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Geilo Winter School 2008

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Optimizing parallel applications

- Low latencies (locality of reference)
  - Cache memories
  - Remote Accesses (NUMA)
- Low parallel overhead
  - Minimize communication and synchronization
- Load Balance
  - Partitioning
  - Each thread has an equal amount of work

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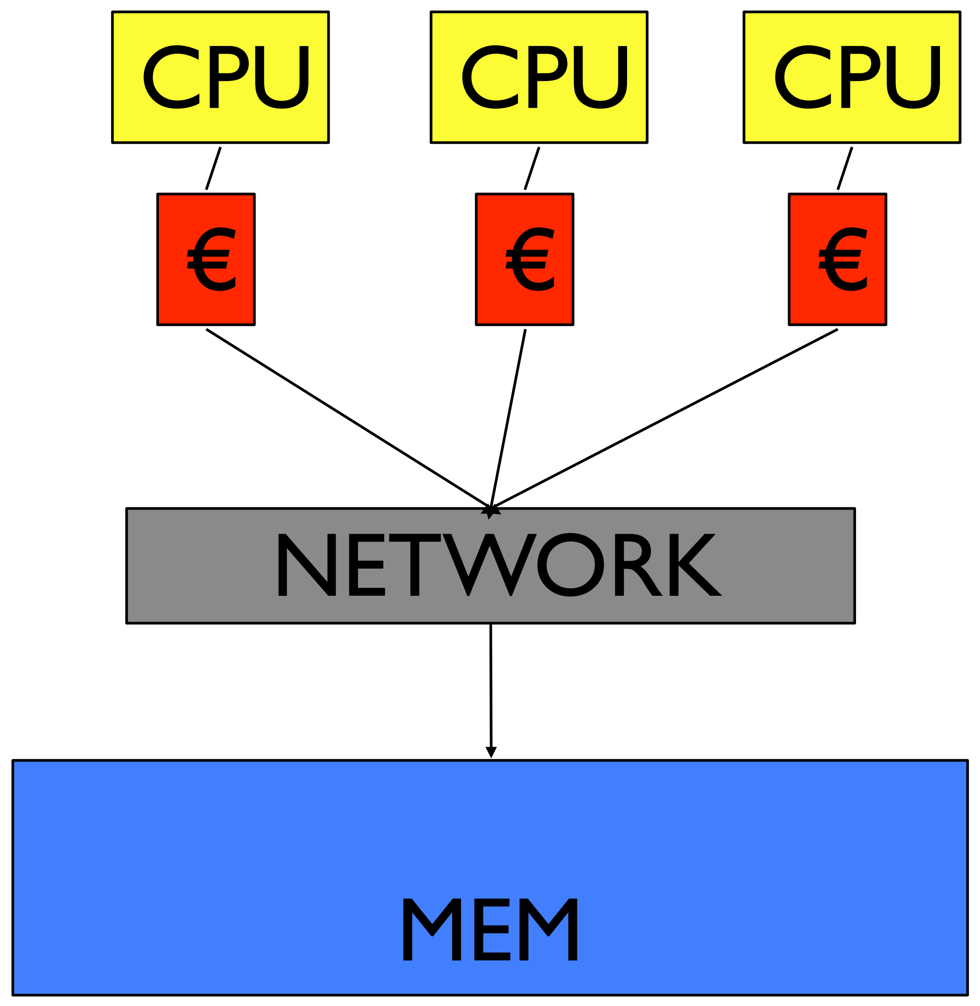
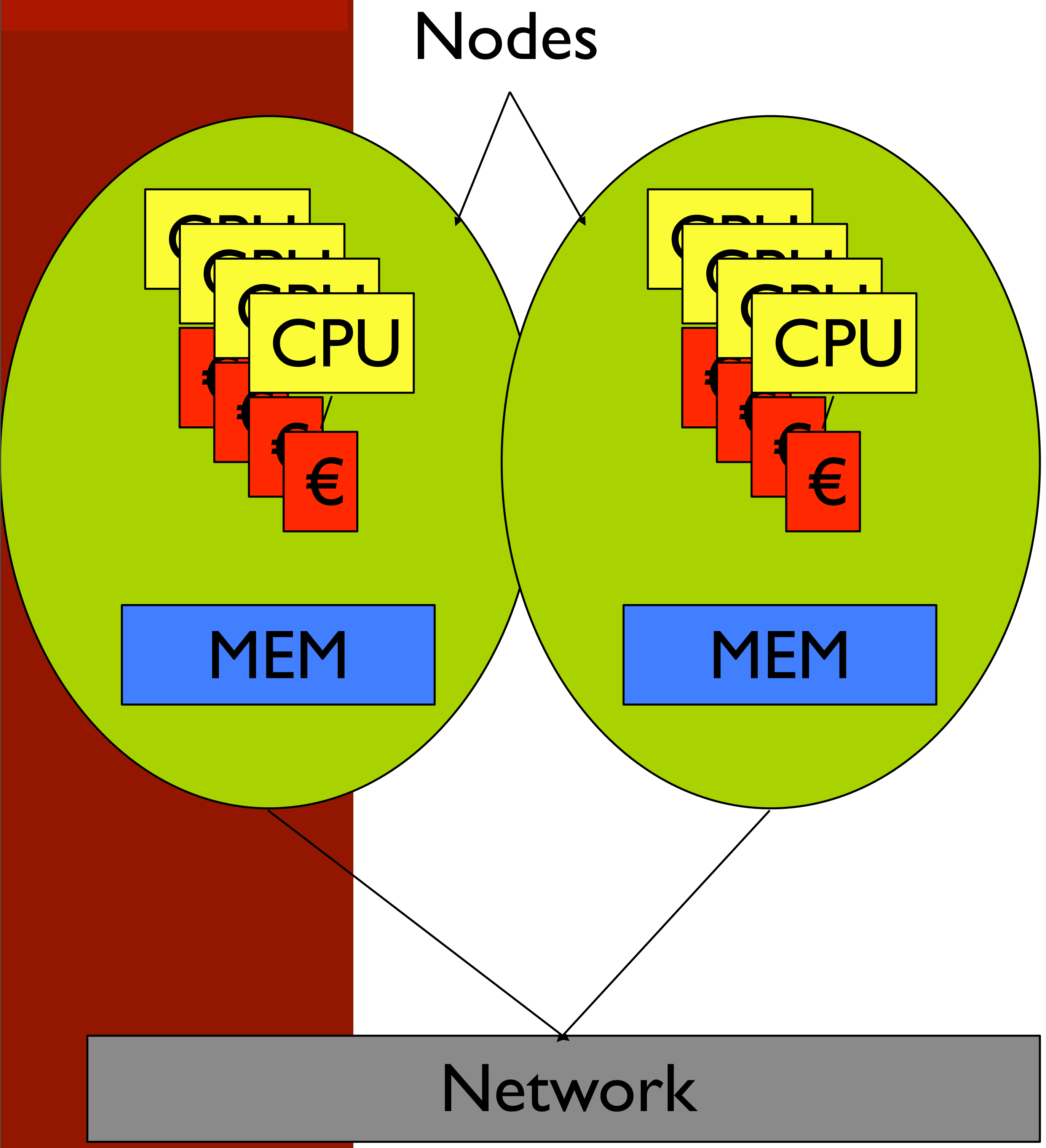
Uniform Memory Access (UMA)



- Server
  - Few dozen CPU/cores
  - Easy to administrate
- Volume product
- Uniform access time
  - "Easy" to understand
- Examples
  - SMP
  - CMP
- Scalability problems
  - Limited interconnect bandwidth

Non-Unifrom Memory Access (NUMA)



- Physically Distributed Memory
  - Shared memory programming
- Cluster of UMA *nodes*
  - CPU boards
  - Multi socket CMP systems
- Better scalability
  - Maintaines simpler shared memory programming model
- Examples
  - Sun Fire 15K, 25K
  - SGI Origin/Altix
  - HyperTransport (AMD)
  - QuickPath (Intel)
- Expensive

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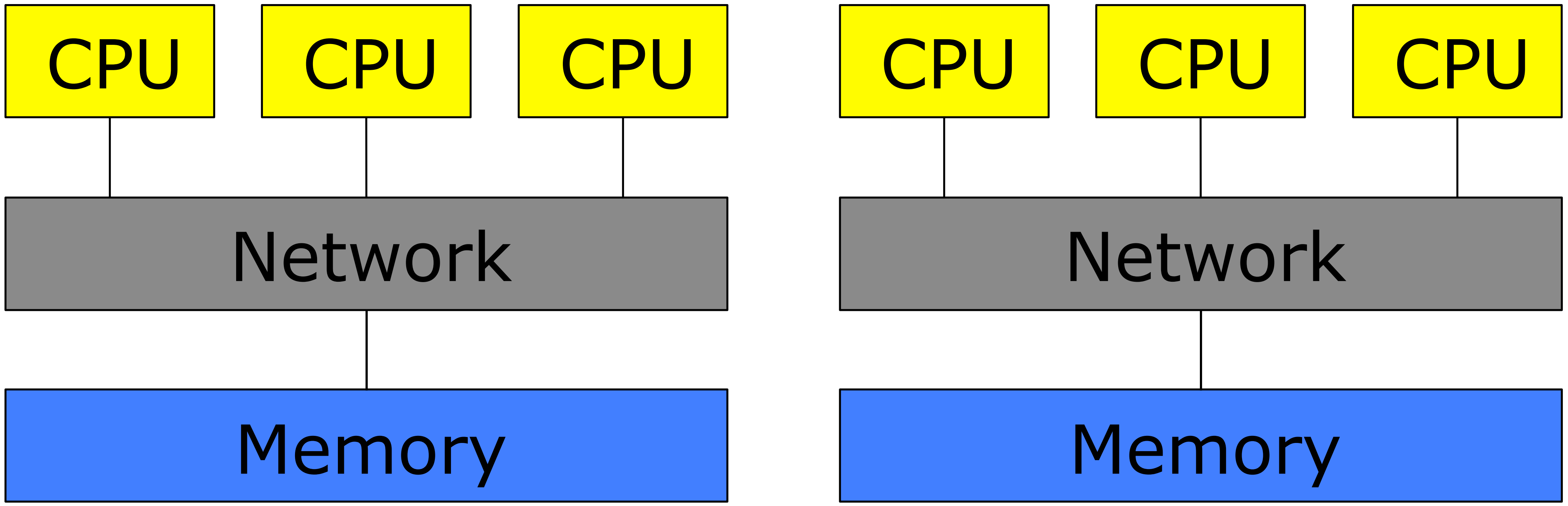
Sun WildFire, a Distributed Shared Memory (DSM) System



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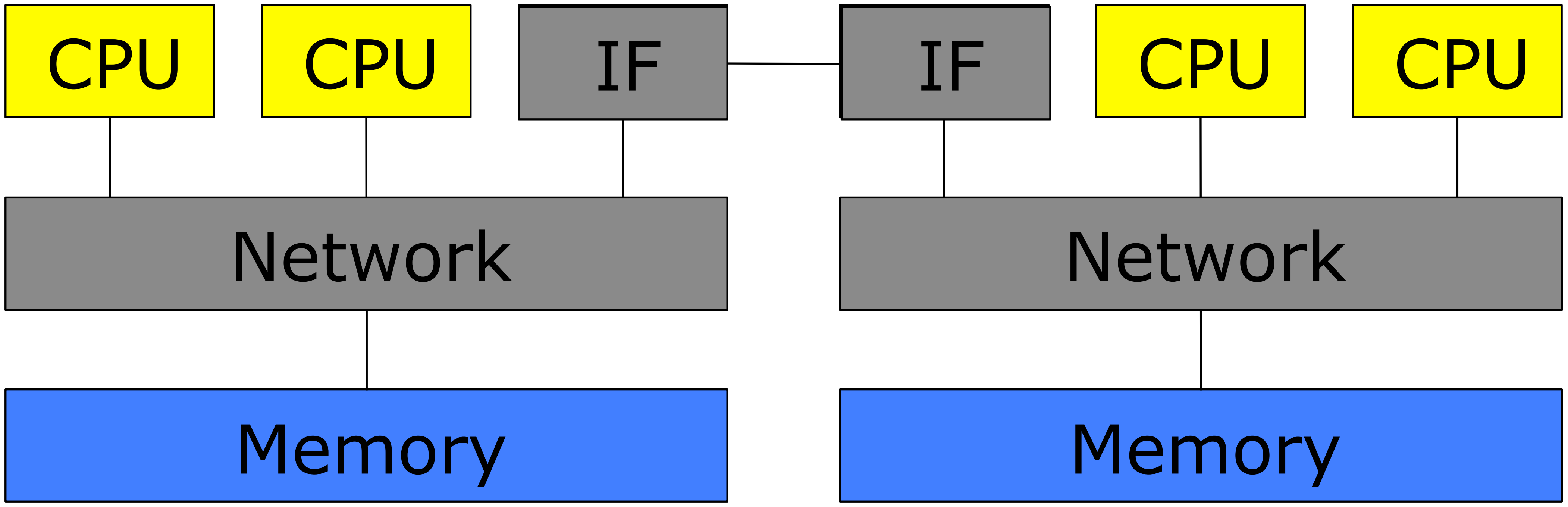
Sun WildFire, a Distributed Shared Memory (DSM) System



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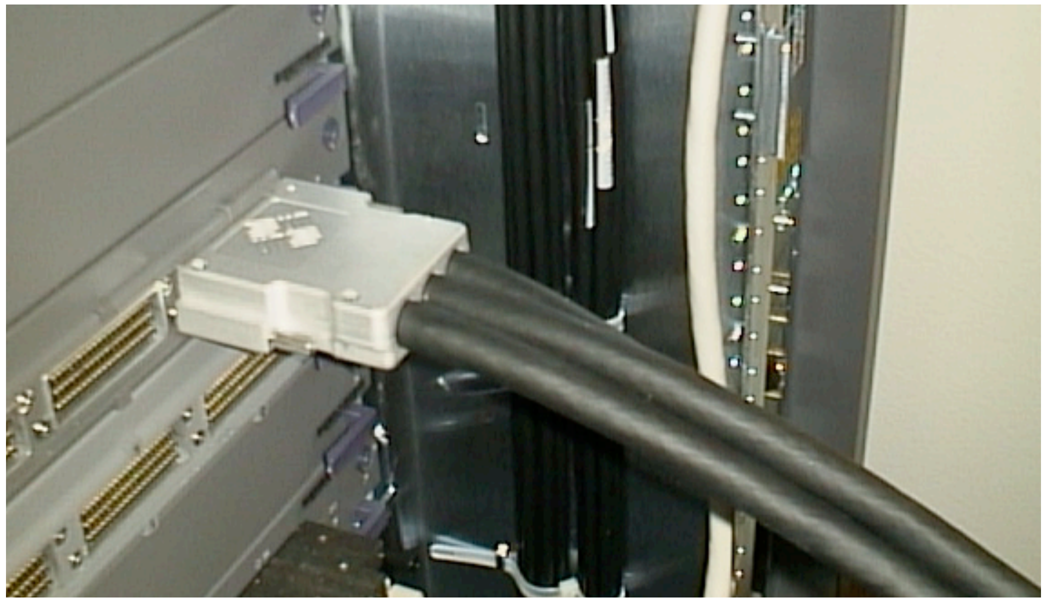
Sun WildFire what it looks like



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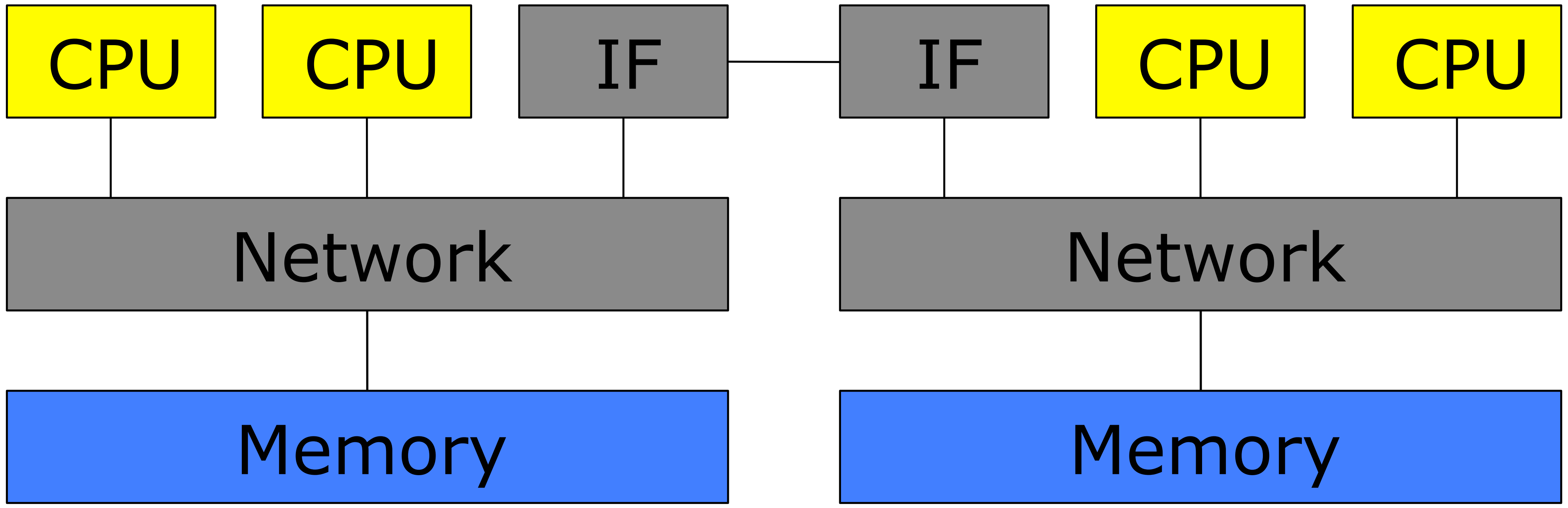
Local access – fast!



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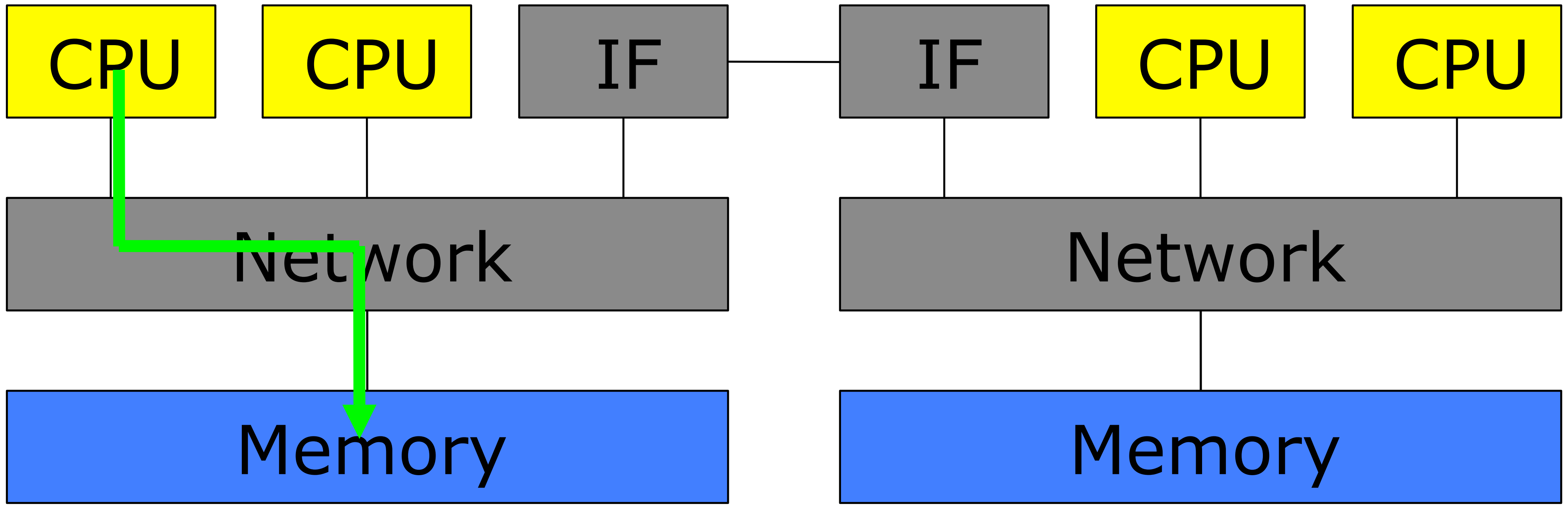
Local access – fast!



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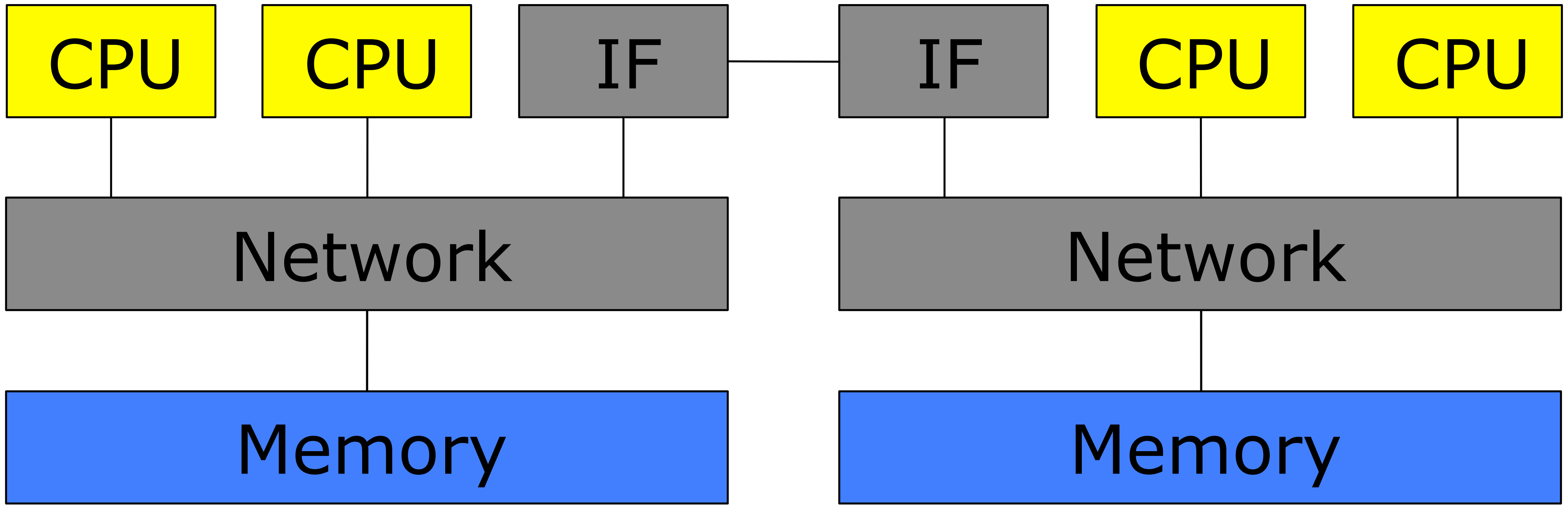
Remote access – slower!



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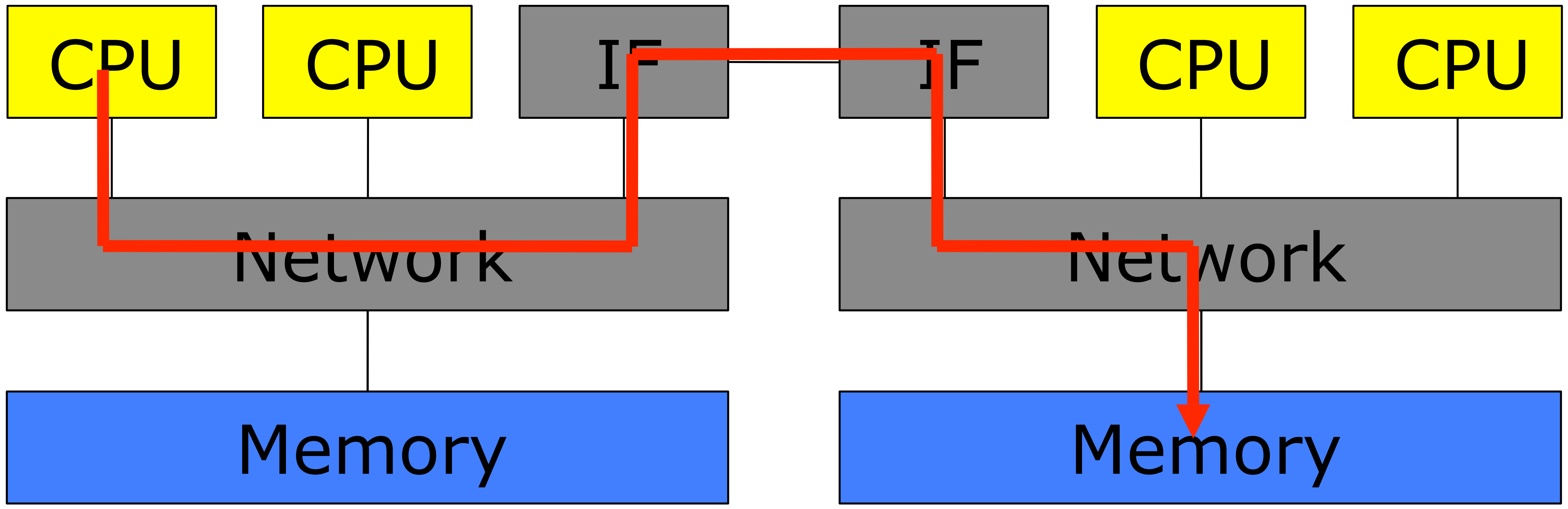
Remote access – slower!



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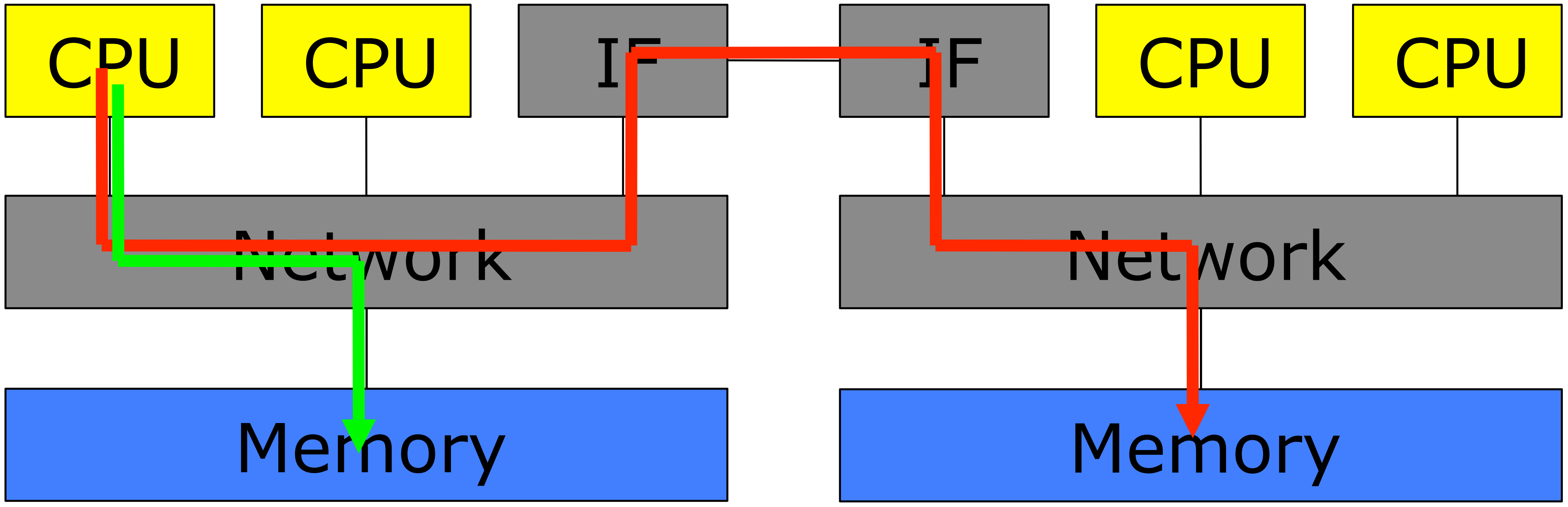
NUMA-ratio



$$NUMA - ratio = \frac{T_{remote}}{T_{local}}$$

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Implementing Conjugate Gradients in OpenMP

Analyzing CG

```
Given an initial guess $x_0$,
store $r_0 = b - Ax_0$ and set $p_0 = r_0$.
do $k = 0,1,\ldots$
(1)    Store $Ap_k$
(2)    Store $\langle p_k, Ap_k \rangle$
(3)    $\alpha_k = \frac{\langle r_k, r_k \rangle}{\langle p_k, Ap_k \rangle}$
(4)    $x_{k+1} = x_k + \alpha_k p_k$
(5)    $r_{k+1} = r_k - \alpha_k A p_k$
(6)    Store $\langle r_{k+1}, r_{k+1} \rangle$
(7)    $\beta_k = \frac{\langle r_{k+1}, r_{k+1} \rangle}{\langle r_k, r_k \rangle}$
(8)    $p_{k+1} = r_{k+1} + \beta_k p_k$
```

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Analyzing CG

```
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(8)    $p_{k+1} = r_{k+1} + \beta_k p_k$
```

3 Vector Ops.

Analyzing CG

Given an initial guess x_0,

store $r_0 = b - Ax_0$ and set $p_0 = r_0$.

do $k = 0,1,K$

(1) Store Ap_k

(2) Store $\langle p_k, Ap_k \rangle$

(3) $\alpha_k = \dfrac{\langle r_k, r_k \rangle}{\langle p_k, Ap_k \rangle}$

(4) $x_{k+1} = x_k + \alpha_k p_k$

(5) $r_{k+1} = r_k - \alpha_k A p_k$

(6) Store $\langle r_{k+1}, r_{k+1} \rangle$

(7) $\beta_k = \dfrac{\langle r_{k+1}, r_{k+1} \rangle}{\langle r_k, r_k \rangle}$

(8) $p_{k+1} = r_{k+1} + \beta_k p_k$

2 Inner Products

3 Vector Ops.

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Analyzing CG

1 Sparse Matrix-Vector Product (SpMxV)

2 Inner Products

3 Vector Ops.

```
Given an initial guess $x_0$,
store $r_0 = b - Ax_0$ and set $p_0 = r_0$.
do $k = 0,1,\mathrm{K}$
    (1)  Store $Ap_k$
    (2)  Store $\langle p_k, Ap_k \rangle$
    (3)  $\alpha_k = \dfrac{\langle r_k, r_k \rangle}{\langle p_k, Ap_k \rangle}$
    (4)  $x_{k+1} = x_k + \alpha_k p_k$
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    (8)  $p_{k+1} = r_{k+1} + \beta_k p_k$
```

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Basic parallelization of CG

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Basic parallelization of CG

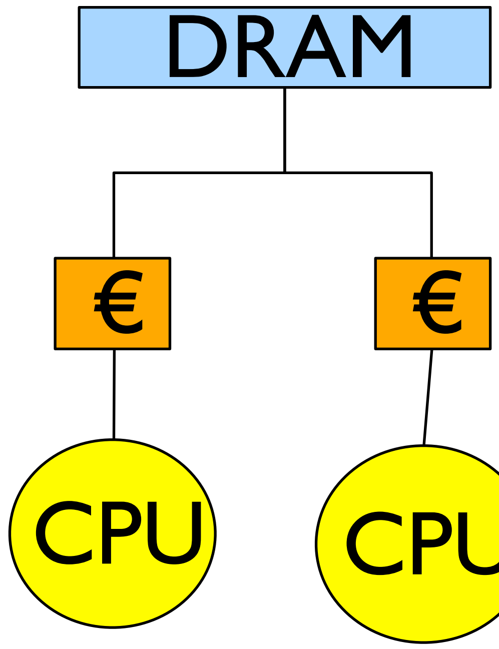
```
 cg_iter_count = 1

do

   call SpMxV(A,p,temp)


   pAp_norm = 0.0_rfp



   do i = 1, matrix_size
      pAp_norm = pAp_norm + p(i)*temp(i)
   end do




   alpha = r_old_norm/pAp_norm



   x(:) = x(:) + alpha * p(:)
   r_new(:) = r_old(:) - alpha * temp(:)

                  .

                  .
```

Basic parallelization of CG

```
!$OMP PARALLEL DEFAULT(SHARED) PRIVATE(cg_iter_count)

   cg_iter_count = 1

  do

      call SpMxV(A,p,temp)

!$OMP SINGLE
      pAp_norm = 0.0_rfp
!$OMP END SINGLE
=====================================
!$OMP DO REDUCTION(+:pAp_norm)
      do i = 1, matrix_size
         pAp_norm = pAp_norm + p(i)*temp(i)
      end do
!$OMP END DO
=====================================

!$OMP SINGLE
      alpha = r_old_norm/pAp_norm
!$OMP END SINGLE
=====================================
!$OMP WORKSHARE
      x(:) = x(:) + alpha * p(:)
      r_new(:) = r_old(:) - alpha * temp(:)
!$OMP END WORKSHARE NOWAIT
```

