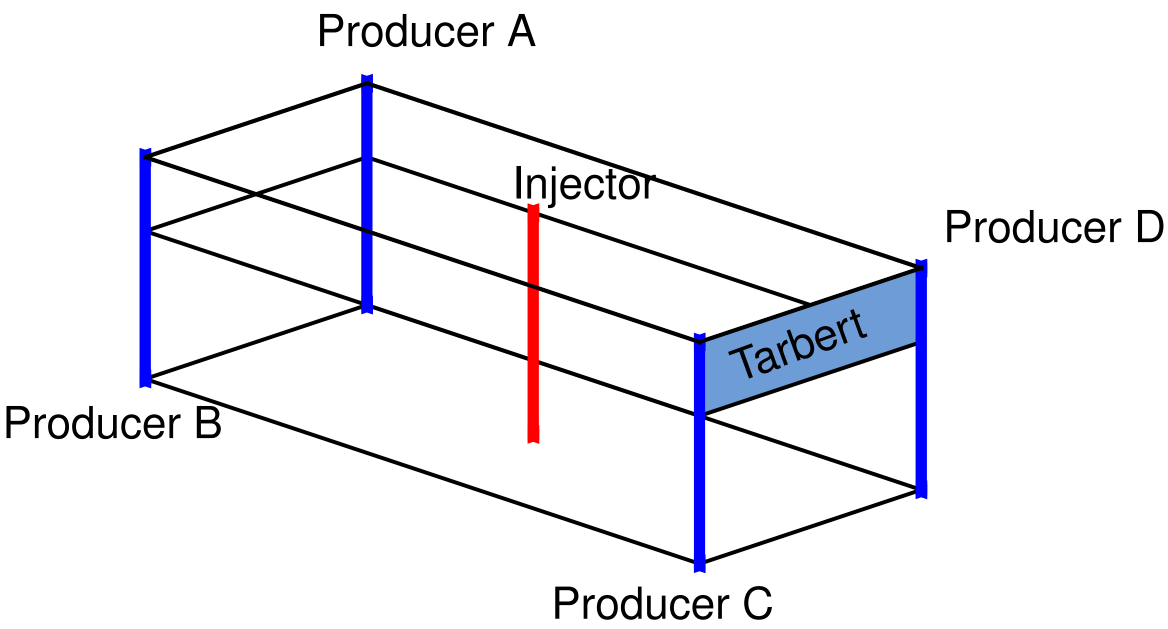
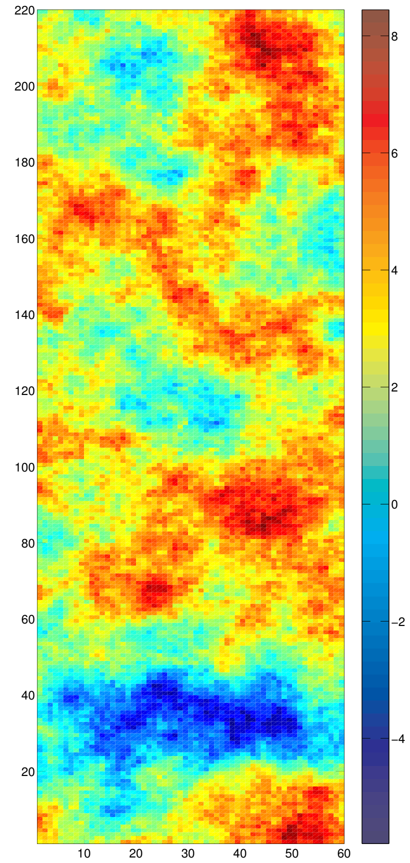


Discretization is 60 x 110 x 85 = 1.122M blocks.

10th SPE Comparative Solution Project - Model 2





The top 35 layers is a Tarbert formation. Both permability and porosity varies greatly, but the variation is relatively smooth.

