Non-uniformly coarsened grid models and a mixed multiscale FEM for reservoir simulation on a geological scale.

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Motivation

We have seen (in earlier talks) that the Mixed Multiscale FEM (MMsFEM) is a robust alternative to upscaling, and has the potential for large geomodels.

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- Motivated from the non-uniform coarsening approach in upscaling.
- Potential of reducing the number of grid blocks needed to obtain satisfactory solutions (increased speed).
- The MMsFEM handles arbitrary gridblocks $\Rightarrow$ (almost) no limitation on grids.

Outline of talk:

- Introduction.

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- Conclusion / further work.

Model equations

Elliptic pressure equation:

$$v = -\lambda(S) K \nabla p$$

$$\nabla \cdot v = q$$

Hyperbolic saturation equation:

$$\phi \frac{\partial S}{\partial t} + \nabla \cdot (v f(S)) = q_w$$

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- Total velocity:

$$v = v_o + v_w$$

- Total mobility:

$$\begin{aligned} \lambda &= \lambda_w(S) + \lambda_o(S) \\ &= k_{rw}(S)/\mu_w + k_{ro}(S)/\mu_o \end{aligned}$$

- Saturation water: S
- Fractional flow water:

$$f(S) = \lambda_w(S)/\lambda(S)$$

Mixed formulation of the pressure equation:

Find $(v, p) \in H_0^{1,\text{div}} \times L^2$ such that

$$\int (\lambda K)^{-1} u \cdot v dx - \int p \nabla \cdot v dx = 0, \qquad \forall u \in H_0^{1,\text{div}},$$

$$\int l \nabla \cdot v dx = \int q l dx, \quad \forall l \in L^2.$$

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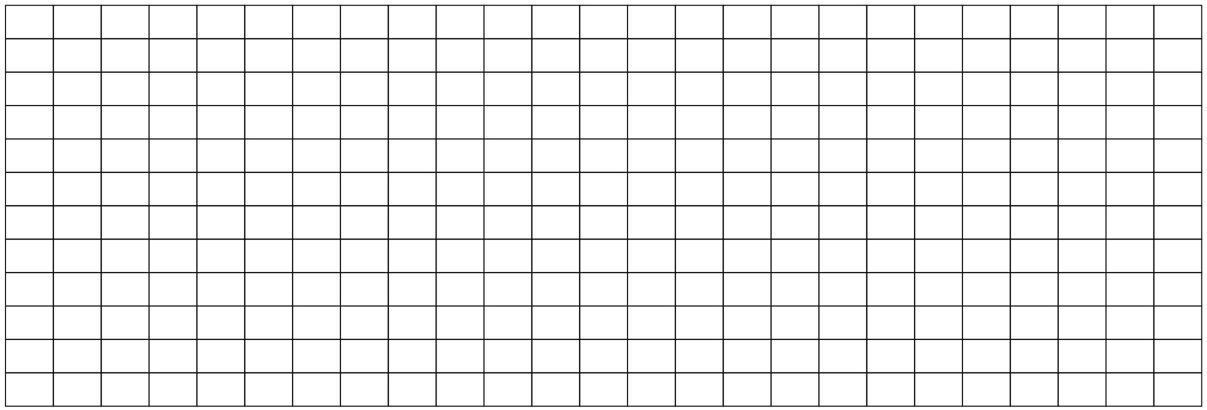
Multiscale discretisation: Seek solutions in low-dimensional subspaces

$$U^{ms} \subset H_0^{1,\text{div}} \text{ and } V \in L^2,$$

where local fine scale properties are incorporated into the basis functions.

Grids and basis functions

We assume we are given a *fine* grid with permeability and porosity attached to each fine grid block.



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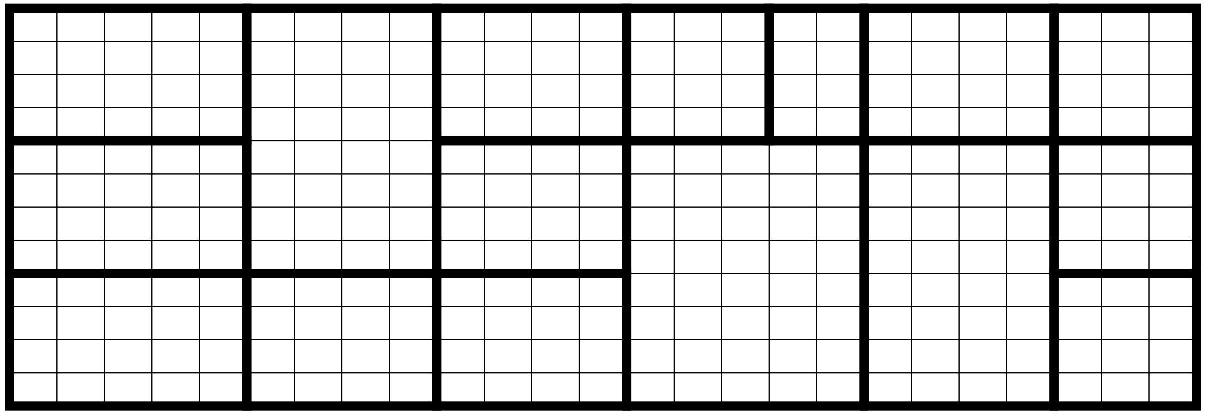

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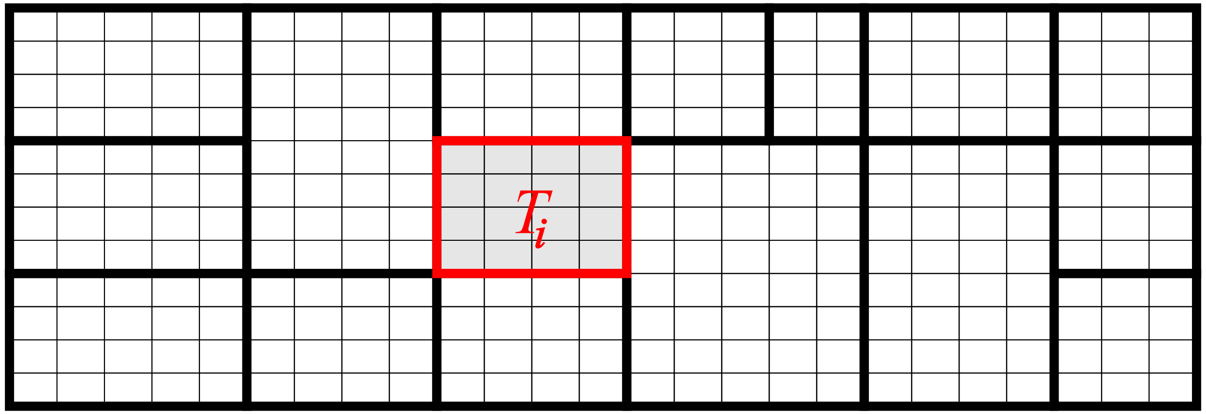

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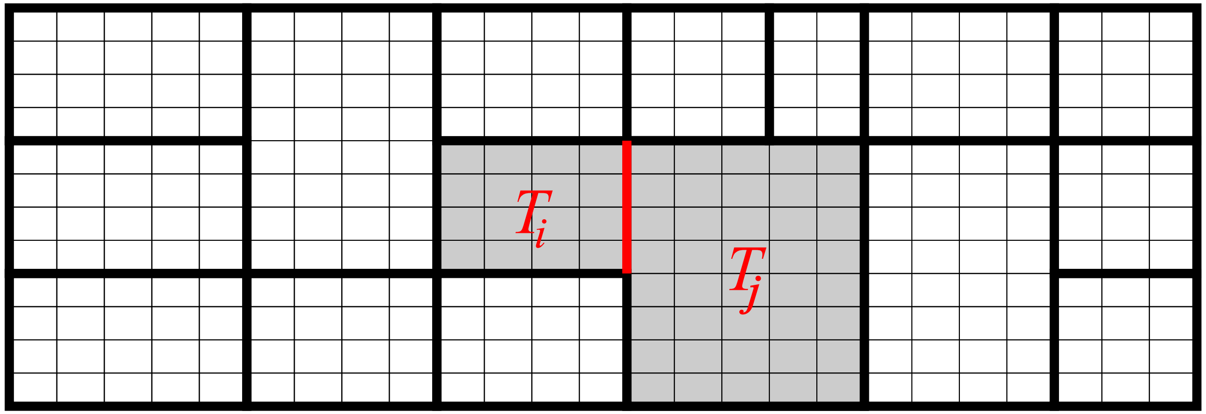

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- For each coarse block T_i,there is a basis function $\phi_i \in V$.
- For each coarse edge Γ_{ij}, there is a basis function $\psi_{ij} \in U^{ms}$.

Basis functions for the velocity field

For each coarse edge Γ_{ij} define a basis function

$$\psi_{ij} : T_i \cup T_j \rightarrow R^2$$

with unit flux through Γ_{ij}, and no flow across $\partial(T_i \cup T_j)$.

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We use $\psi_{ij} = -\lambda K \nabla \phi_{ij}$ with

$$\nabla \cdot \psi_{ij} = \begin{cases} f_i(x) / \int_{T_i} f_i(x)dx & \text{for } x \in T_i, \\ -f_j(x) / \int_{T_j} f_j(x)dx & \text{for } x \in T_j, \\ 0 & \text{otherwise,} \end{cases}$$

with BCs $\psi_{ij} \cdot n = 0$ on $\partial(T_i \cup T_j)$.