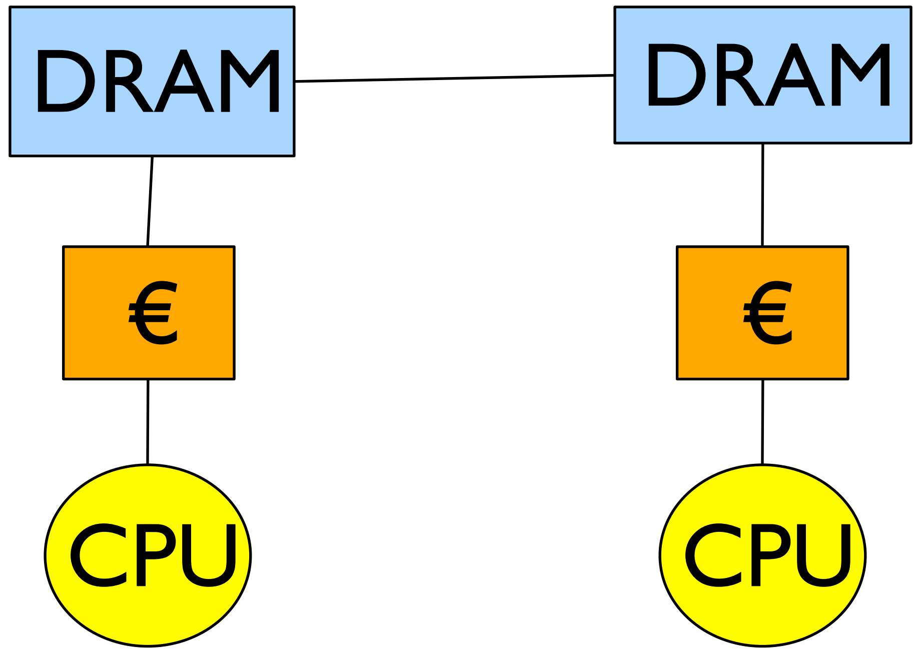
Parallel CG

Sun E10K



400MHz US-II + UMA

Sun SF15K

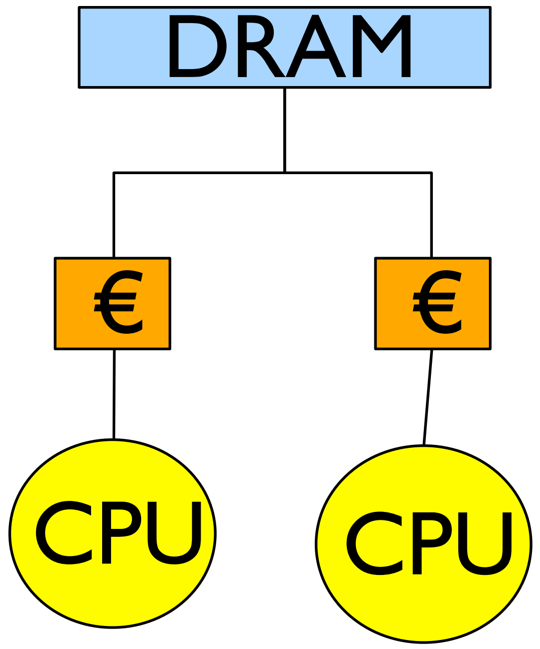


900MHz US-III + NUMA

Parallel CG

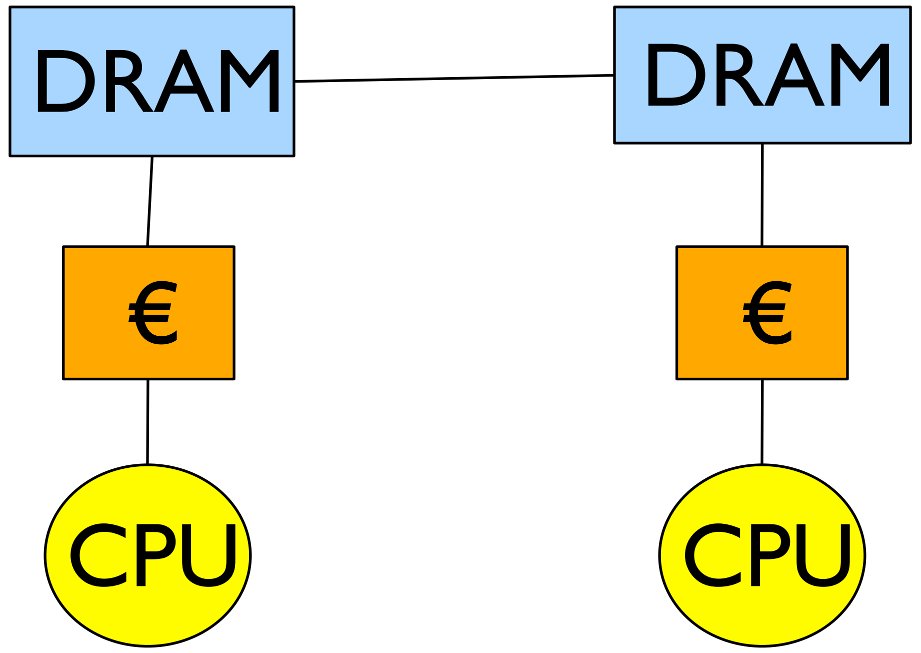
Serial

Sun E10K



400MHz US-II + UMA

Sun SF15K

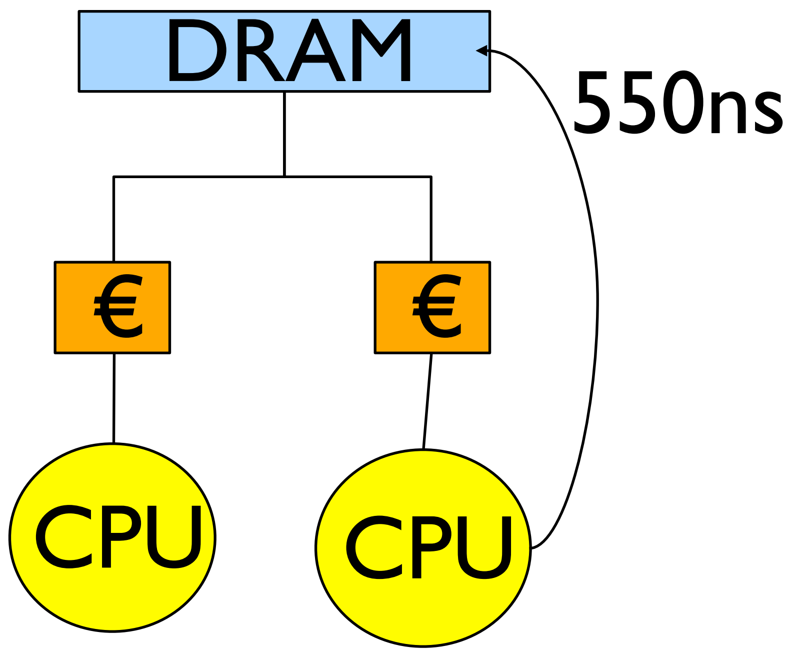


900MHz US-III + NUMA

Parallel CG

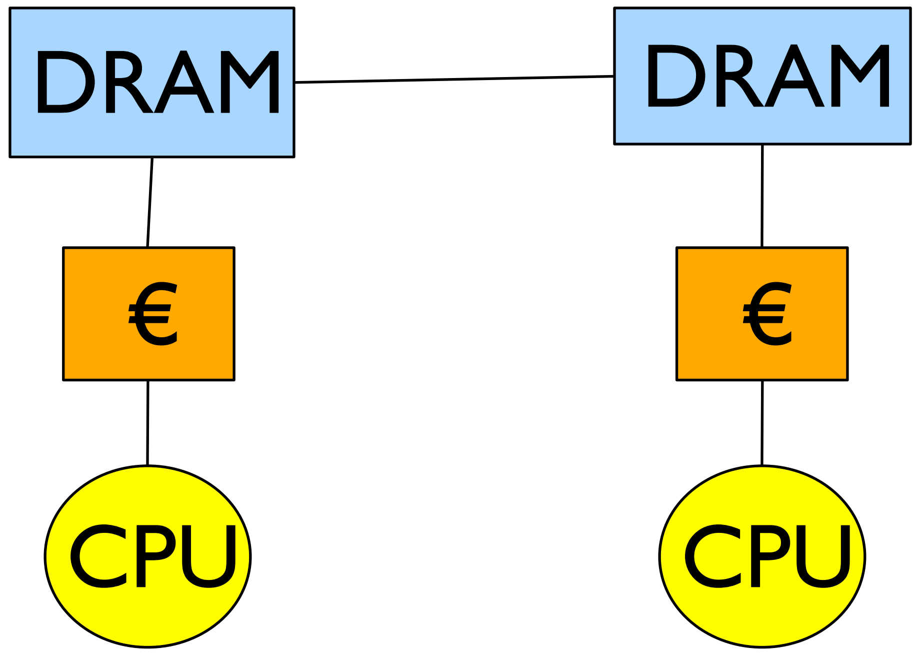
Serial 220s

Sun E10K



400MHz US-II + UMA

Sun SF15K



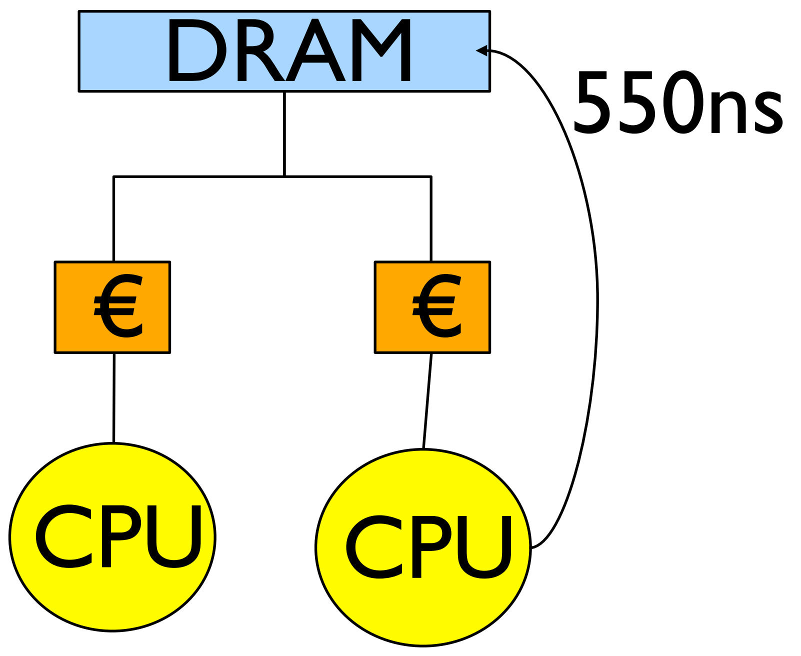
900MHz US-III + NUMA

Parallel CG

Serial 220s

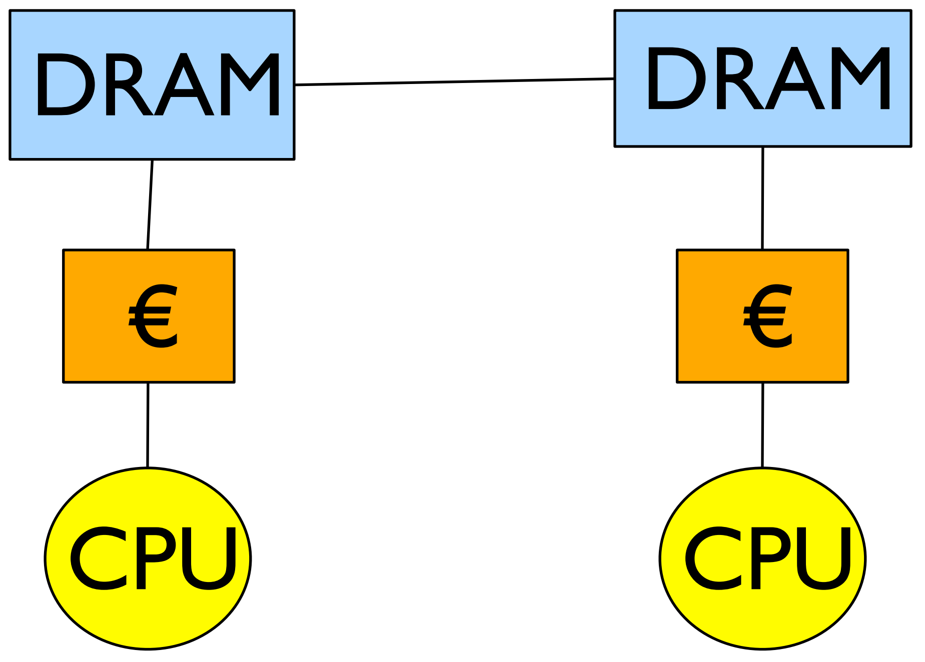
Serial

Sun E10K



400MHz US-II + UMA

Sun SF15K



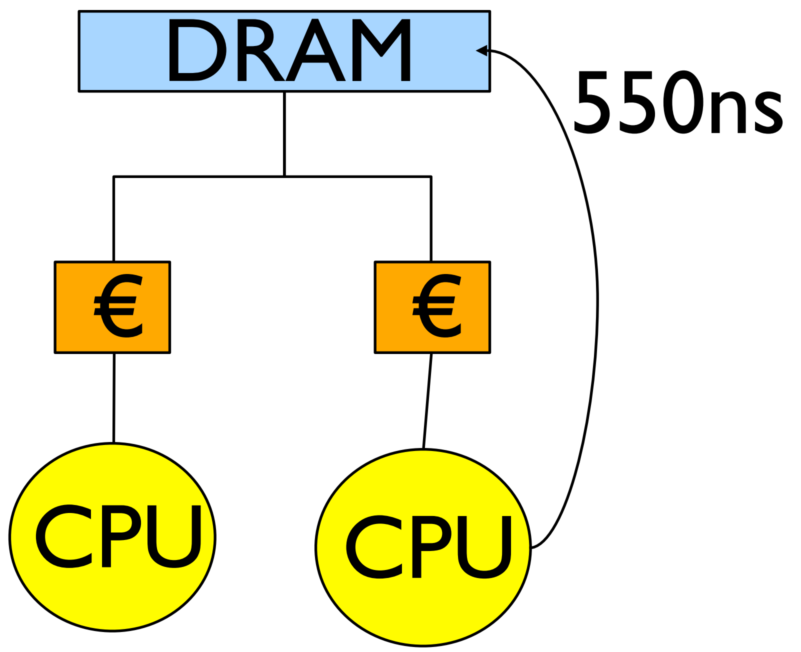
900MHz US-III + NUMA

Parallel CG

Serial 220s

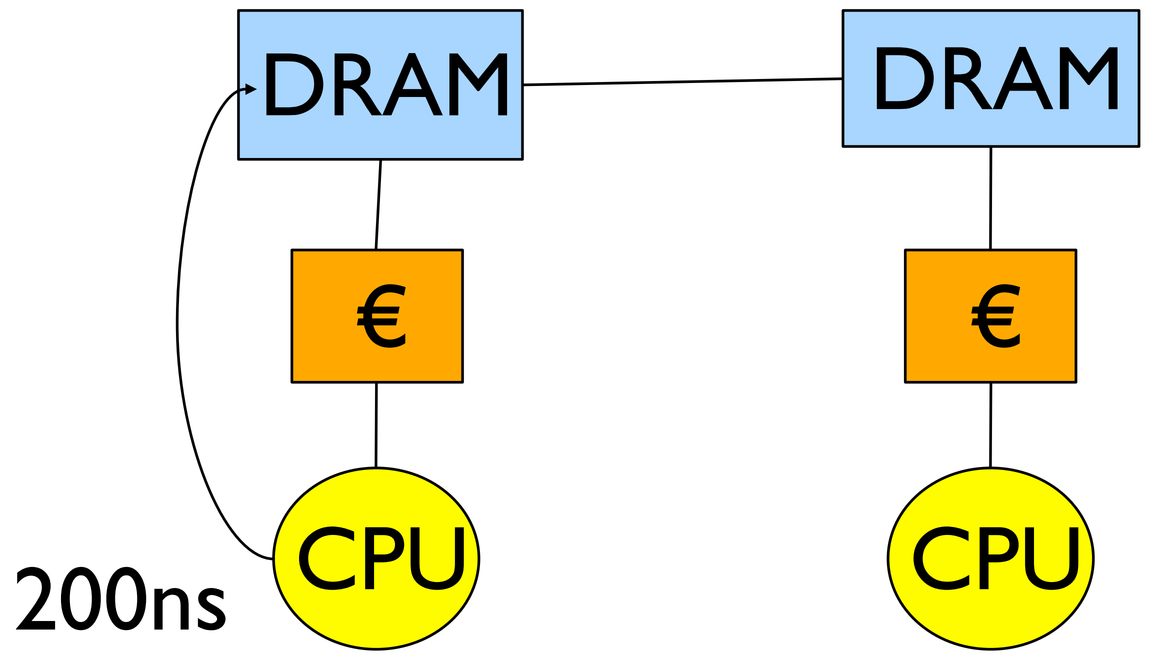
45s Serial

Sun E10K



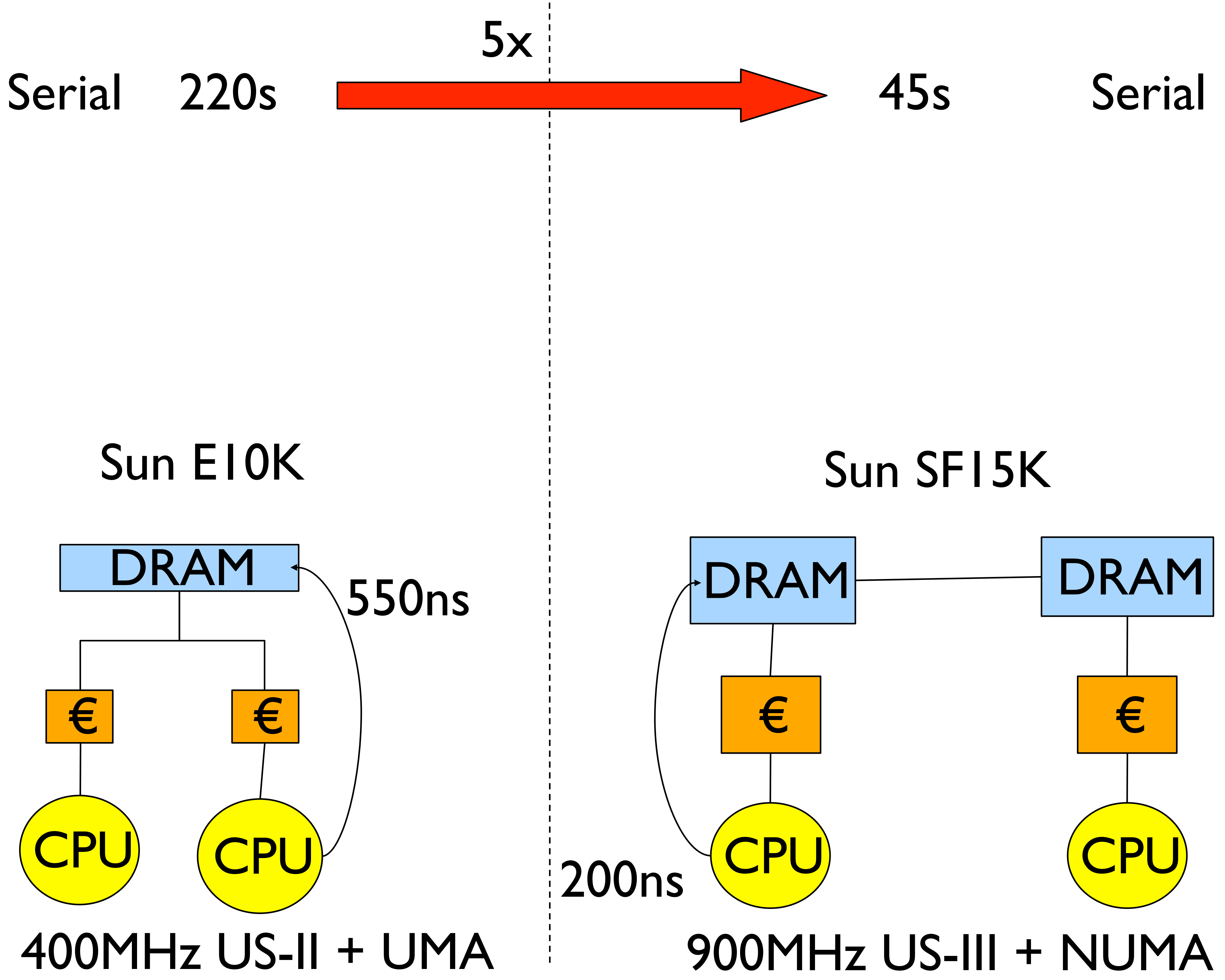
400MHz US-II + UMA

Sun SF15K



900MHz US-III + NUMA

Parallel CG

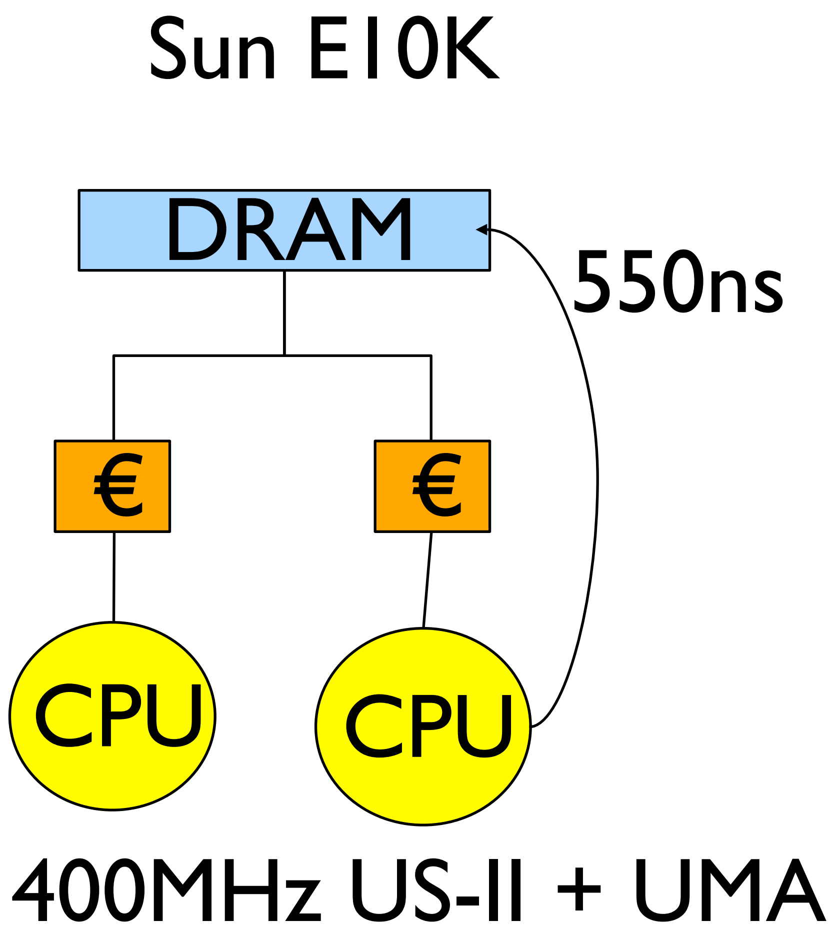


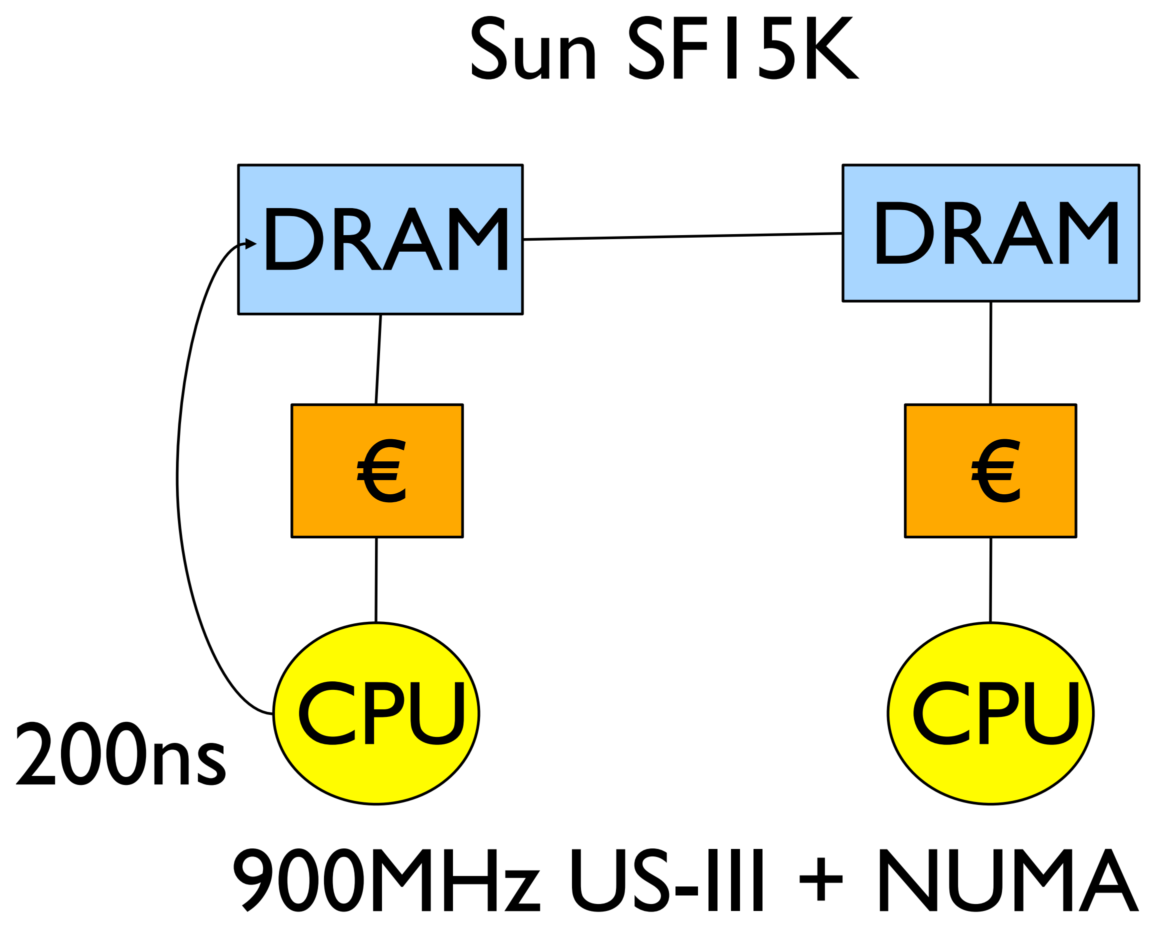
5x
Serial
220s
45s
Serial
Sun E10K
DRAM
550ns
€
€
CPU
CPU
400MHz US-II + UMA
Sun SF15K
DRAM
DRAM
€
€
CPU
CPU
200ns
900MHz US-III + NUMA

Parallel CG

Serial 220s → 5x → 45s Serial

Parallel,
28 threads



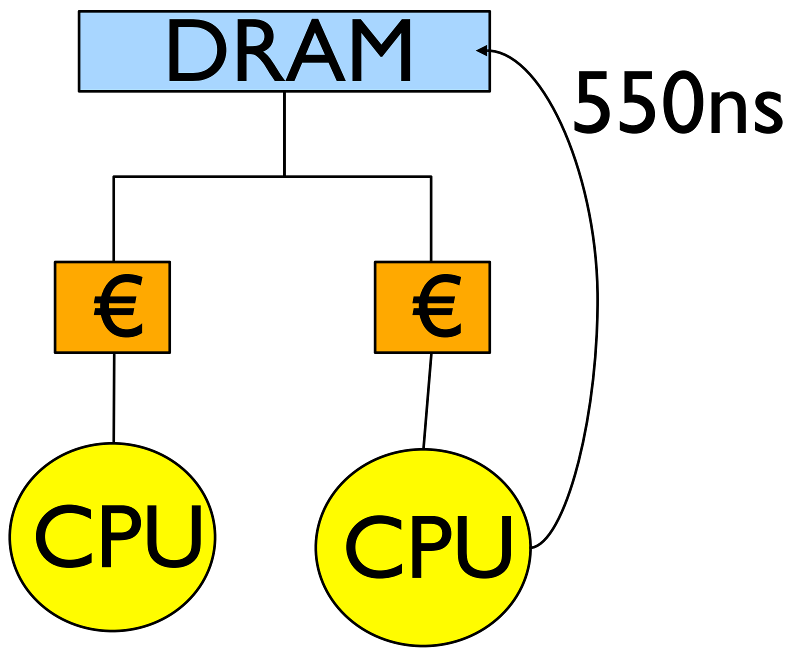


Parallel CG

Serial 220s → 5x → 45s Serial

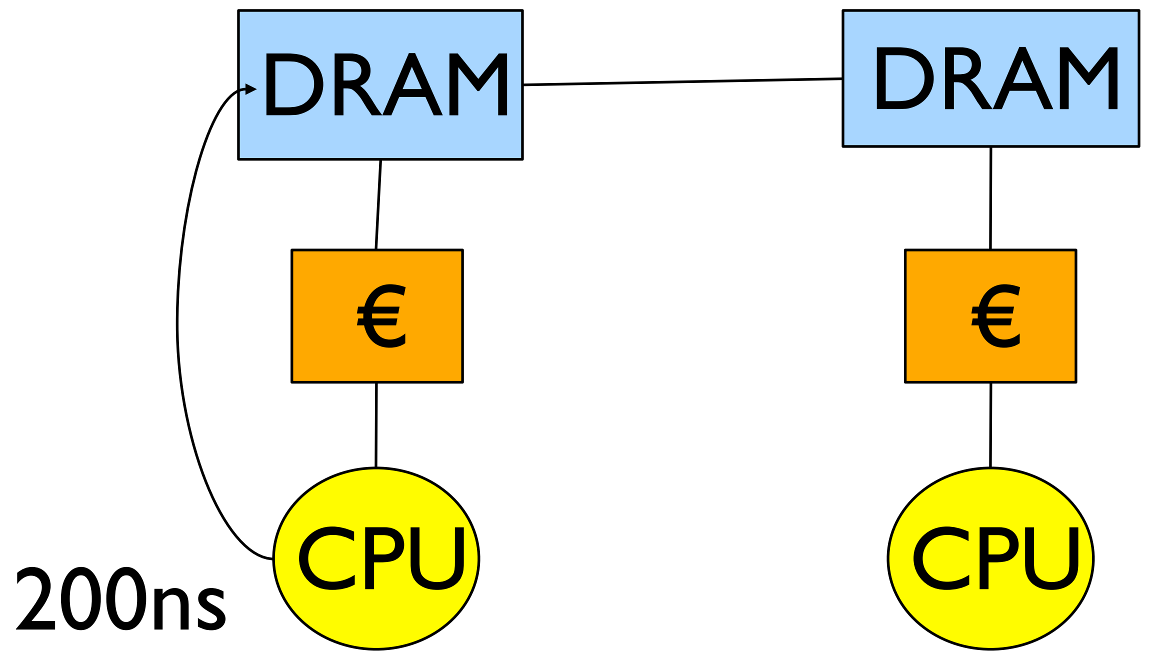
Parallel, 28 threads 8s

Sun E10K



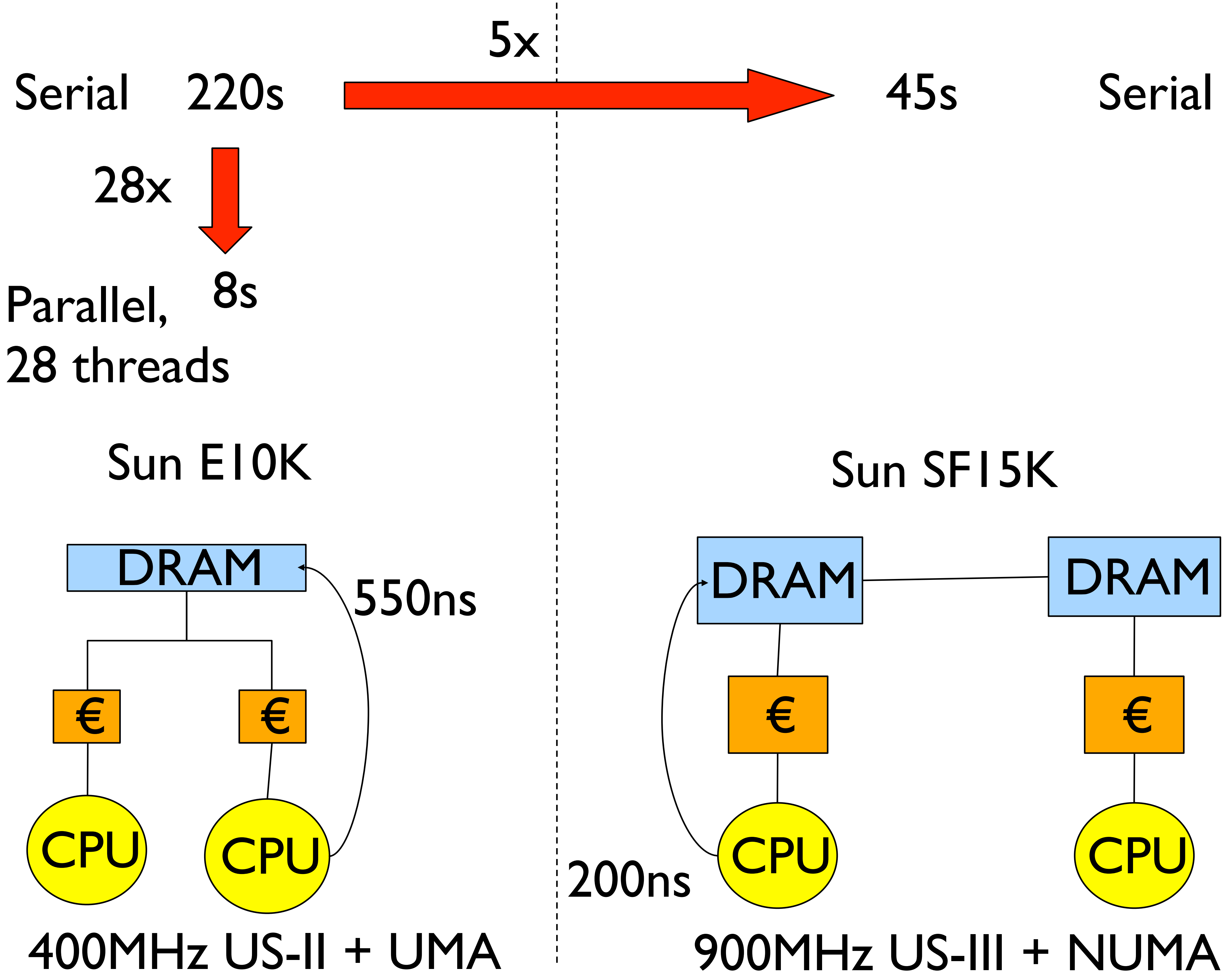
400MHz US-II + UMA

Sun SF15K



900MHz US-III + NUMA

Parallel CG



5x
Serial 220s
45s Serial
28x
8s
Parallel,
28 threads
Sun E10K
DRAM
550ns
€
€
CPU
CPU
400MHz US-II + UMA
Sun SF15K
DRAM
DRAM
€
€
200ns
CPU
CPU
900MHz US-III + NUMA

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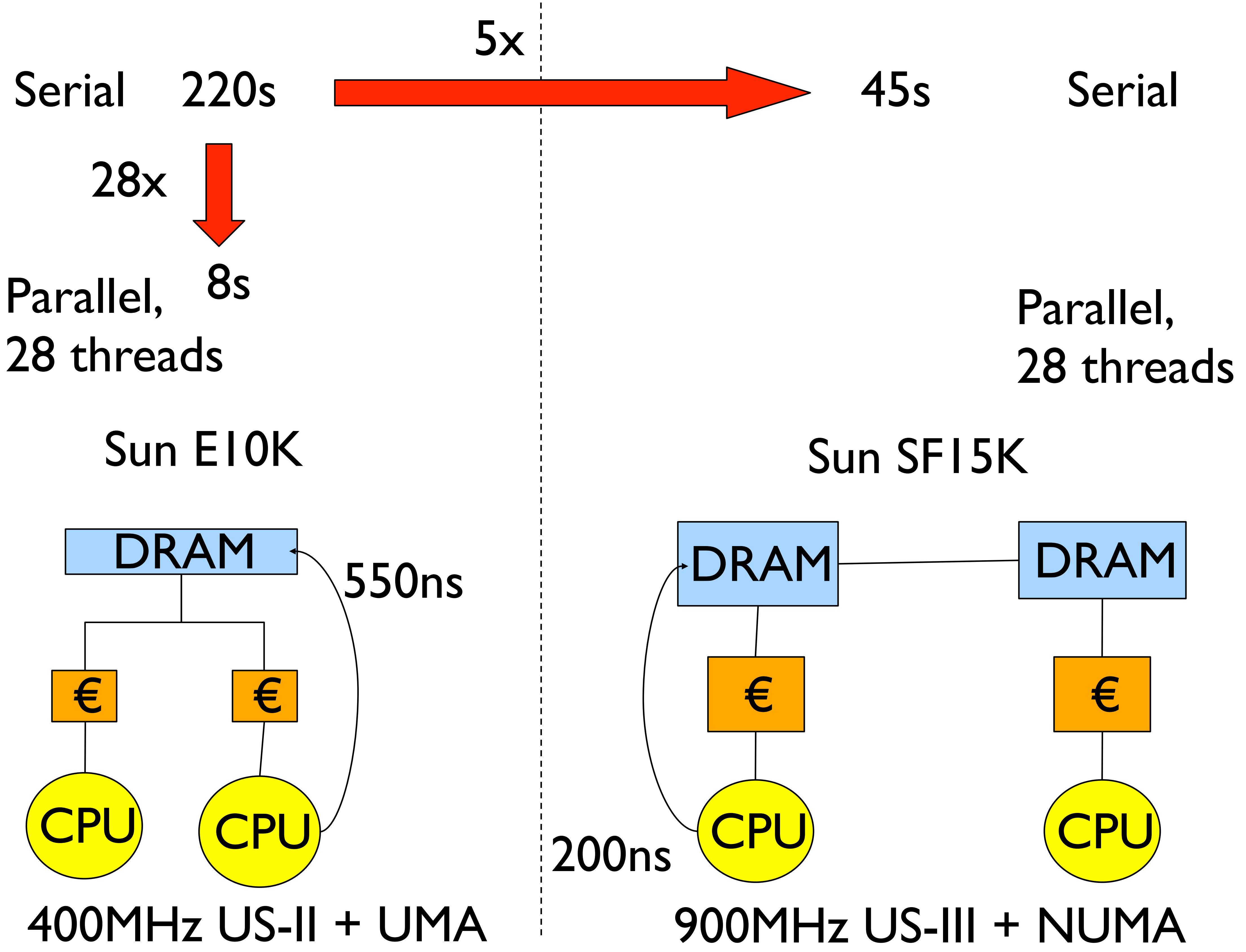
Parallel CG



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Parallel CG

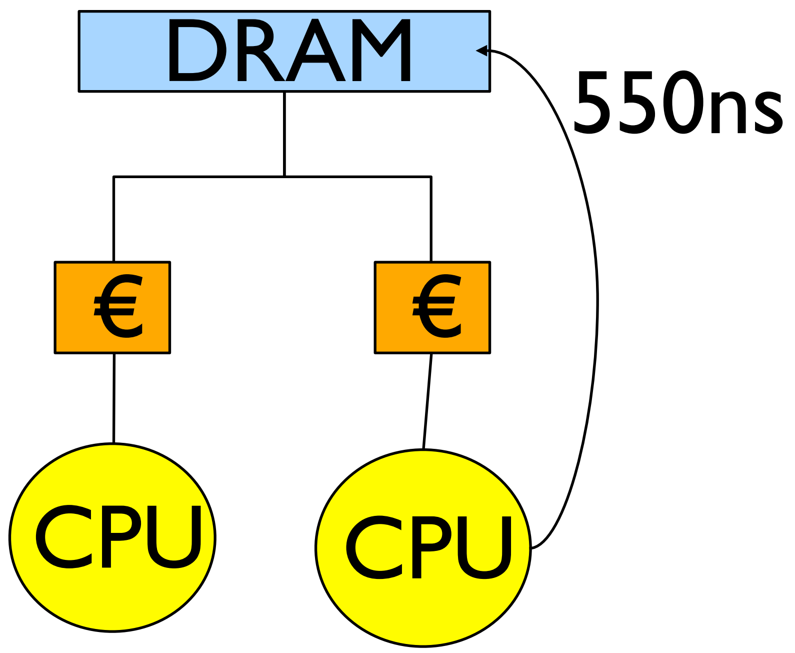
Serial 220s $\xrightarrow{5\times}$ 45s Serial

28x ↓

Parallel, 28 threads 8s

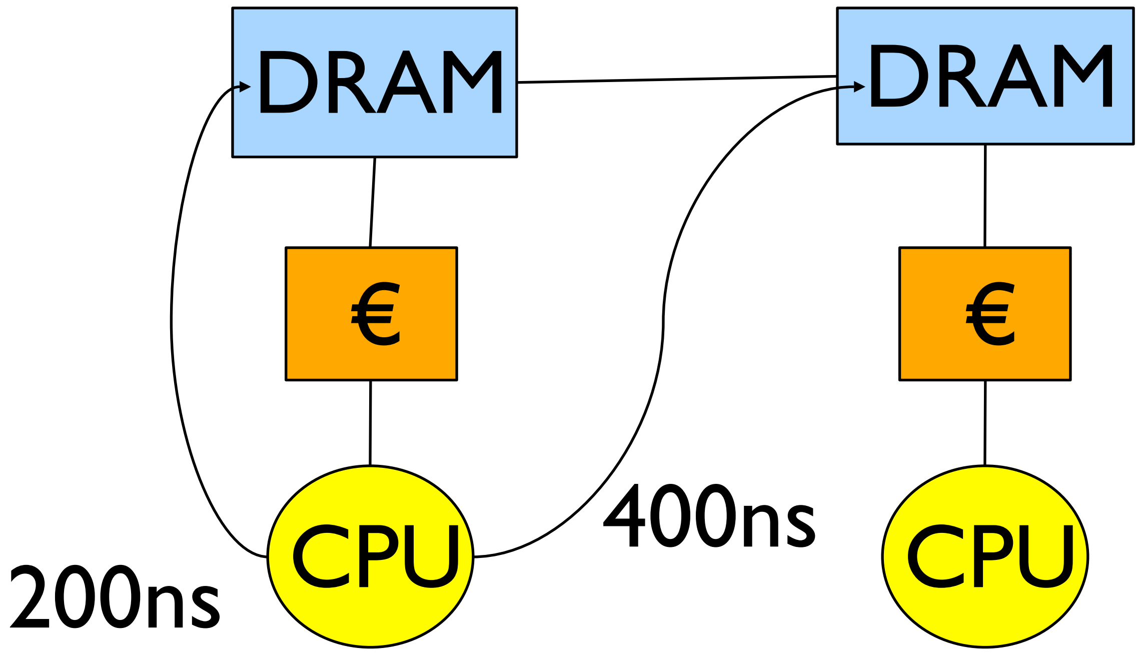
Parallel, 28 threads

Sun E10K



400MHz US-II + UMA

Sun SF15K



900MHz US-III + NUMA

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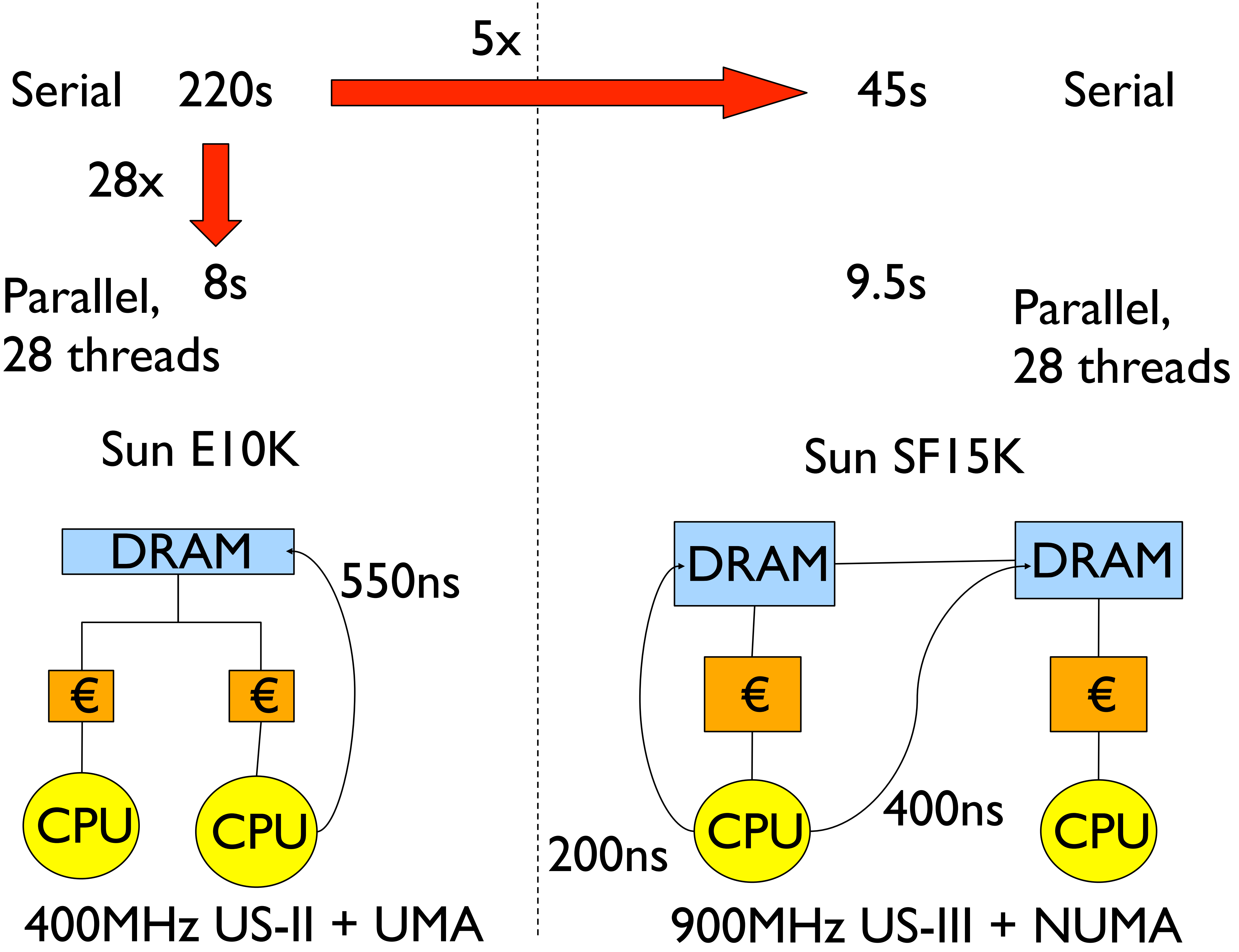
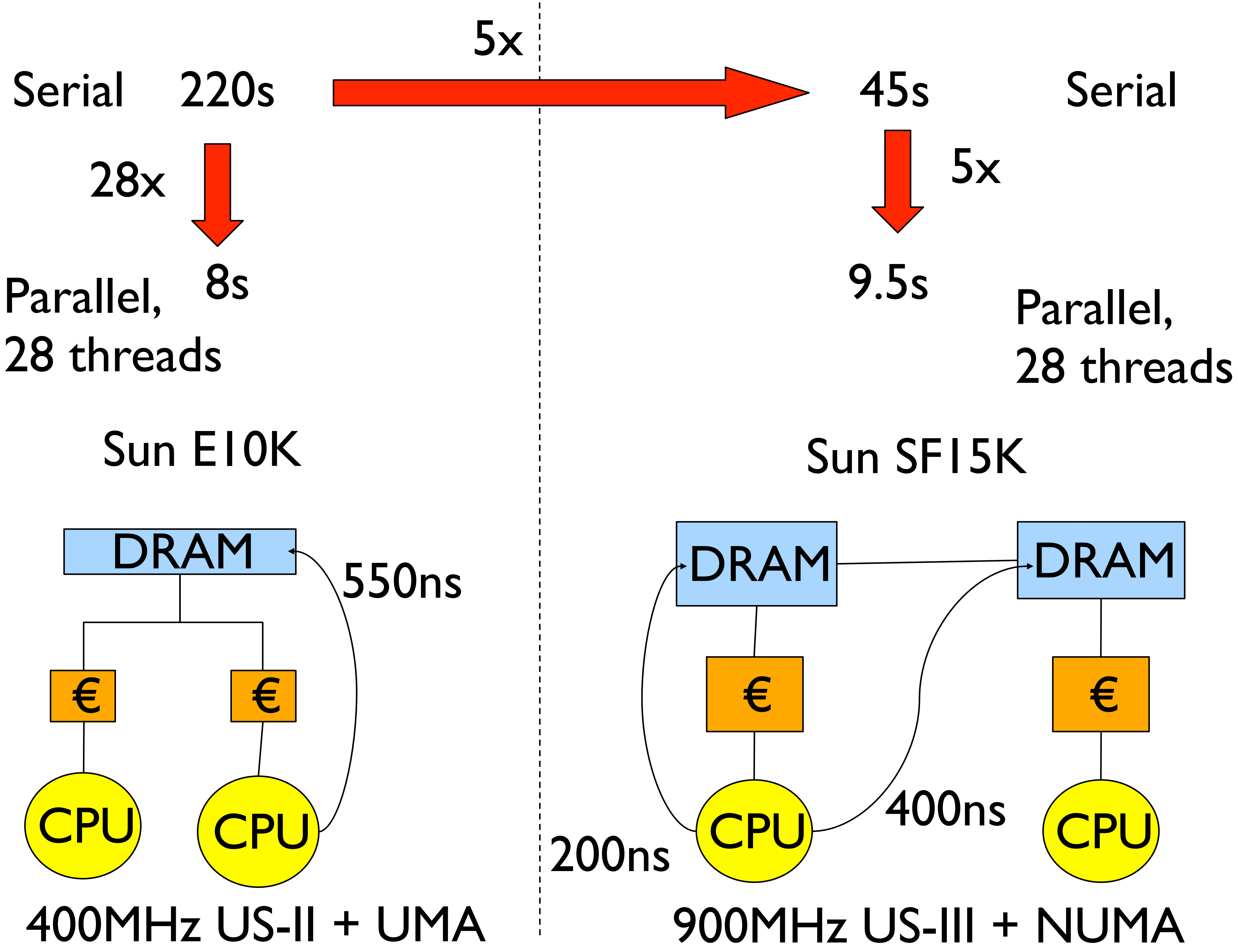
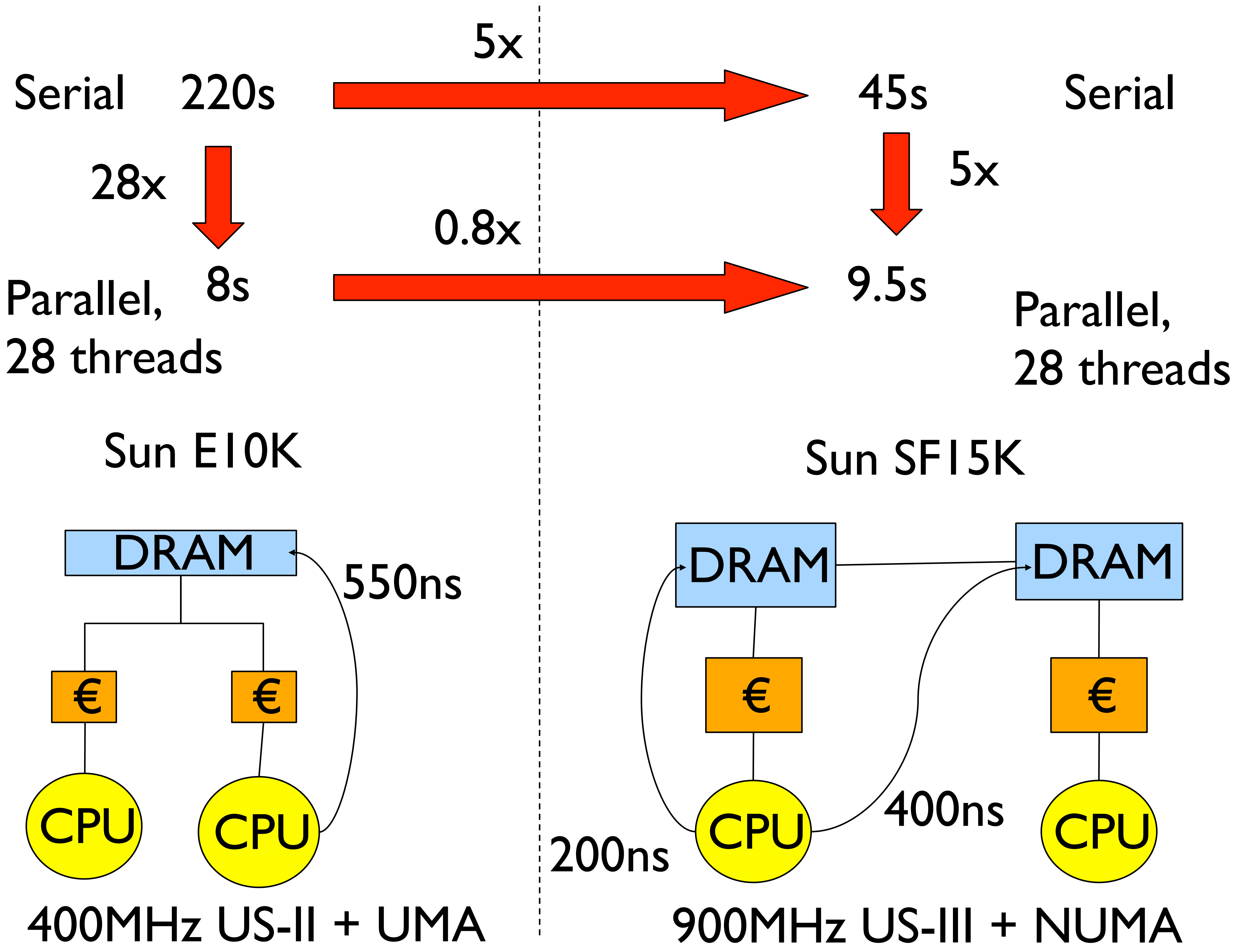
Parallel CG



Parallel CG


5x
Serial 220s
45s Serial
28x
5x
8s
Parallel, 28 threads
9.5s
Parallel, 28 threads
Sun E10K
Sun SF15K
DRAM
550ns
€
€
CPU
CPU
DRAM
DRAM
€
€
CPU
CPU
200ns
400ns
400MHz US-II + UMA
900MHz US-III + NUMA

Parallel CG


5x
Serial
220s
45s
Serial
28x
5x
0.8x
Parallel, 28 threads
8s
9.5s
Parallel, 28 threads
Sun E10K
Sun SF15K
DRAM
550ns
€
€
CPU
CPU
DRAM
DRAM
€
€
CPU
CPU
200ns
400ns
400MHz US-II + UMA
900MHz US-III + NUMA

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Geographical Locality

- Property of an application
  - ”How many memory accesses are node-local?”
  - Communication
- Dependent on many things
  - Data distribution (source code/OS)
  - Thread scheduling (OS)
- Thread-data affinity: *Minimize the amount of remote accesses by co-location of threads and data*

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Problems of initialization

- FEM assembly process serial
- First-touch strategy
  - All data allocated on a single node
- Access pattern stored in the data
  - Compressed Sparse Row (CSR)
  - Efficient parallel initialization not possible
- We need ways of redistributing data during runtime

Code modifications

- Inserted affinity-on-next-touch call before the first CG iteration
- Access pattern is static cross the iterations
  - Only need to redistribute once
- Used hardware counters to quantify effect