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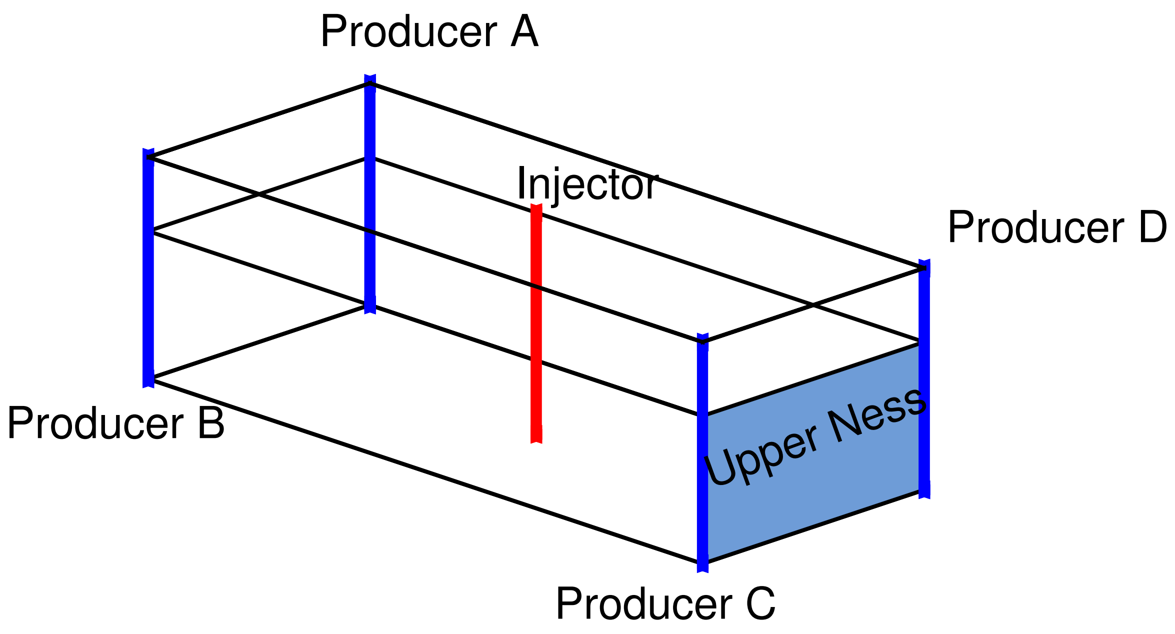
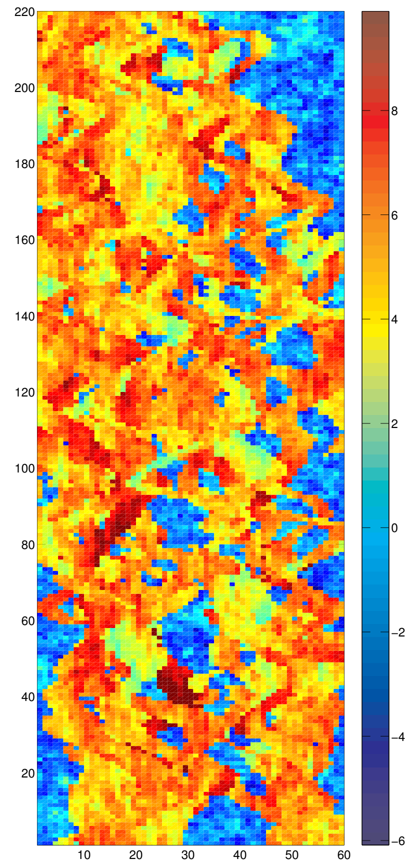




The bottom 50 layers is an Upper Ness sequence. This part is characterized by numerous high-flow channels.