Basis functions for the velocity field cont.

If $\int_{T_i} q dx \neq 0$ (T_i contains a source), then

$$f_i(x) = q(x).$$

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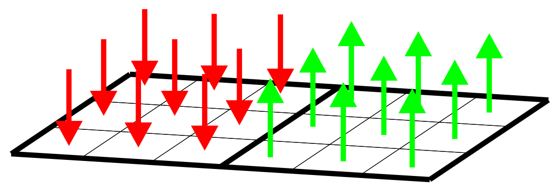

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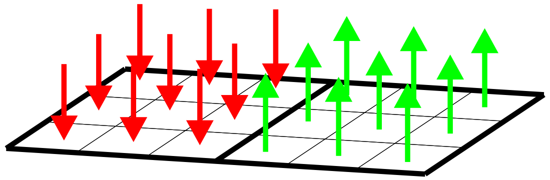
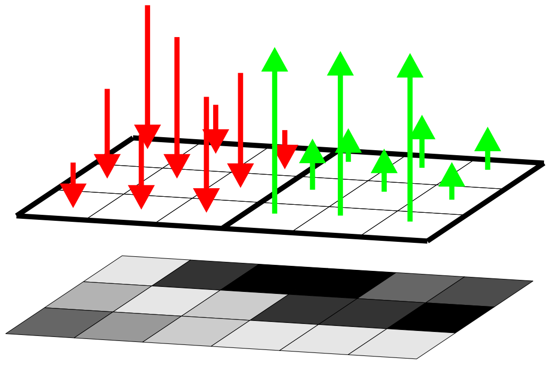


or to avoid high flow through low-perm regions

$$f_i(x) = (\mathrm{det}(K(x)))^{\frac{1}{d}}.$$



Non uniform grids - for upscaling and the MMsFEM

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- **However**: the MMsFEM can represent such regions correctly even on a very coarse scale.
- Why bother refining?

When is refinement required?

***Case 1*: Non uniform direction of flux across coarse edges.**

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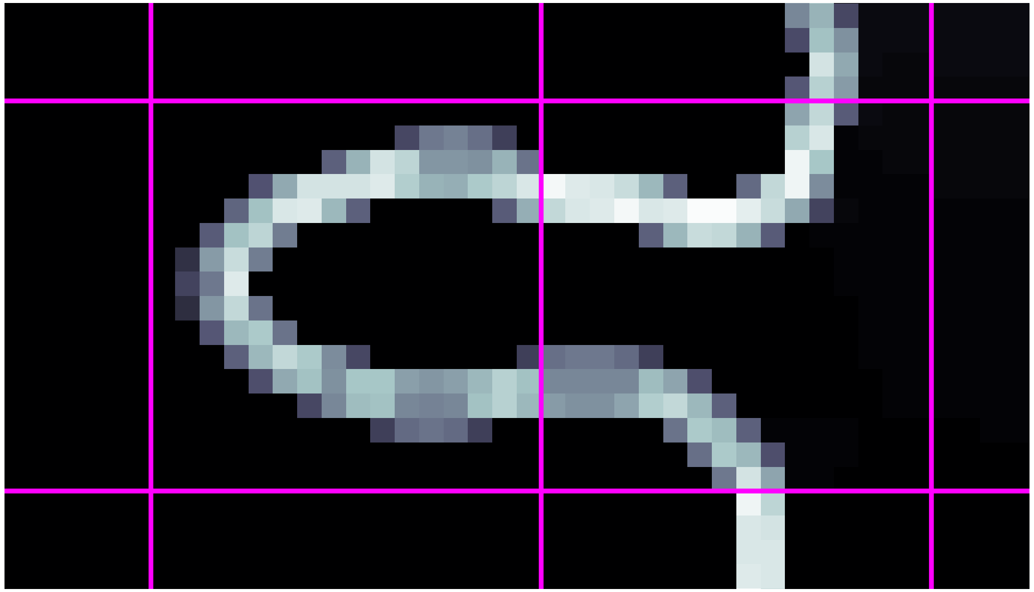
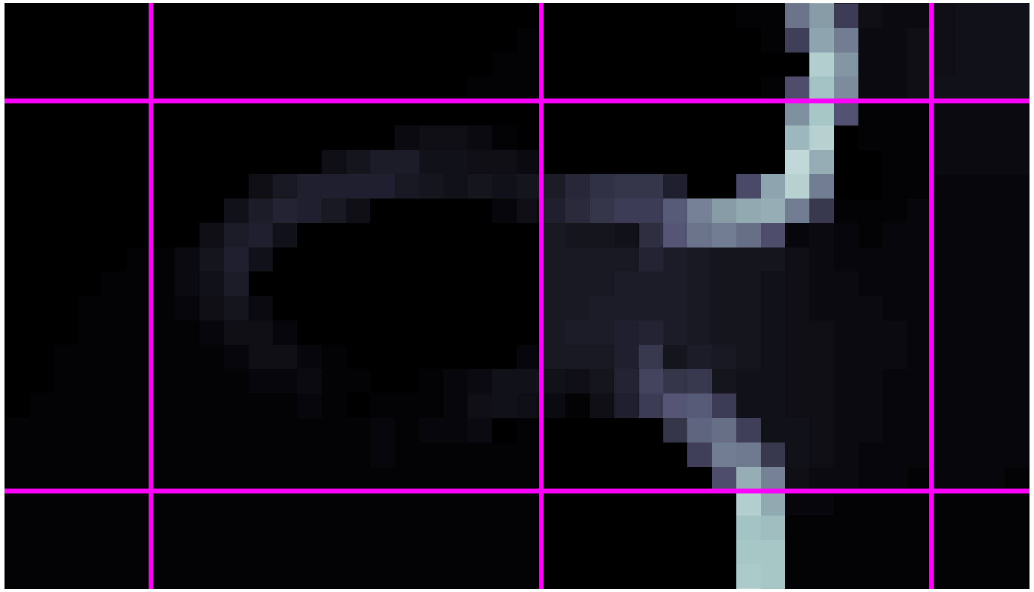
Consider the following fine grid velocity field:



When is refinement required?

***Case 1*: Non uniform direction of flux across coarse edges.**
Solving on the coarse grid, we obtain



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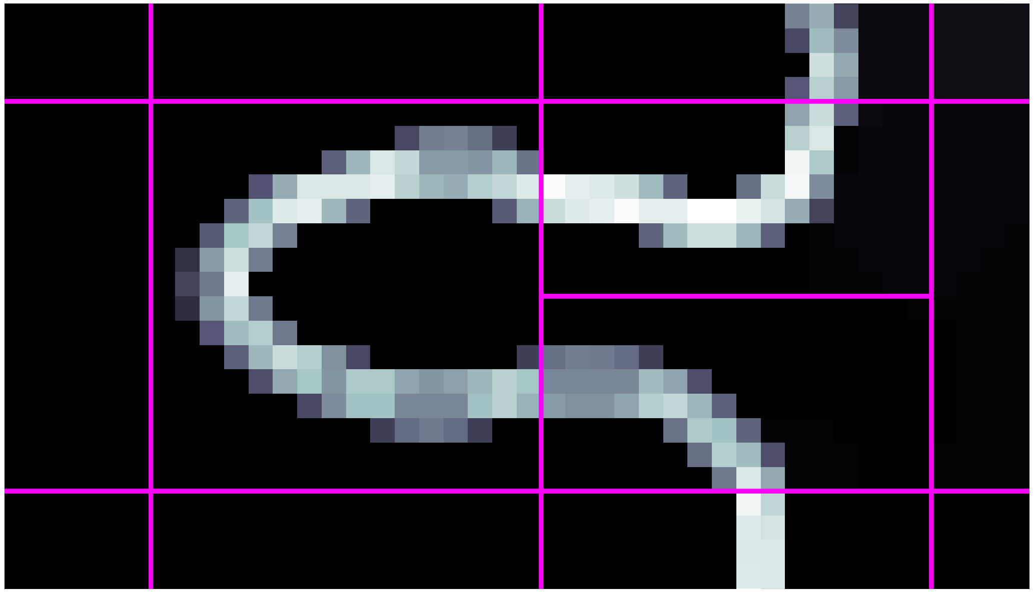
After a local refinement , we obtain



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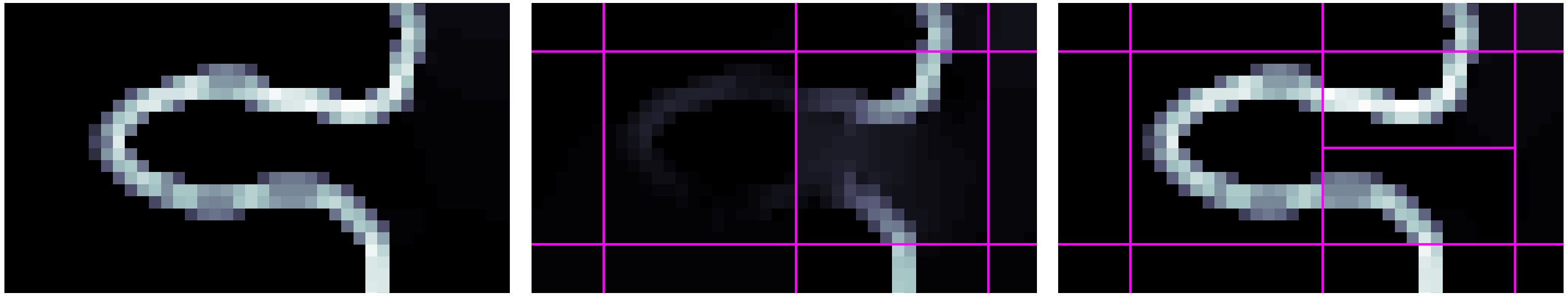