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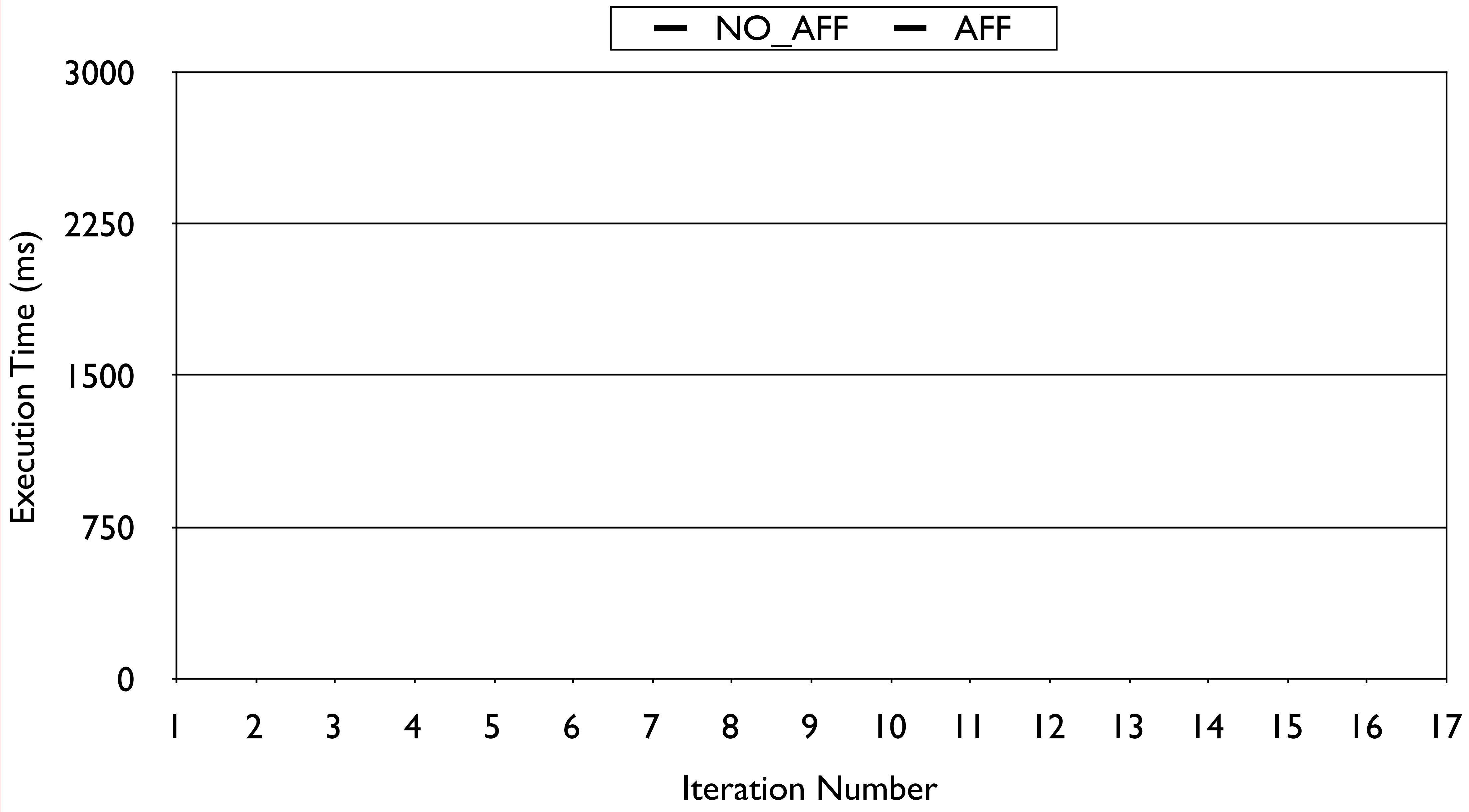
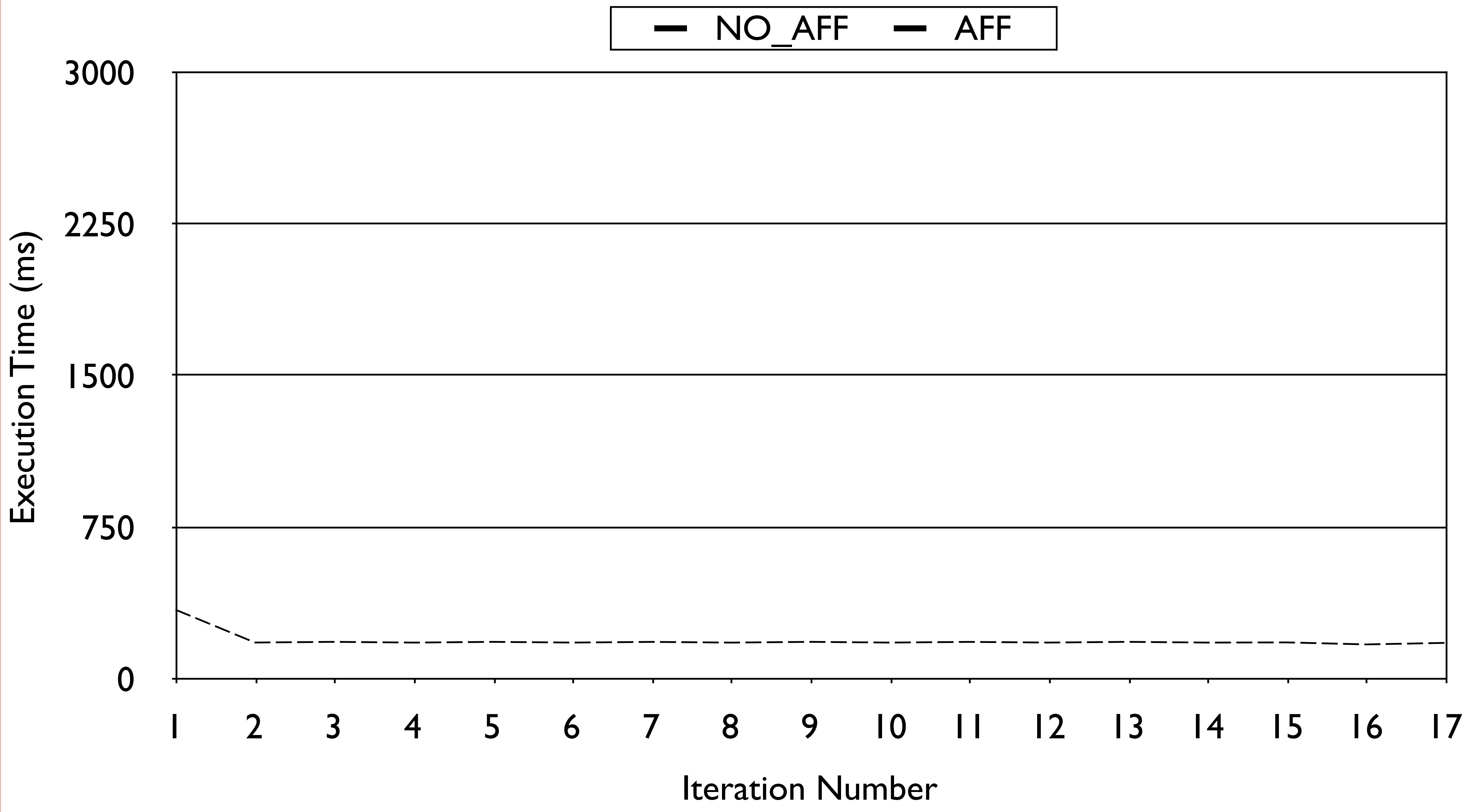
Effect of affinity-on-next-touch



16 threads

Effect of affinity-on-next-touch



16 threads

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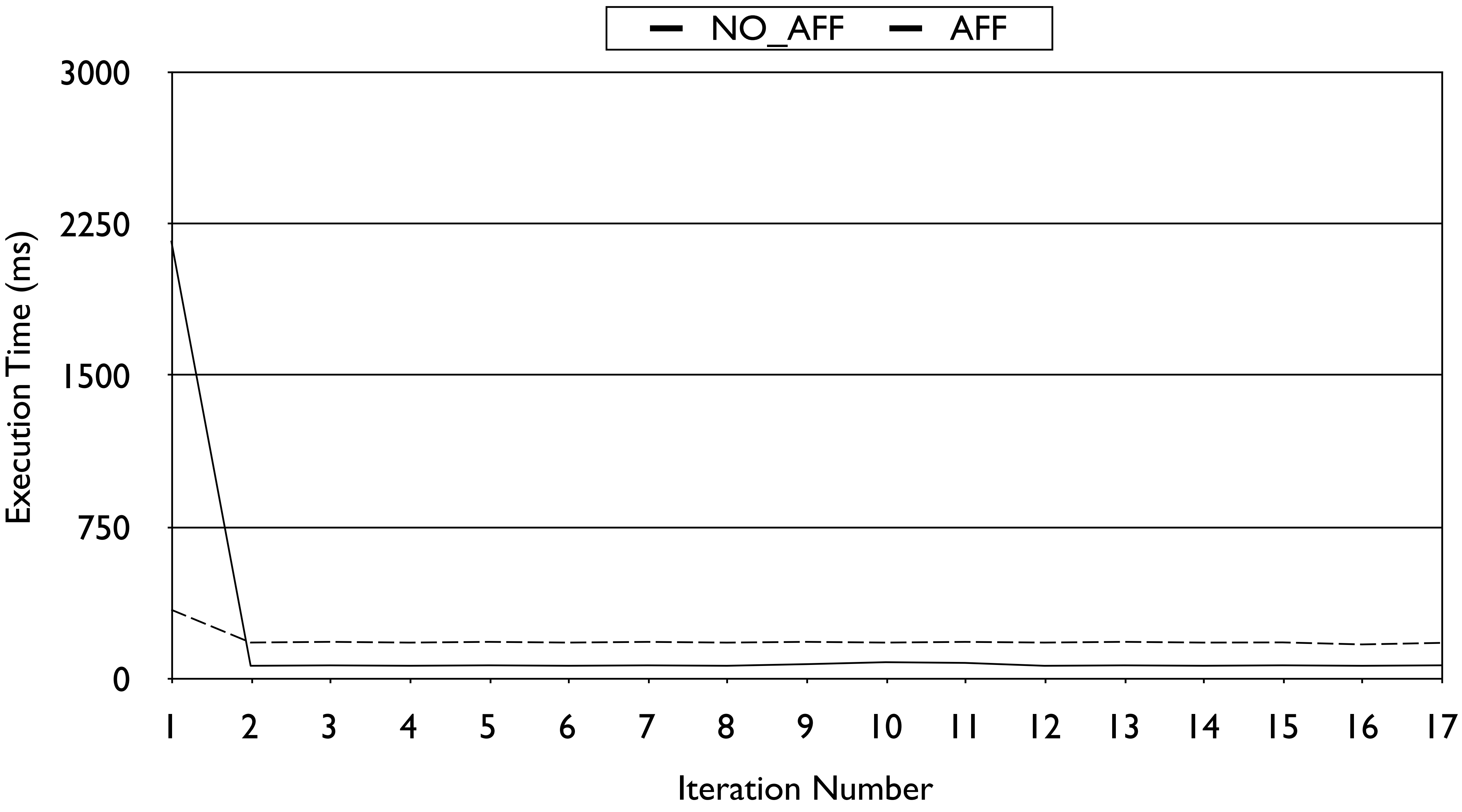
Effect of affinity-on-next-touch



16 threads

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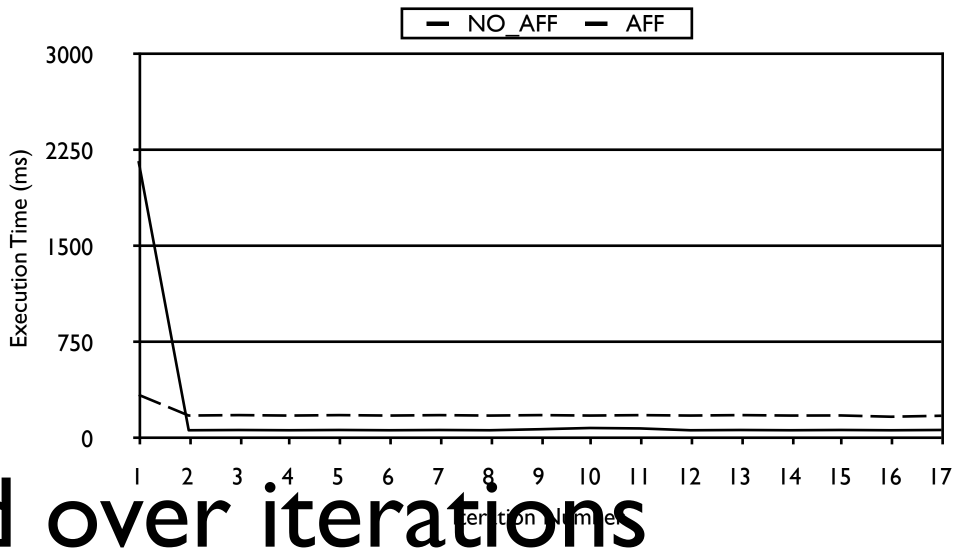
Analysis, affinity-on-next-touch

- Proper data distribution important
  - Every iteration 2.61 times faster
- Overhead
  - Cold start effects
  - Cost of migration



- Overhead is amortized over iterations
- Scalability better, still poor
  - Speedup 9 on 28 threads

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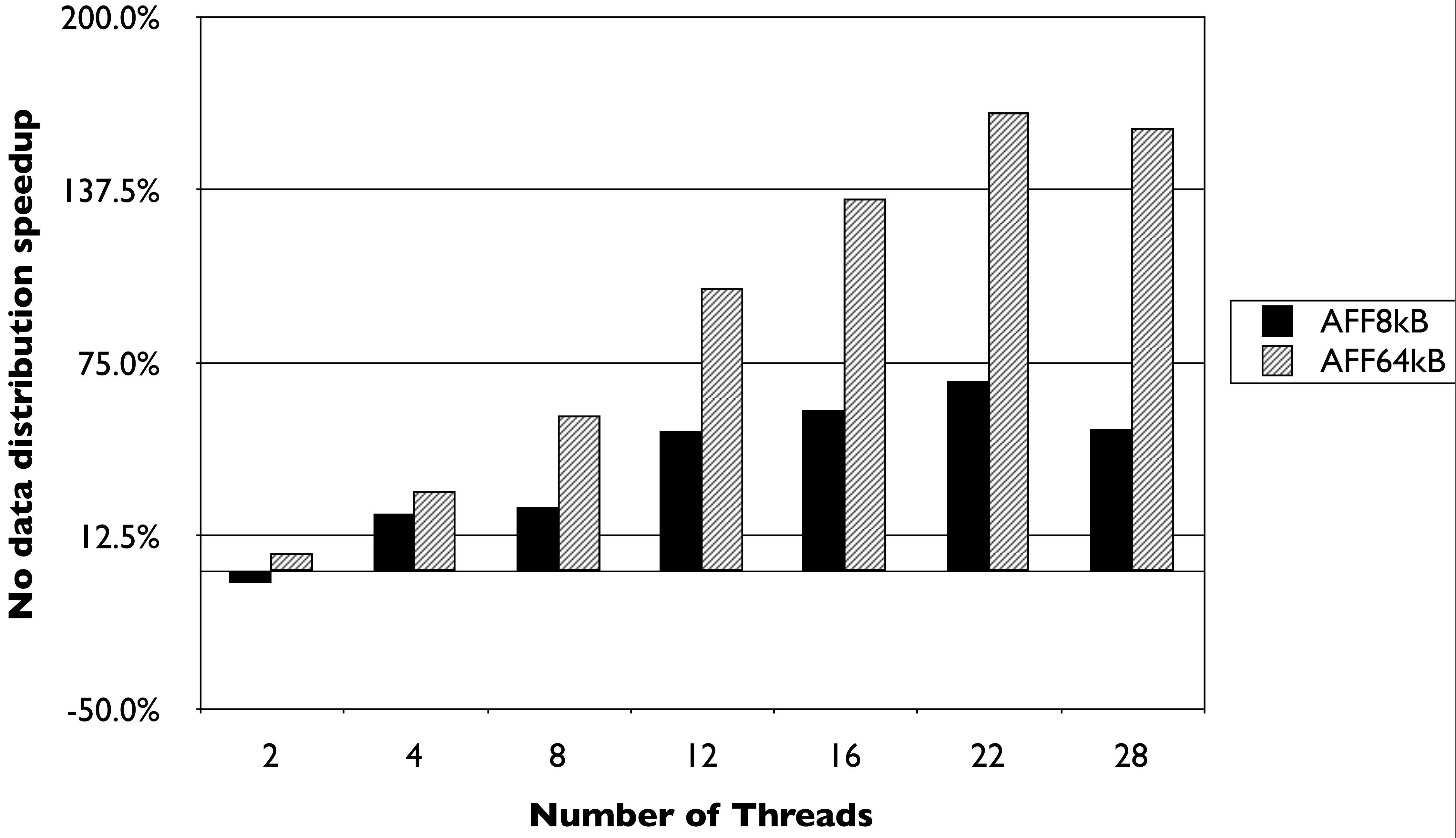
Overall effect of affinity-on-next-touch



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Two generations, again

$$\begin{tikzcd}
\text{Serial} & 220\text{s} \arrow[r, "5\text{x}"] \arrow[d, "28\text{x}"'] & 45\text{s} \arrow[d, "5\text{x}"] & \text{Serial} \\
\text{Parallel} & 8\text{s} \arrow[r, "0.8\text{x}"] & 9.5\text{s} & \text{Parallel}
\end{tikzcd}$$

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Two generations, again

$$\begin{tikzcd}
\text{Serial} & 220\text{s} \arrow[r, "5\text{x}"] \arrow[d, "28\text{x}"'] & 45\text{s} \arrow[d, "5\text{x}"] & \text{Serial} \\
\text{Parallel} & 8\text{s} \arrow[r, "0.8\text{x}"] & 9.5\text{s} & \text{Parallel}
\end{tikzcd}$$

affinity-on-next-touch + 64Kb

Two generations, again

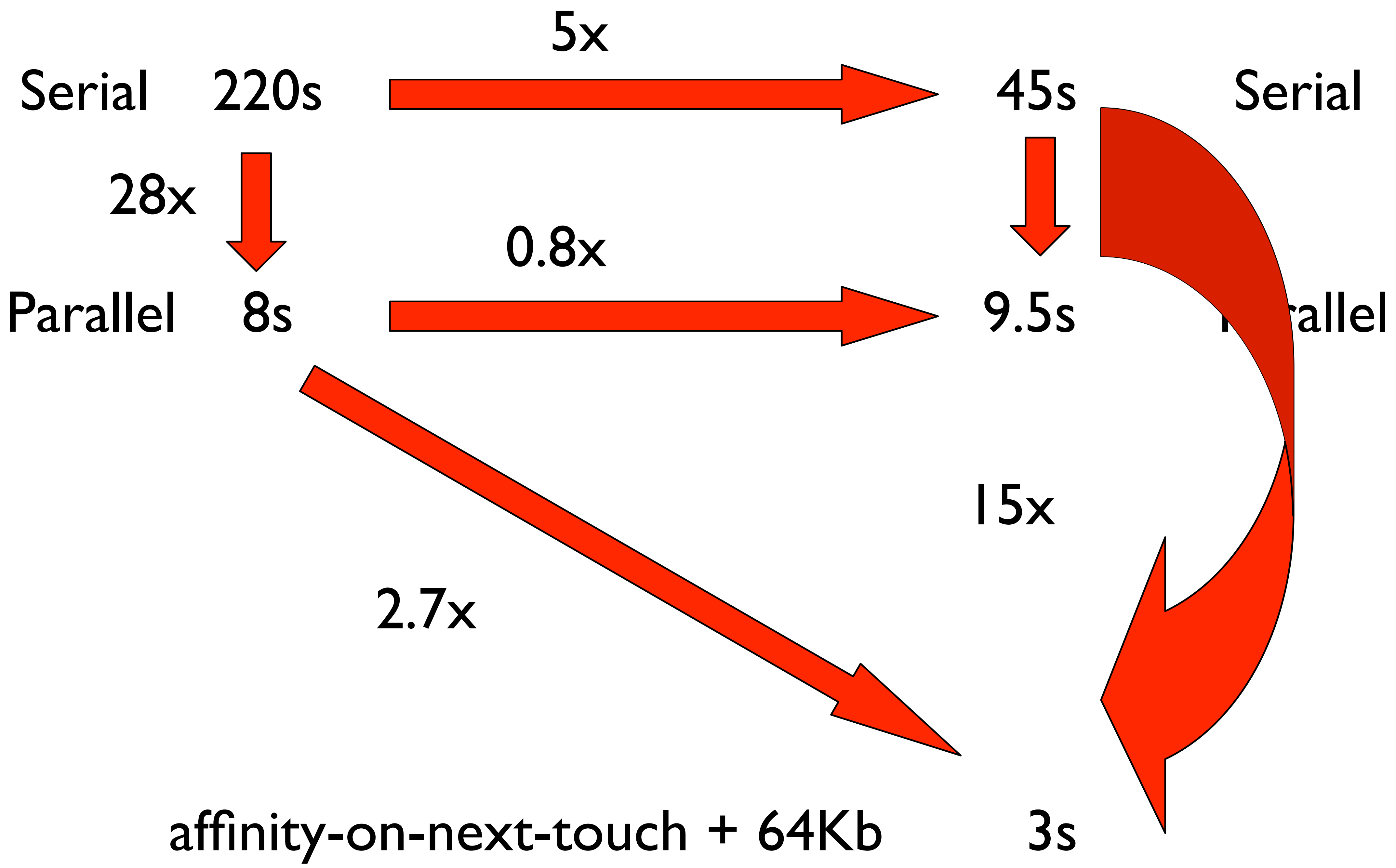
$$\begin{tikzcd}
\text{Serial} & 220\text{s} \arrow[rr, "5\text{x}"] \arrow[d, "28\text{x}"'] & & 45\text{s} \arrow[d, "5\text{x}"] & \text{Serial} \\
\text{Parallel} & 8\text{s} \arrow[rr, "0.8\text{x}"] & & 9.5\text{s} & \text{Parallel}
\end{tikzcd}$$

affinity-on-next-touch + 64Kb 3s

Two generations, again

$$\begin{tikzcd}
\text{Serial} & \text{220s} \arrow[r, "\text{5x}"] \arrow[d, "\text{28x}"'] & \text{45s} \arrow[d] \arrow[dd, bend left, "\text{15x}"'] & \text{Serial} \\
\text{Parallel} & \text{8s} \arrow[r, "\text{0.8x}"] & \text{9.5s} & \text{Parallel} \\
\text{affinity-on-next-touch + 64Kb} & & \text{3s} &
\end{tikzcd}$$

Two generations, again



Algorithmic Optimizations (CG)

- Bandwidth minimization
  - Increases cache utilization
- Load Balance
  - Graph partitioning (MeTiS)
- Removing barriers
  - Reduces parallel overhead
- How do they interact with data distributions?

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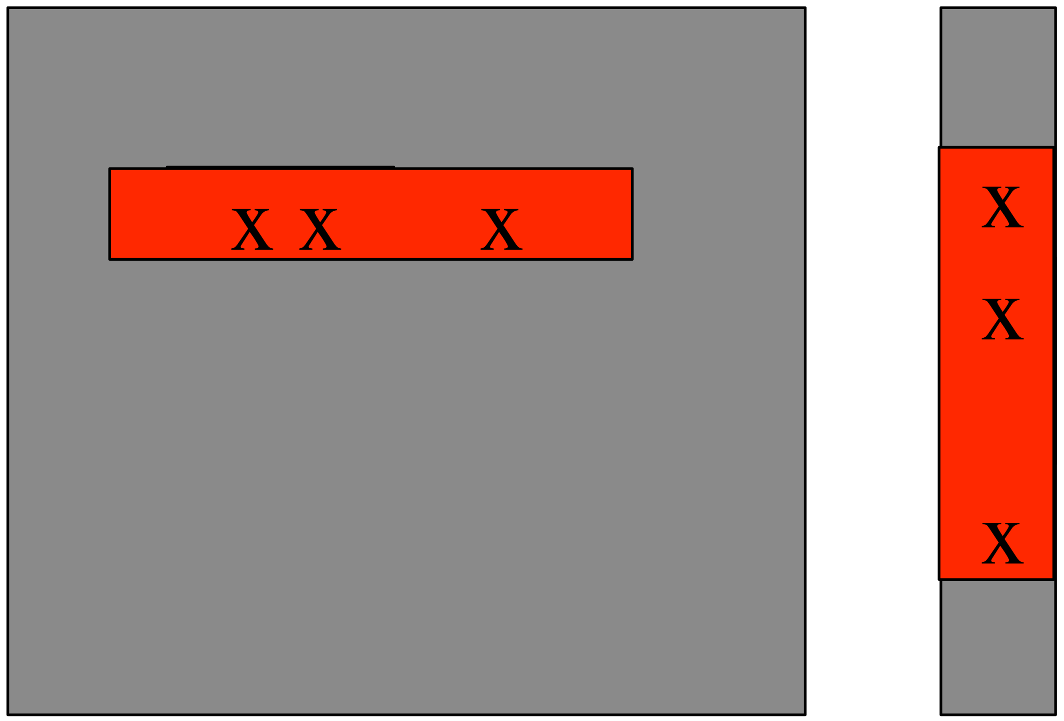
Removing Barriers

- Standard implementation uses 7 barriers
- 4 barriers can be removed
  - Privatizing scalars
  - Reording initializations of reduction variables
  - 3 global barriers in total
- S-step methods (Chronopoulos/Gear)
  - Introduces another vector to calculate two iterations simultaneously
  - Only one reduction necessary
  - 2 global barriers in total
  - No numerical problems

Bandwidth Minimization

- Graph theoretical method
  - Reverse Cuthill-McKee
  - Gibbs-Poole-Stockmayer (GPS)
- Permutes matrix to minimize bandwidth
- Small bandwidth increases locality
  - Larger part of RHS cache blocks utilized



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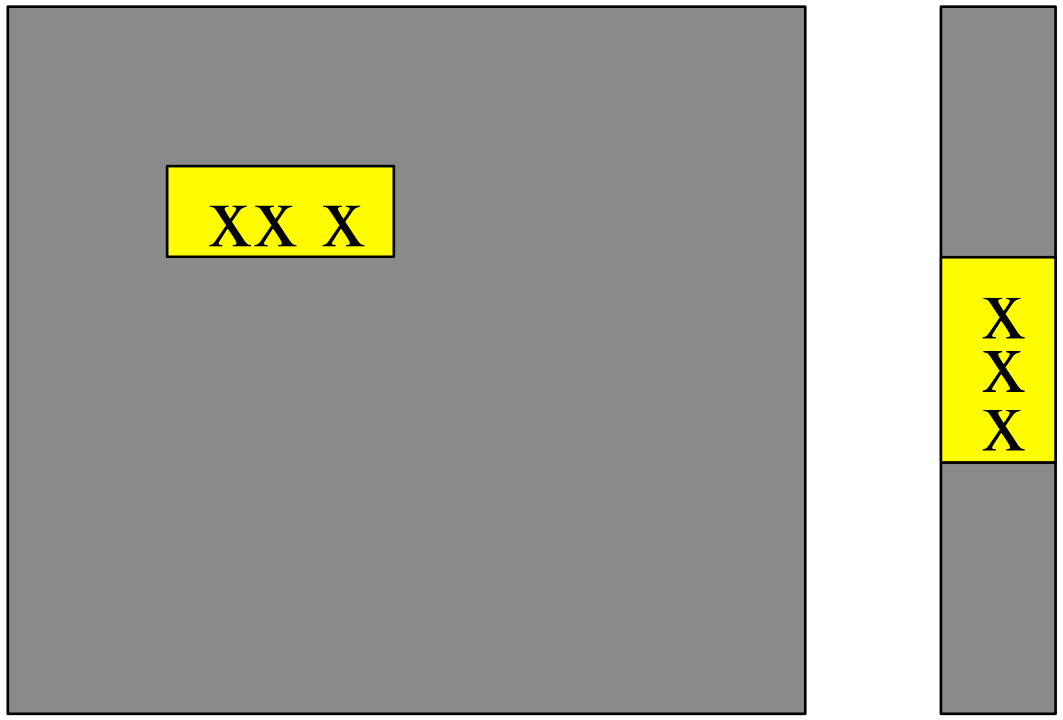

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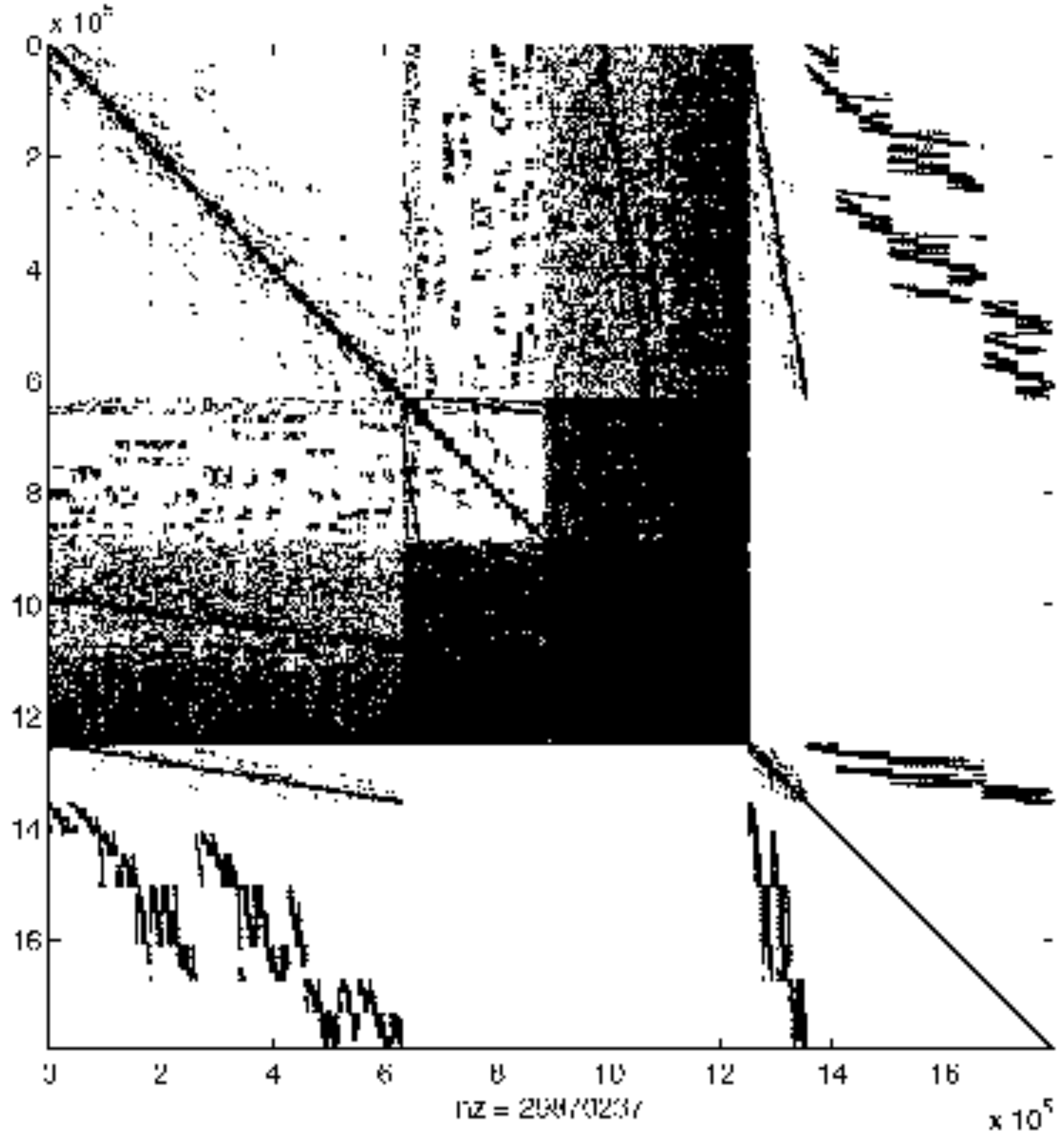
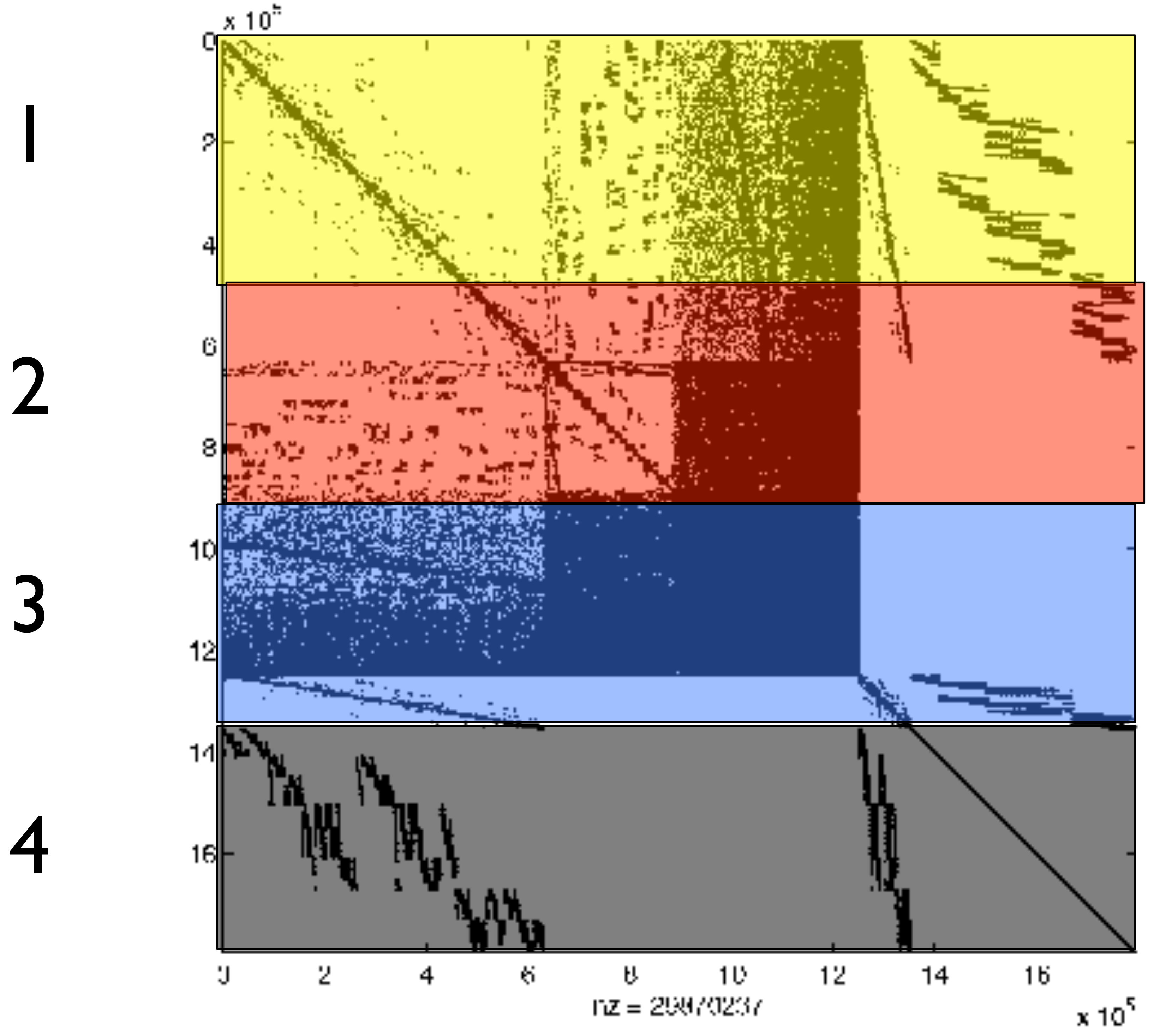
Load Balance, example



Load Balance, example



Load Balance and Partitioning

- In OpenMP partitions are linear
  - A chunk of rows (indeces) is associated with a thread
  - No care is taken to the number of non-zeros
- Graph partitioning partitions the non-zeros more evenly
  - Used in message-passing applications
- Results:
  - Graph partitioning increased matrix bandwidth which led to poor locality
  - Bandwidth minimization toghether with standard OpenMP produced very good partitions

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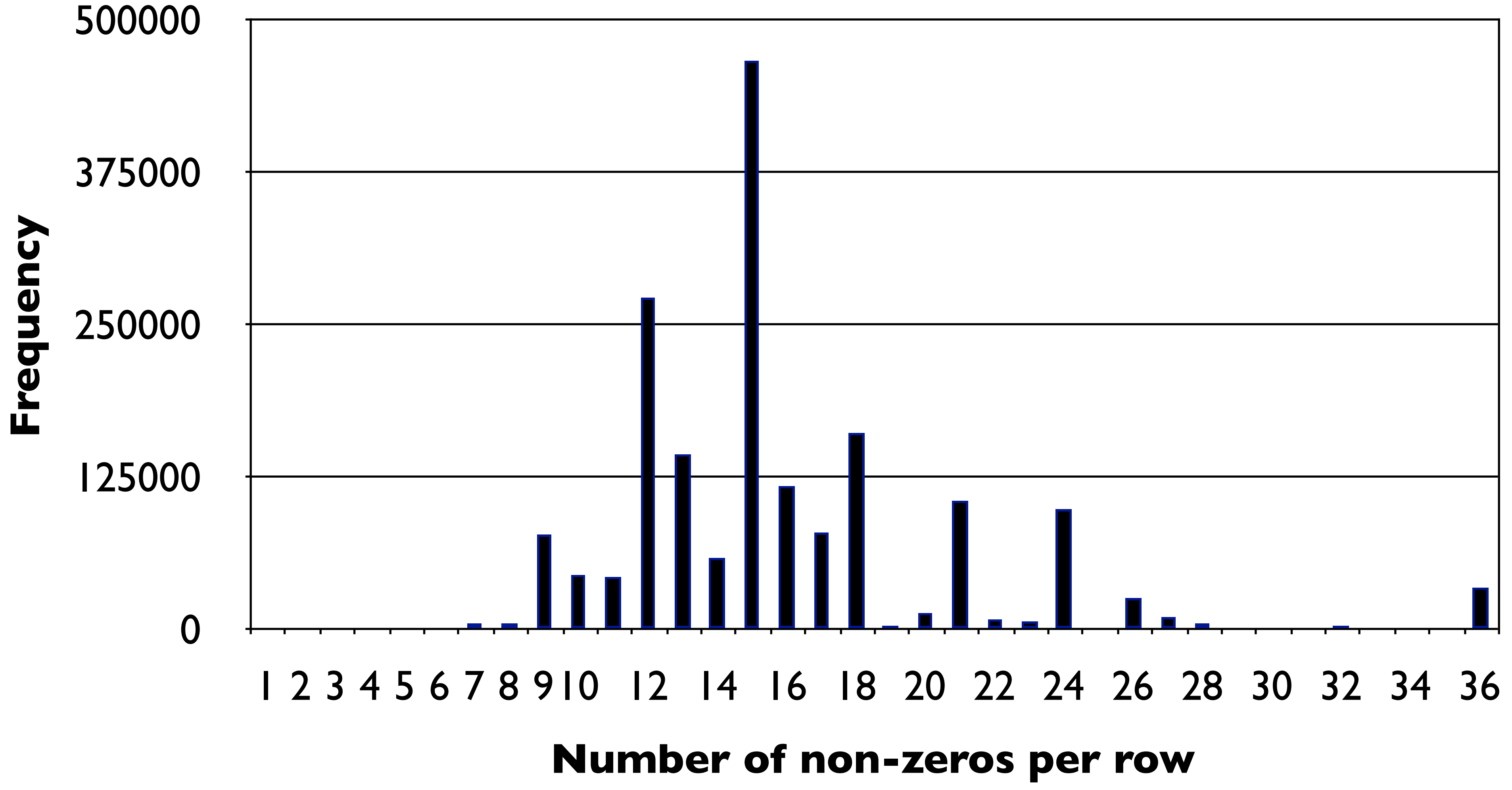
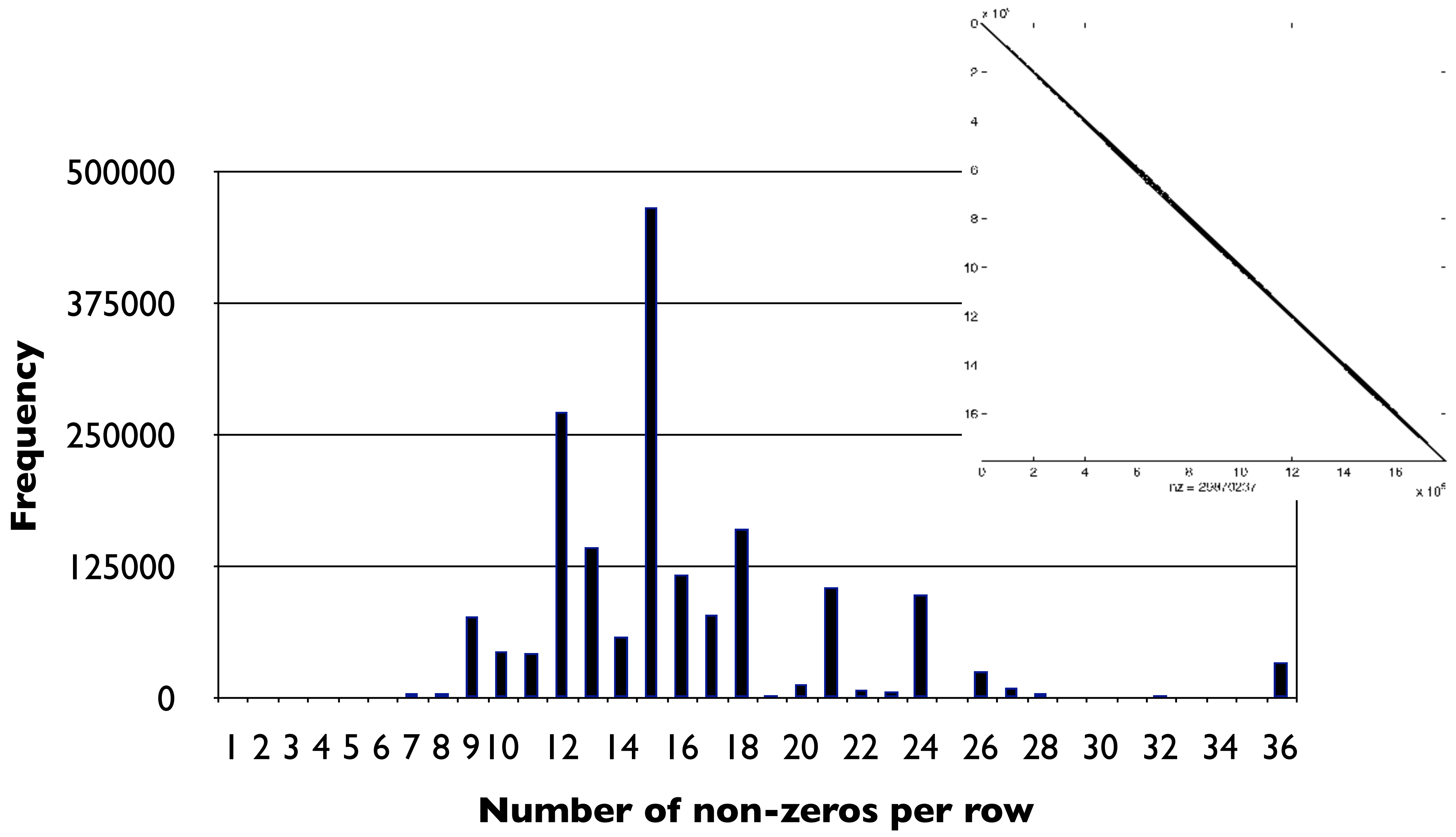
Load balance of SpMxV



16 partitions	Base	GPS	MeTiS
Max(non_zeros)/Avg(non_zeros)	1.24	1.01	1.15

Load balance of SpMxV



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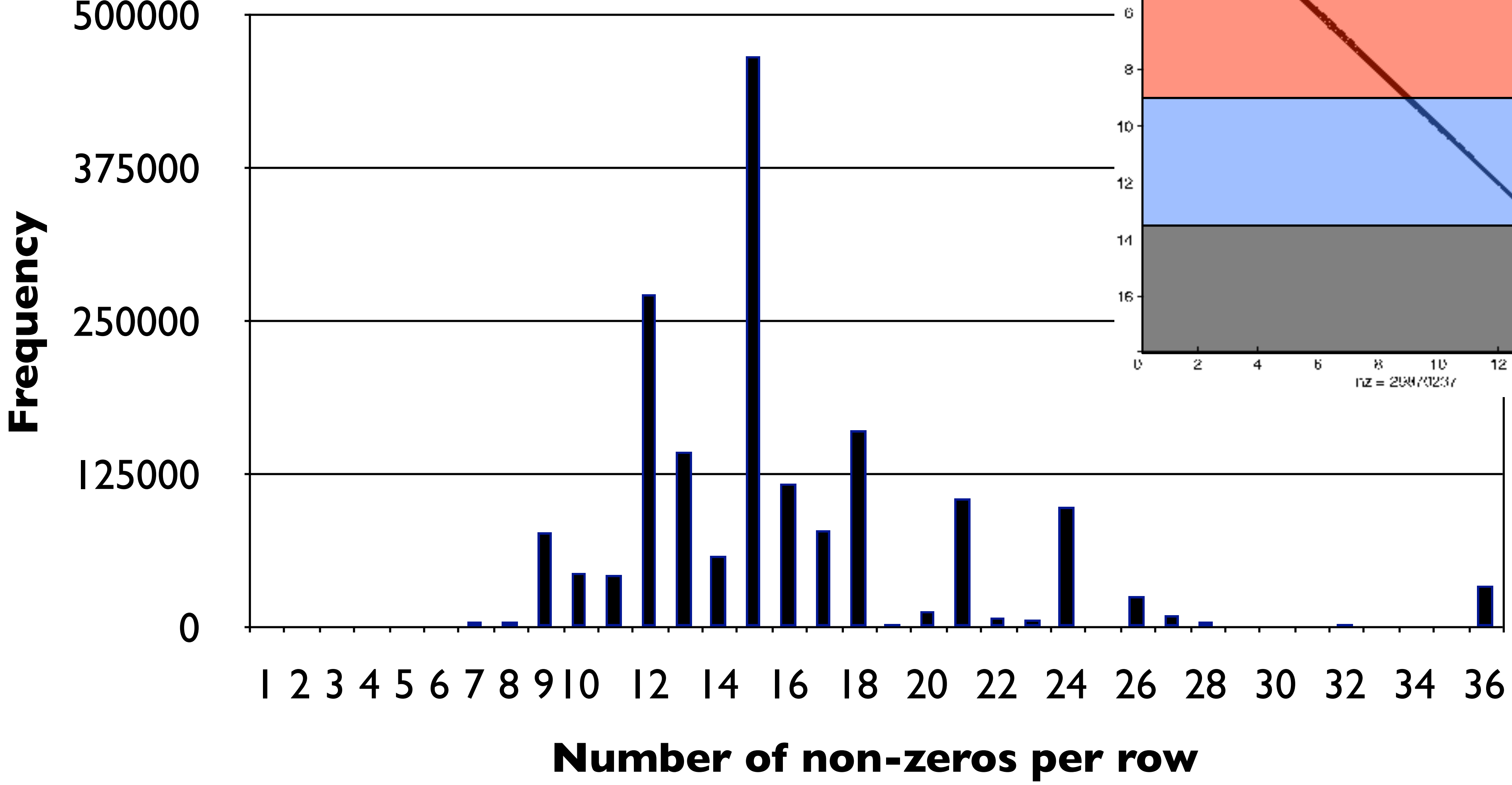