Physical Parameters

Relative Permeabilities:

- $k_{ro} = (1 - S^*)^2$
- $k_{rw} = (S^*)^2$
- $S^* = \frac{S - S_{\text{wc}}}{1 - S_{\text{wc}} - S_{\text{or}}}$
- $S_{\text{or}} = S_{\text{wc}} = S_{\text{wi}} = 0.2$

PVT Data:

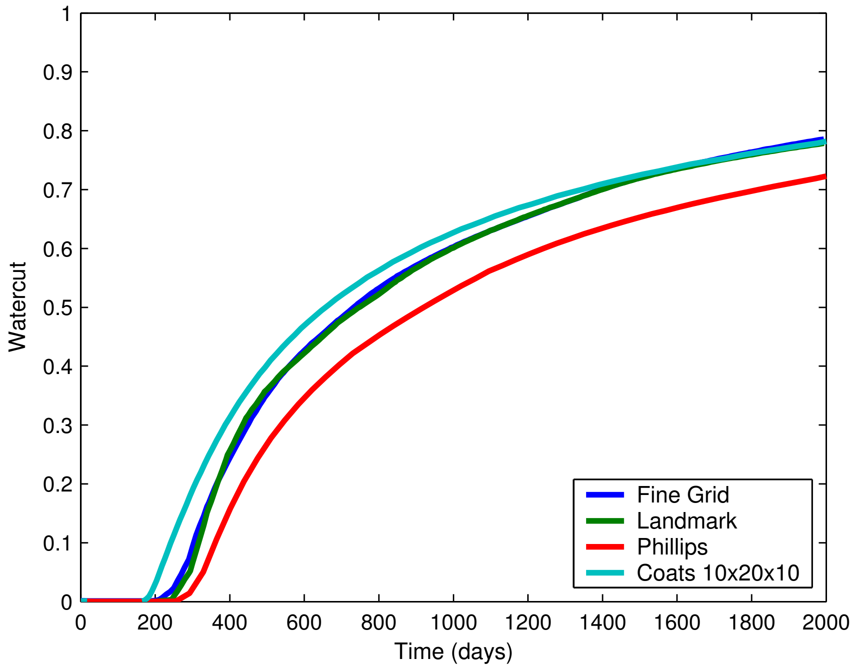
- $\mu_o = 3.0$ cP
- $\mu_w = 0.3$ cP

Model assumptions:

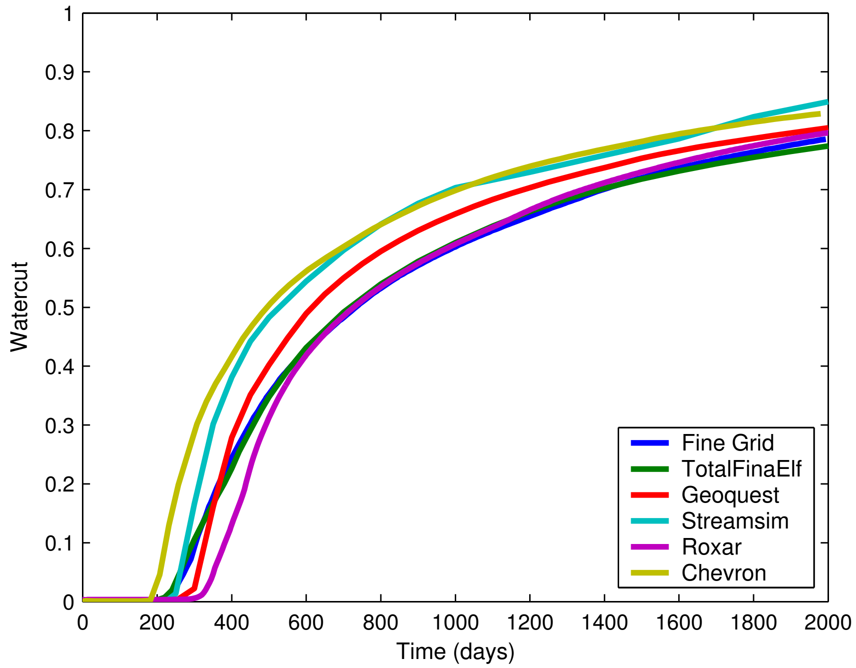
- Immiscible
- Isothermal
- Non-reactive
- Incompressible
- Ignorable gravity effects

Upscaling

Producer A:



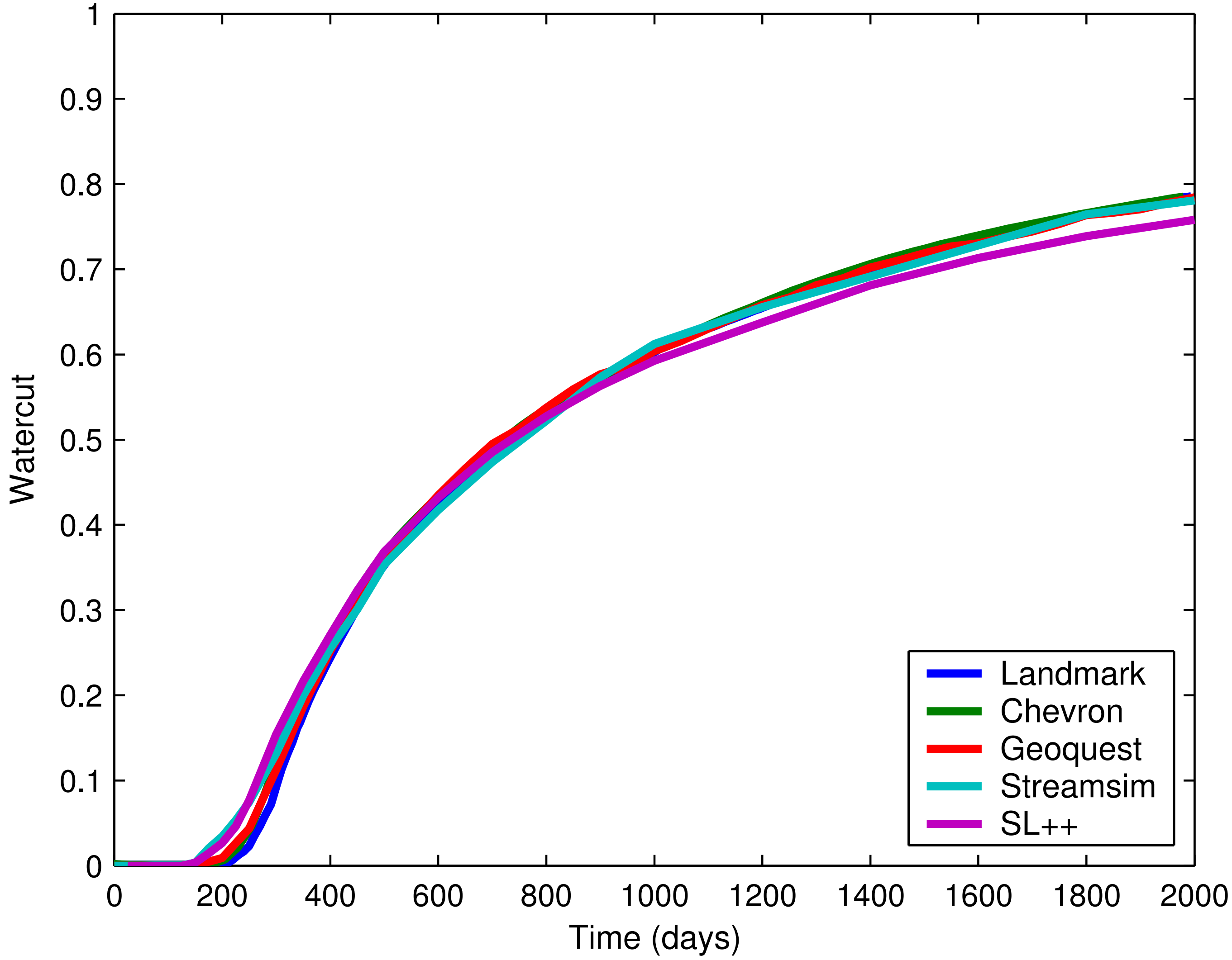
Pseduo



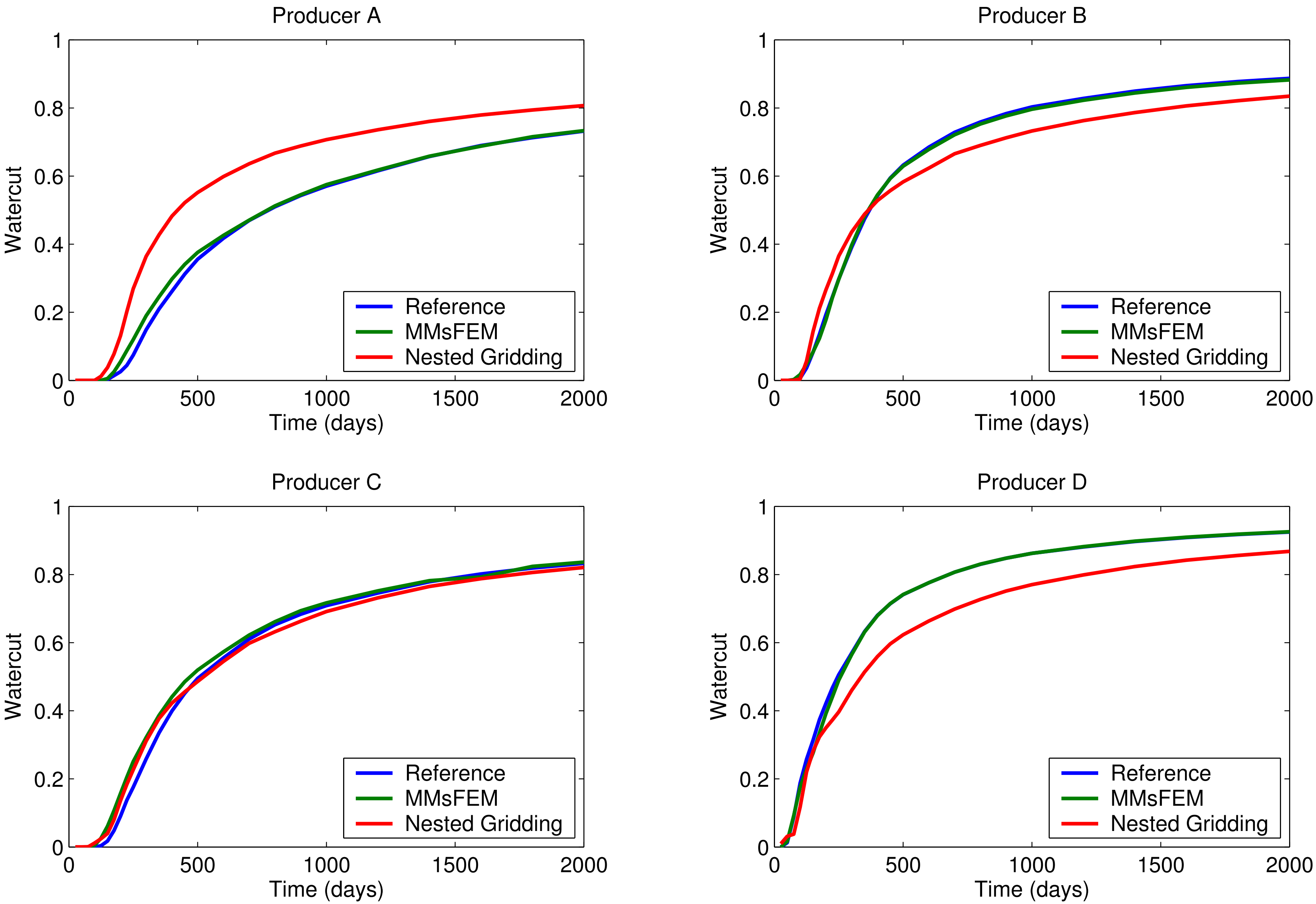
Non-pseudo

Our Reference Solution

Producer A:



MMsFEM Results (Coarse Grid: 5 x 11 x 17)



Concluding Remarks

Conclusions

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Extenstions and further work

- Non-orthogonal (corner-point) and unstructured grids
- Three-phase and multi-component flows
- Efficiency and parallellization