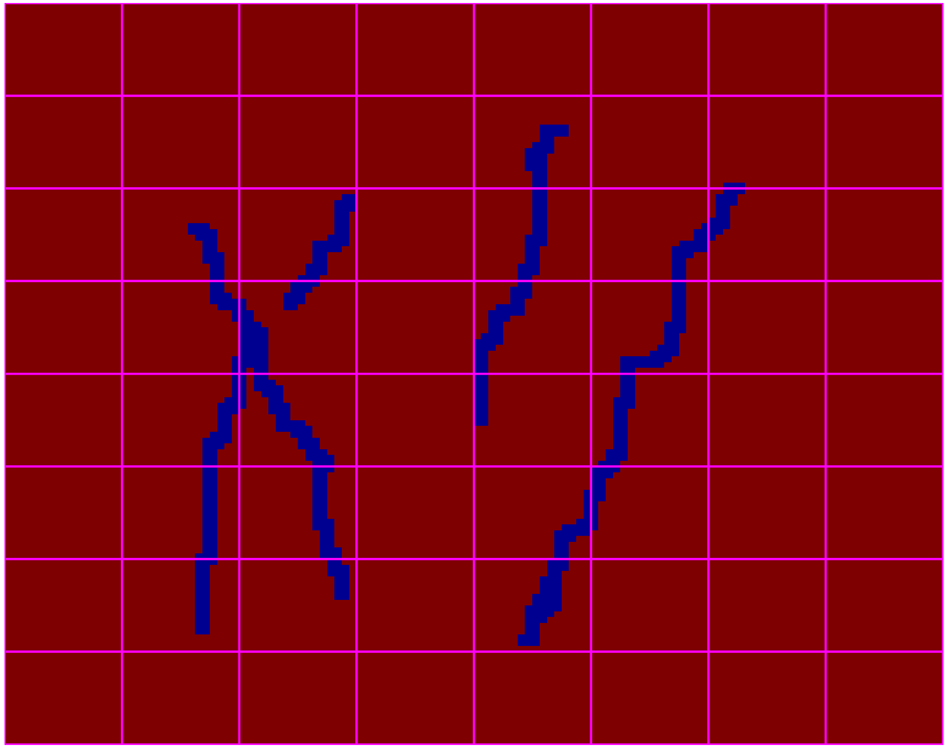
Criteria: Given an initial fine grid velocity field v_0, we modify the coarse grid such that

$$\frac{\int_\Gamma |v_0 \cdot n| \, ds}{\left| \int_\Gamma (v_0 \cdot n) ds \right|}$$

is close to 1 for every coarse edge Γ.

When is refinement required?

***Case2*: Flow of basis functions is forced through barriers.**
Consider the following permeability field with everywhere $K = 1$, except barriers (blue) with $K = 10^{-10}$.



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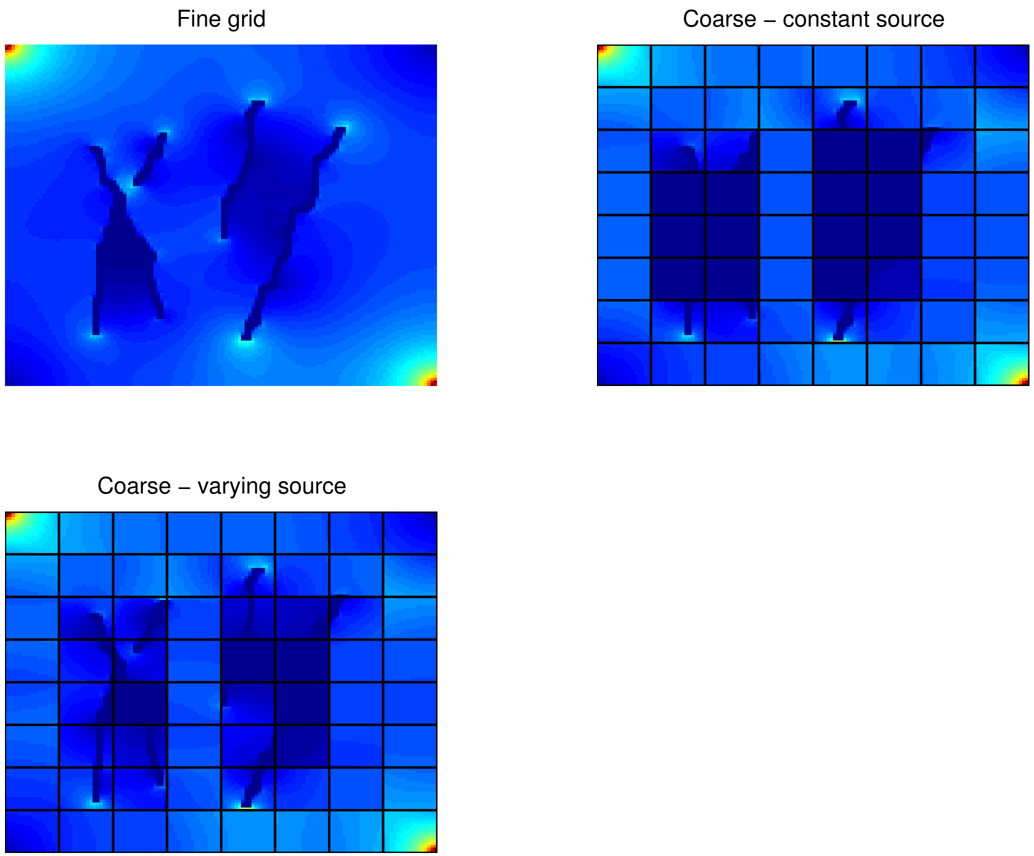
***Case2*: Flow of basis functions is forced through barriers.**
With the MMsFEM on the coarse grid we obtain the following velocity fields:

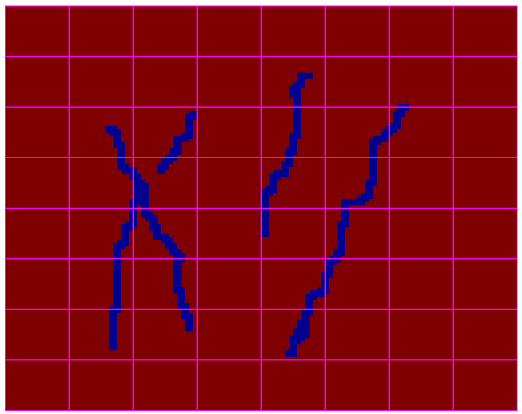


When is refinement required?

***Case2*: Flow of basis functions is forced through barriers.**
Criteria for refinement: For every basis function ψ_{ij} we monitor

$$\psi_{ij}^T K^{-1} \psi_{ij}.$$

If for some $x \in T_i$, say, $\psi_{ij}(x) K(x)^{-1} \psi_{ij}(x)$ achieves an *unnatural* high value, then $\psi_{ij}(x)$ is trashed and T_i is split in two new blocks.



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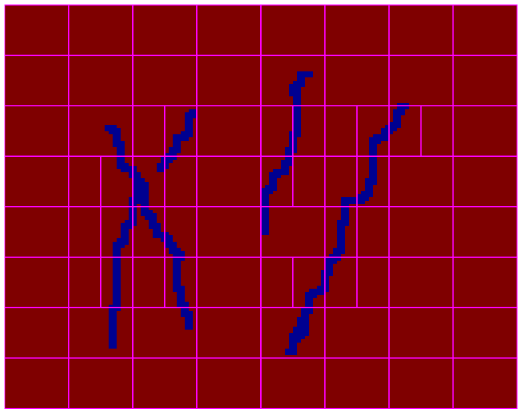

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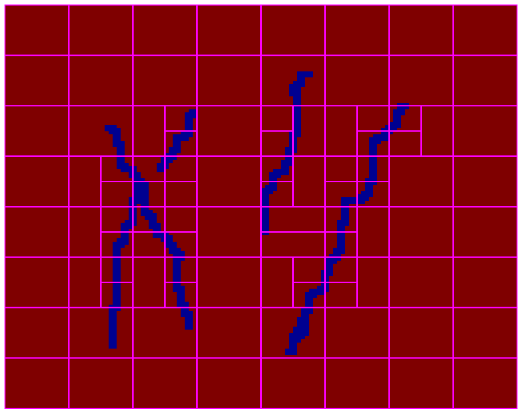