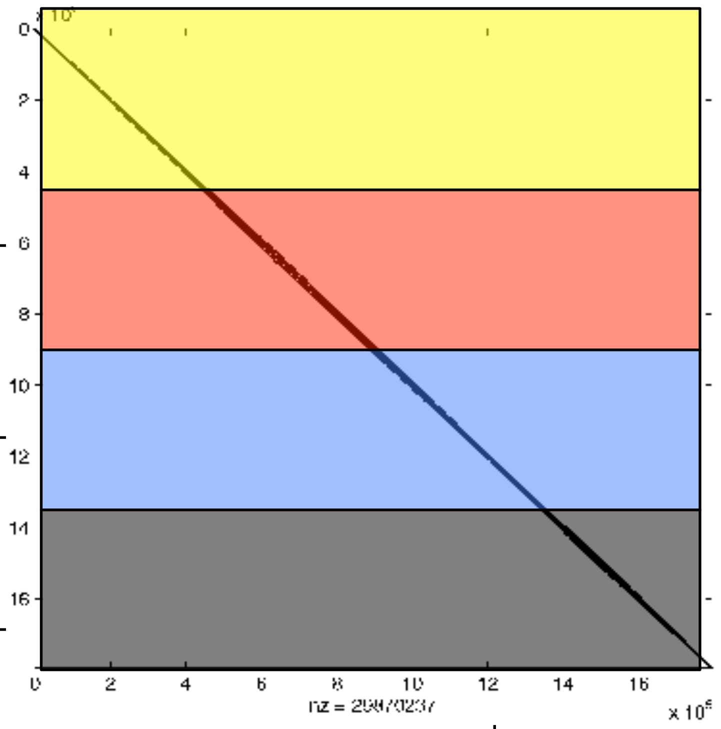

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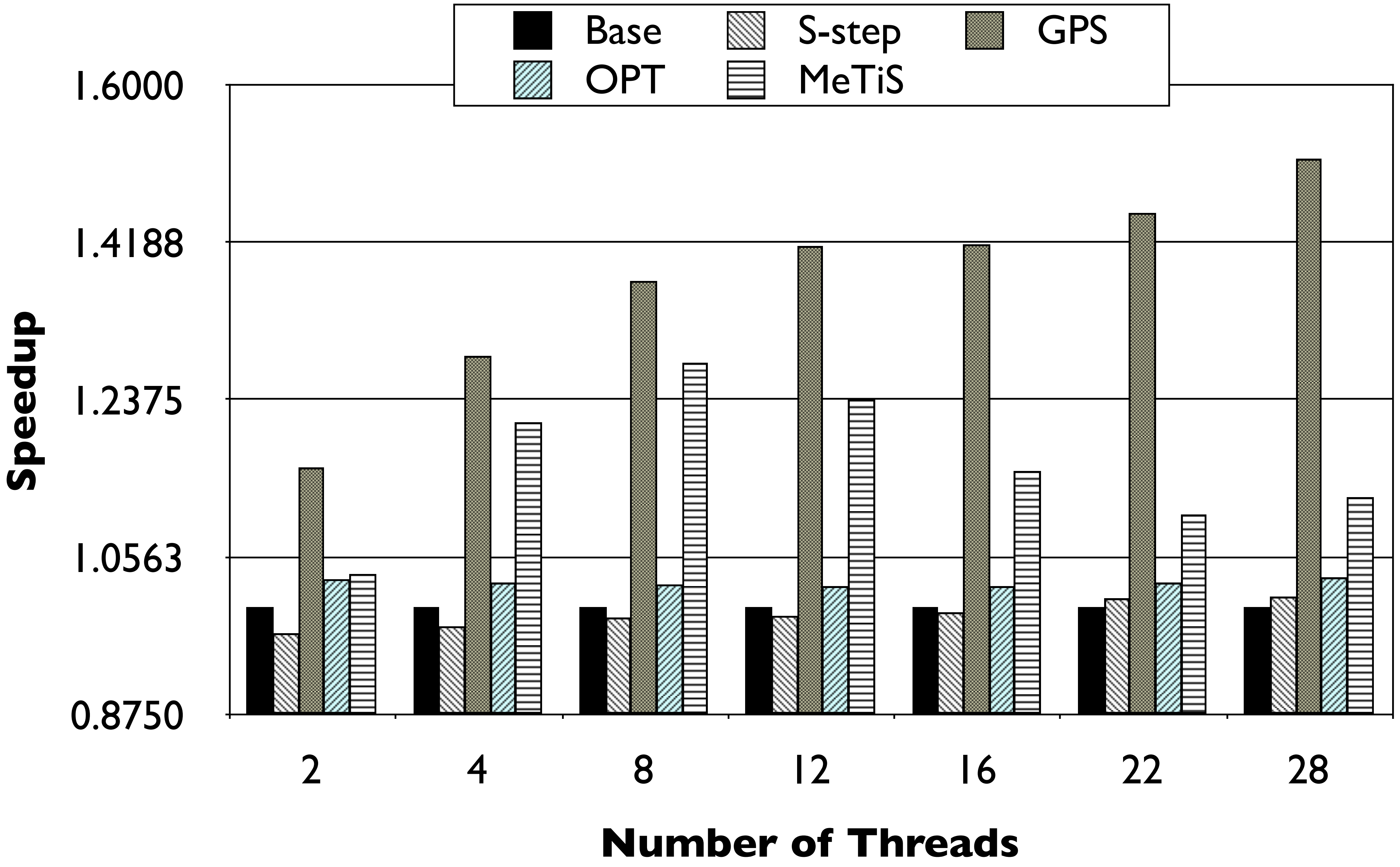
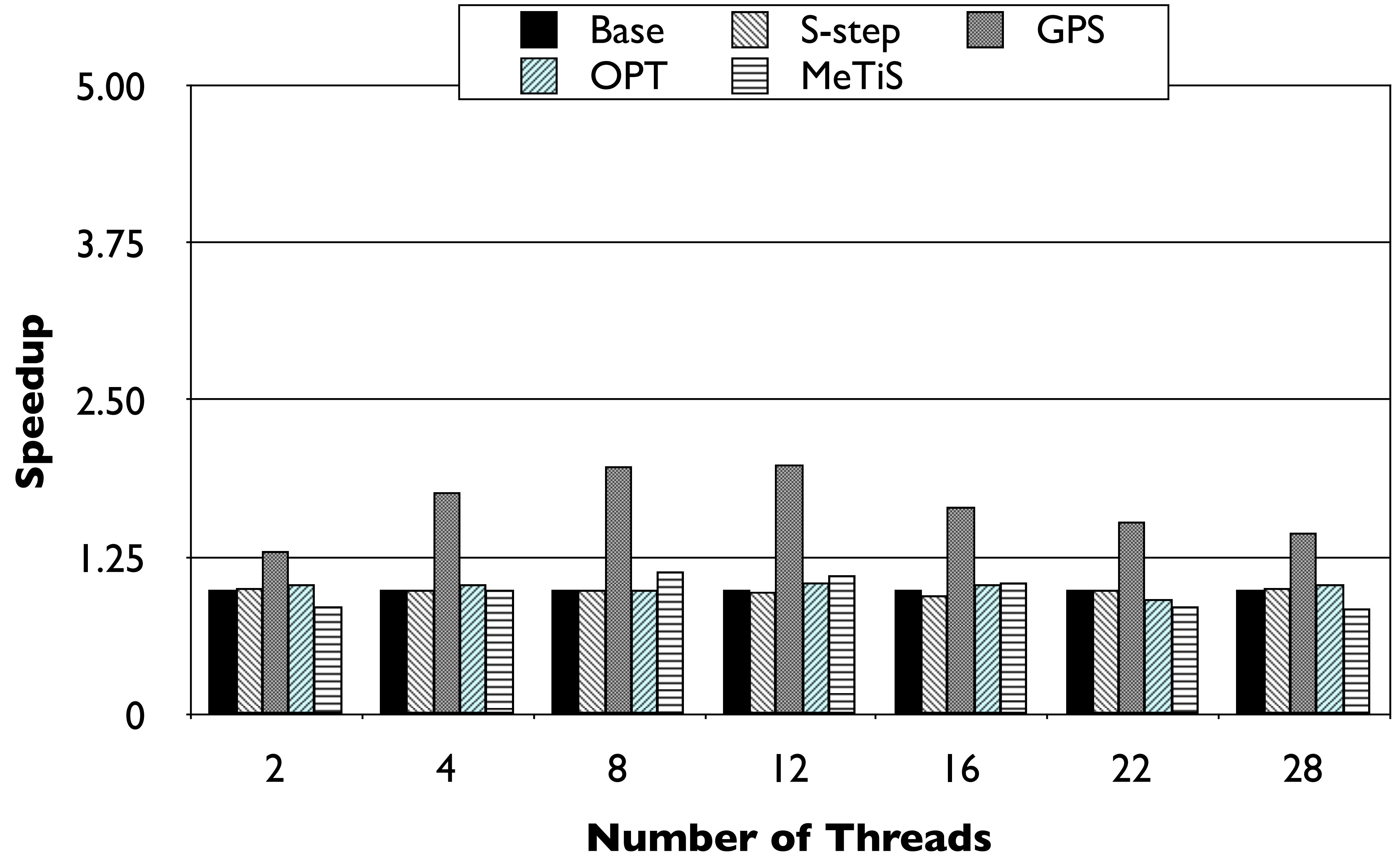
Uniform system, E10K



Non-uniform system (SF15K) with data distribution



Conclusions, algorithmical optimizations

- Locality most important
  - Bandwidth minimization
- Bandwidth minimization also produced load balanced partitions
- Graph partitioning increased the amount of remote accesses
- Load balance and synchronization overheads are of secondary importance

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Dynamic access patterns

- Many applications exhibit dynamic and/or unstructured access patterns
  - Commercial Software
  - Adaptive Mesh Refinement (AMR)
- For these applications, clever initializations will not be optimal
  - We need some kind of migration/replication mechanism

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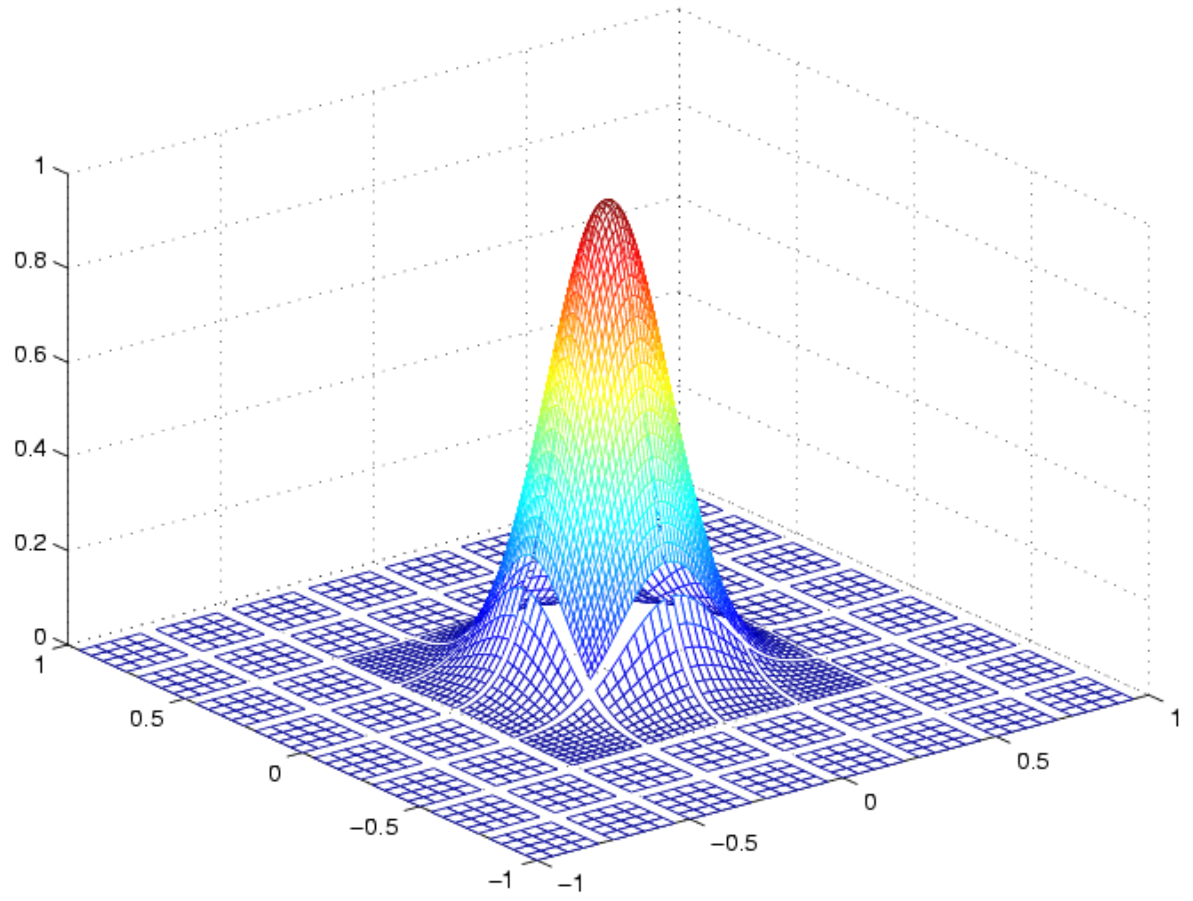
Model problem

- $$\frac{\partial u}{\partial t} = a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y}$$
- Periodic boundaries
- Finite differences
  - – 2nd order in space
  - – 4th order R-K in time
- Blockwise AMR
- Written in Fortran 90/95
- Parallelized using OpenMP



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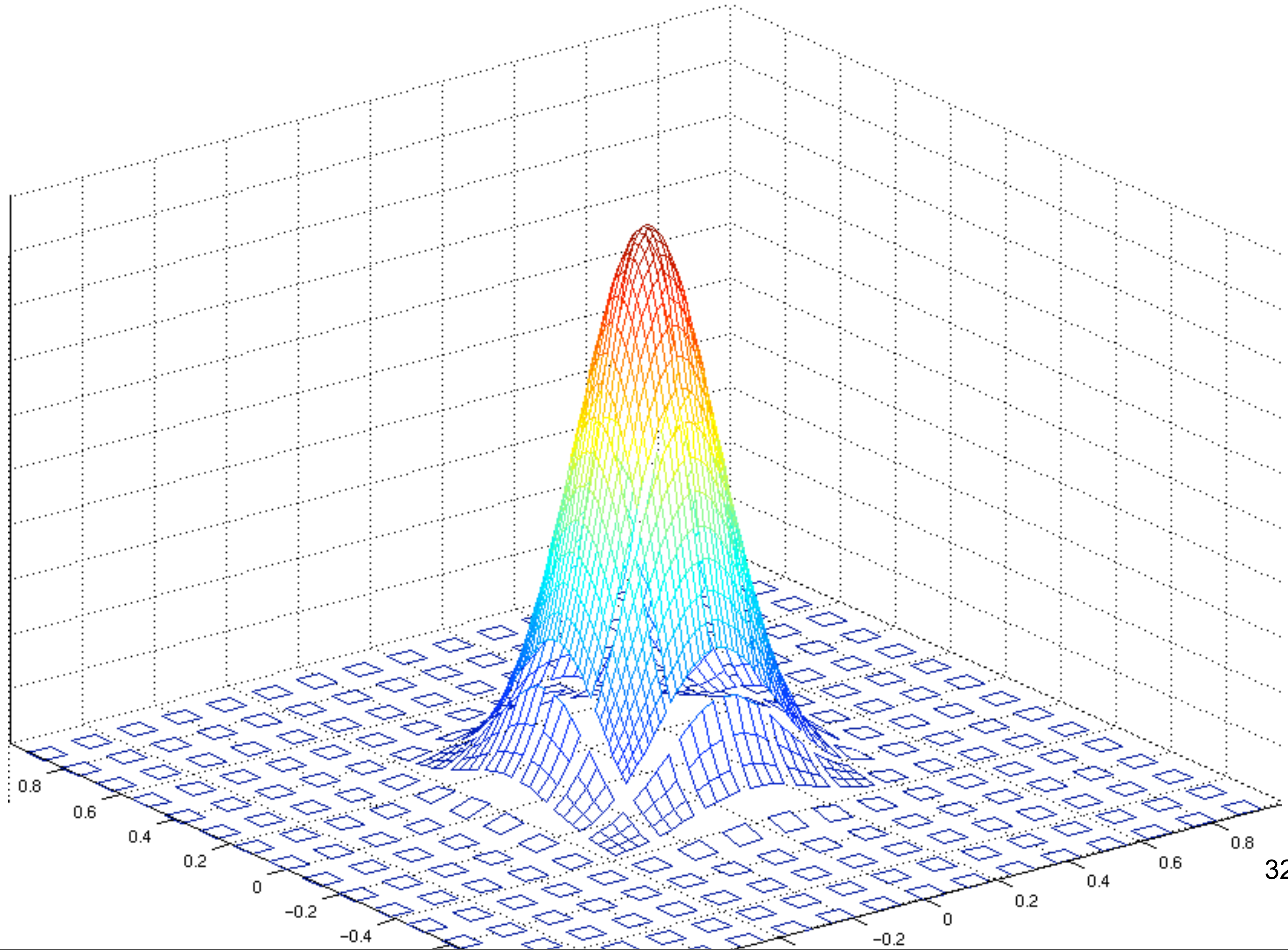
AMR movie



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Solver Algorithm

```
do t=1,Nt
   if (t mod adaptInterval=0) then
       Estimate error per block.
       Adapt blocks with inappropriate resolution.
       Repartition the grid.
       Migrate blocks (if migration is activated).
   end if
   F1=Diff(u);
   F2=Diff(u+k/2*F1)
   F3=Diff(u+k/2*F2)
   F4=Diff(u+k*F3)
   u=u+k/6*F1+k/3*F2+k/3*F3+k/6*F4
end do
```

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Partitioning and locality

- The blocks needs to be assigned to threads in a way that balance the work (partitioning)
  - Domain based partitioning won't do the job since the pulse moves across the domain
  - We used the Jostle diffusion partitioner
- Our problem:
  - *We need to make sure that the data in each partition is physically allocated on the node where the corresponding thread executes*

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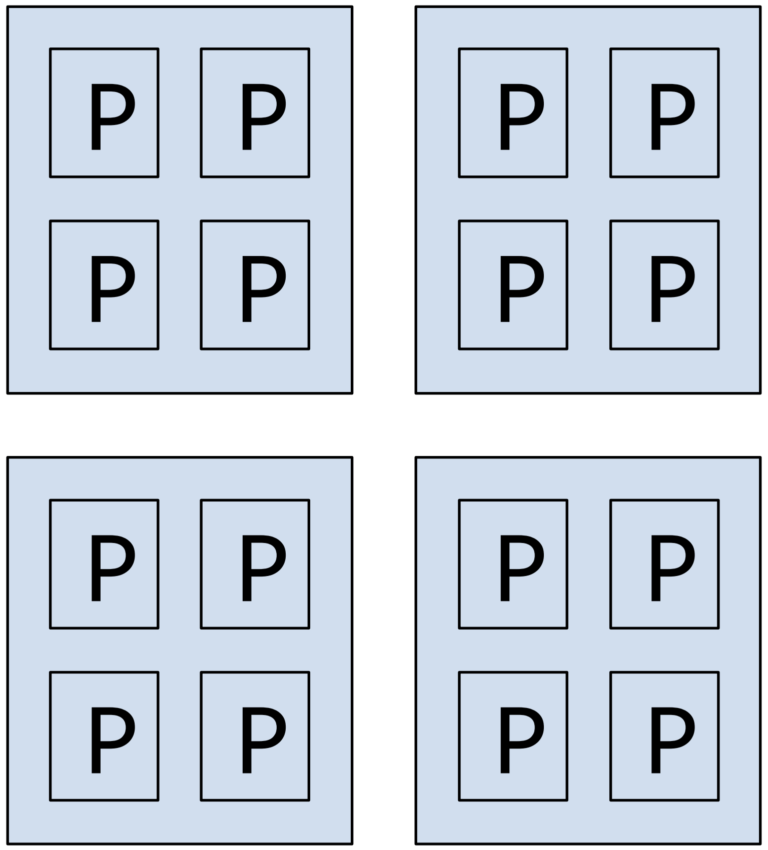
Experimental Setup

- Four nodes
  - Four CPUs per node
- Bound threads
  1. UMA
  2. NUMA
  3. NUMA-MIG
- The amount of remote accesses quantifed using CPU hardware counters



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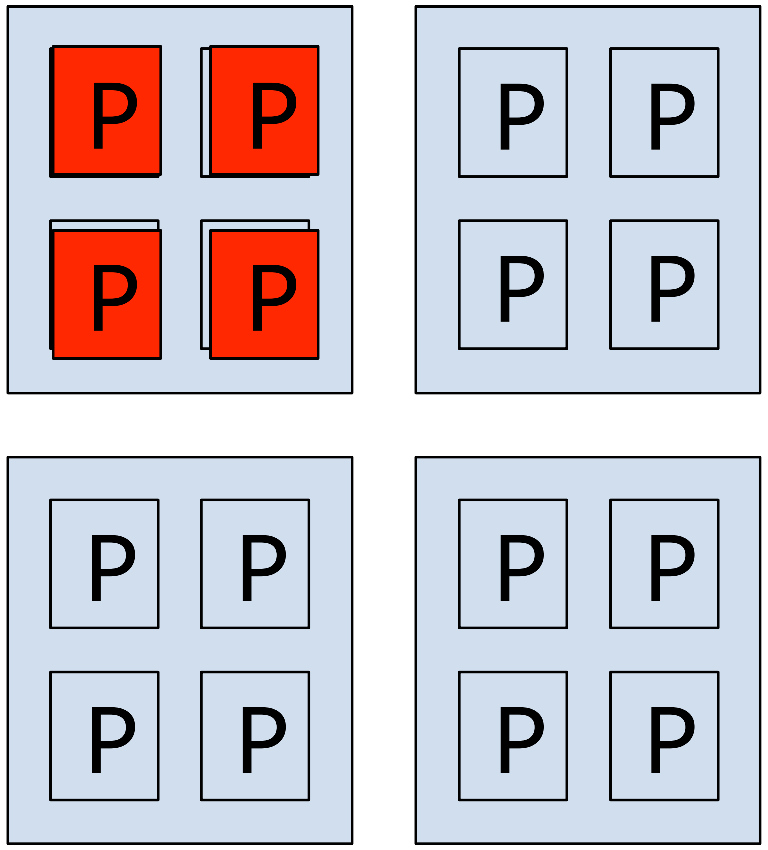

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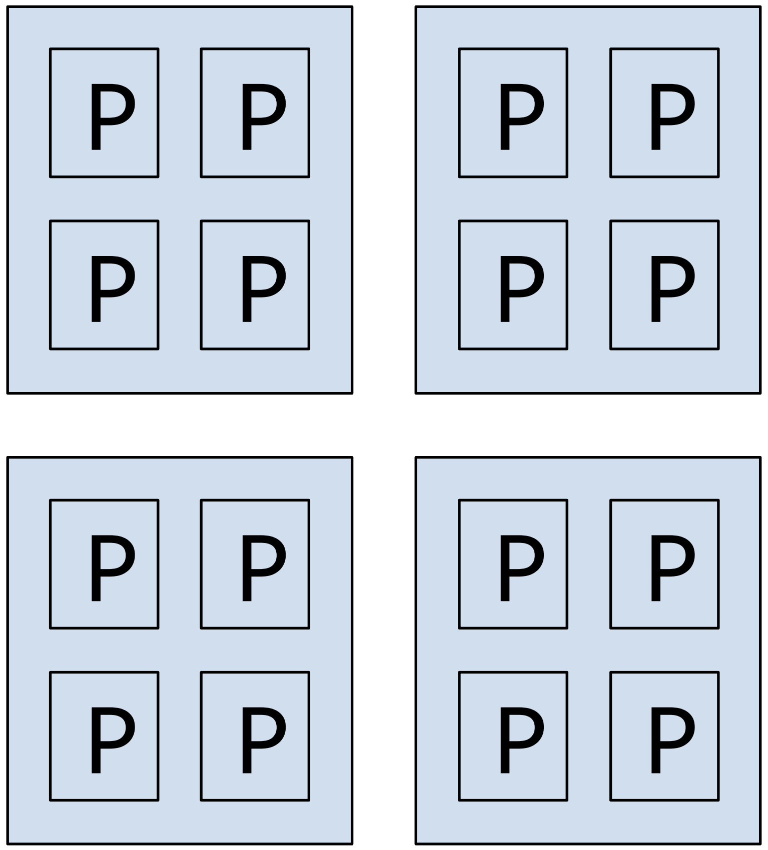

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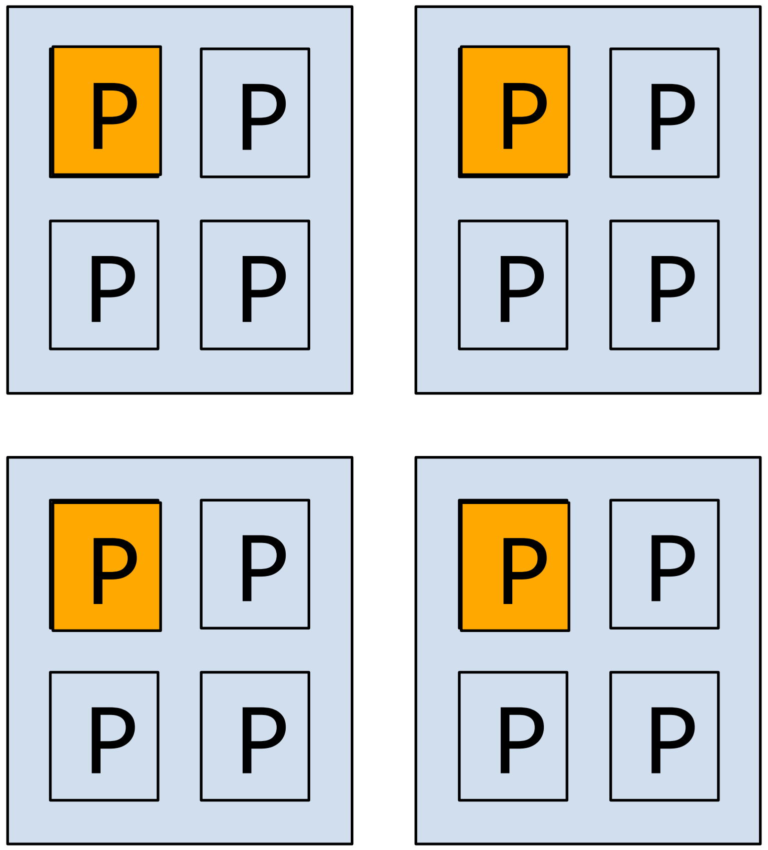

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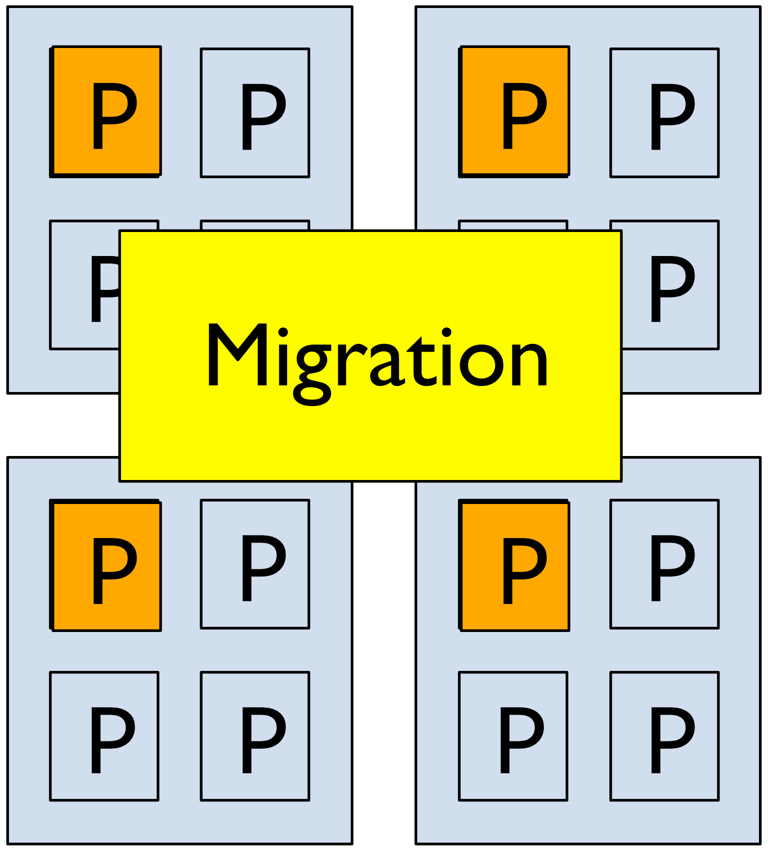

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Results

	UMA	NUMA	NUMA-MIG
Total time	4.09 h	6.64 h	3.99 h
L2 miss rate	4.3%	3.9%	4.2%
Remote accesses	0.2%	62.9%	8.1%

- Geographical locality has a significant effect!
- Dynamic page migration works!

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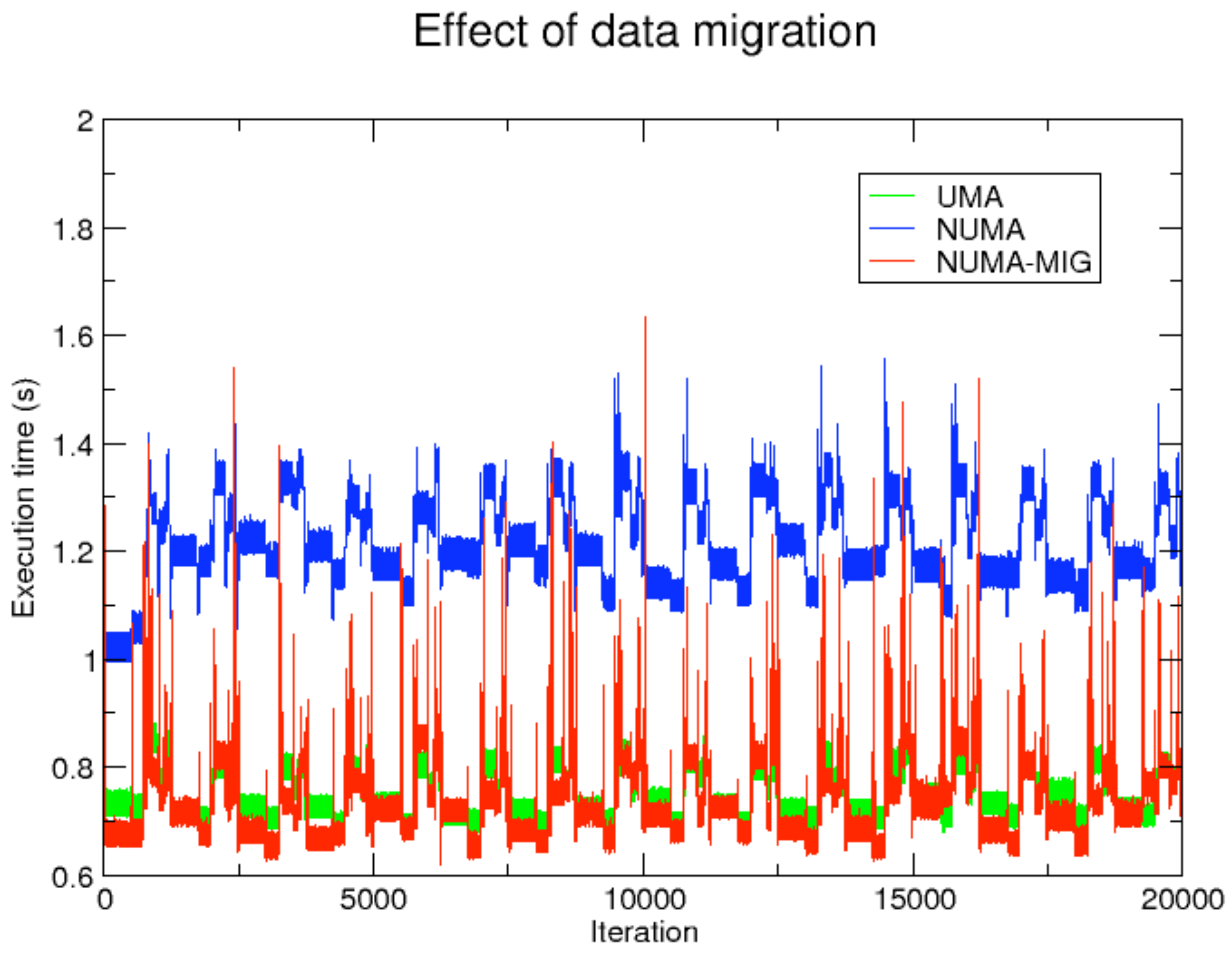
Execution Time



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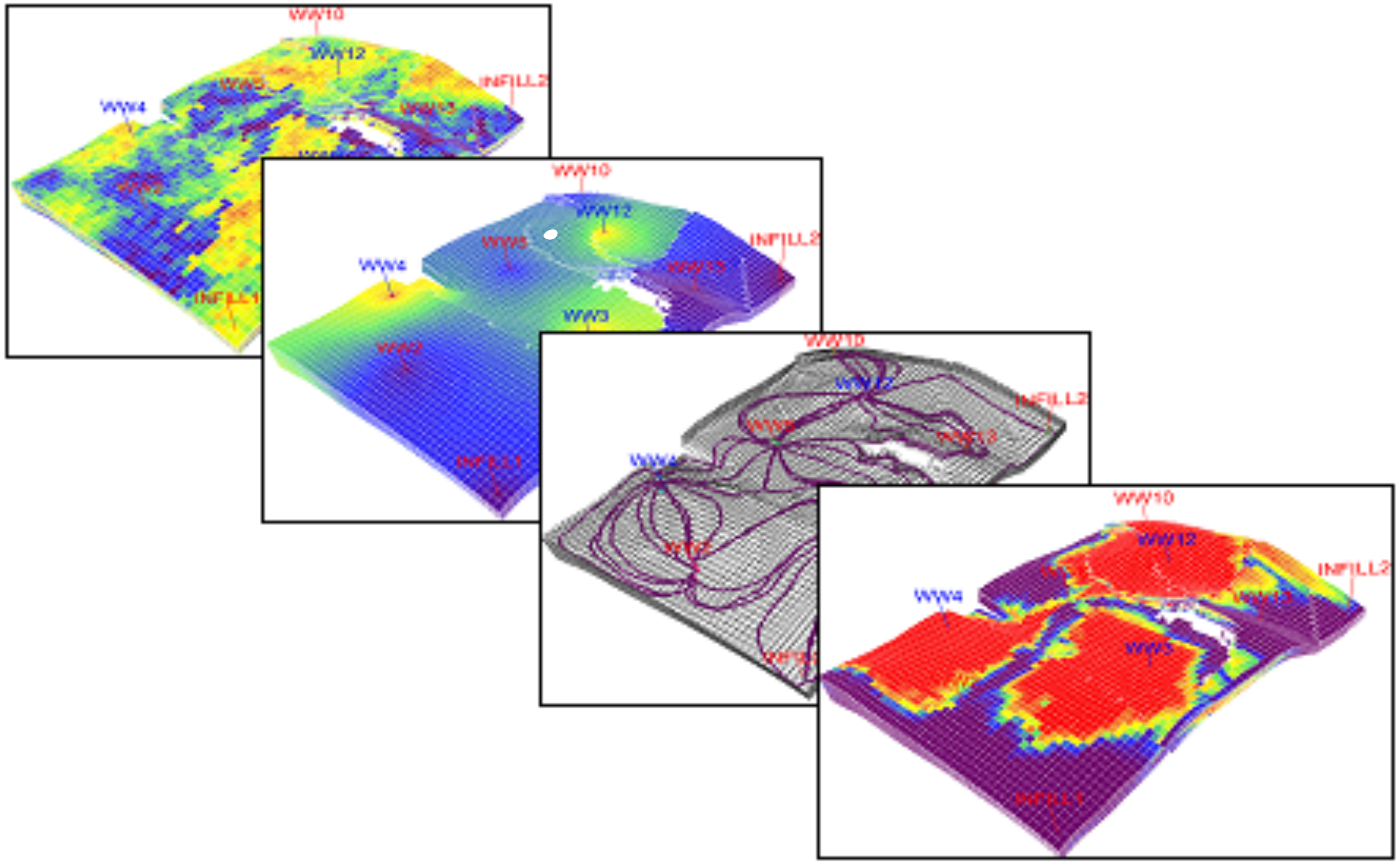
Streamline Simulation



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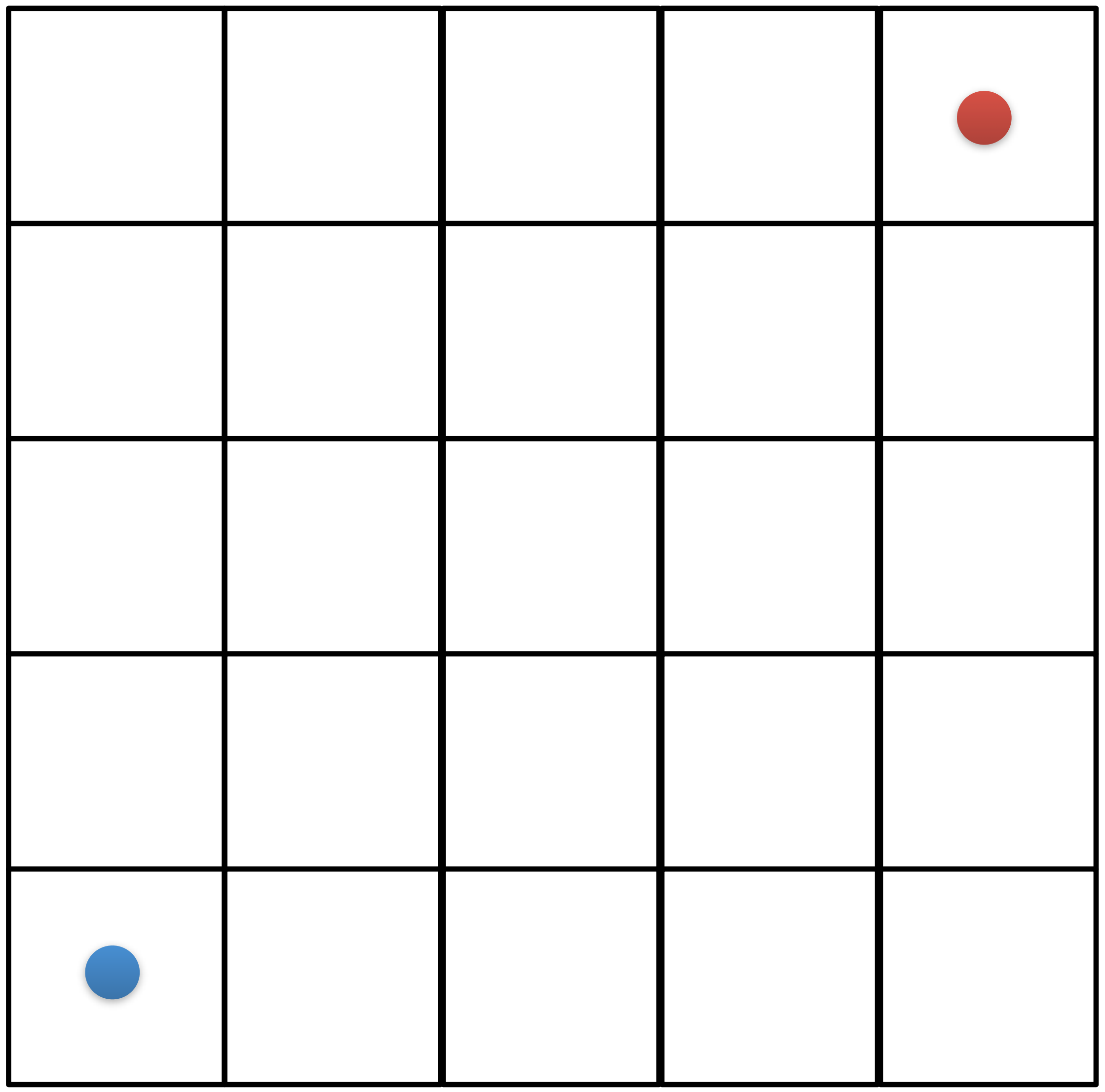
Streamline Tracing

Segments



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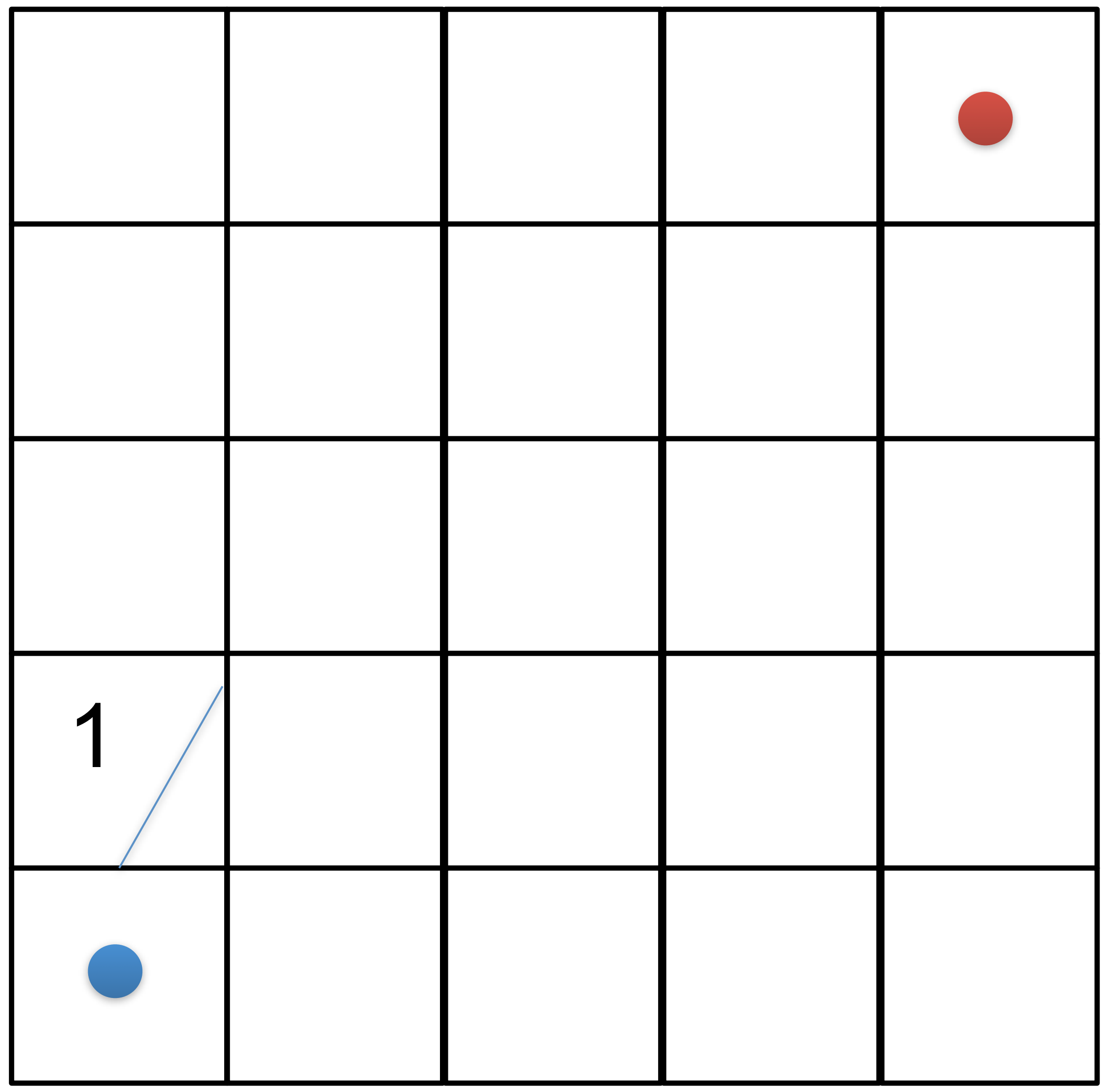

Segments

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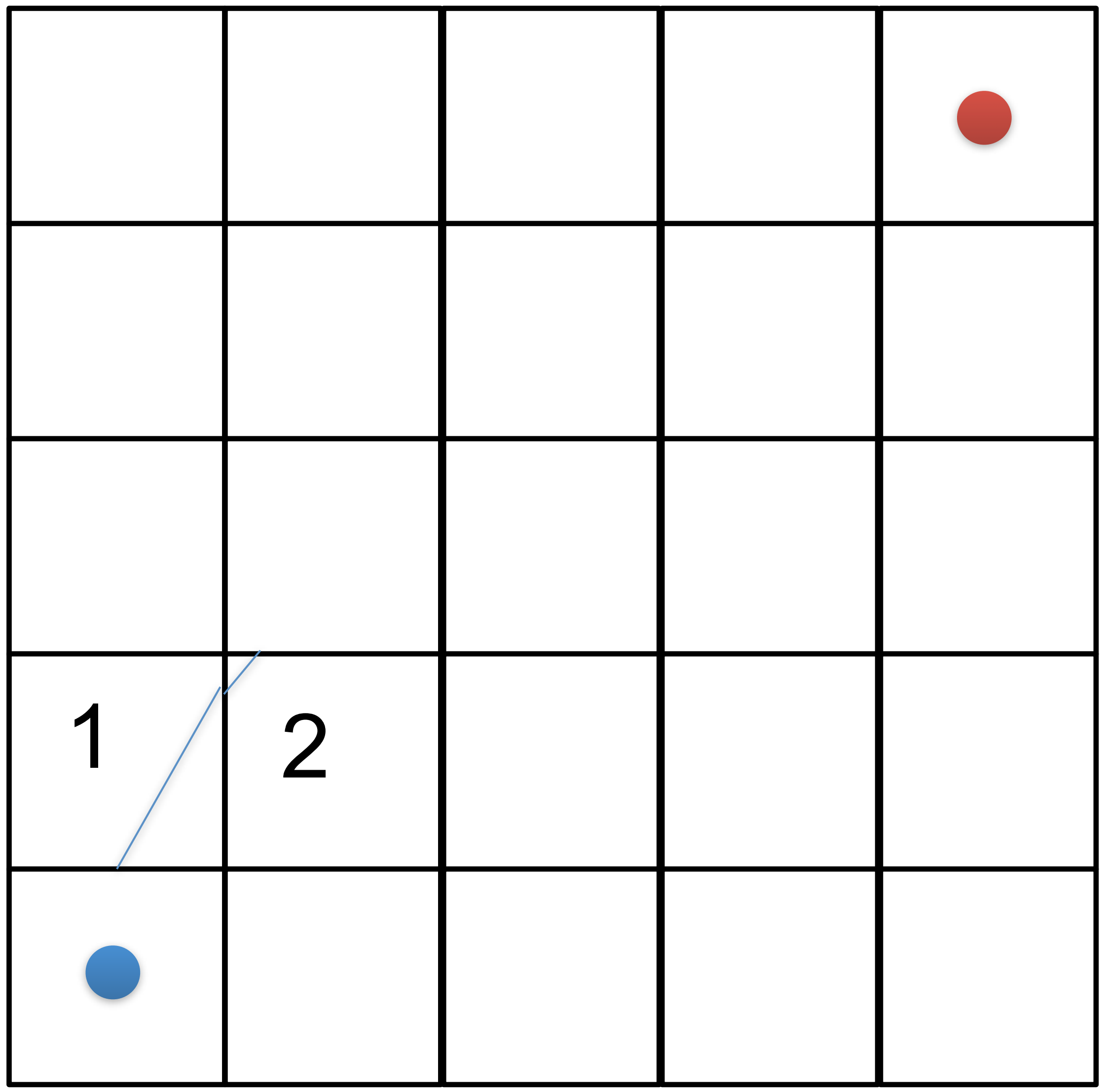
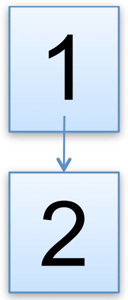


Segments



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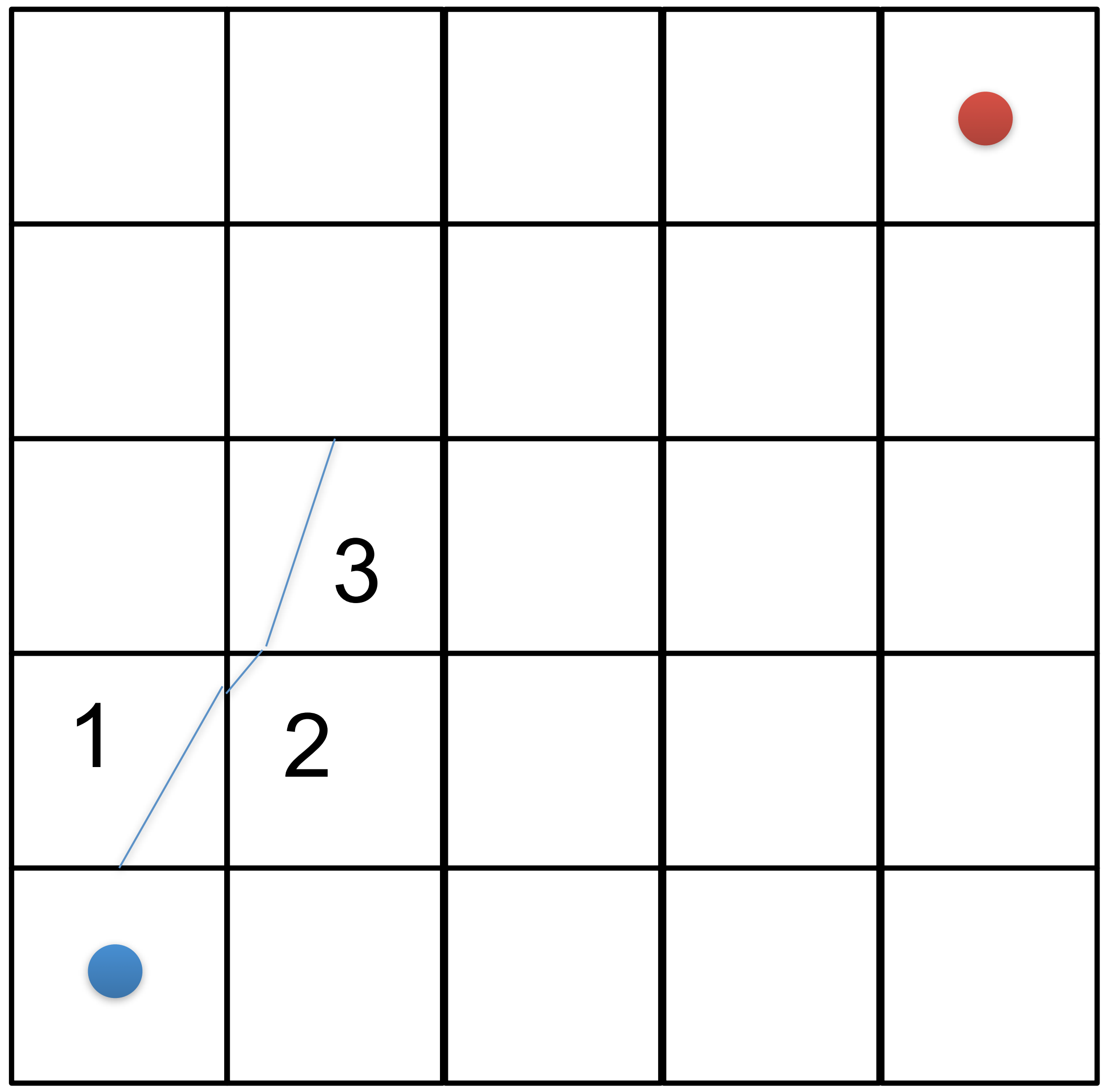

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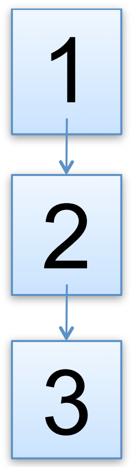

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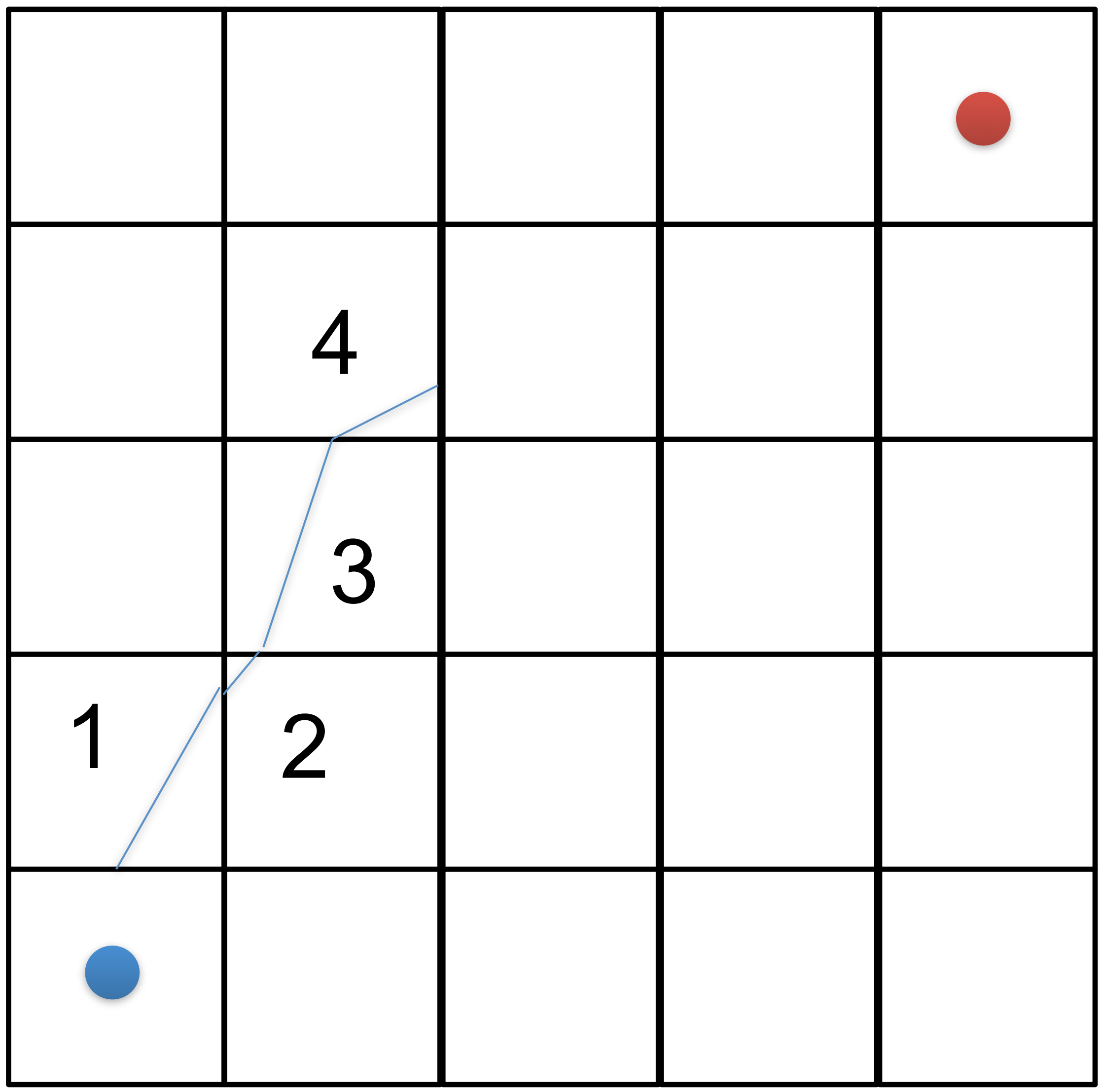

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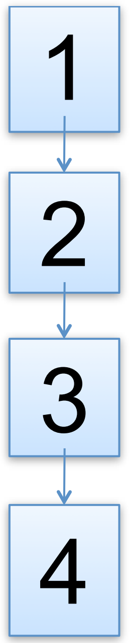

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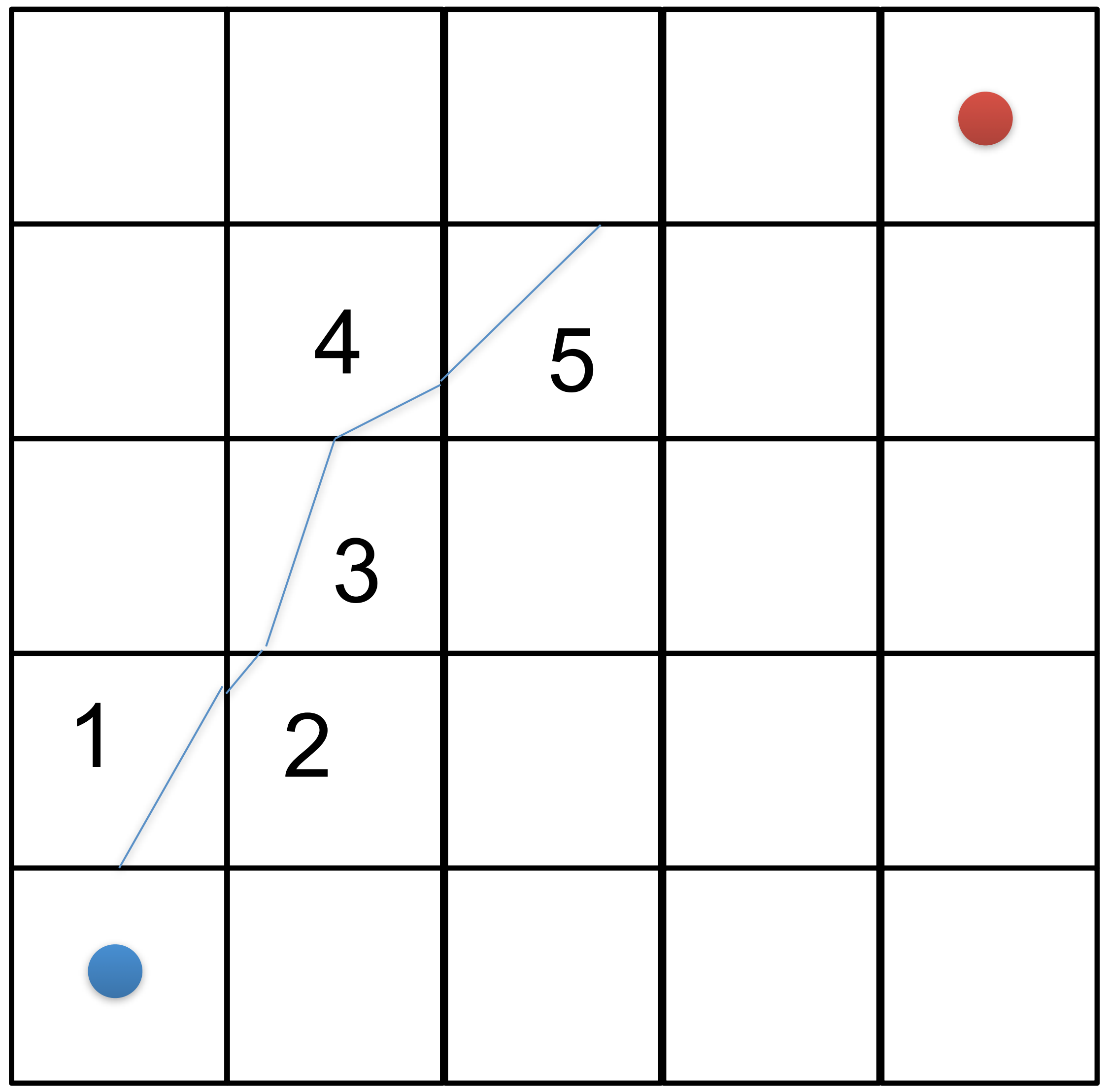
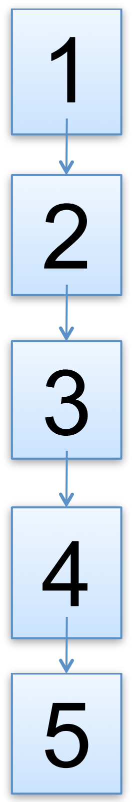


Segments



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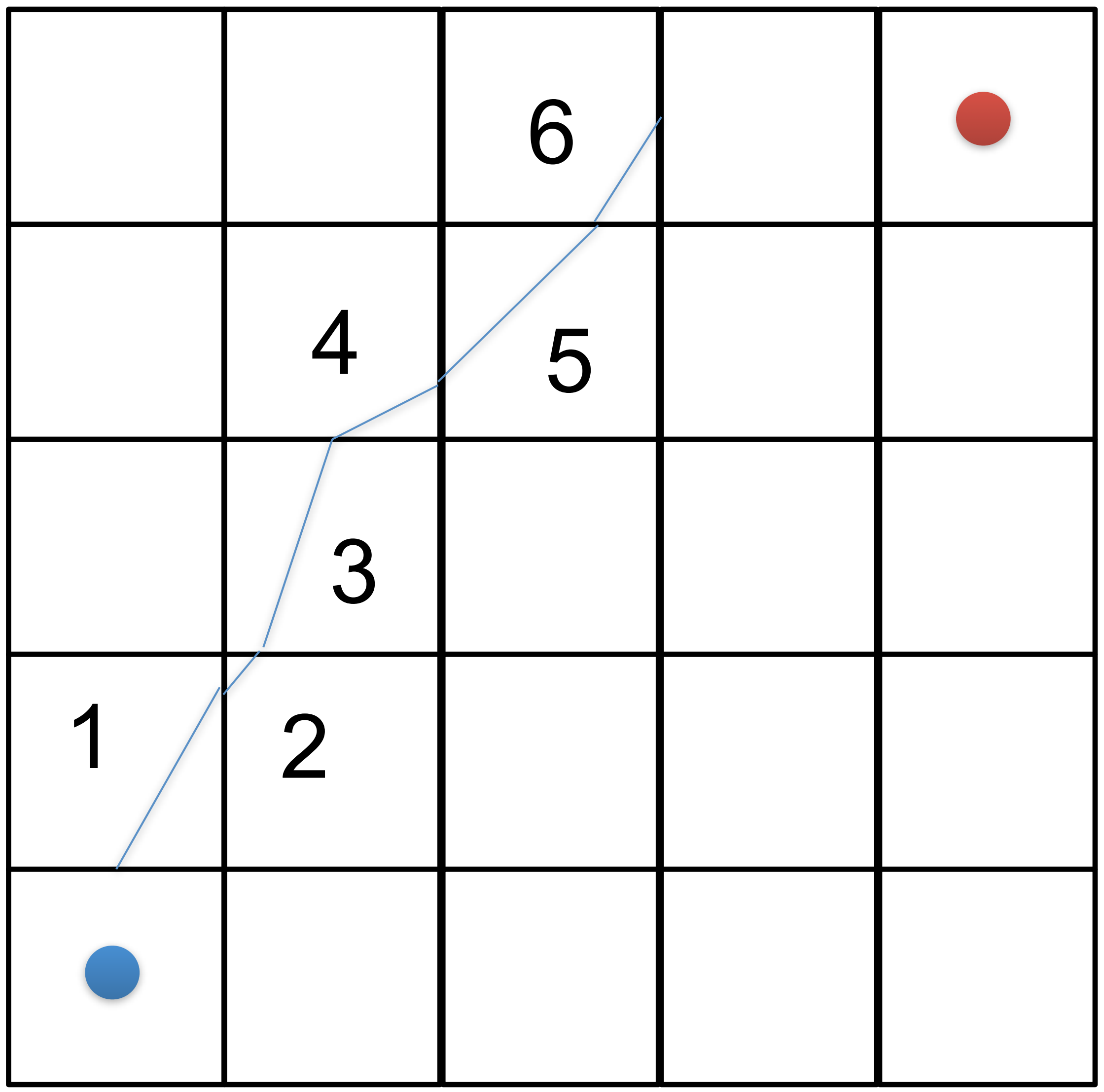

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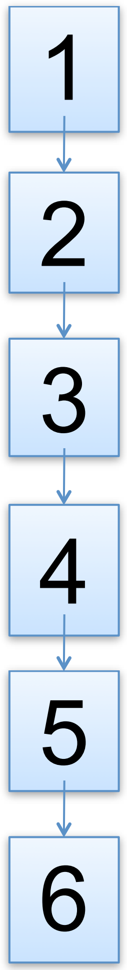

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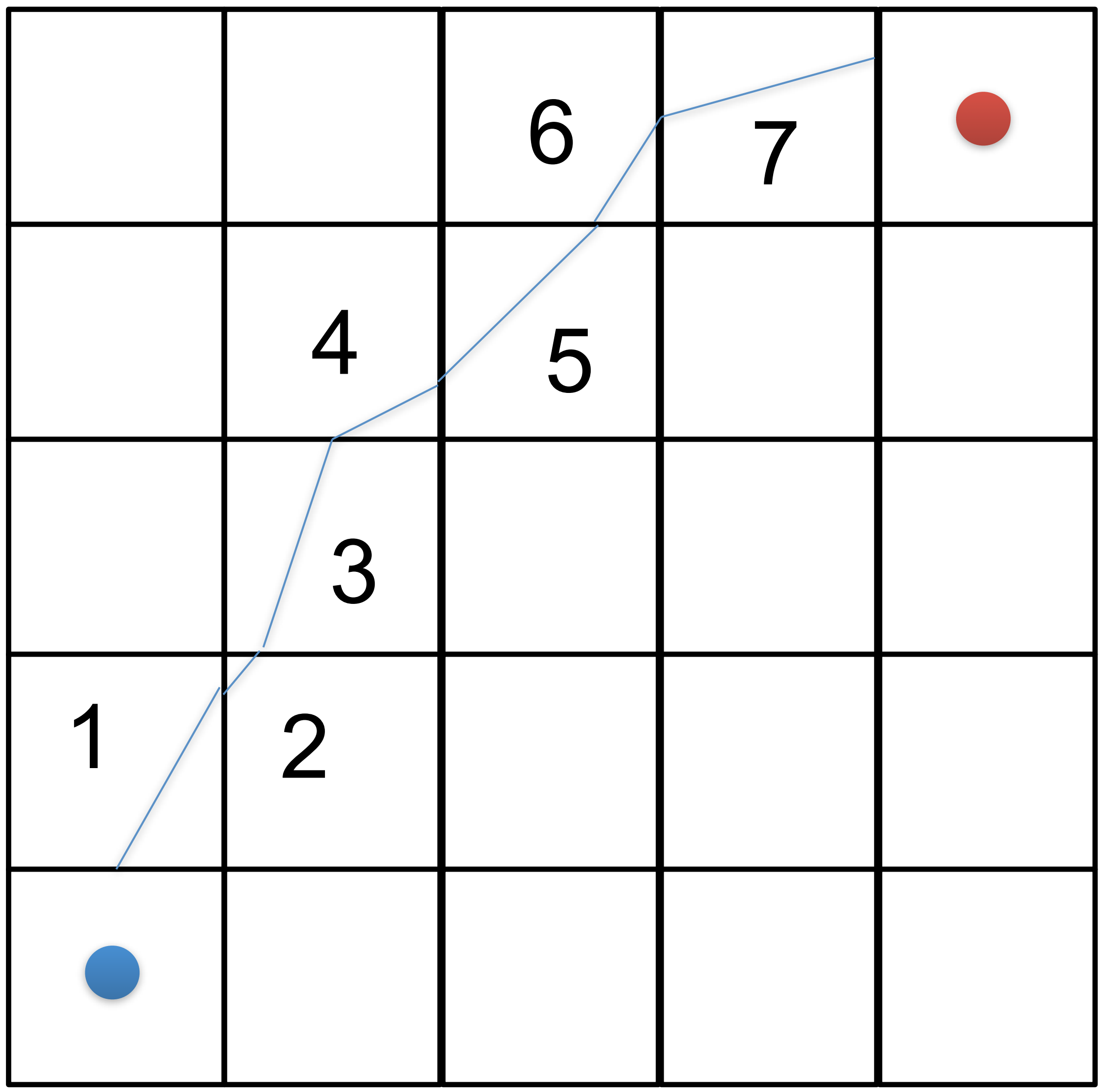
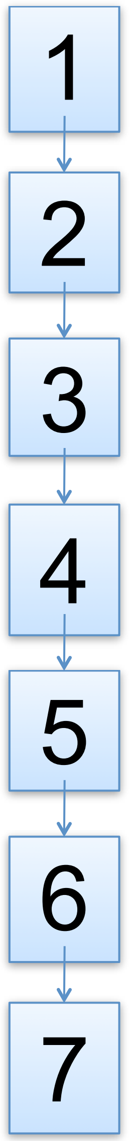


Segments



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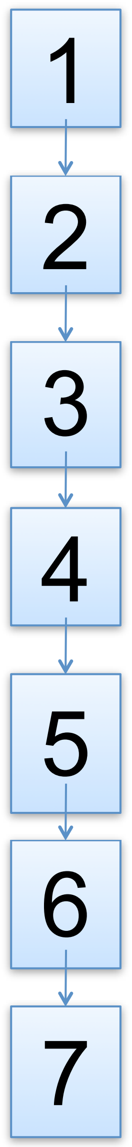
Streamline Storage

- The geometry of the streamlines is stored as a sequence of coordinates
- To store the streamlines we have several options

1. Allocate huge array and hope for the best
2. Use a dynamic array or a linked list
3. Trace once to count the segments, allocate an array and trace again



1-D Solves. Illustration

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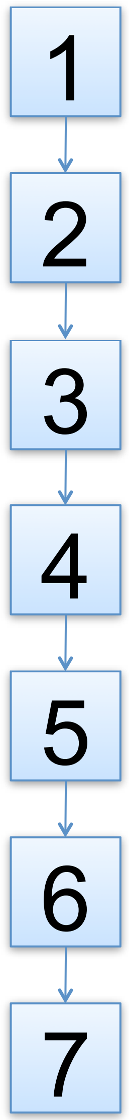
1-D Solves. Illustration

Segments



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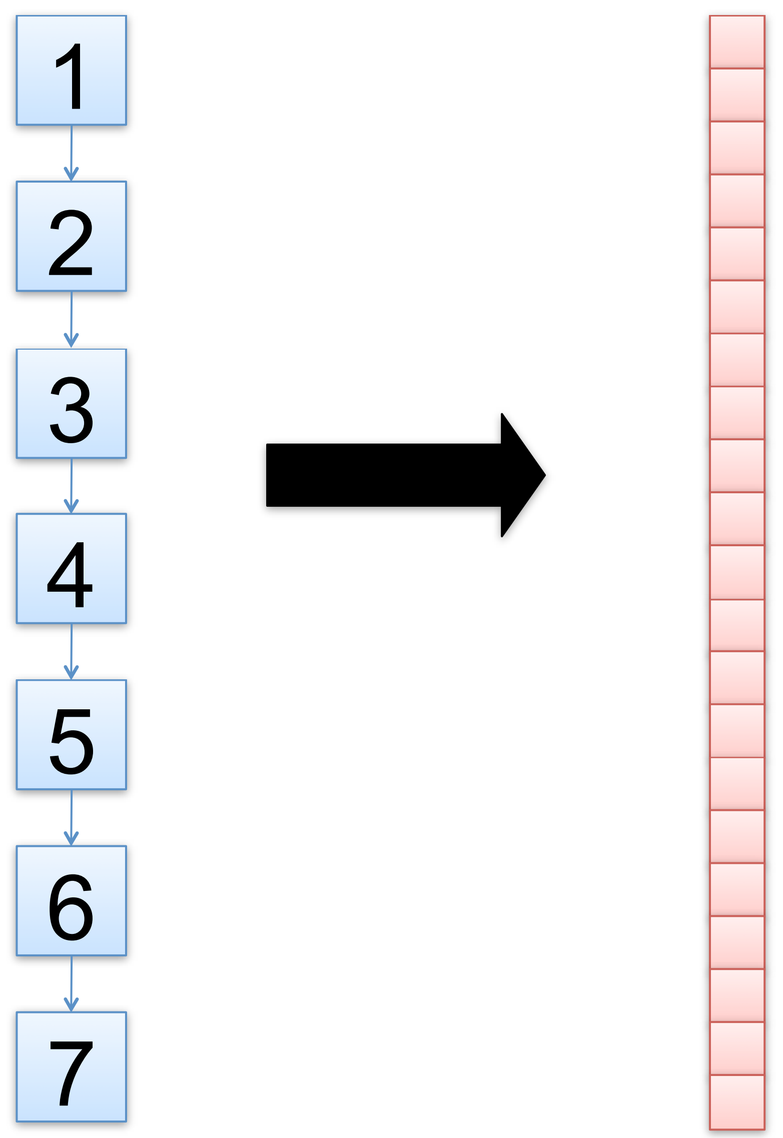
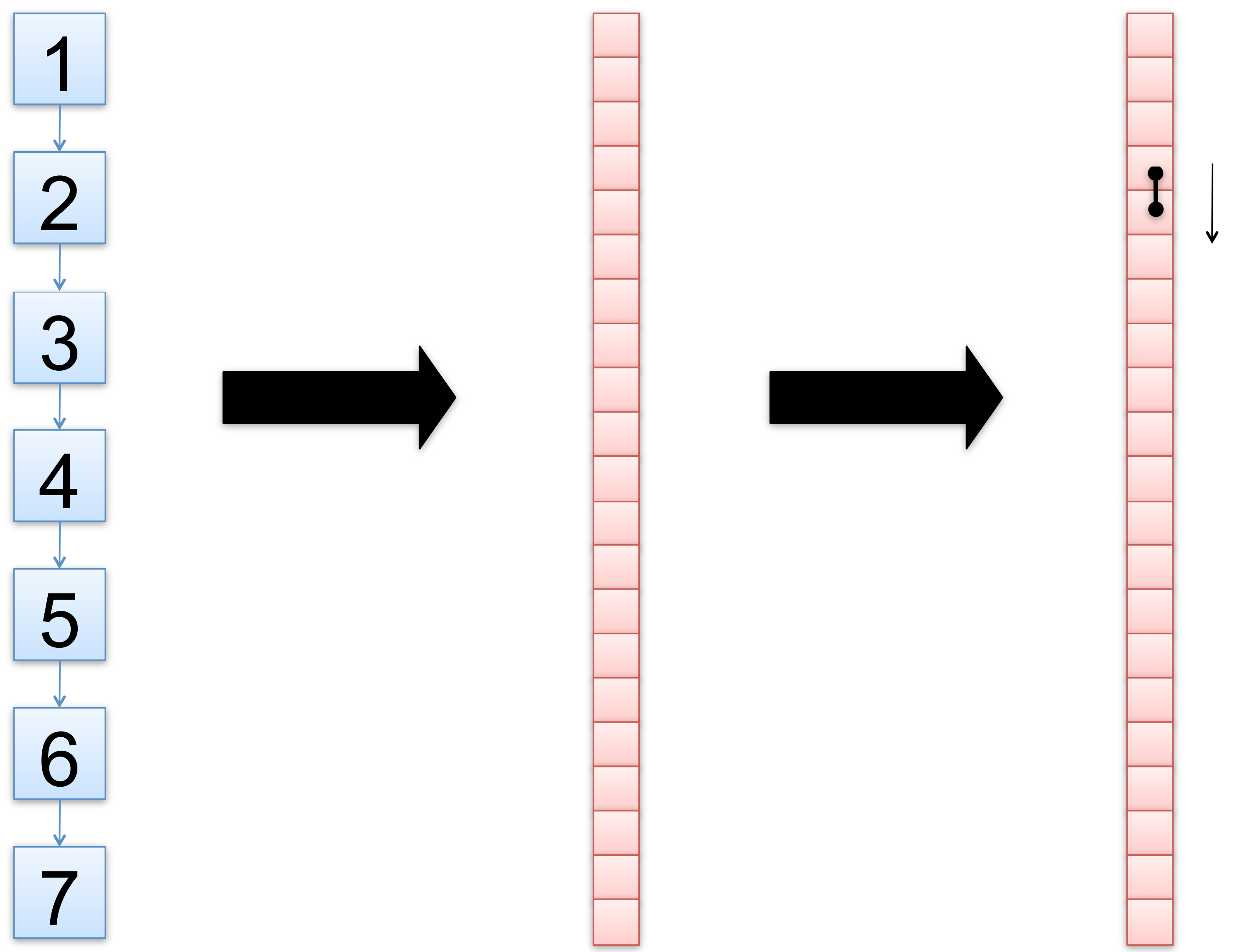
1-D Solves. Illustration

Segments Regularized Grid



1-D Solves. Illustration

Segments Regularized Grid Upwinding Solve



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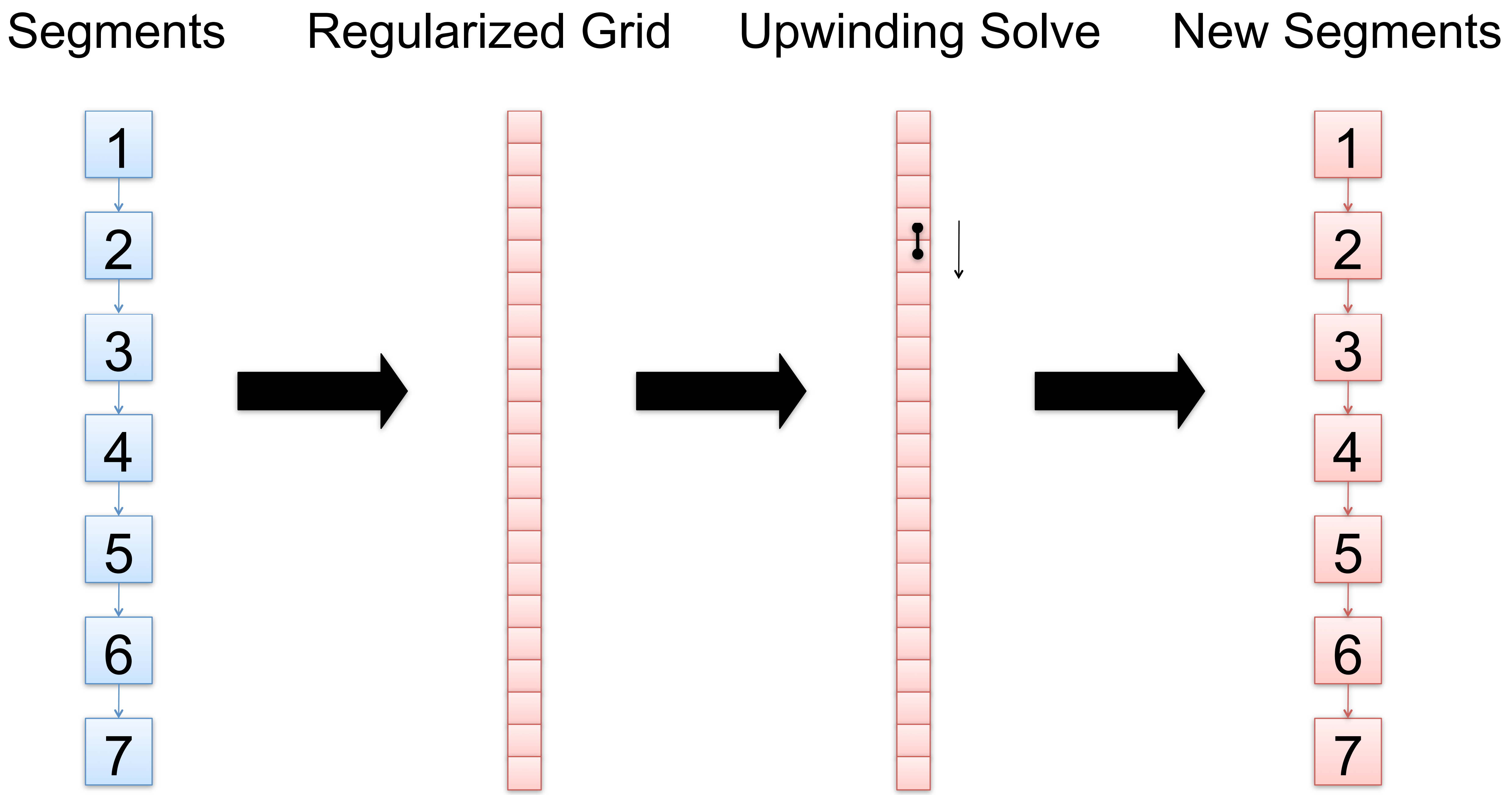

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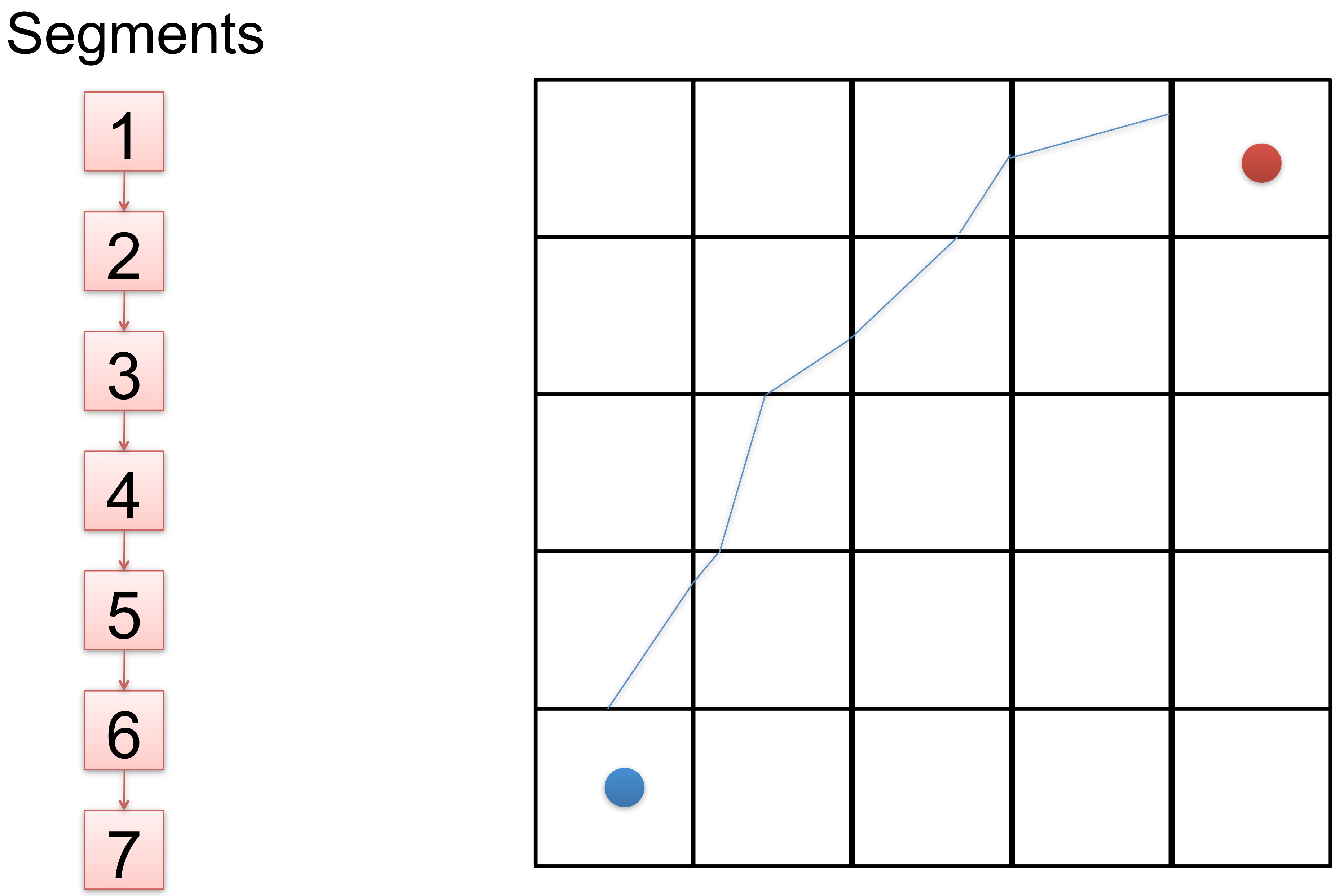
Segment Mapping, illustration



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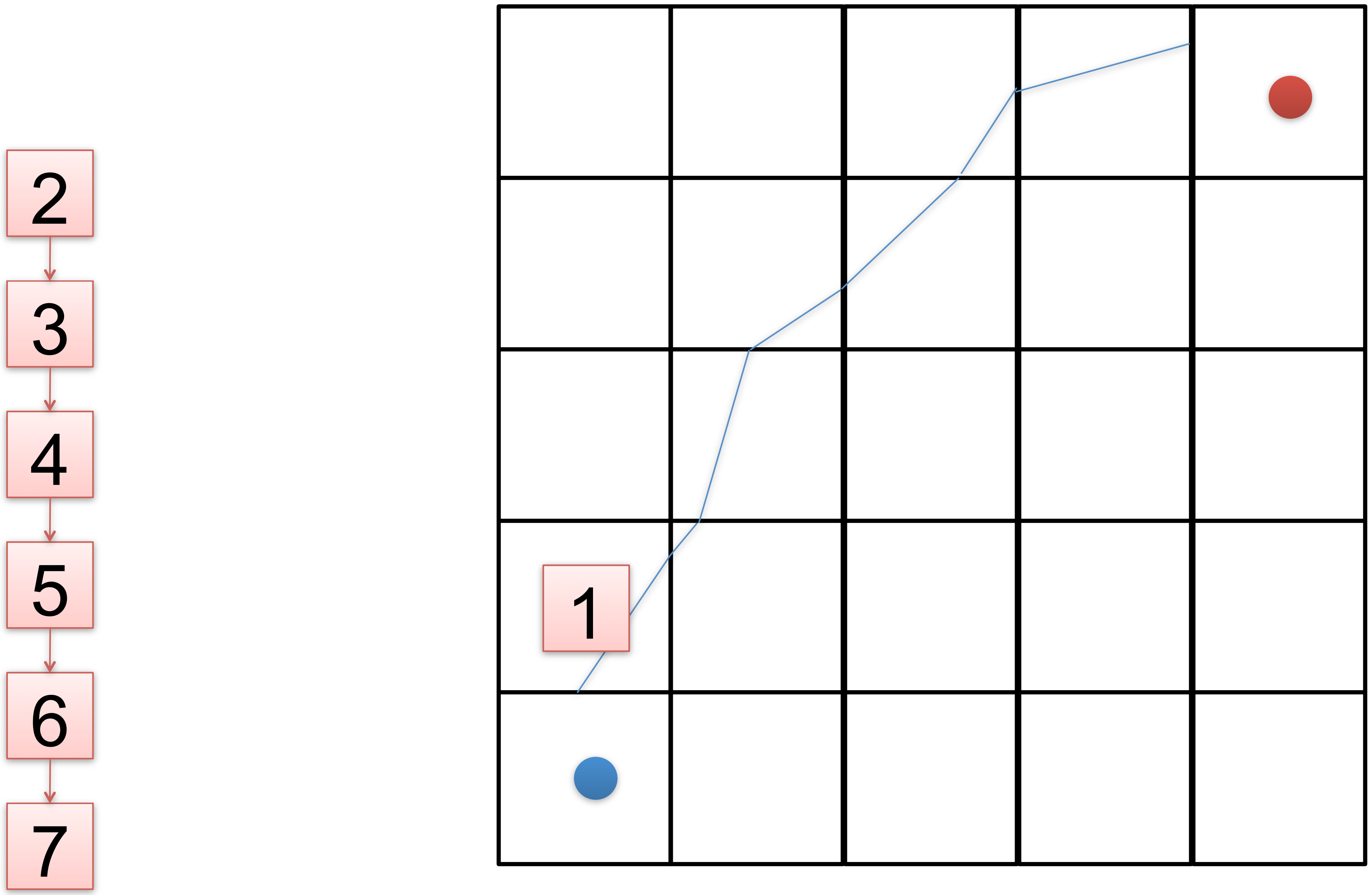