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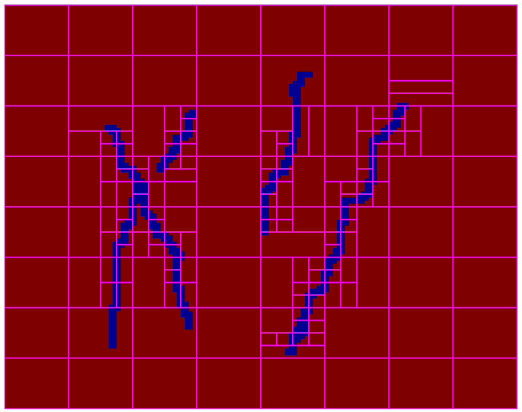
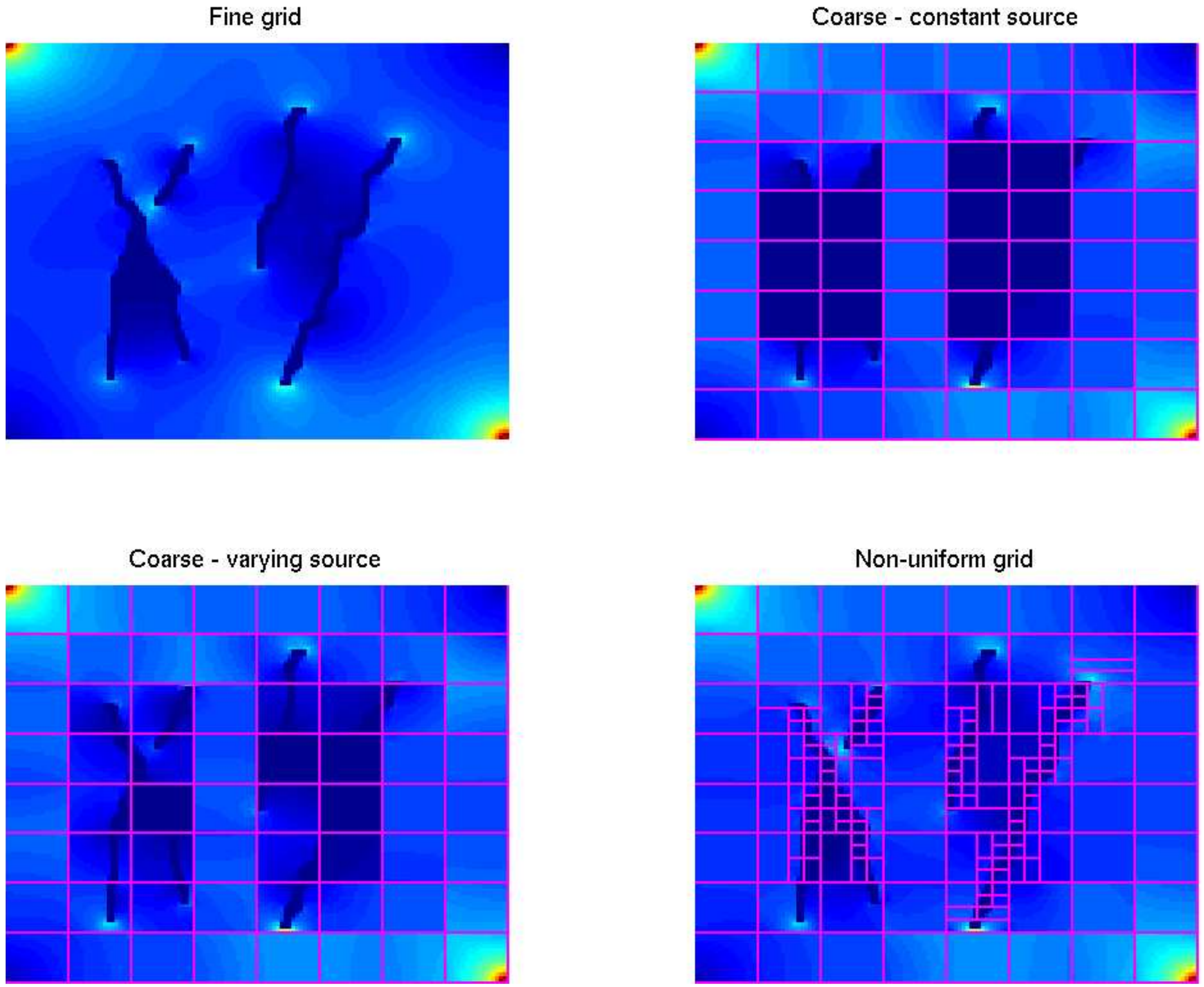


When is refinement required?

***Case2*: Flow of basis functions is forced through barriers.**
Velocity fields for all four cases:



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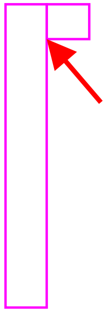
Regularization

- Long/thin grid blocks might give rise to nonphysical flow.



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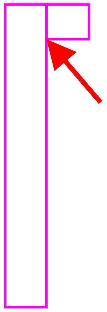
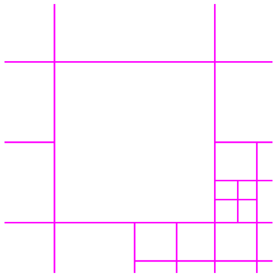
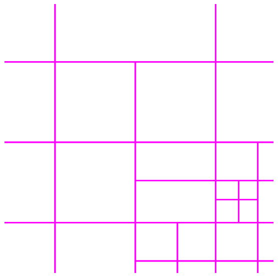


- The number of edges associated with each grid block determines the sparsity pattern of the discretization matrix.





Numerical experiments-grid construction

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- For any coarse edge Γ if

$$\left[\max_{\Gamma}(v_0 \cdot n) - \min_{\Gamma}(v_0 \cdot n)\right] \frac{\int_\Gamma |v_0 \cdot n|}{\left|\int_\Gamma (v_0 \cdot n)\right|} > \text{condition},$$

then one of the neighboring (randomly chosen) blocks are split.

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  then one of the neighboring (randomly chosen) blocks are split.
- Further splitting is performed according to

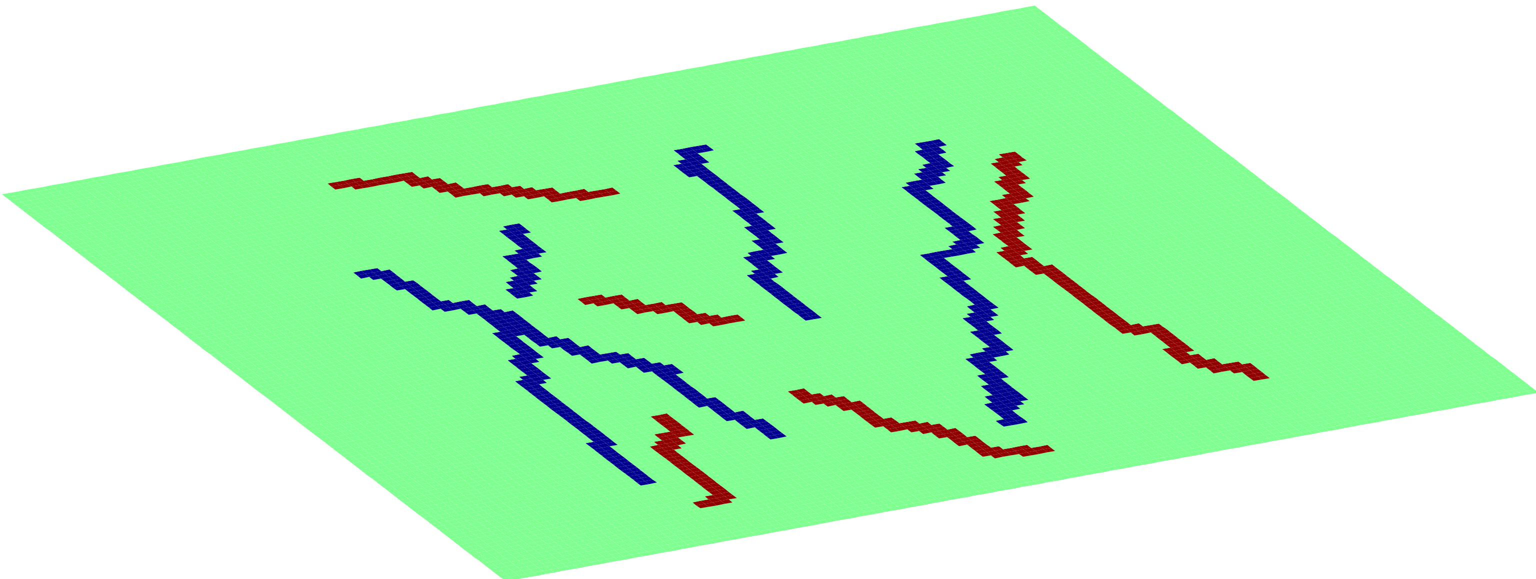
$$\psi^T K^{-1} \psi > \text{condition}$$

Numerical experiments 1

A model problem with barriers/ high permeability channels

The fine scale (128×128) permeability field consists of

- channels: $K = 10^4$,
- barriers: $K = 10^{-4}$,
- everywhere else $K = 1$.