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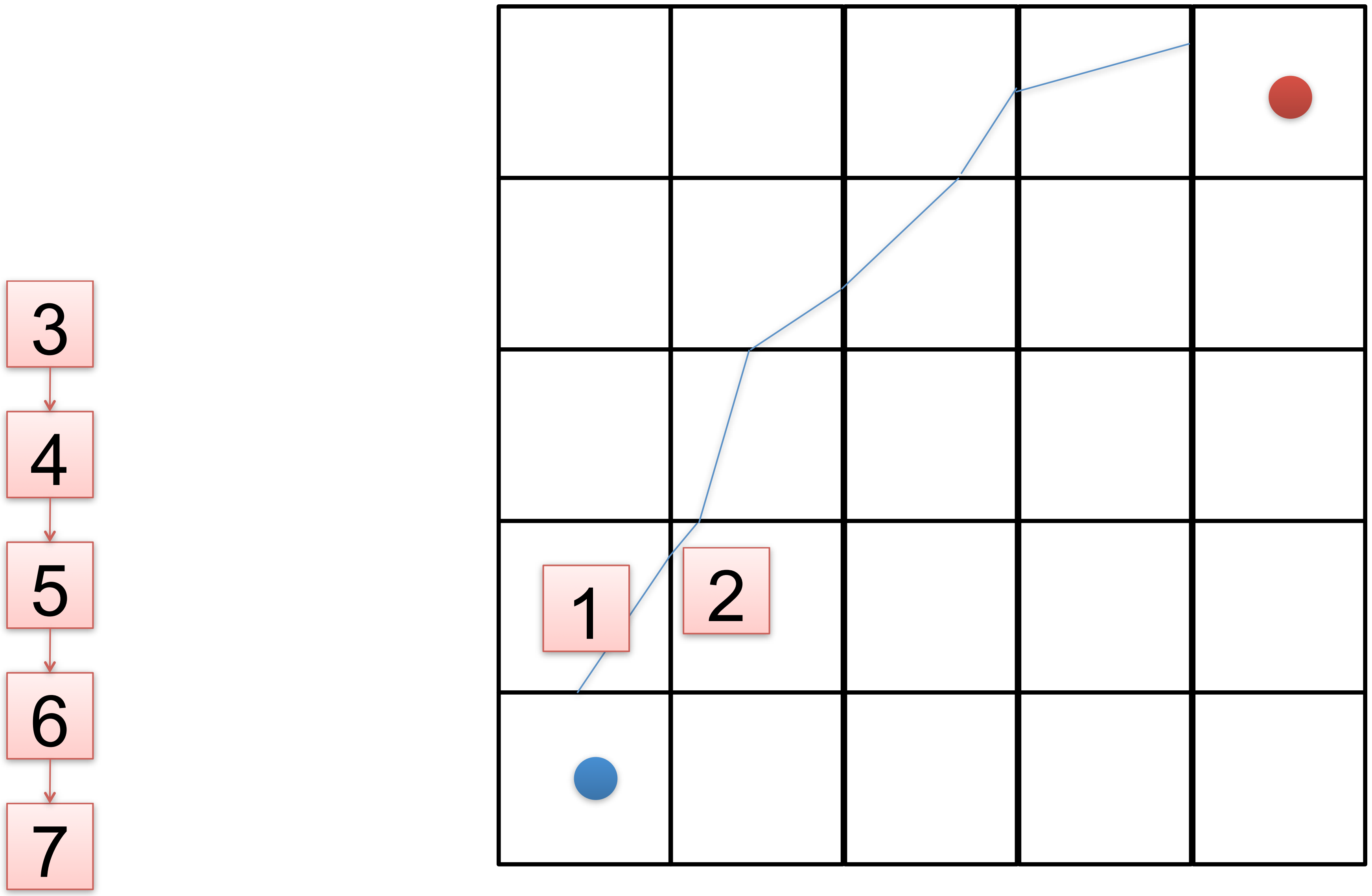

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Segment Mapping, illustration

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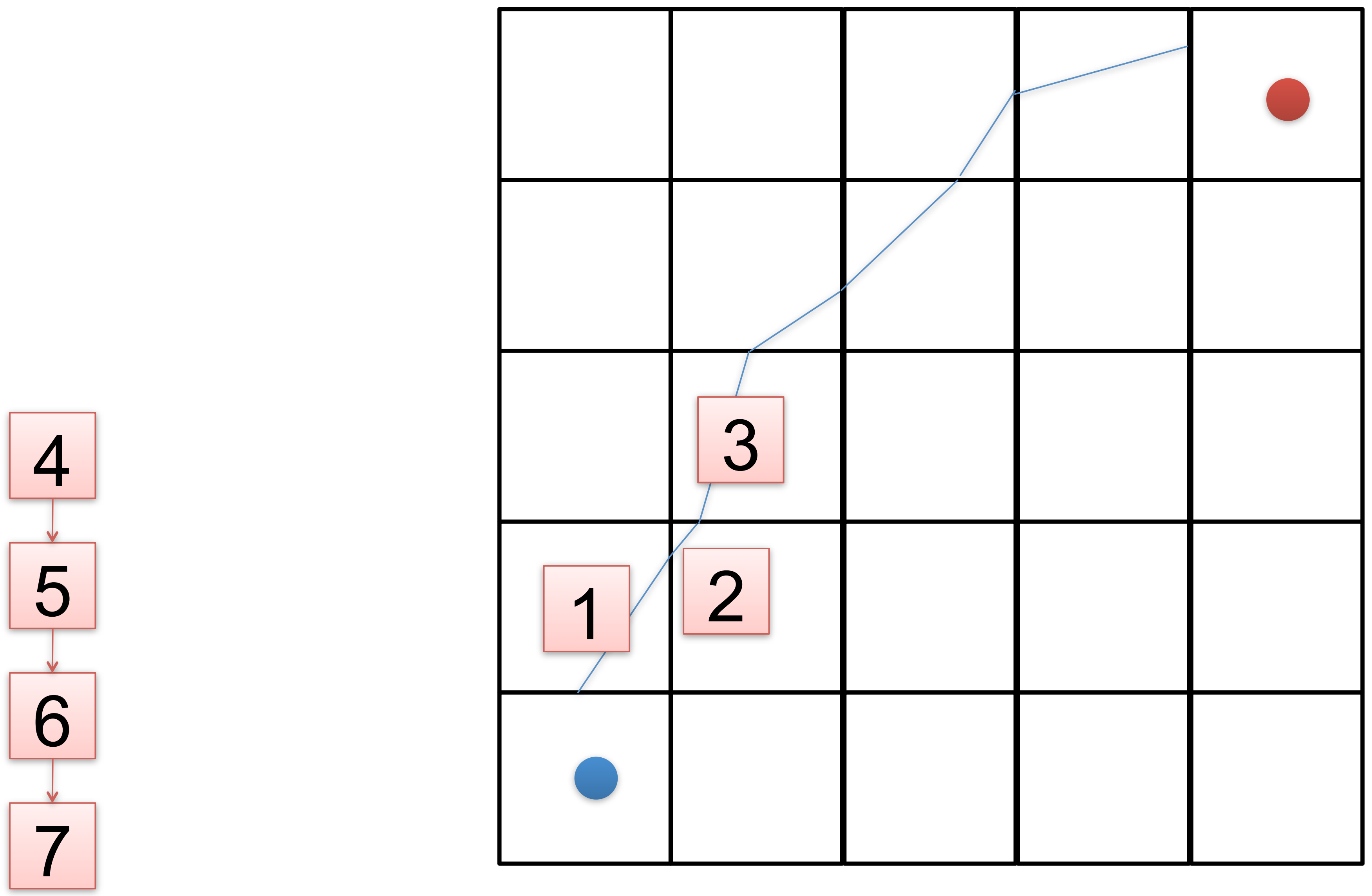

Segment Mapping, illustration

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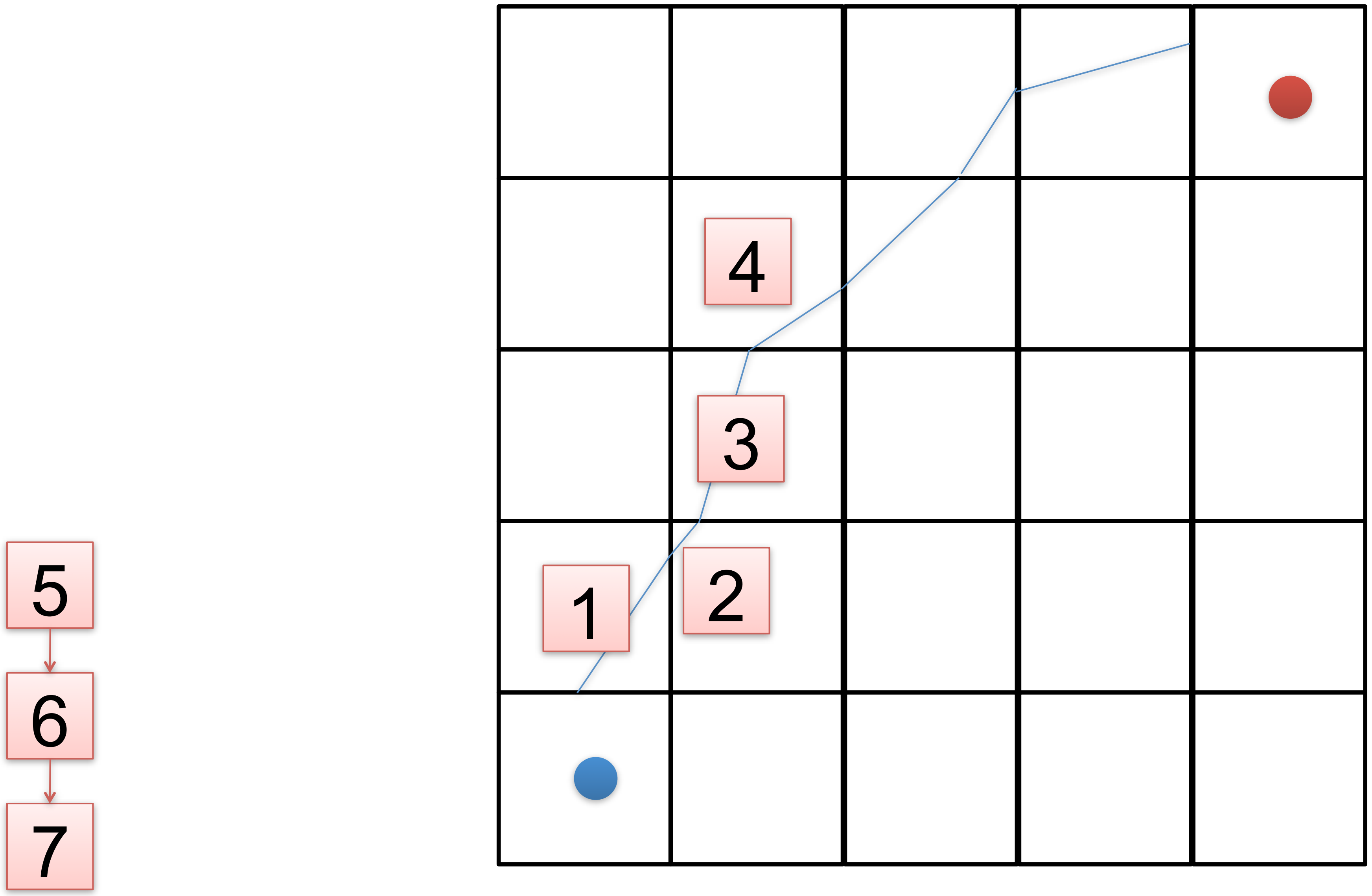

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Segment Mapping, illustration

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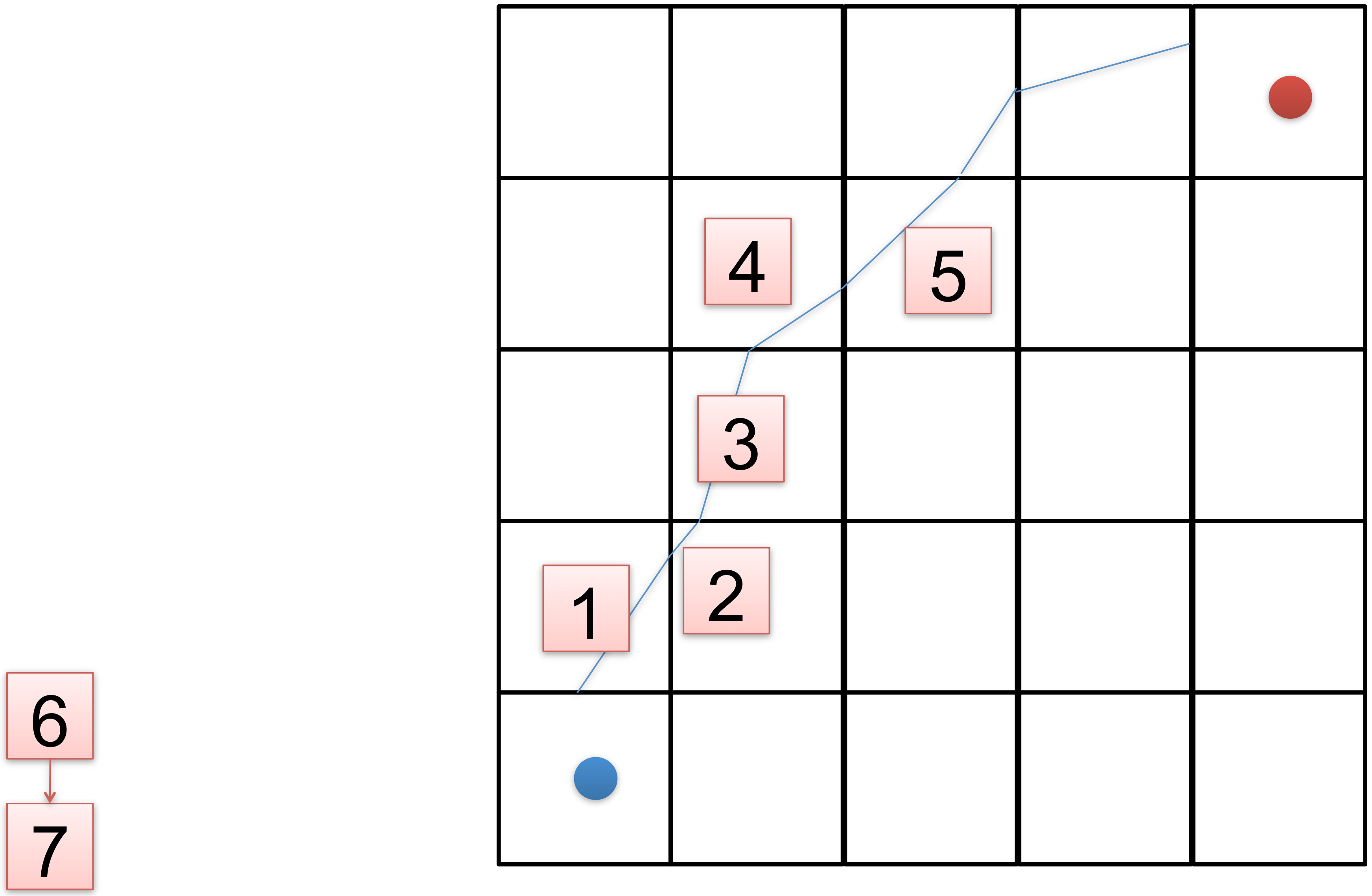

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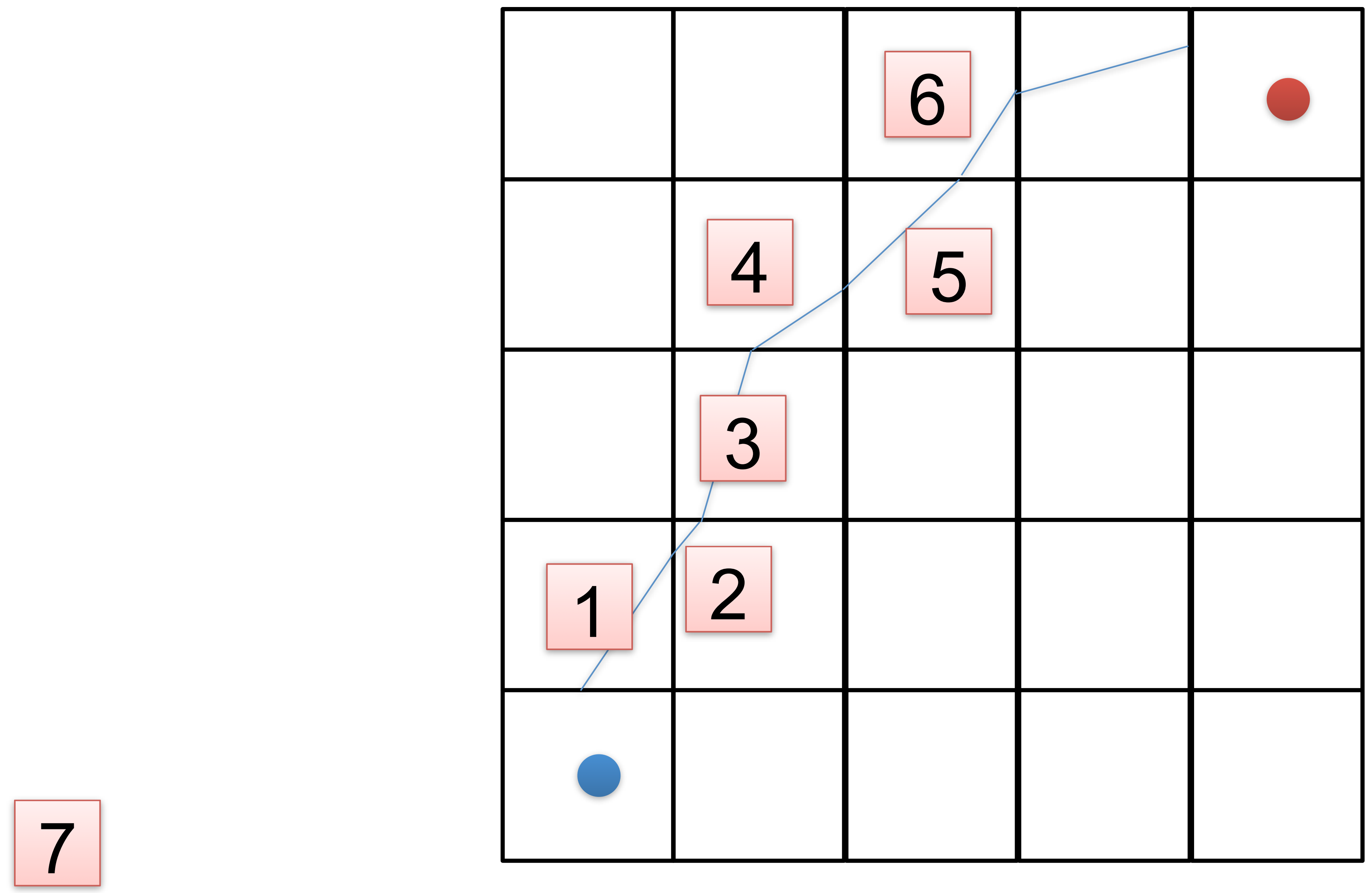

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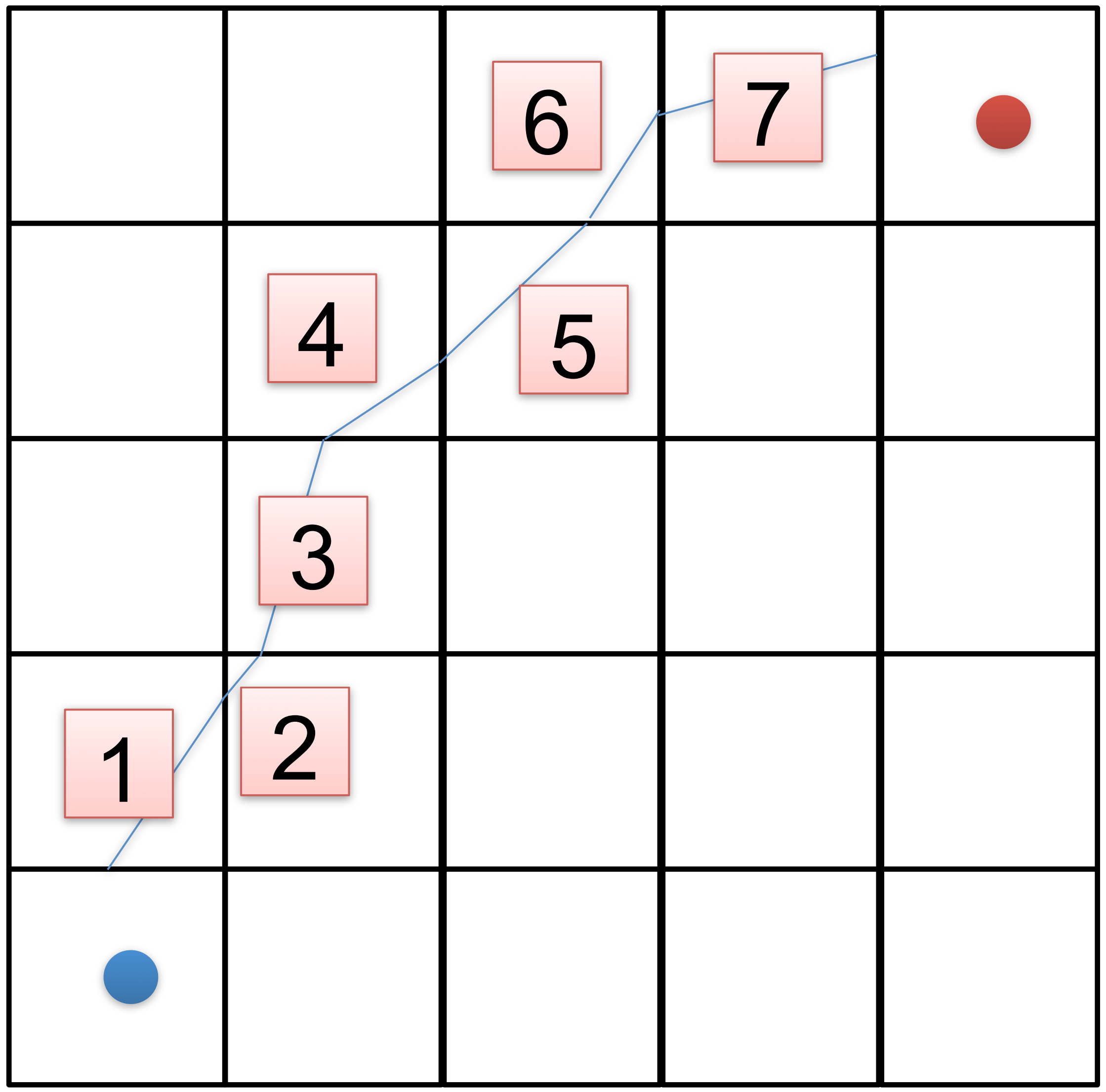

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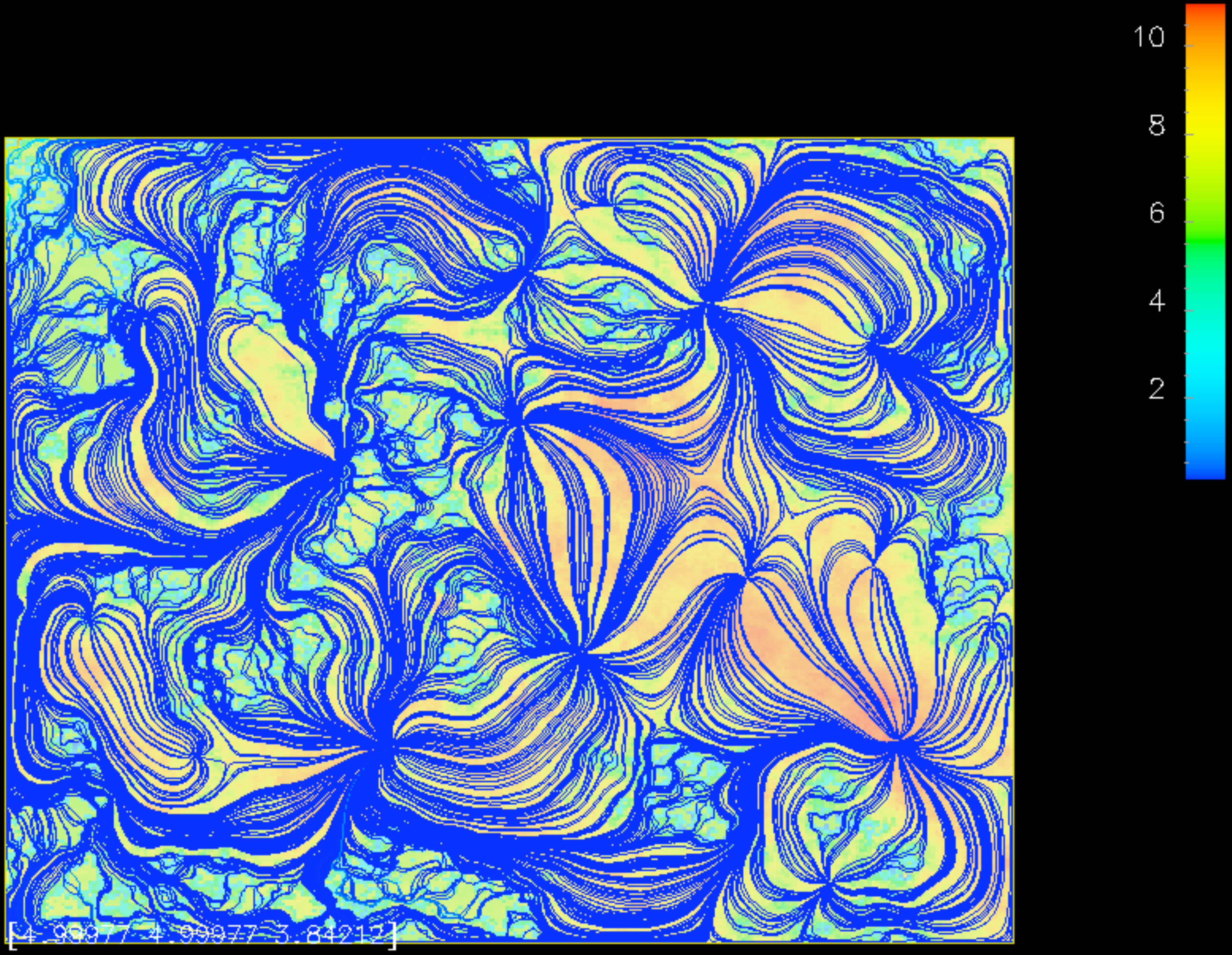
Streamlines and Shared Memory

- To extract parallelism we trace, solve and map for several streamlines concurrently
  - You can extract fine-grained parallelism from a single streamline
- We call a set of streamlines a **streamline bundle** or simply **bundle**
- We seek a parallel algorithm that minimizes parallel overhead
  - Communication
  - Load imbalance
  - Synchronization


10
8
6
4
2
[4.99877 4.99877 3.84212]

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Parallel Algorithm

Algorithm 1 Parallel Streamline Simulation

```
Require: $T_{end} \leftarrow$ simulation end time
Require: $dt \leftarrow$ global timestep
Require: $T \leftarrow dt$
 1: repeat
 2:     Solve pressure equation
 3:     Calculate velocity field
 4:     repeat
 5:         Select launch points for streamlines and build bundle
 6:         Assign launch points in bundle to threads
 7:         for all streamlines in bundle do
 8:             1:st trace, count segments
 9:             2:nd trace, pick up pressure grid data, store the segments
10:             Build streamline grids from segment and pressure grid data
11:             Solve the corresponding 1-D transport equations
12:         end for
13:         Map new values of the transport variables to the pressure grid
14:     until Domain is sufficiently covered
15:     $T \leftarrow T + dt$
16: until $T \geq T_{end}$
```

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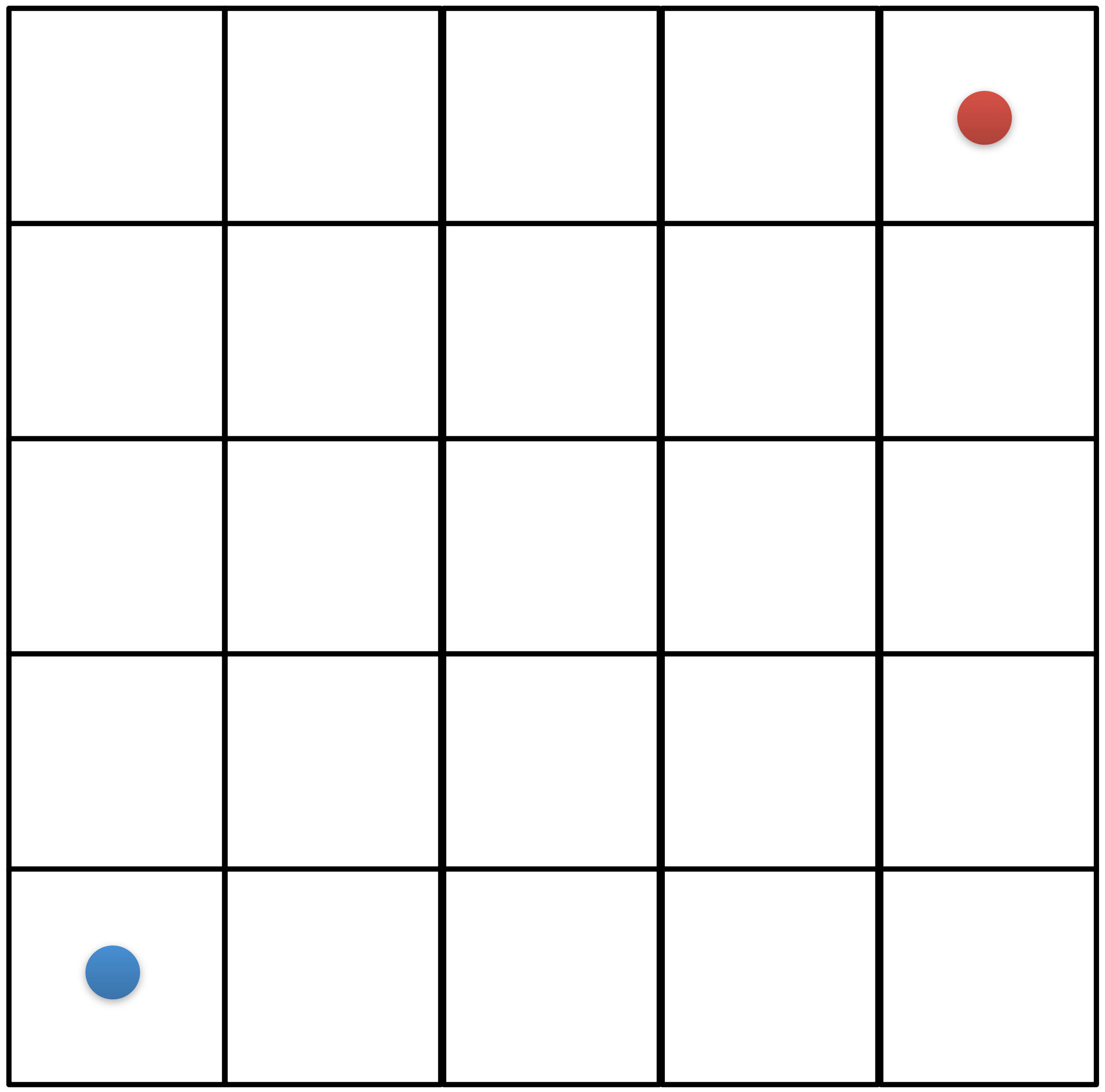
Static Assignment

Thread 1 Thread 2



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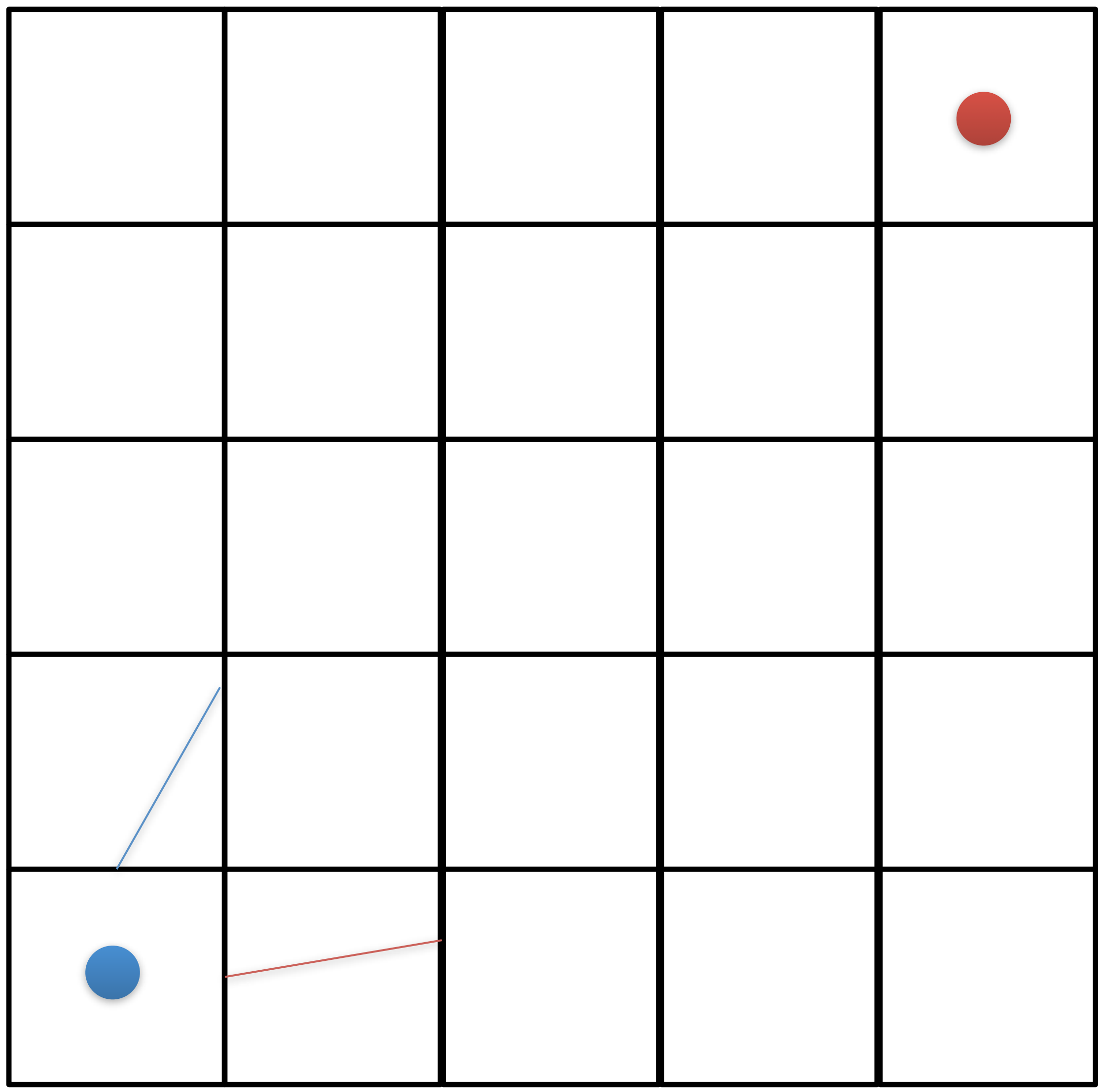

Thread 1
1
Thread 2
1

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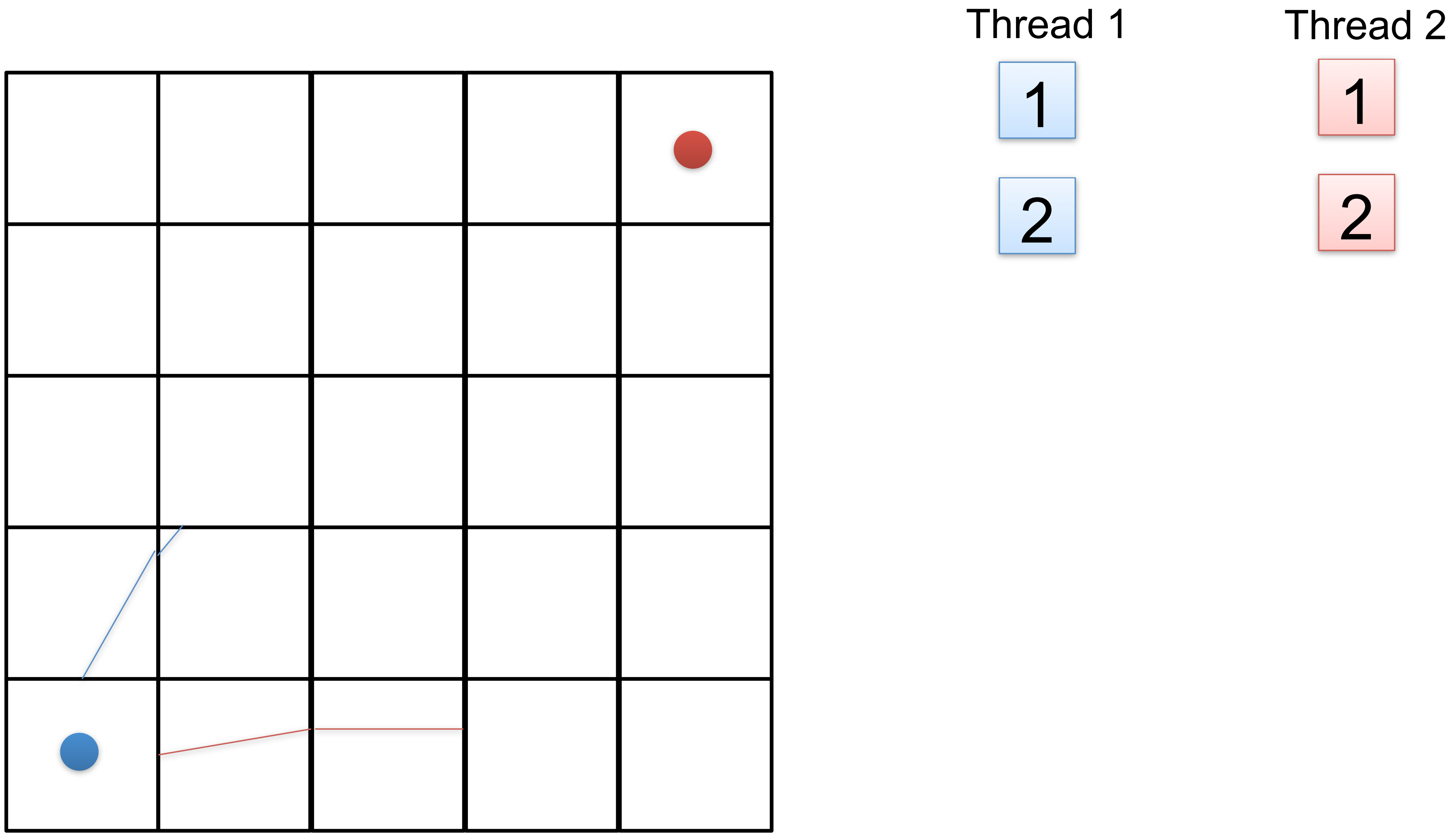
Static Assignment



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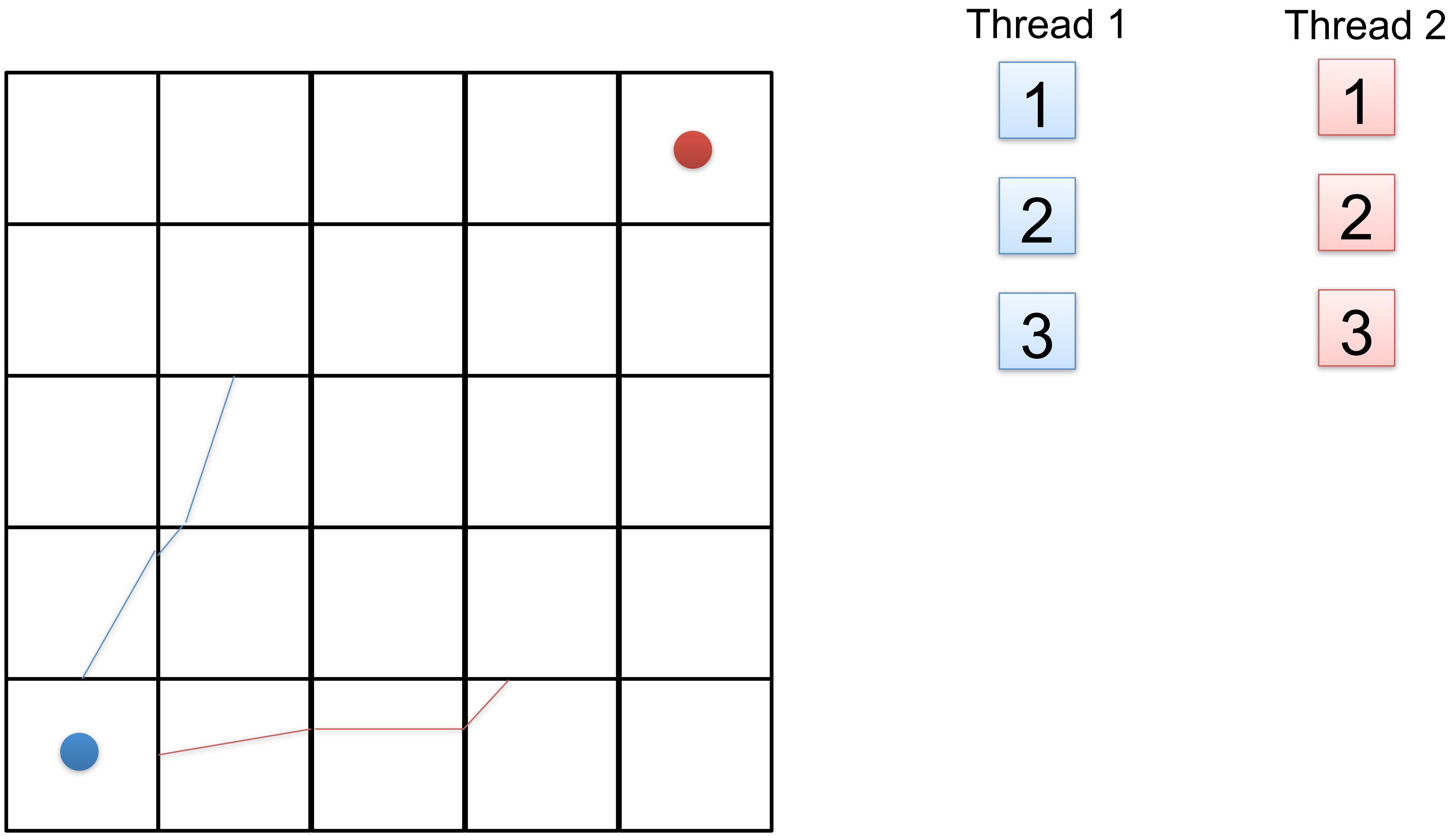

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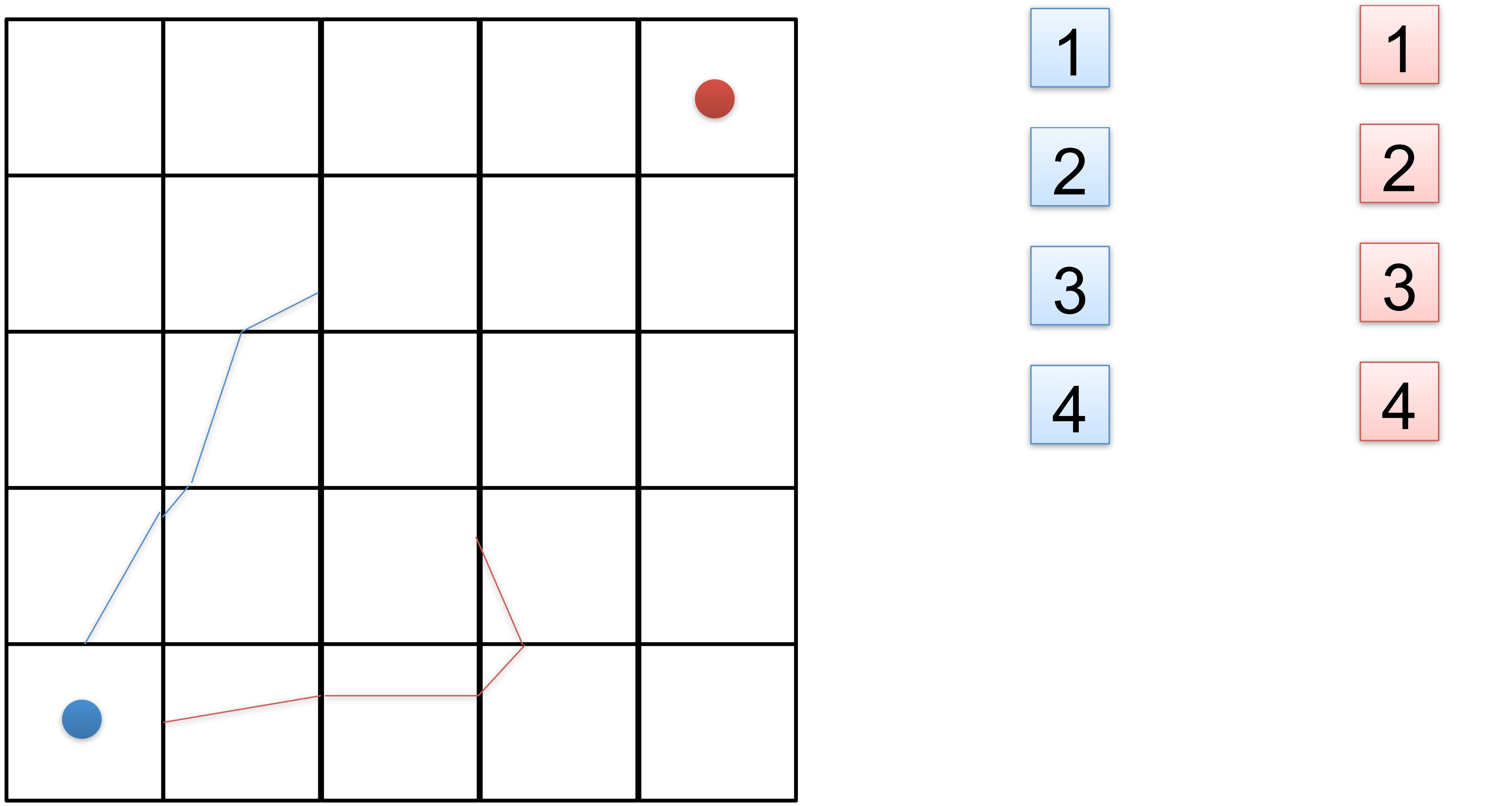

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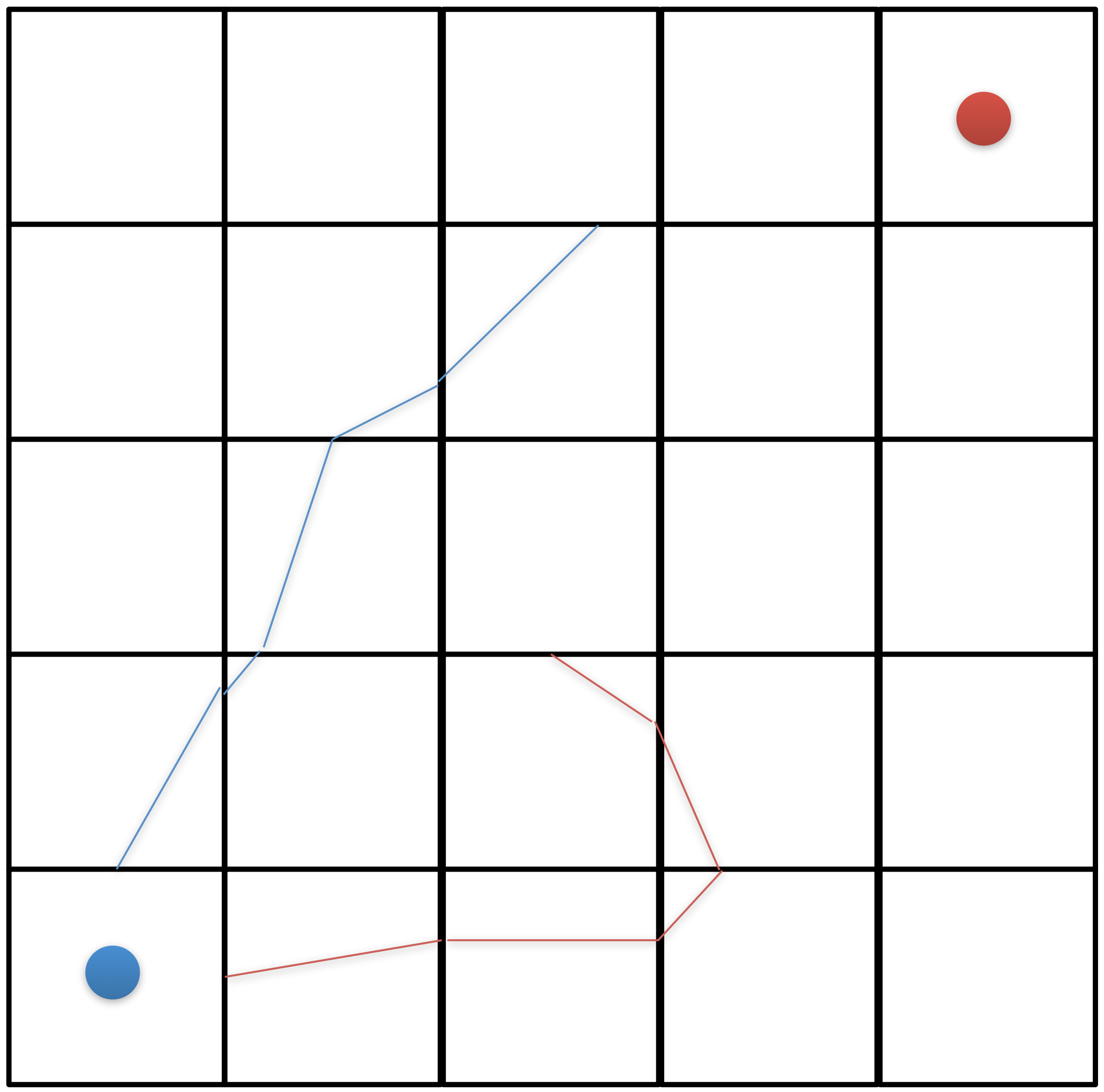
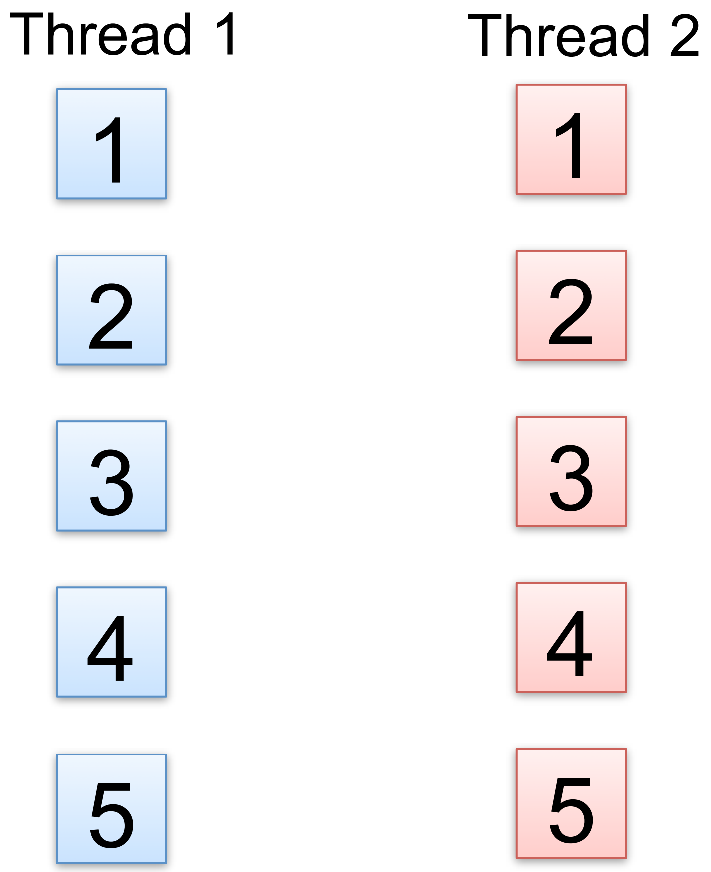
Static Assignment





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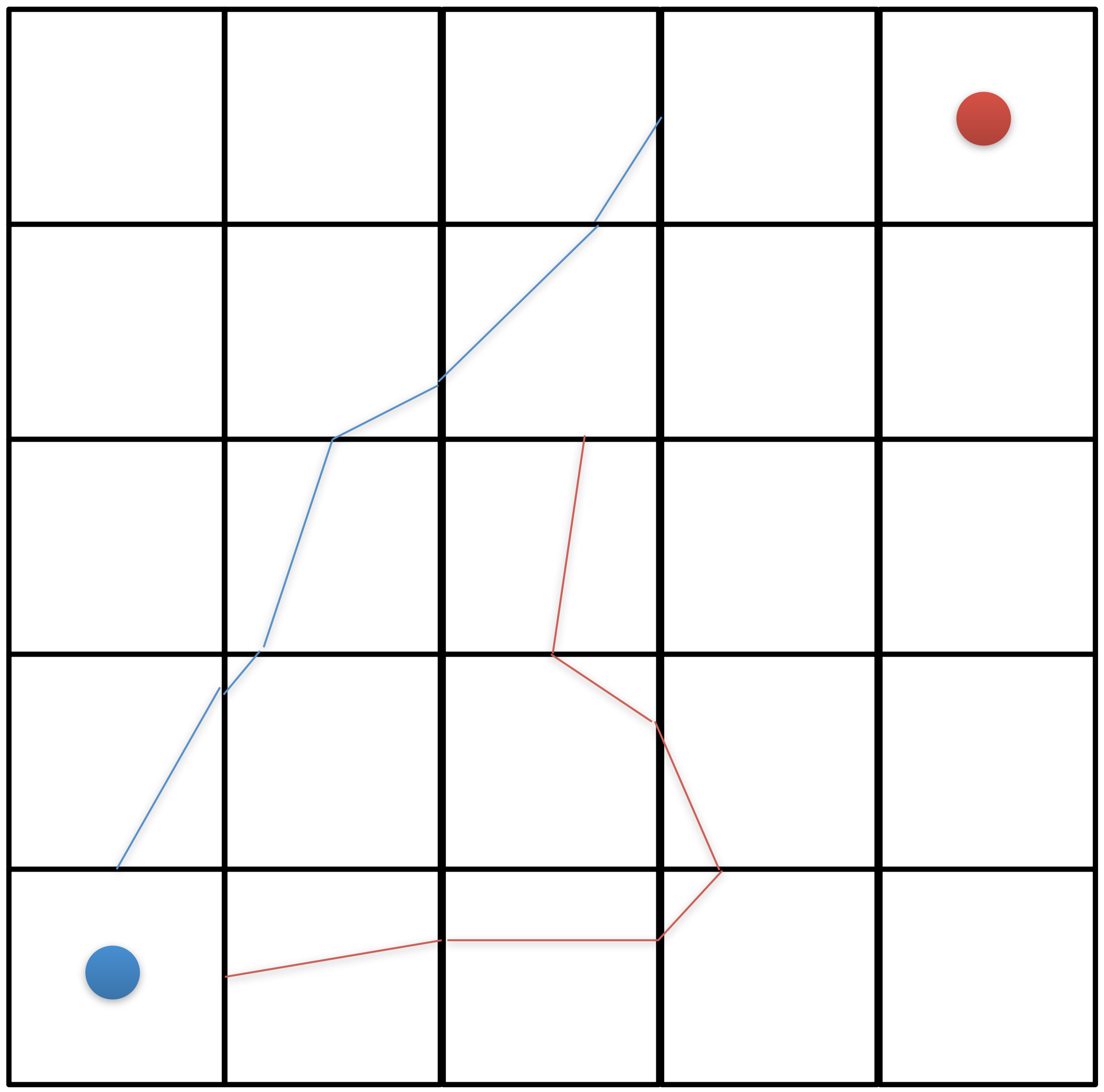

Thread 1	Thread 2
1	1
2	2
3	3
4	4
5	5
6	6

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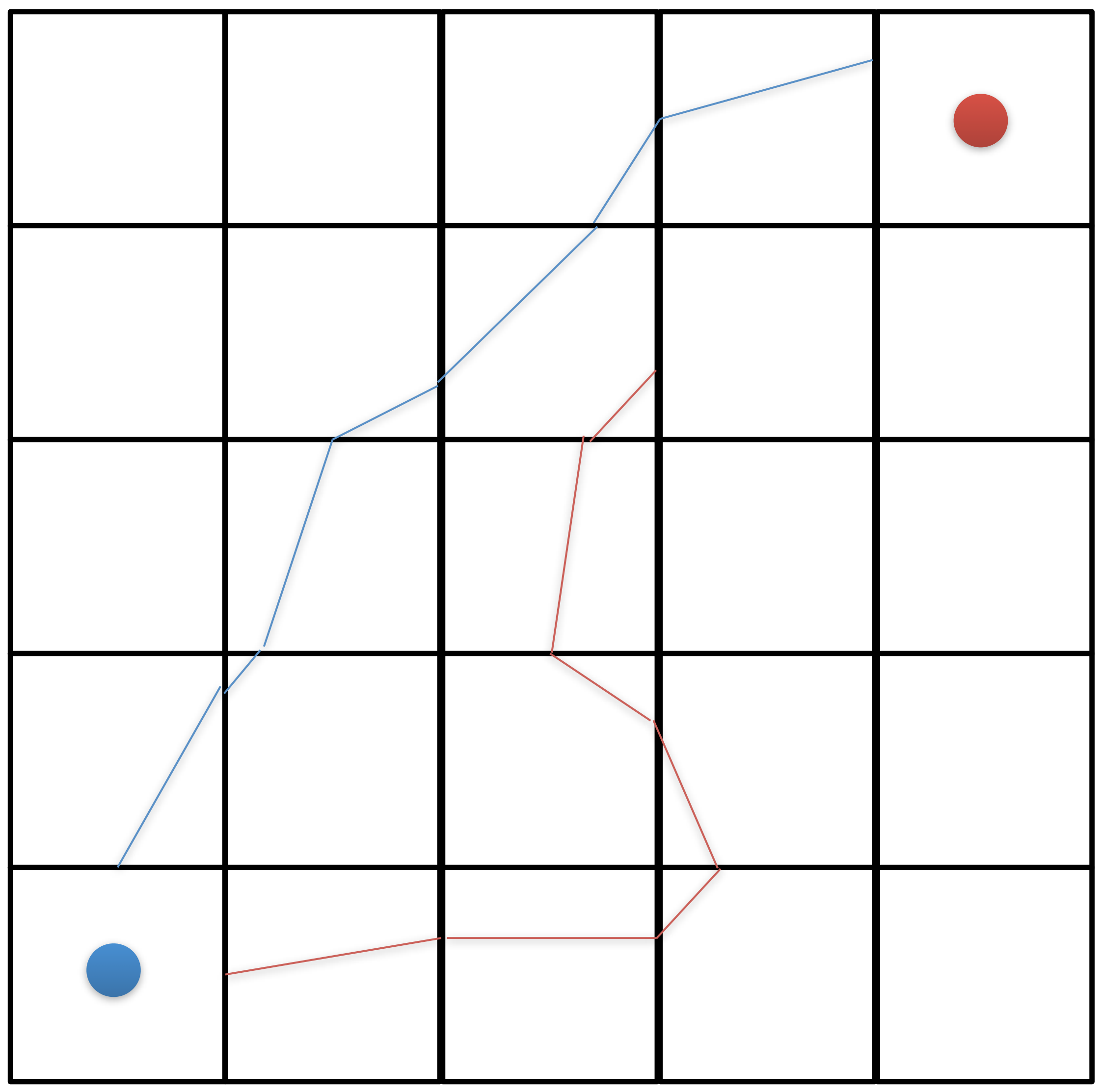
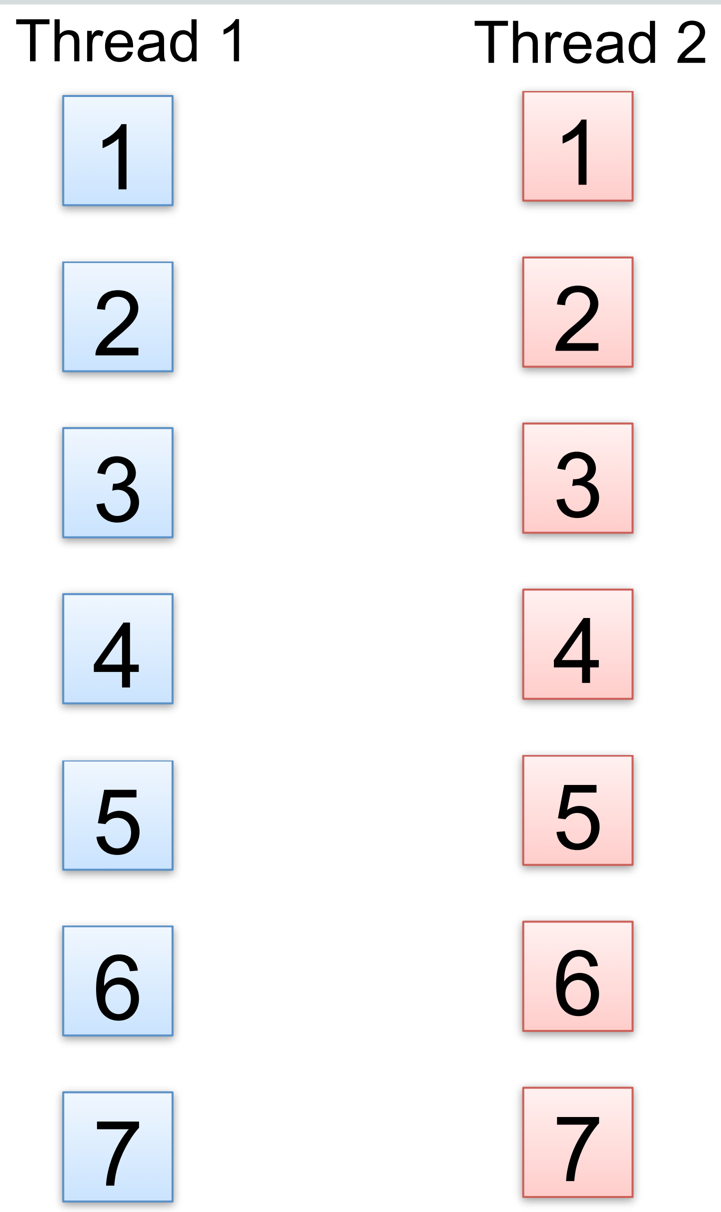
Static Assignment





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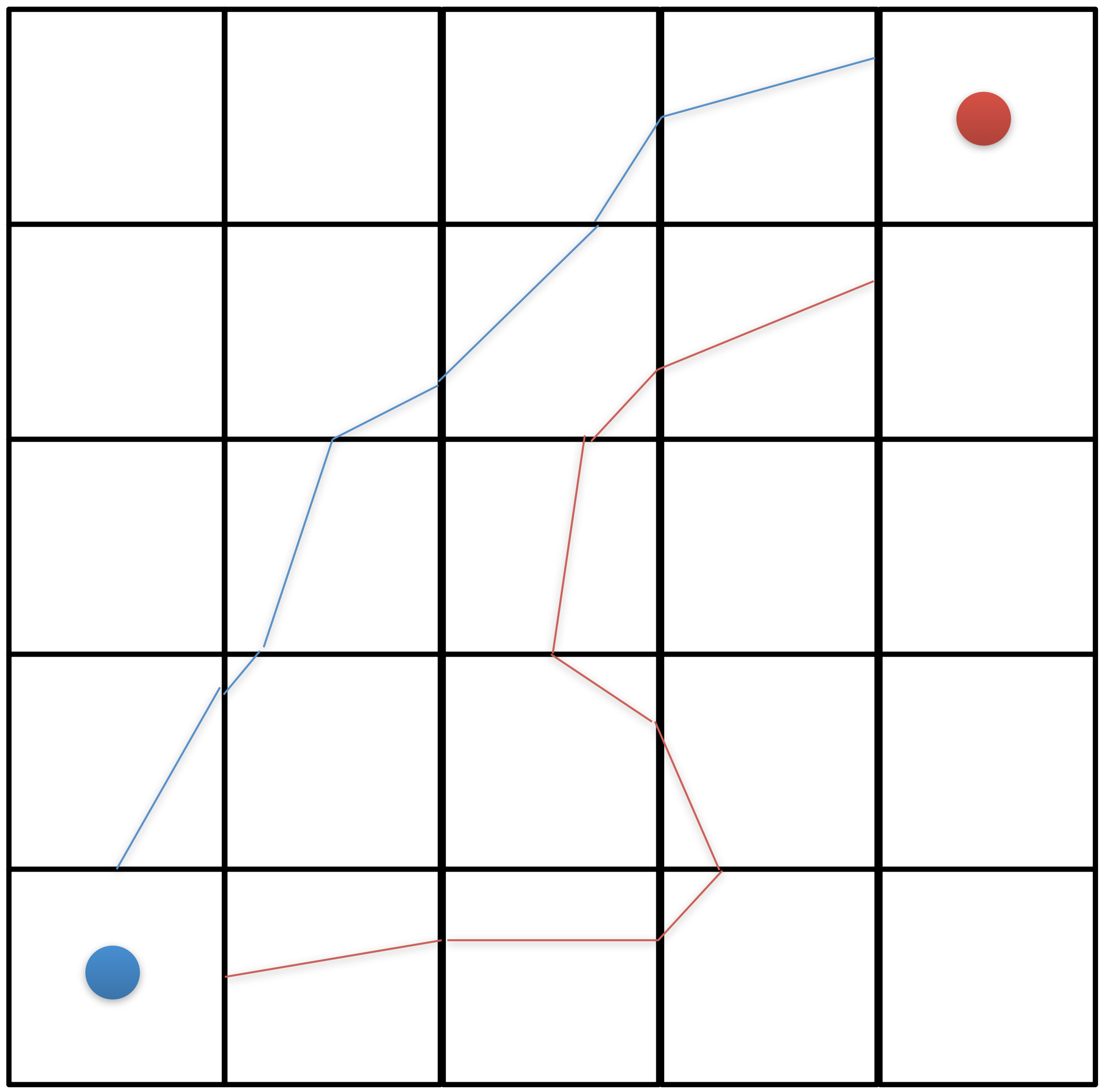

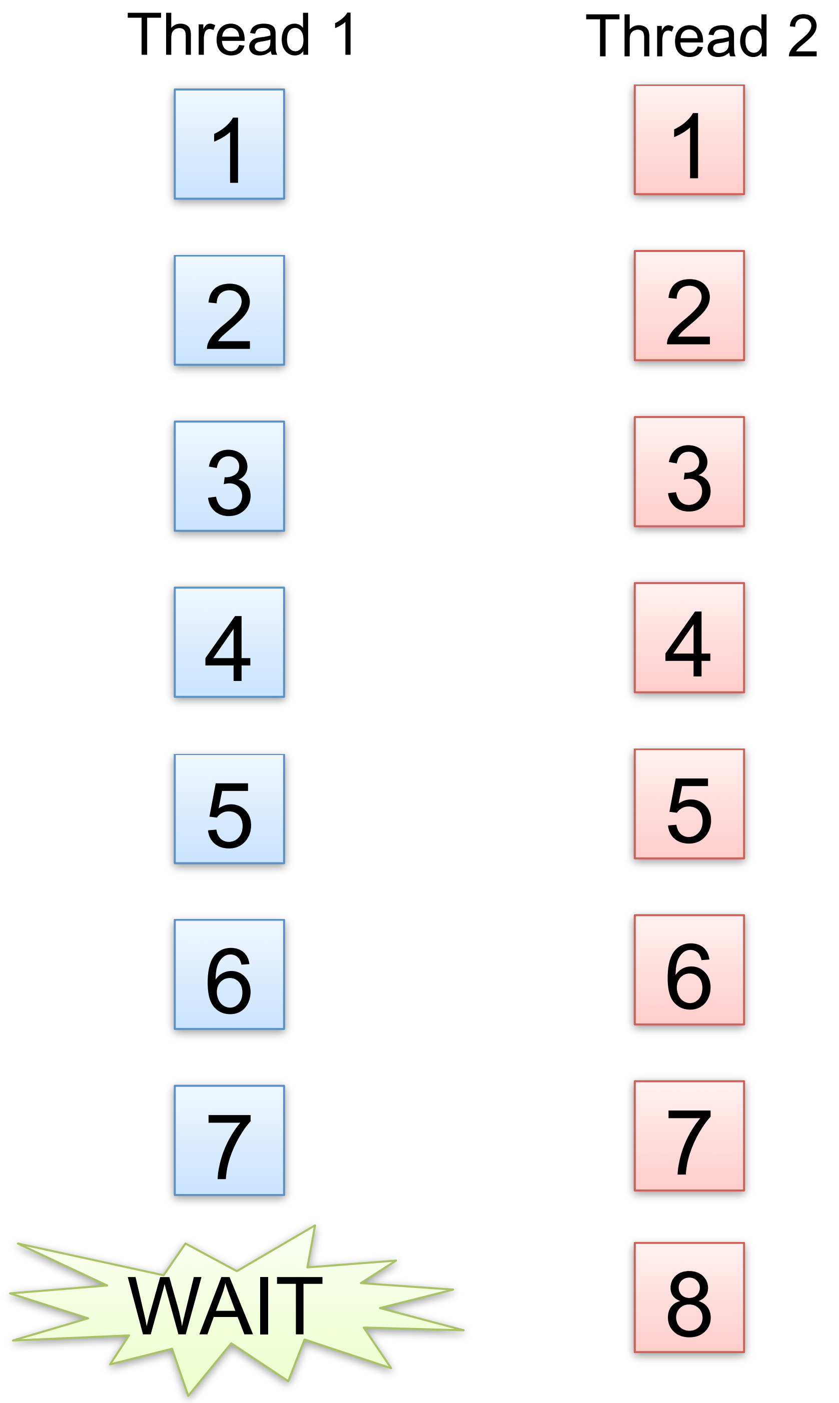

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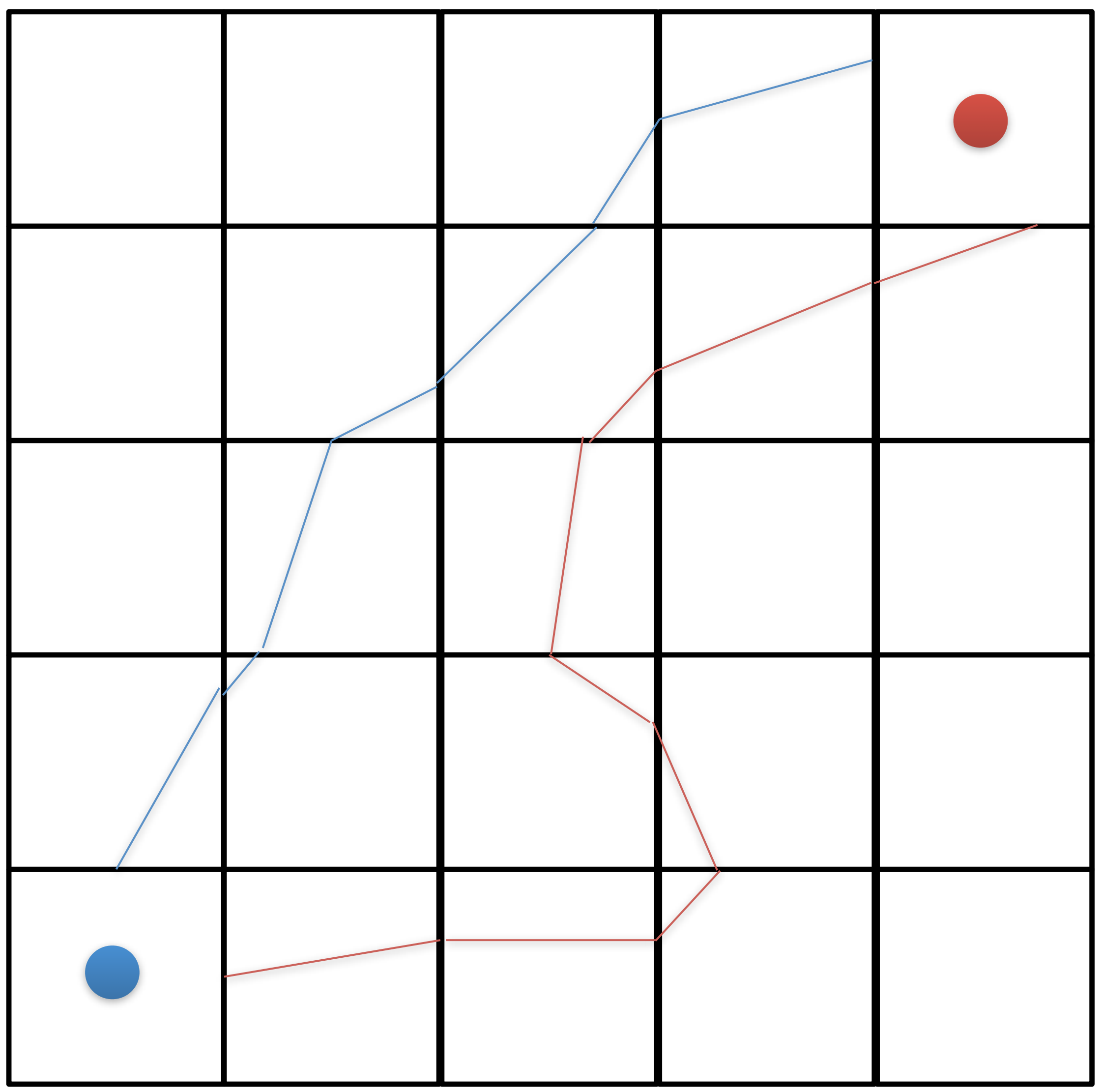
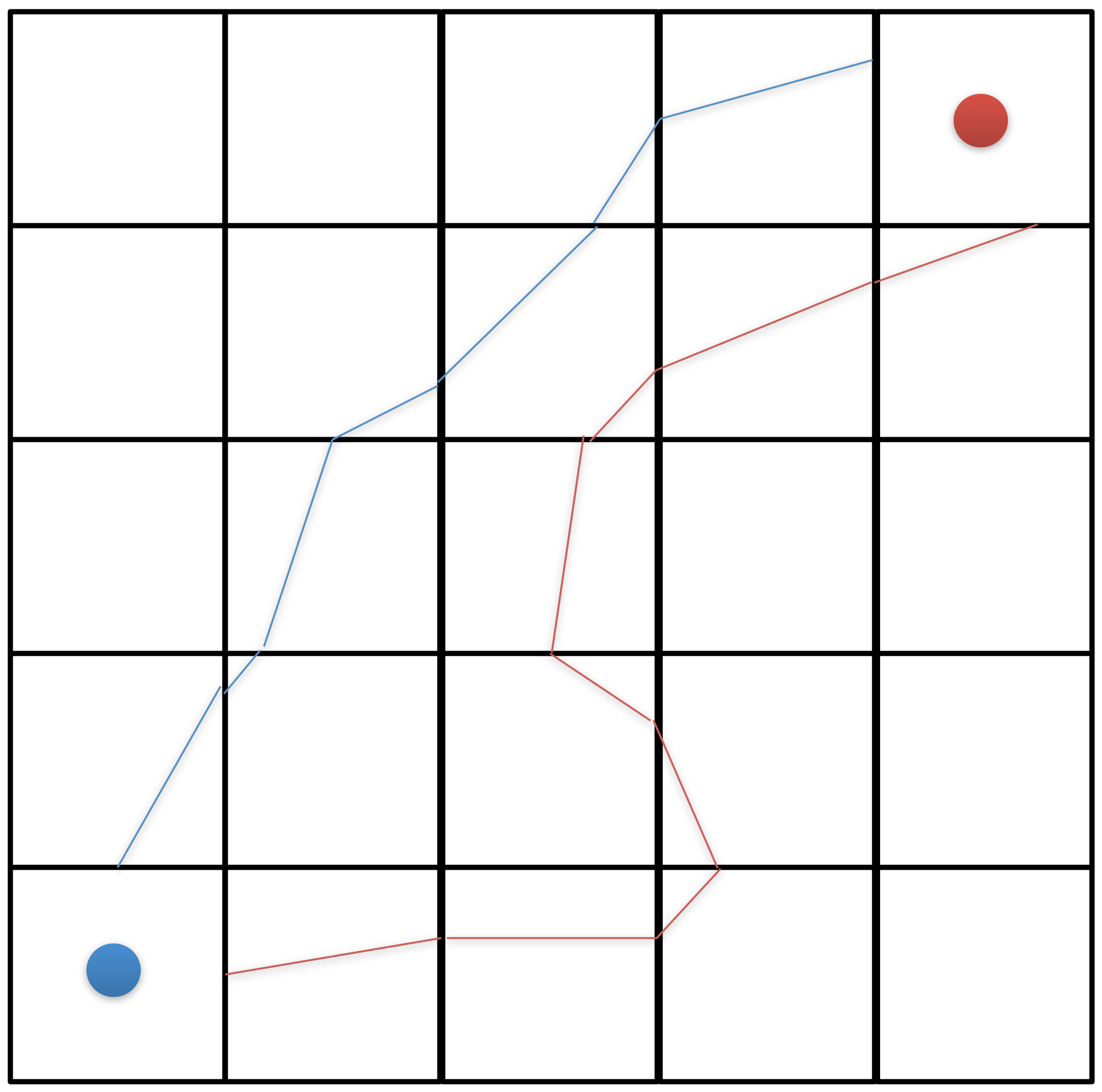
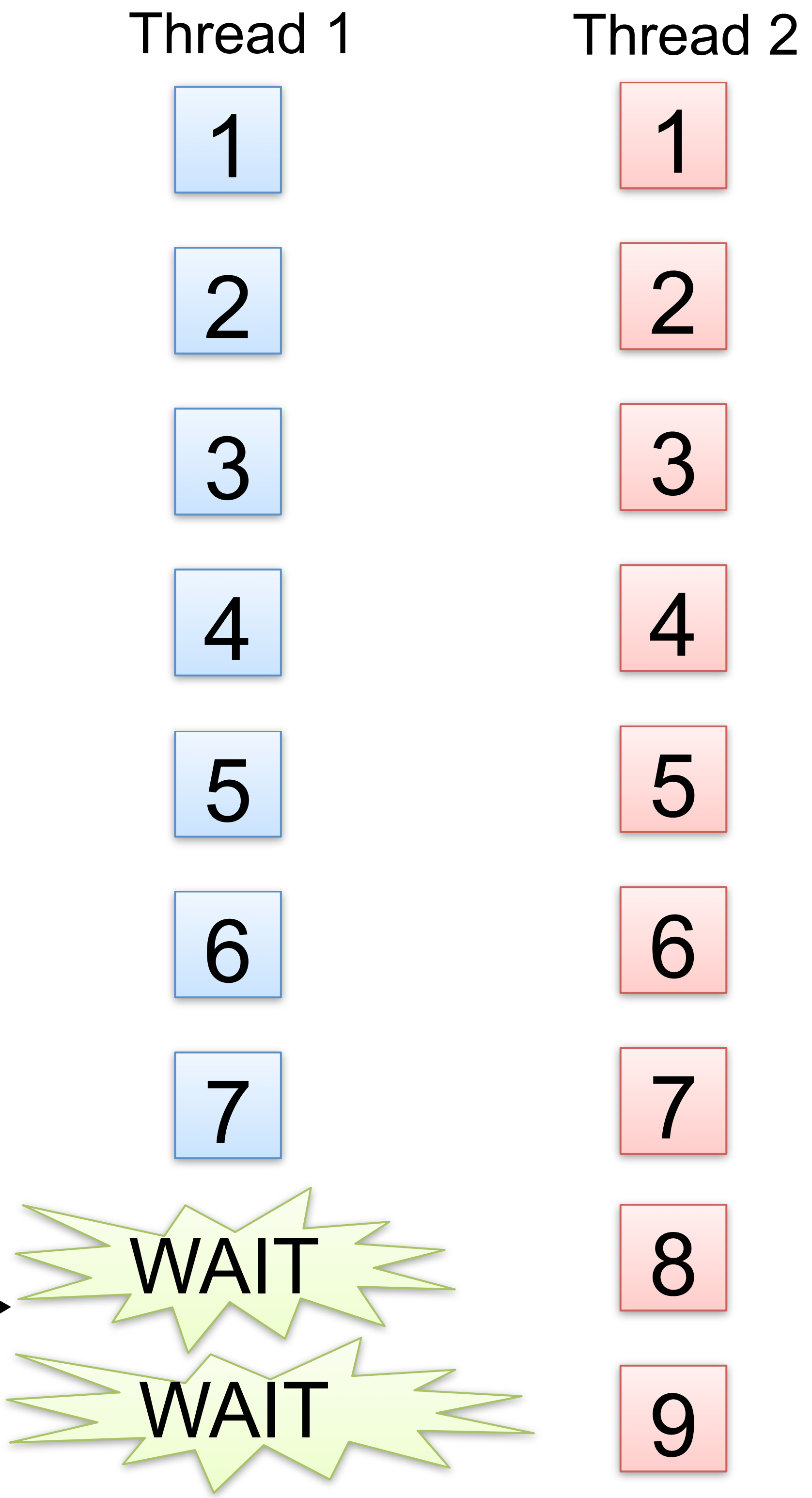


Thread 1	Thread 2
1	1
2	2
3	3
4	4
5	5
6	6
7	7
WAIT	8
WAIT	9

Static Assignment





Waiting will limit scalablility

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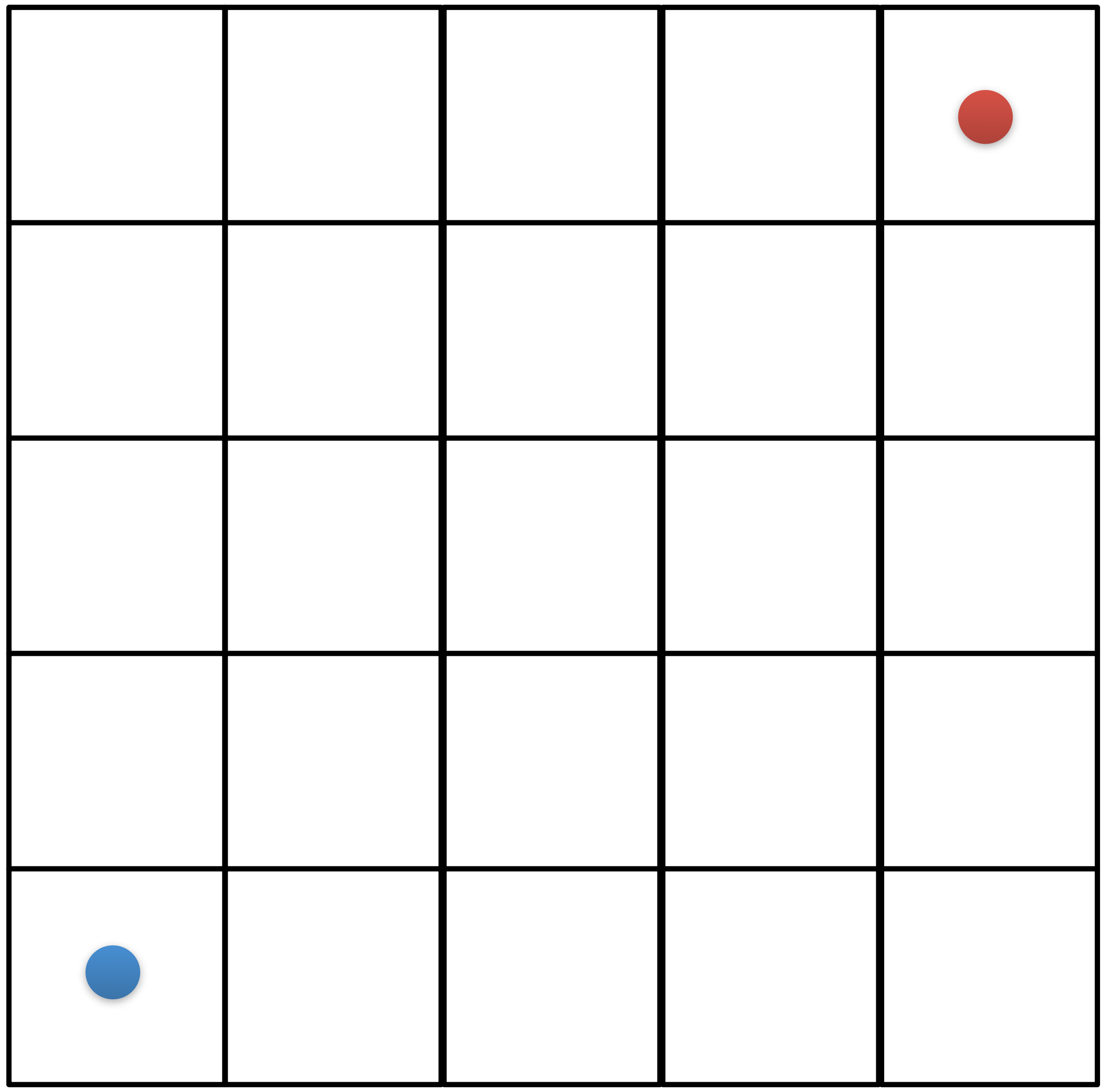
Dynamic Assignment

Thread 1 Thread 2



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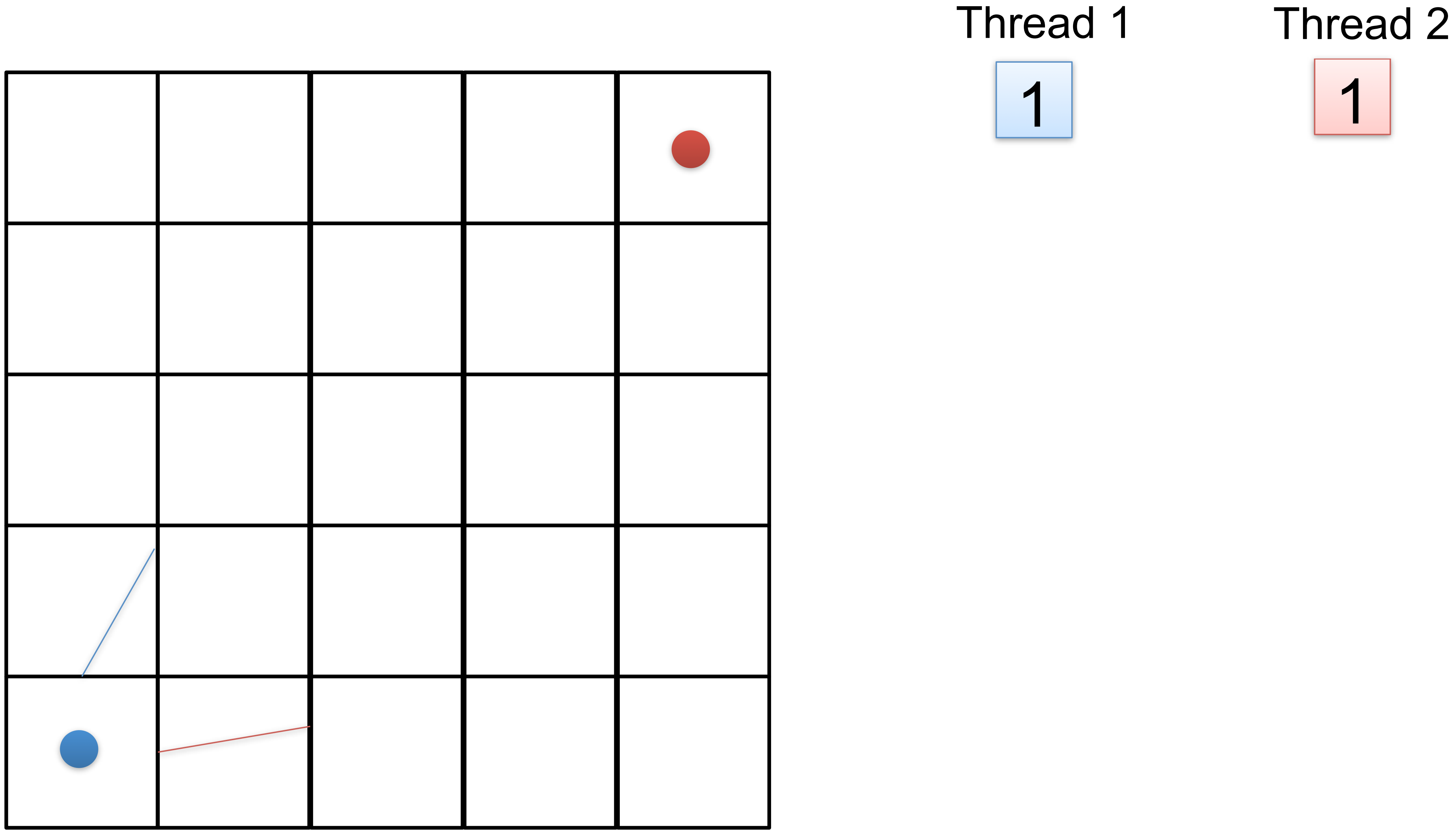

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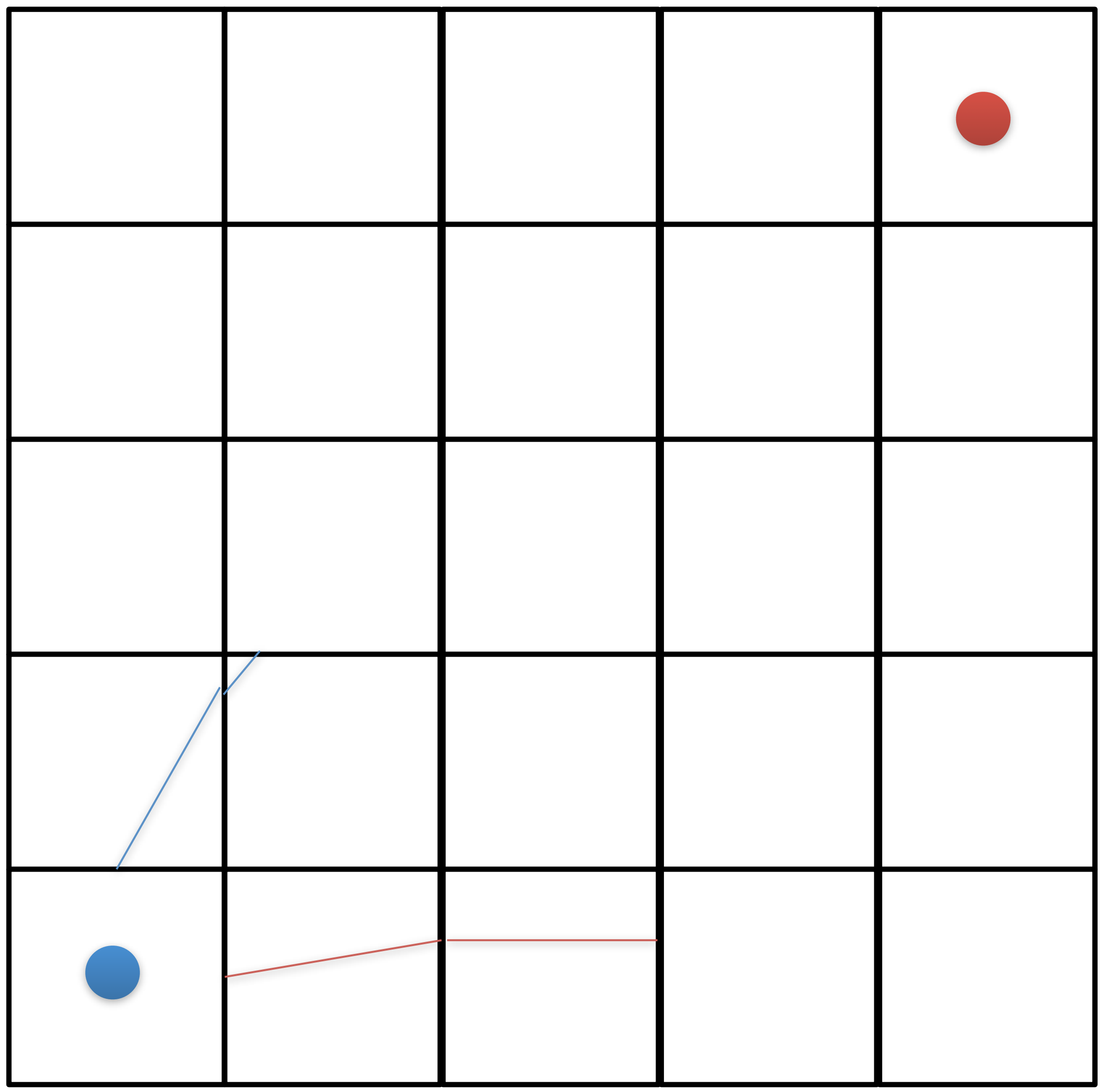