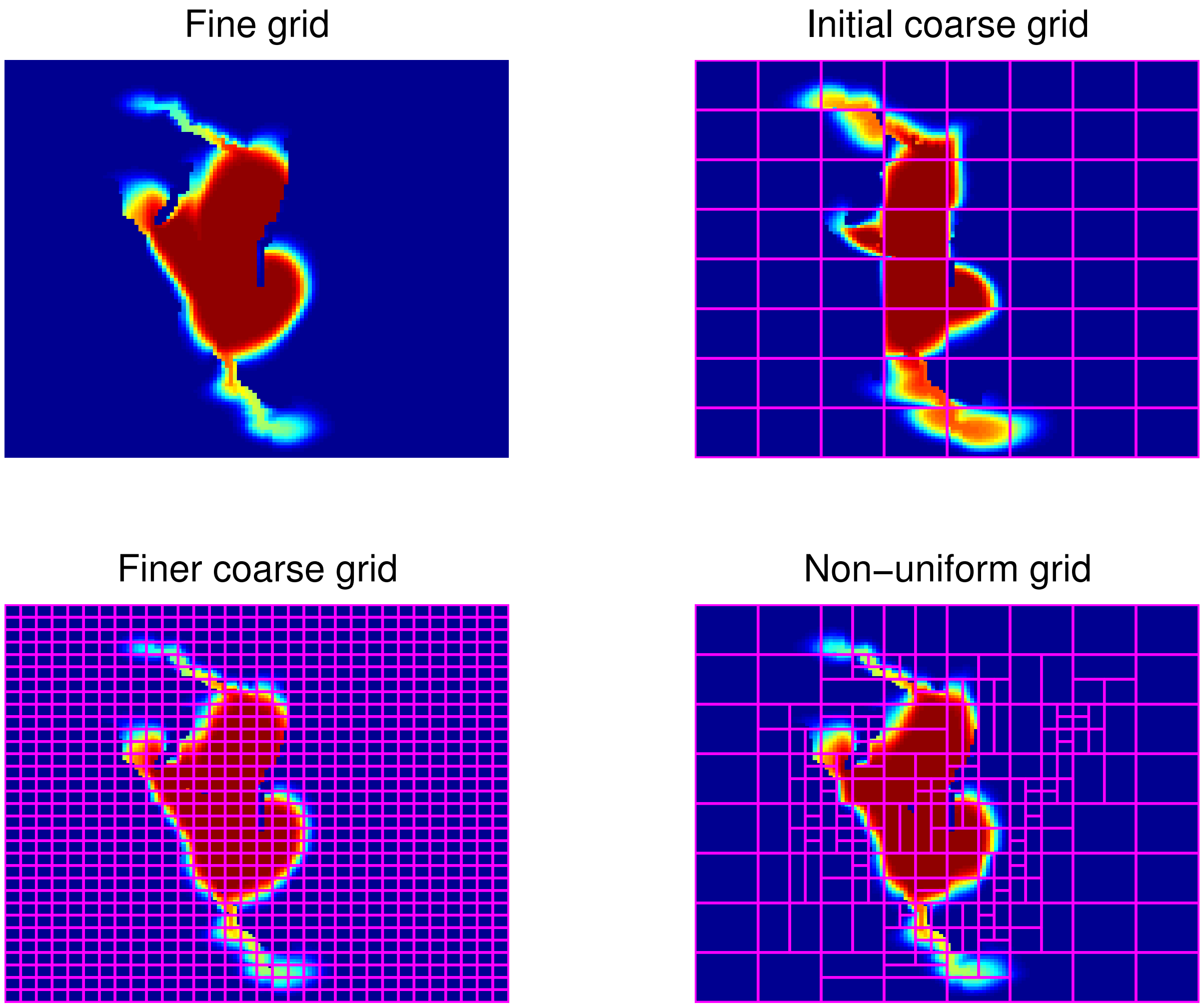
Numerical experiments 1, cont.

The simulation is run with unit mobility $\lambda = 1 \Rightarrow f(S) = S$ (the velocity is computed only once).
We apply four different grids:

- Fine grid, 128×128 blocks.
- Coarse grid, 8×8.
- Finer coarse grid, 32×32
- Non-uniform grid, 230 blocks.

Numerical experiments 1, cont.

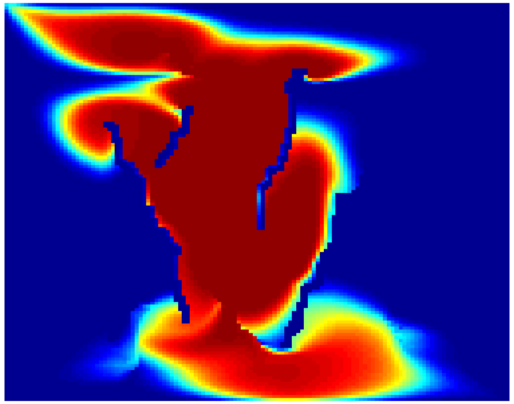
Saturation profiles at $t = 0.12$.



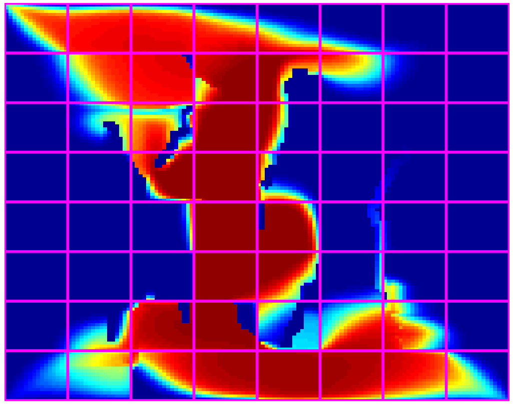
Numerical experiments 1, cont.

Saturation profiles at $t = 0.36$.

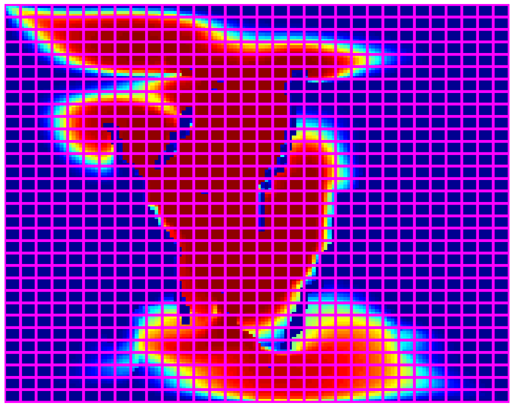
Fine grid



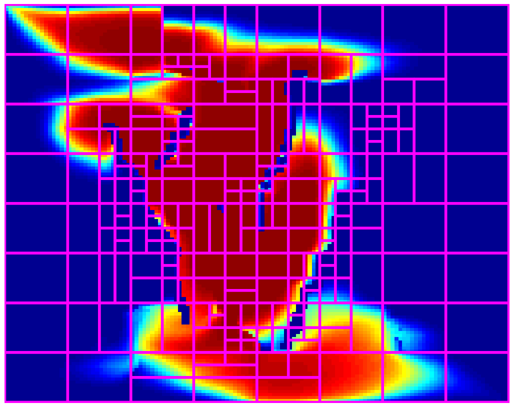
Initial coarse grid



Finer coarse grid

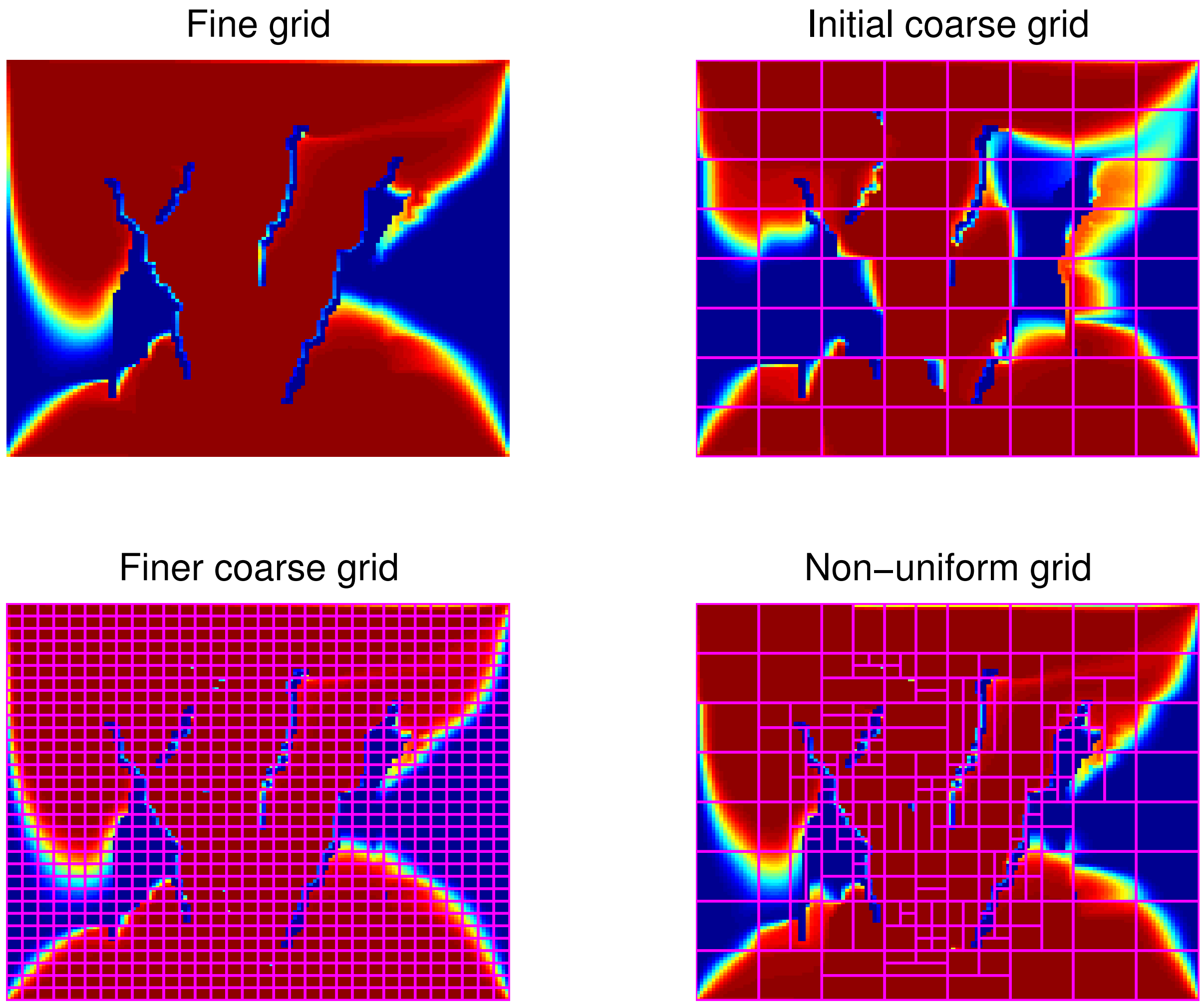


Non–uniform grid



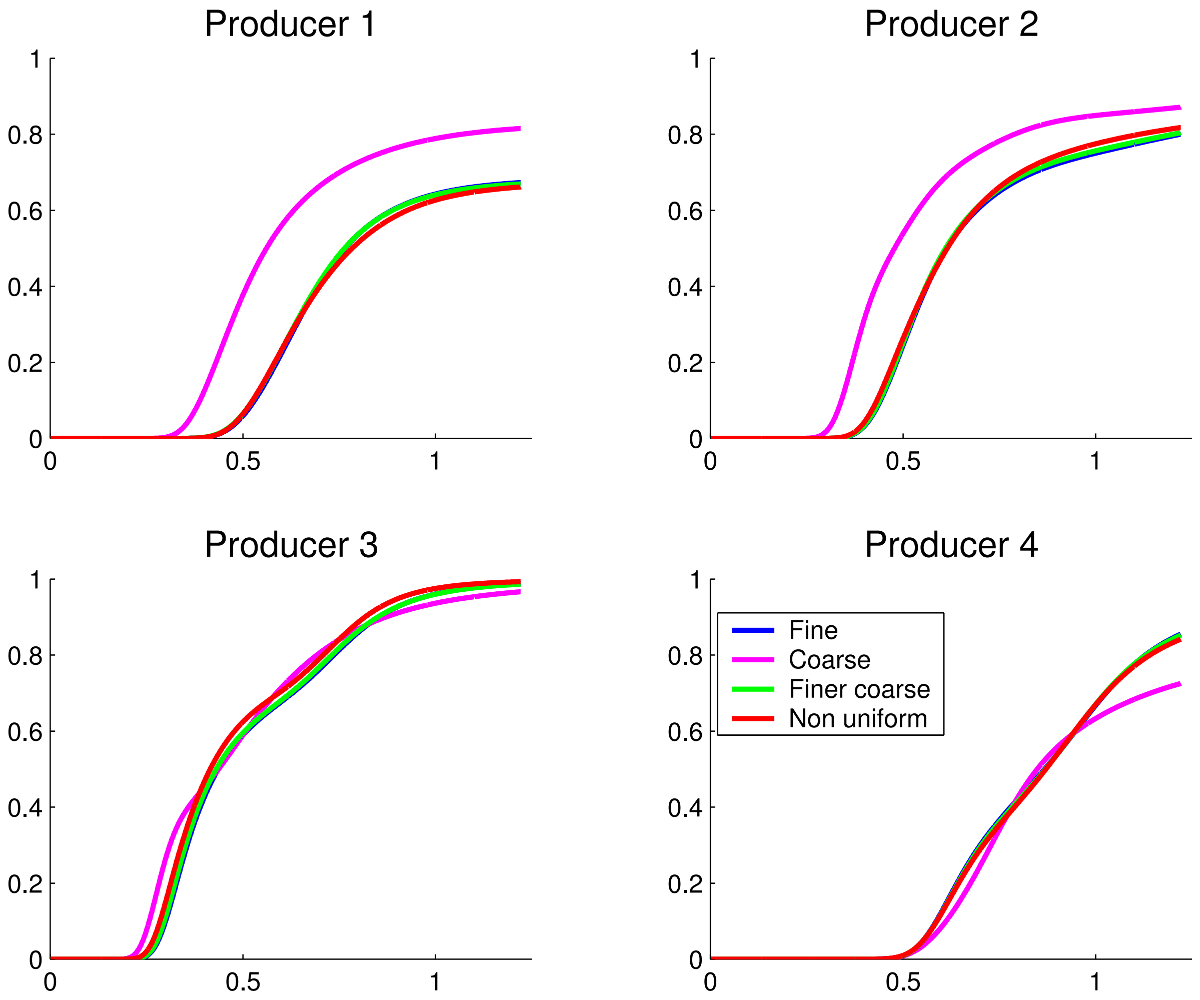
Numerical experiments 1, cont.

Saturation profiles at $t = 1.2$.



Numerical experiments 1, cont.

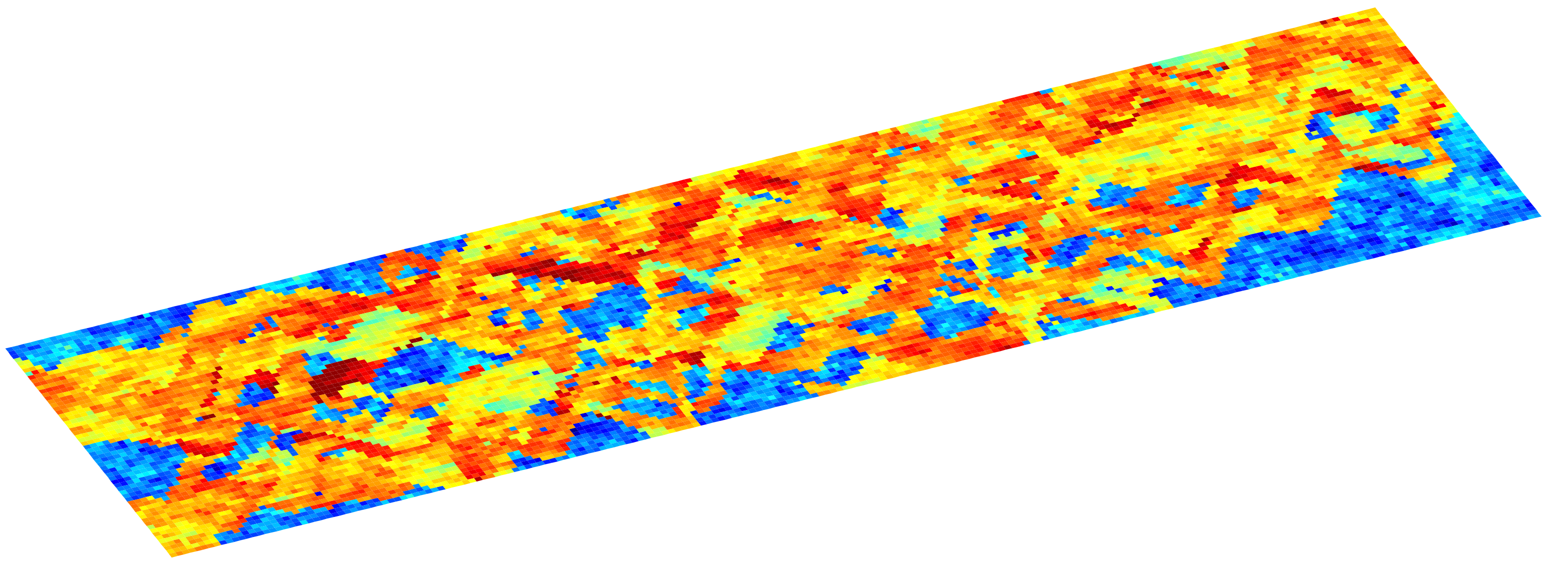
Water cut curves:



Numerical experiments 2

The bottom layer of the 10. SPE comparative solution project.

Note: The second criteria does not apply to this case (it doesn't find any barriers).



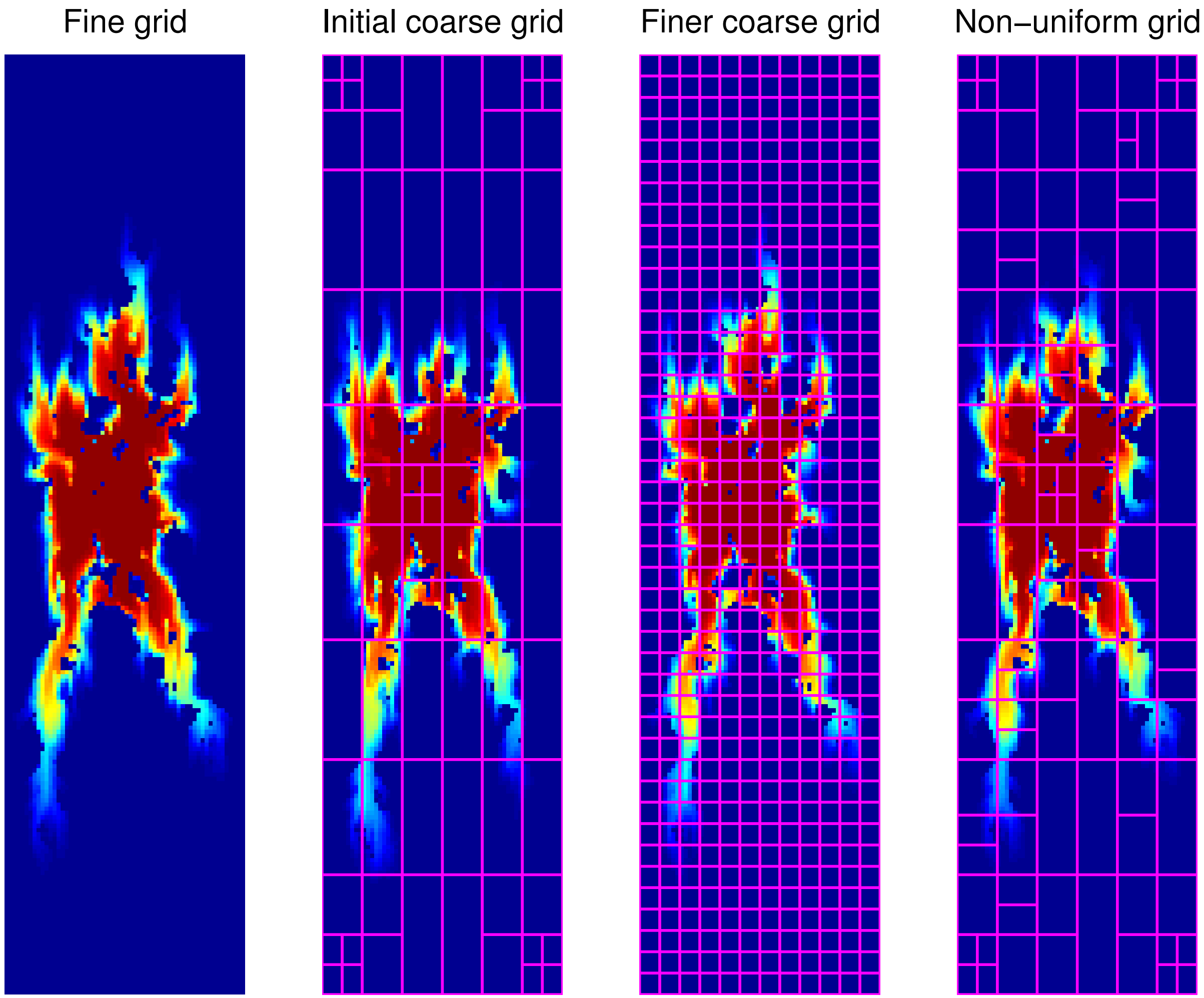
Numerical experiments 2, cont.

We apply four different grids:

- Fine grid, 60×220 blocks.
- Coarse grid,with some refinement around the wells $6 \times 8 + 12$ blocks.
- Finer coarse grid, 12×44 blocks.
- Non-uniform grid, 110 blocks.

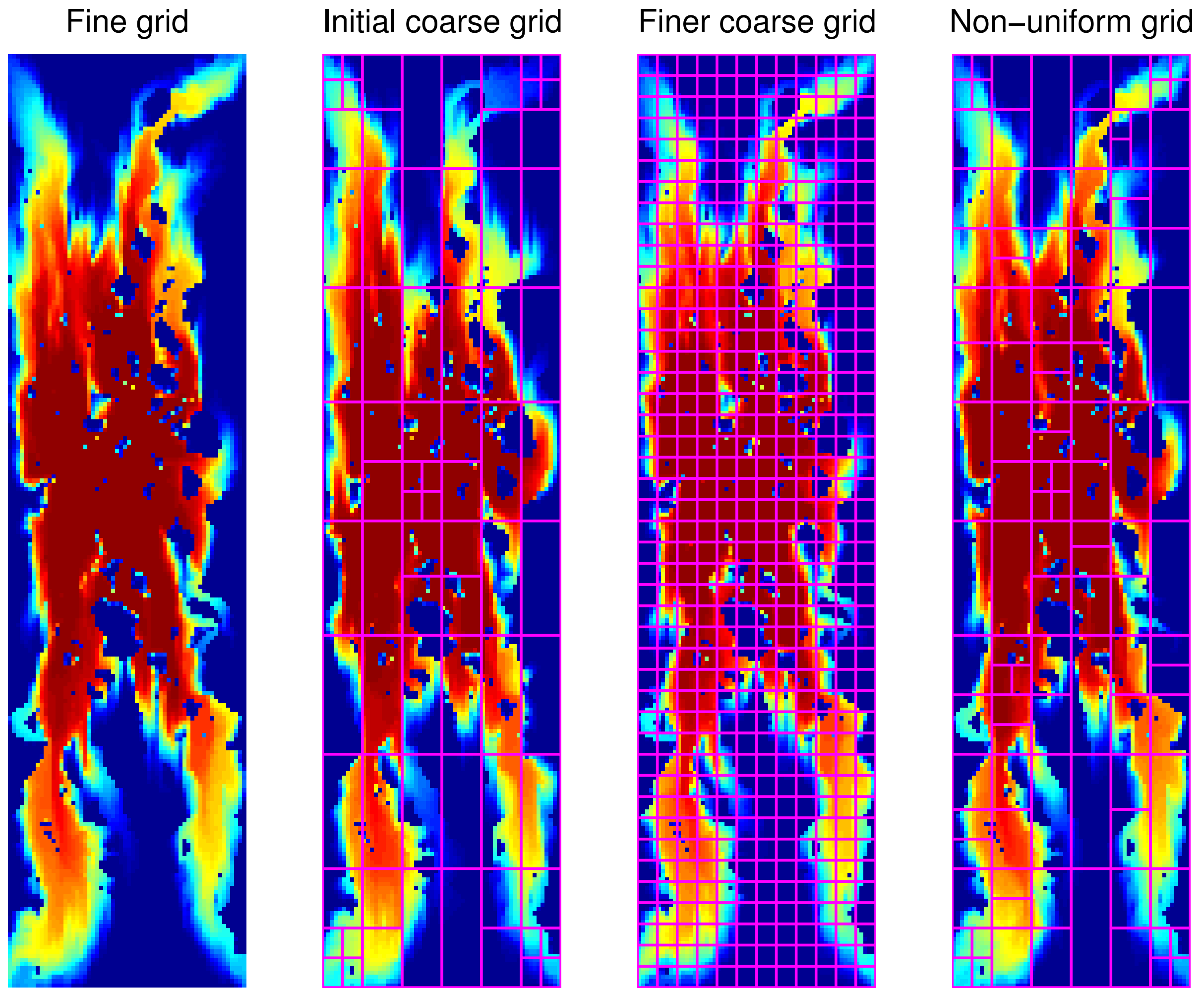
Numerical experiments 2, cont.

Saturation profiles at $t = 0.15$.



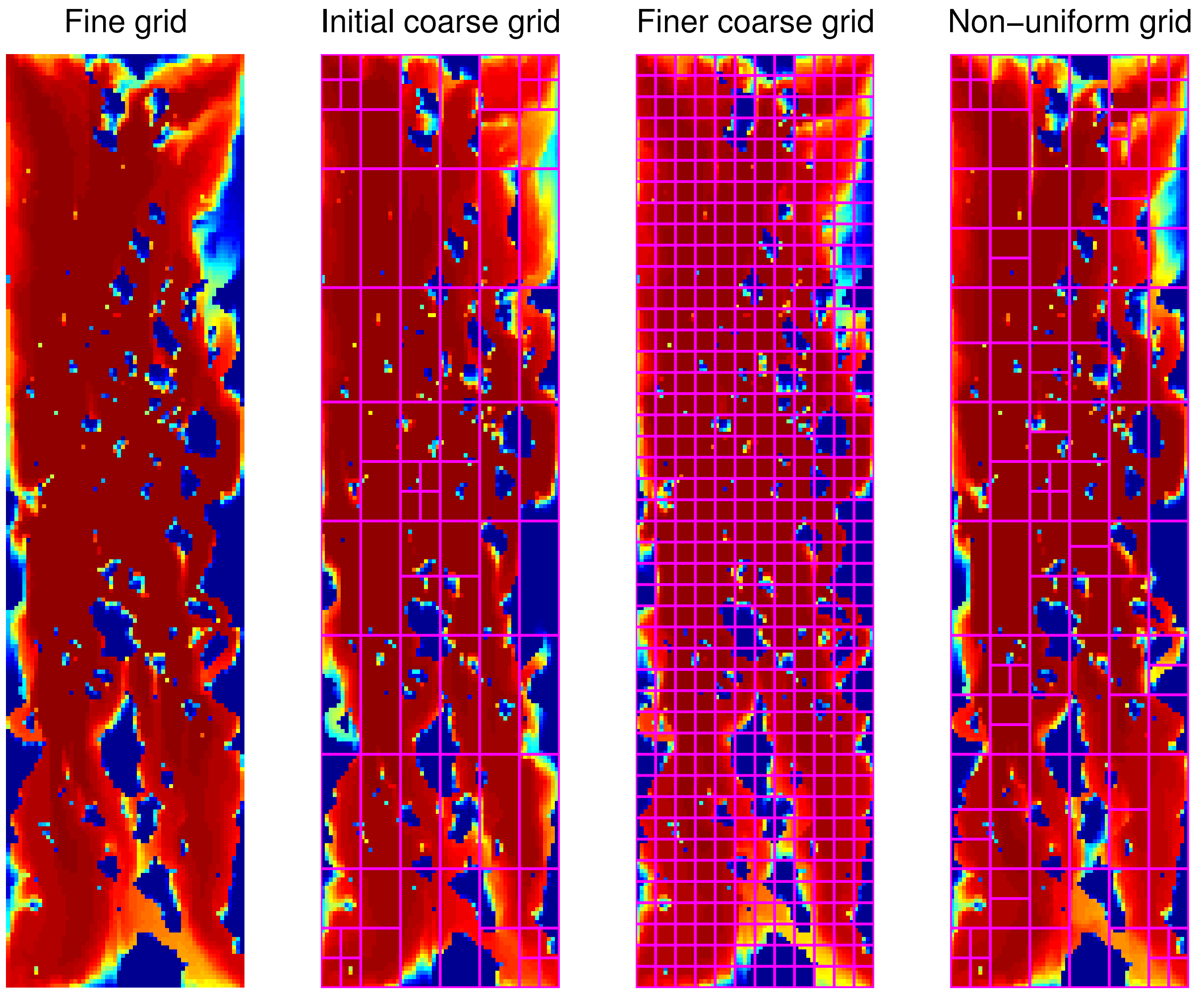
Numerical experiments 2, cont.

Saturation profiles at $t = 0.45$.



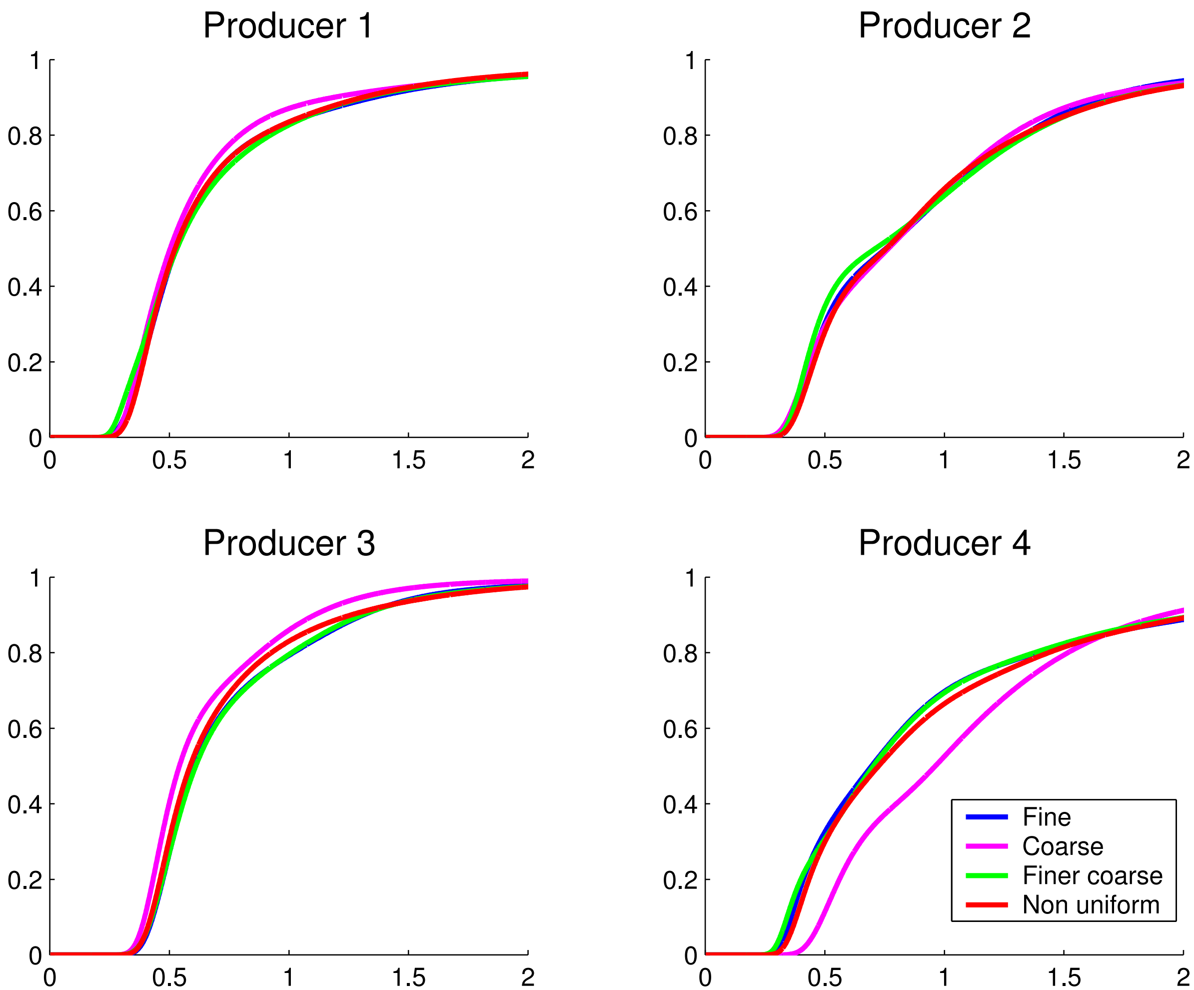
Numerical experiments 2, cont.

Saturation profiles at $t = 1.5$.



Numerical experiments 2, cont.

Water cut curves ($\lambda = 1$, $f(S) = S$):



Numerical experiments 2, cont.

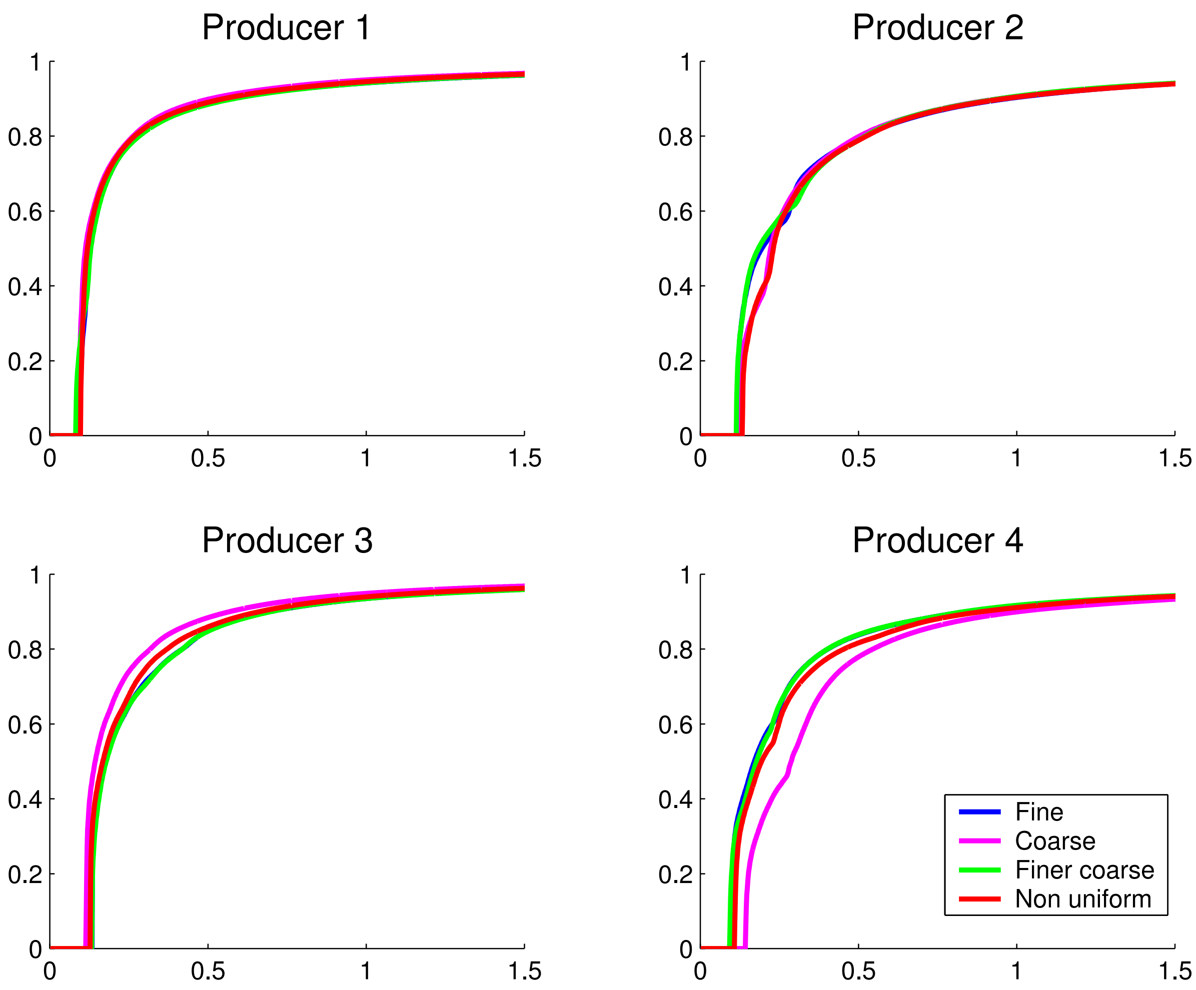
Water cut curves for

$$\lambda_w = \frac{(S^*)^2}{\mu_w}, \lambda_o = \frac{(1 - S^*)^2}{\mu_o},$$

where $S^* = (S - S_{wc})/(1 - S_{wc} - S_{or})$, $S_{wc} = S_{or} = 0.2$ and $\mu_o/\mu_w = 10$.

Numerical experiments 2, cont.

Water cut curves for (non linear mobility)



Final remarks

- We have seen that the use of non-unform grid models for the MMsFEM can reduce the number of required grid blocks, and thus has a potential for computational speed-up.

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- Solving the coarse system (preconditioning).
- Adaptivity.
- Theory...