Thread 1	Thread 2
1	1
2	2

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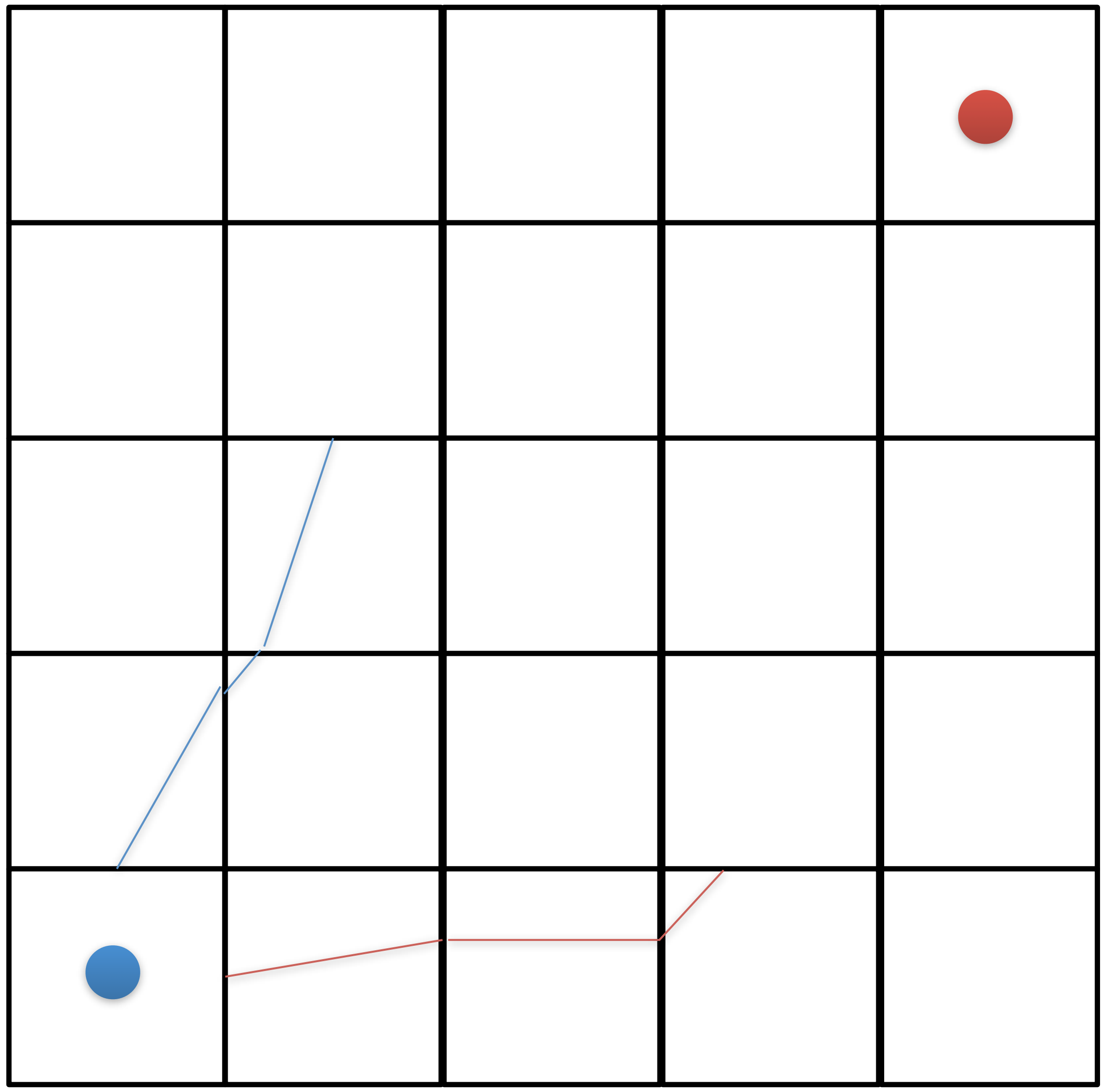

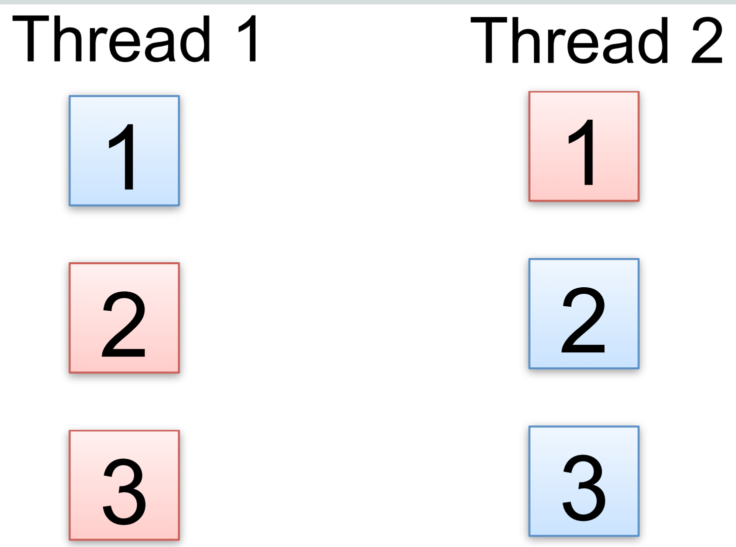

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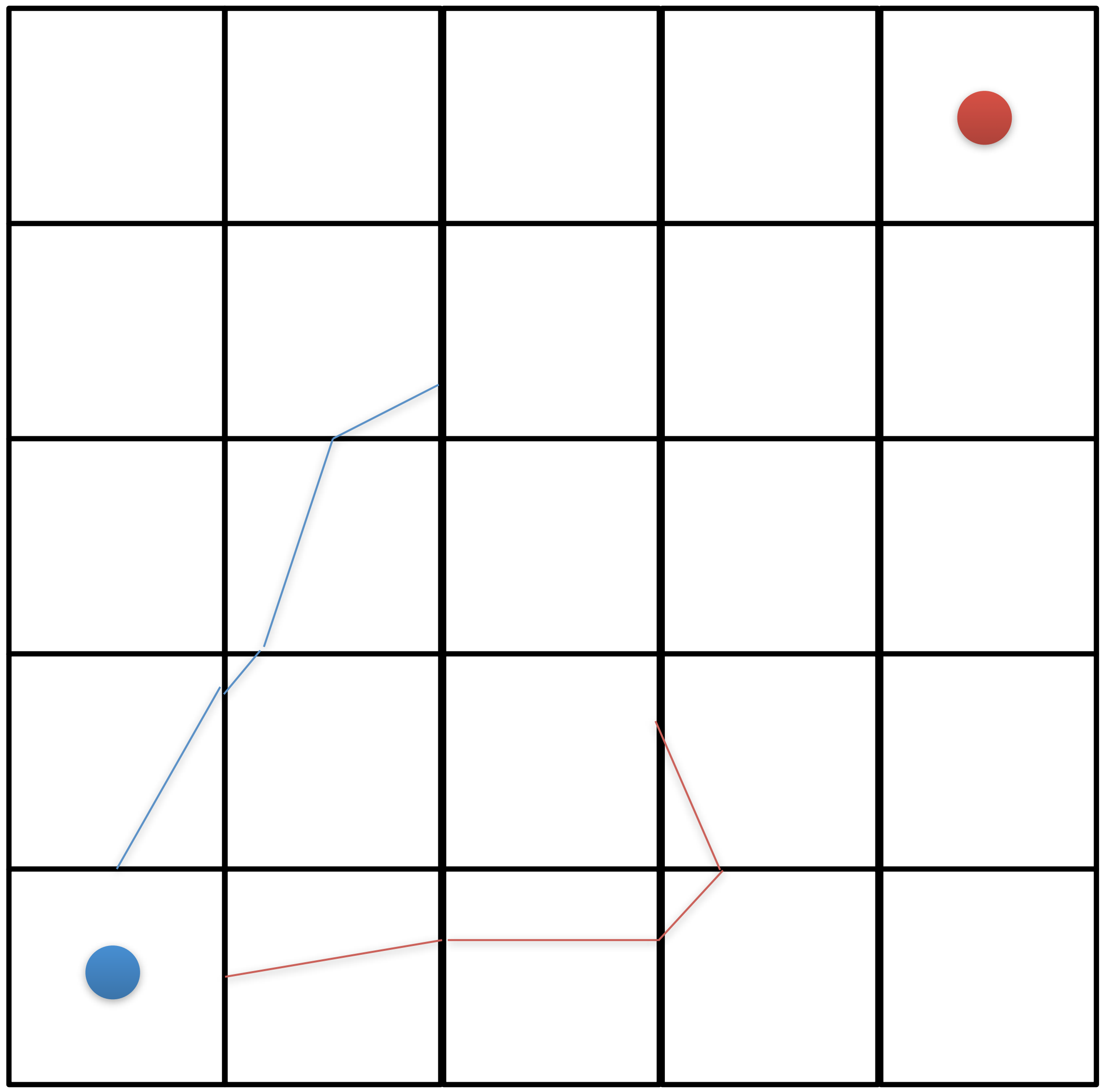

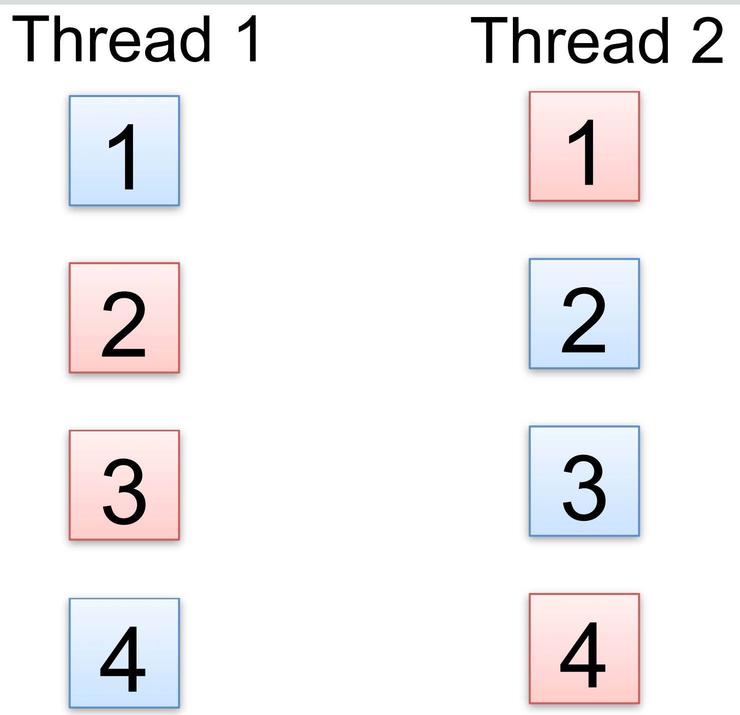

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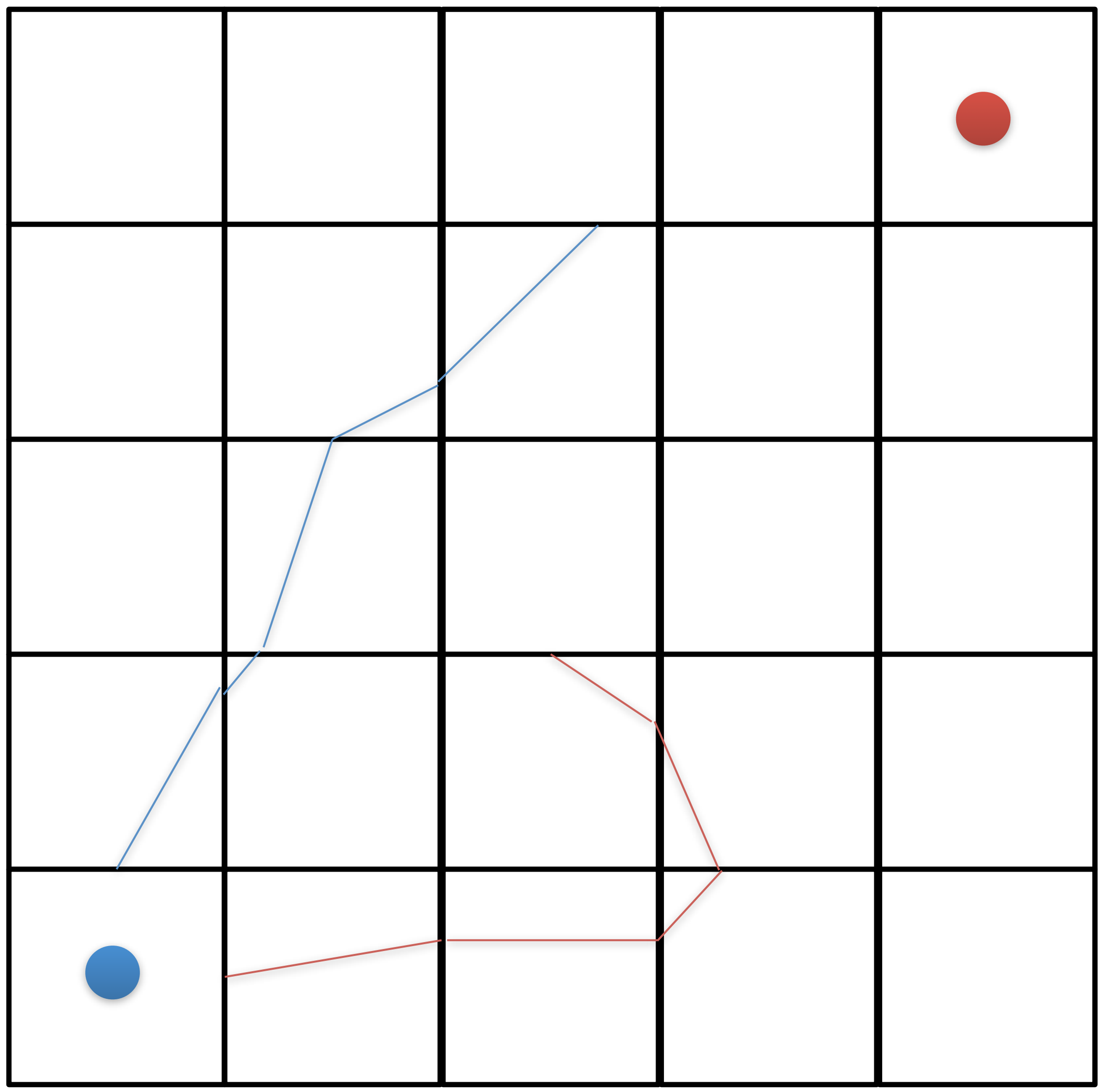
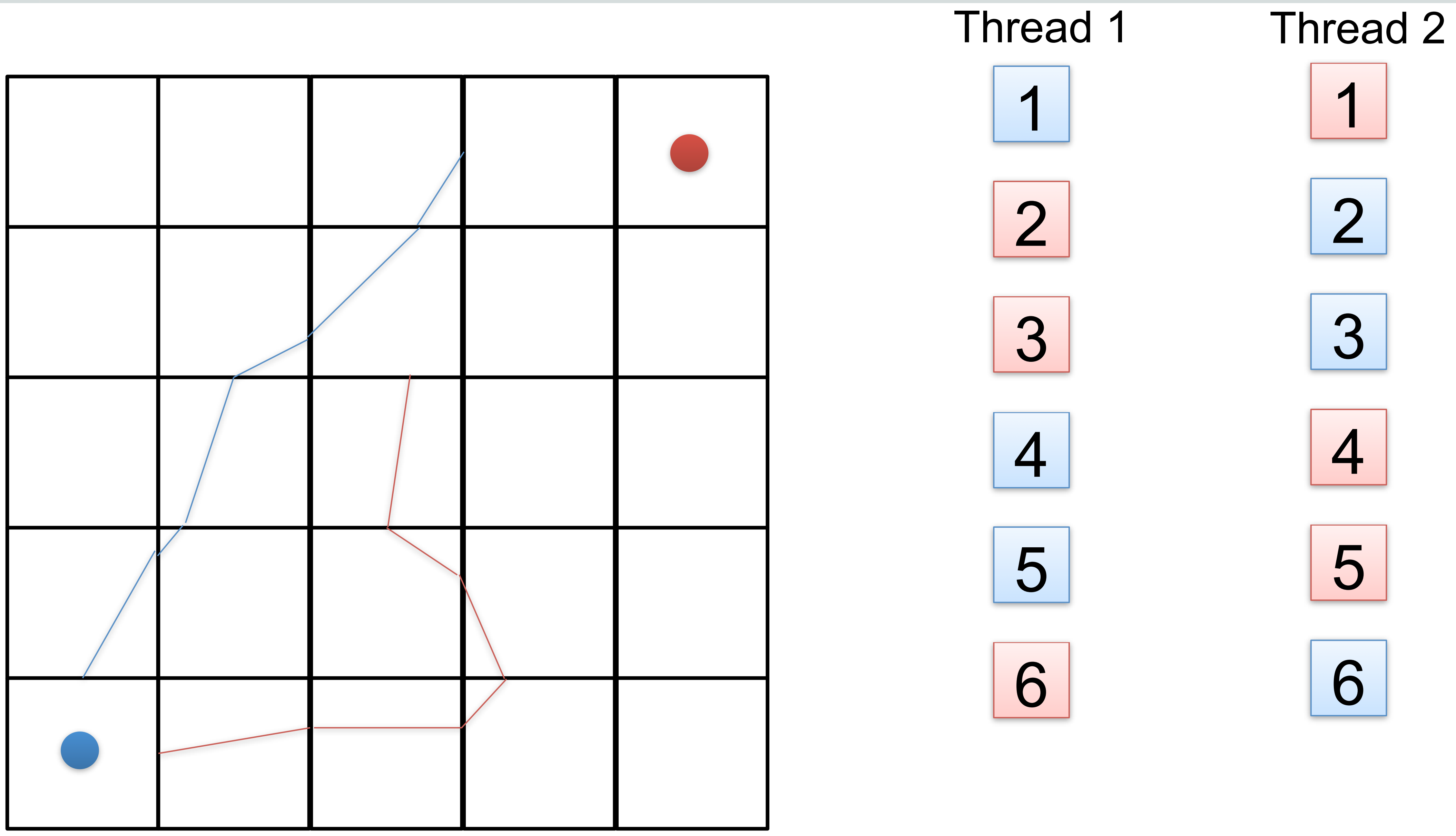
Dynamic Assignment



Thread 1	Thread 2
🟦 1	🟥 1
🟥 2	🟦 2
🟥 3	🟦 3
🟦 4	🟥 4
🟦 5	🟥 5

Dynamic Assignment



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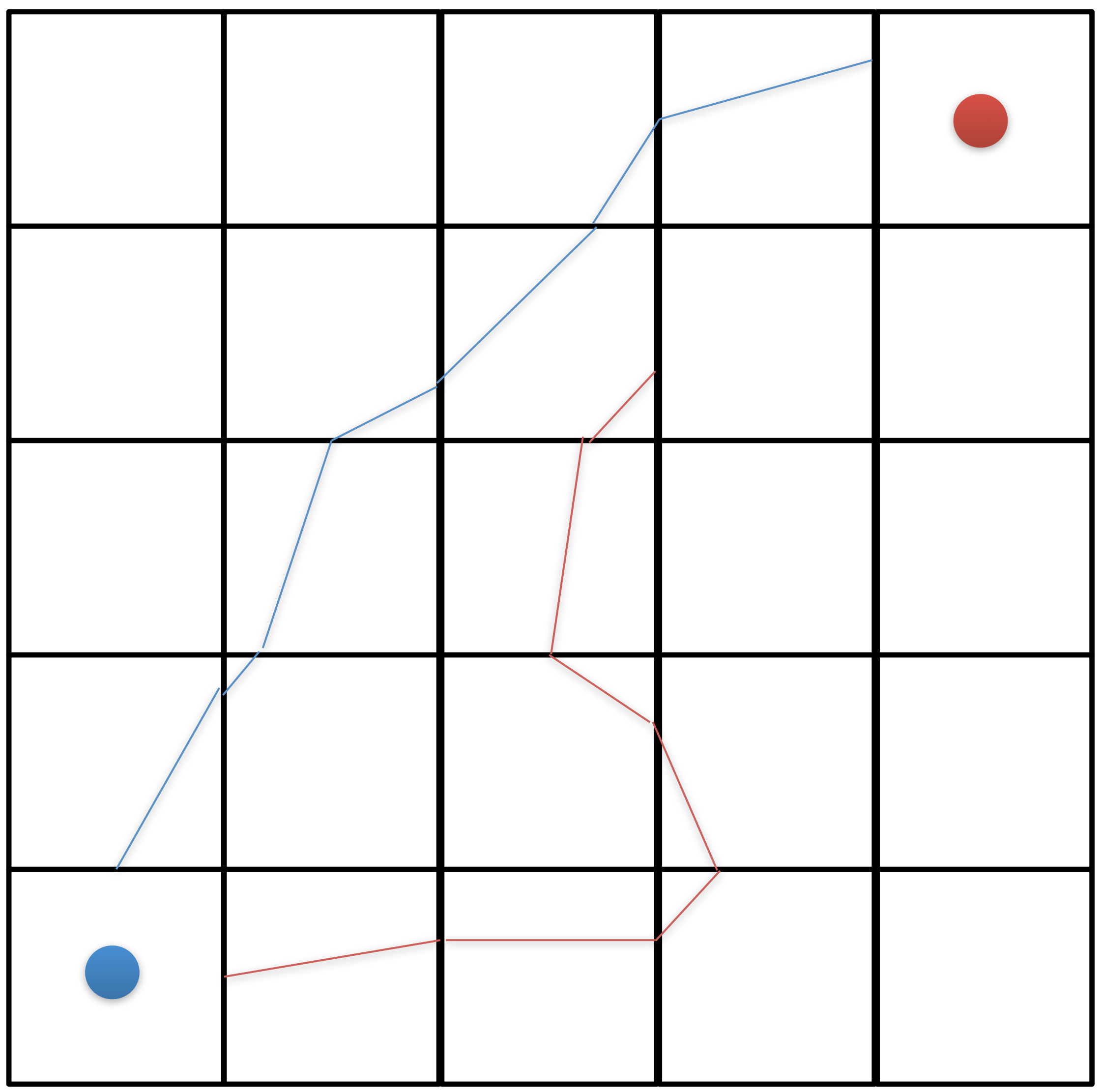

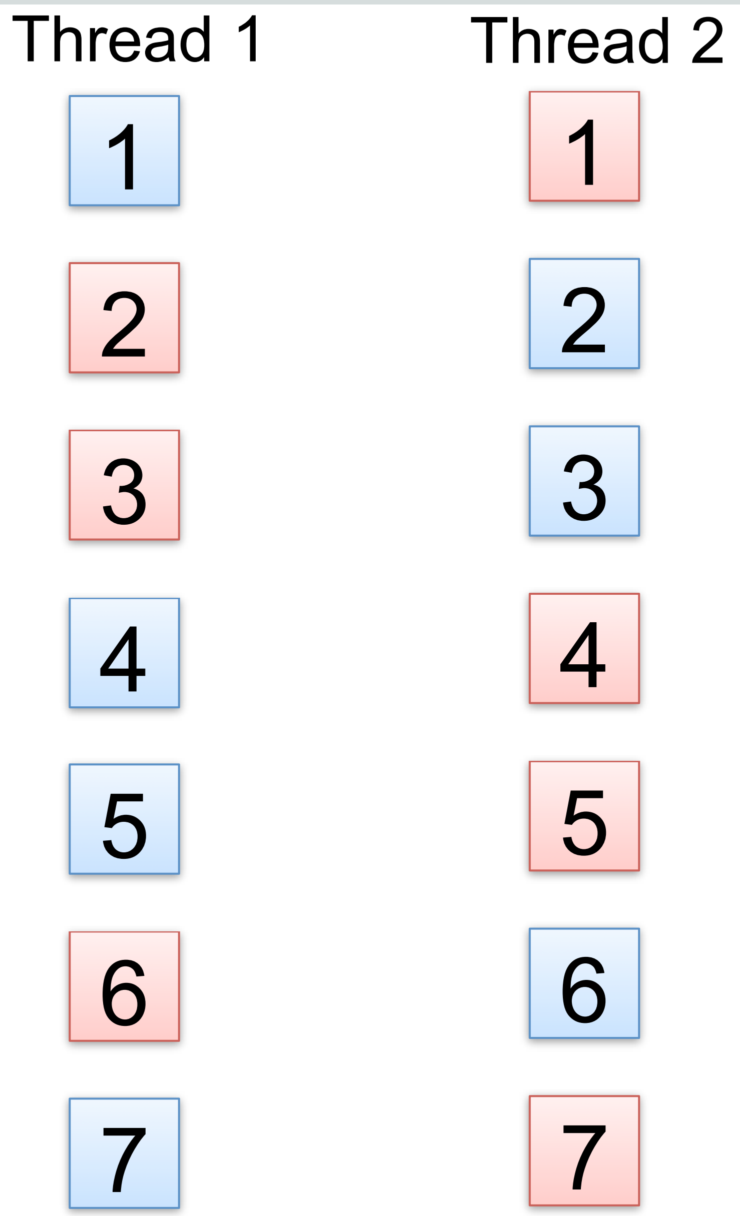

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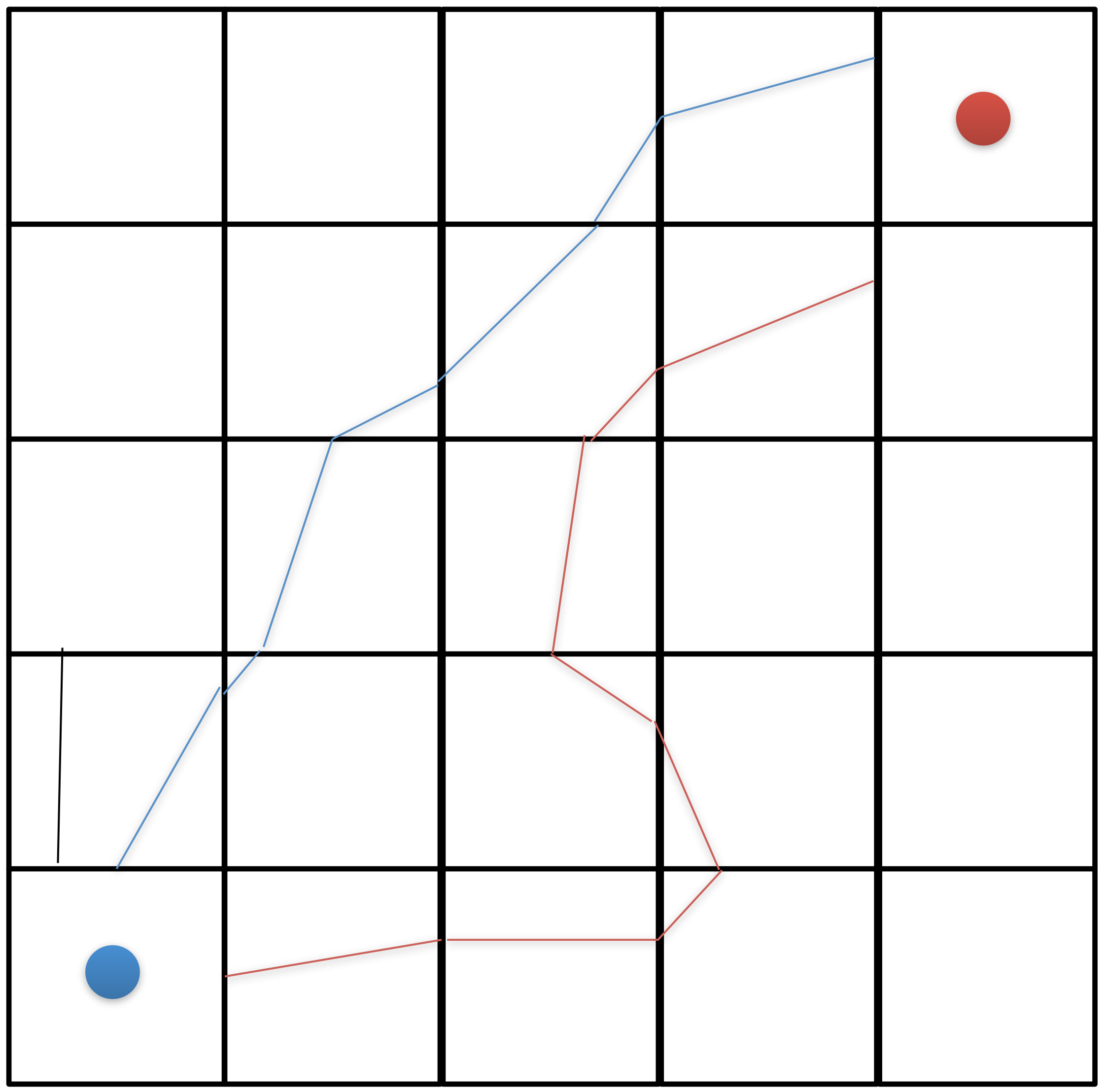
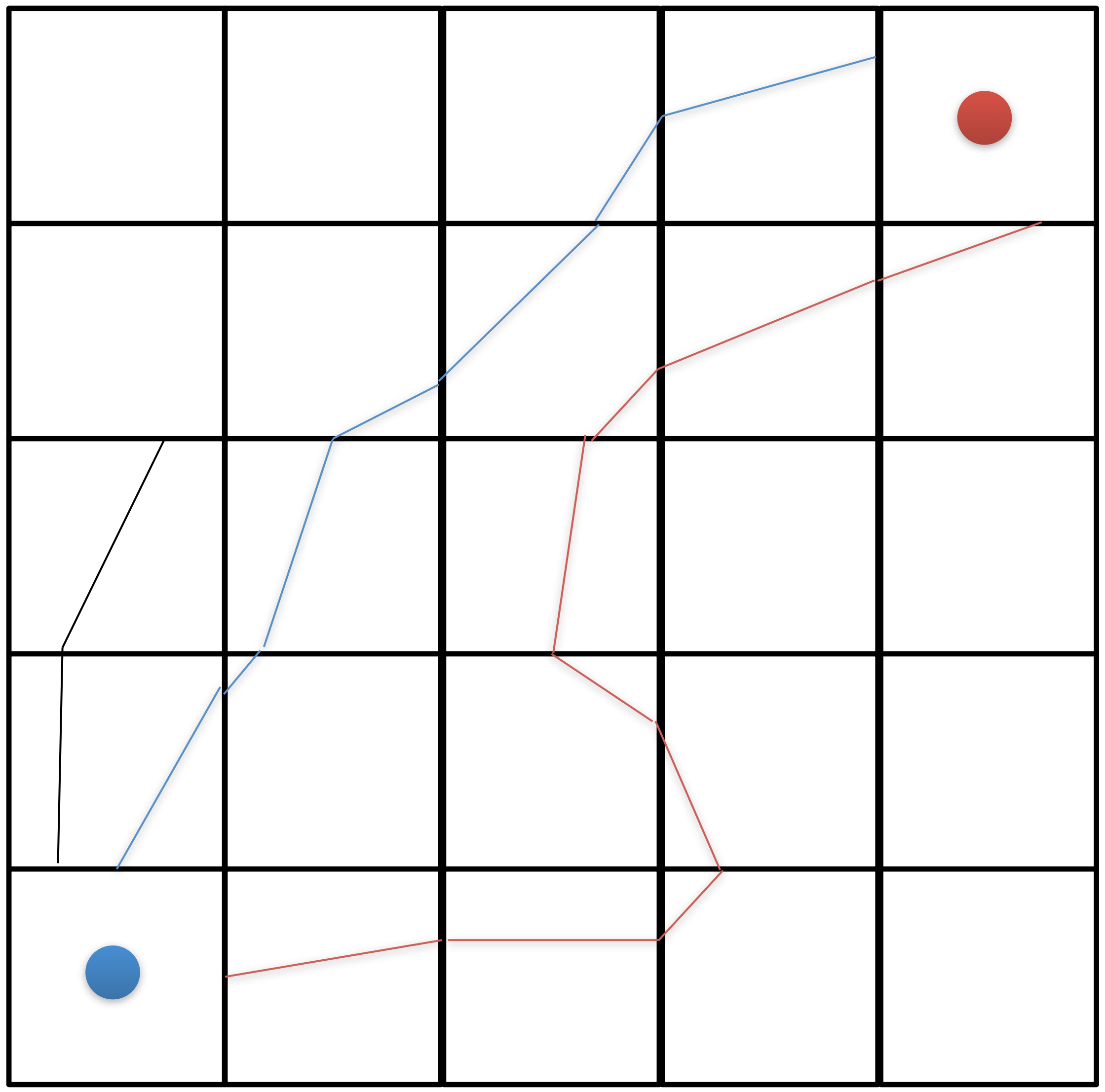


Thread 1	Thread 2
1	1
2	2
3	3
4	4
5	5
6	6
7	7
1	8

Dynamic Assignment



Thread 1	Thread 2
1	1
2	2
3	3
4	4
5	5
6	6
7	7
1	8
2	9

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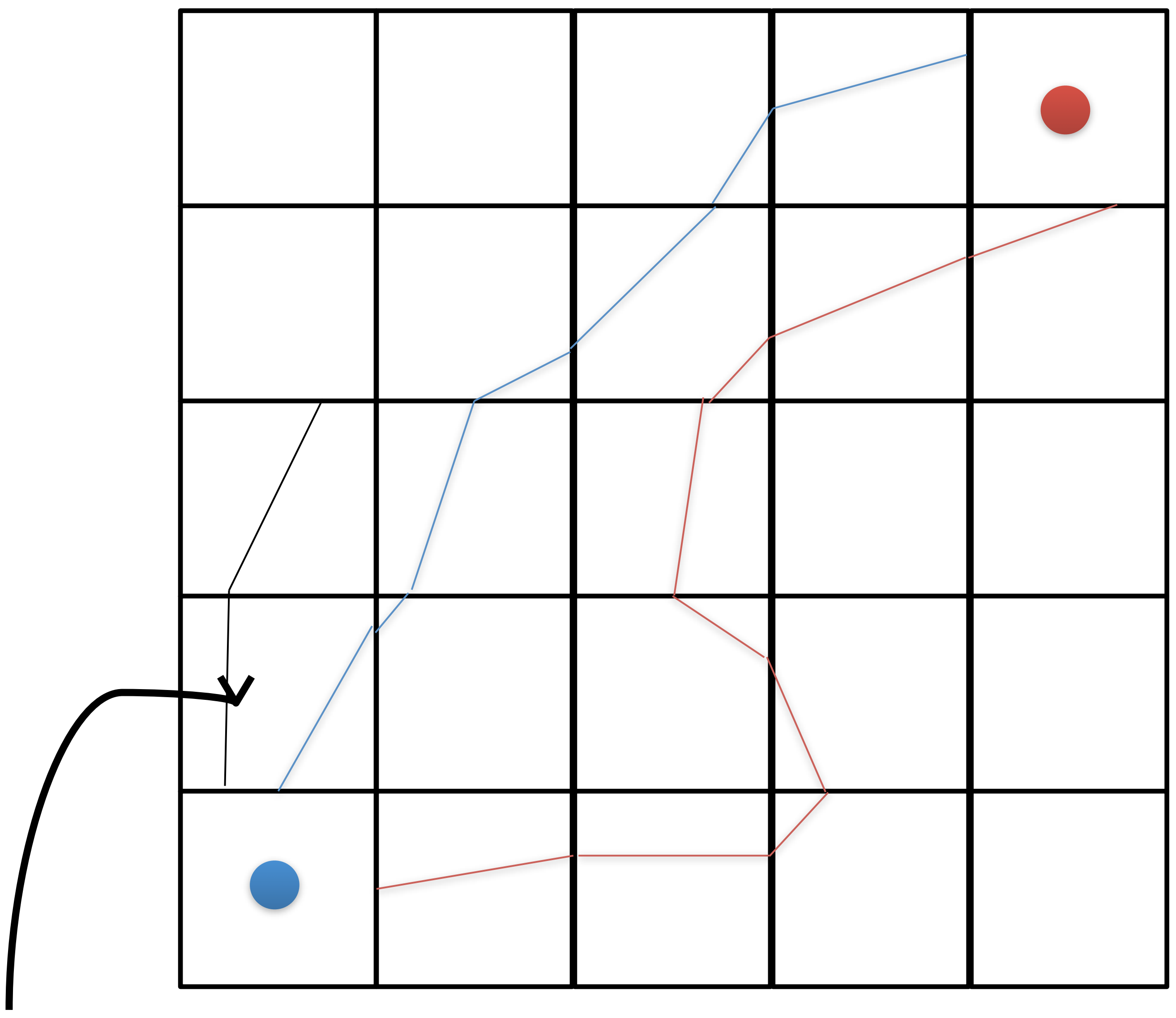
Dynamic Assignment



Thread 1	Thread 2
1	1
2	2
3	3
4	4
5	5
6	6
7	7
1	8
2	9

No waiting. We can start on the next SL directly

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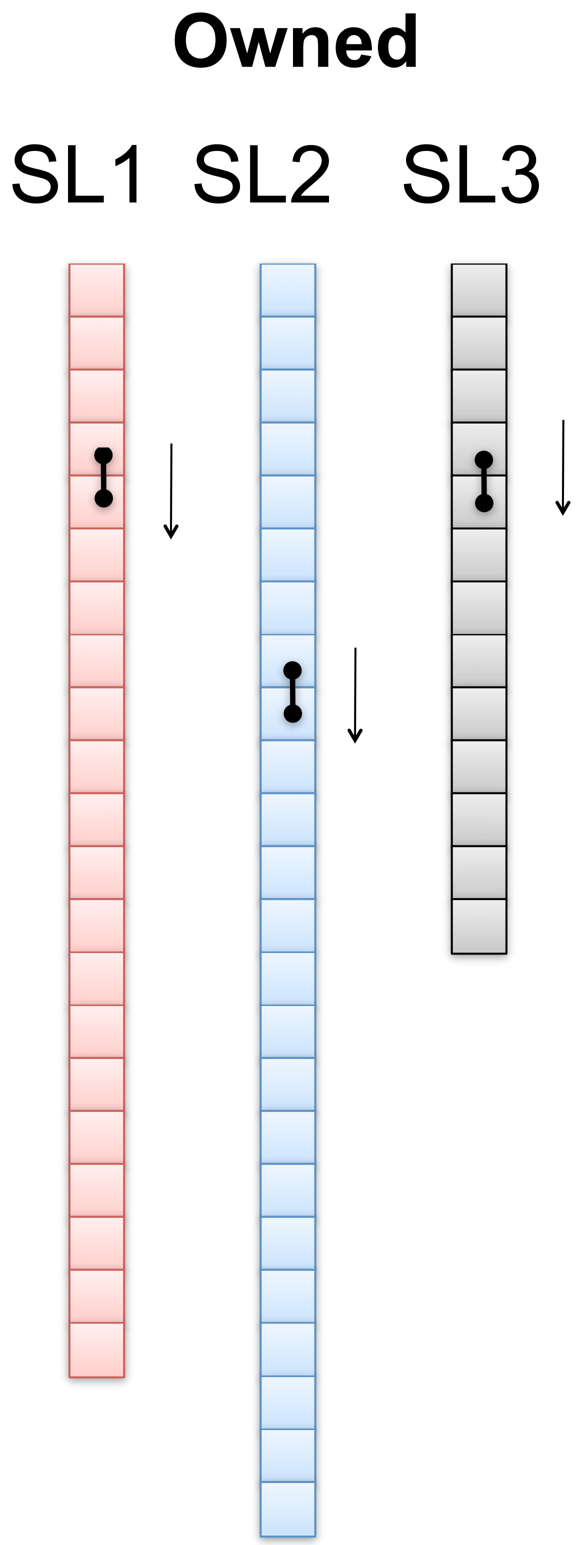
Parallel 1-D Solves, illustration



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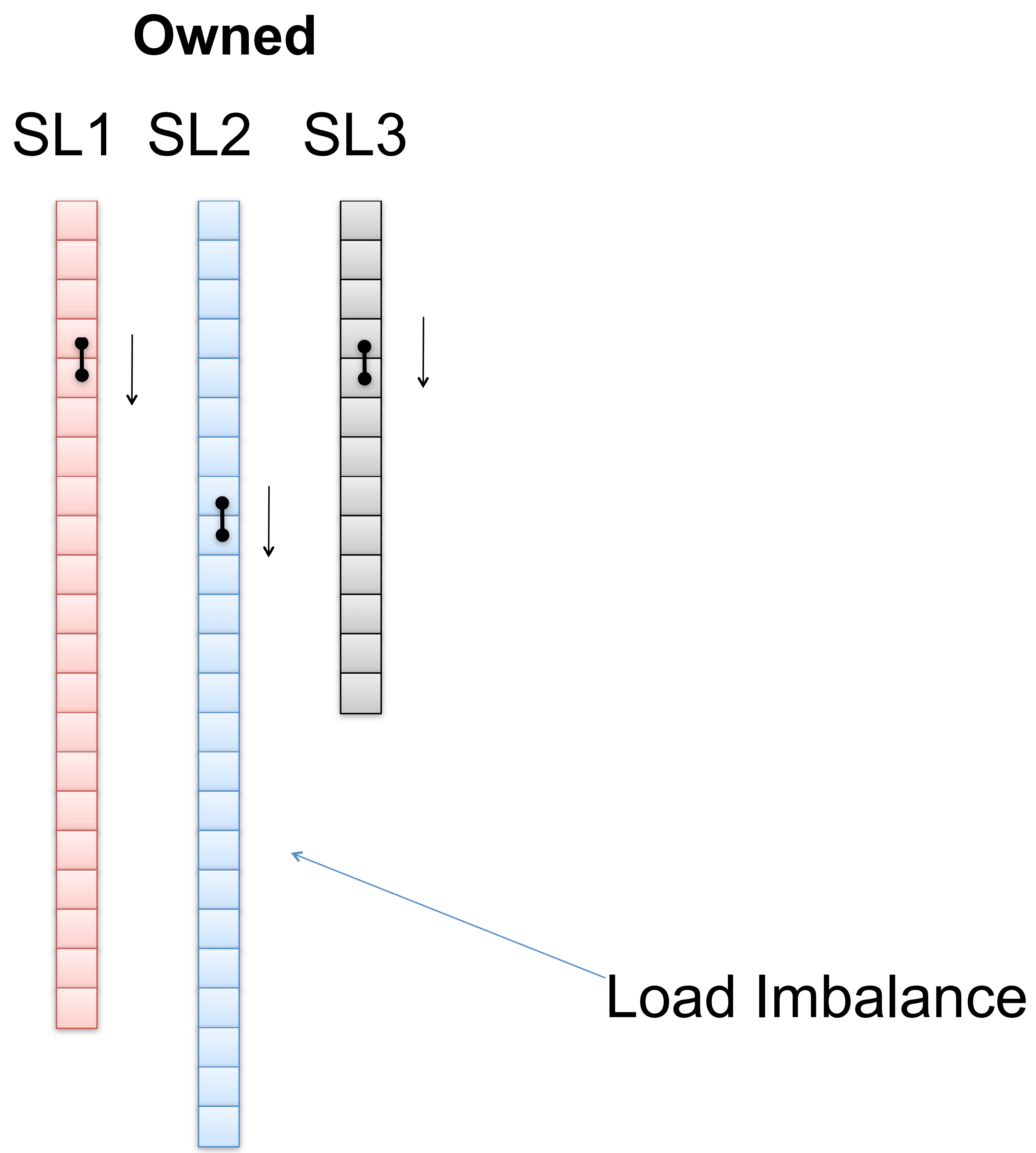
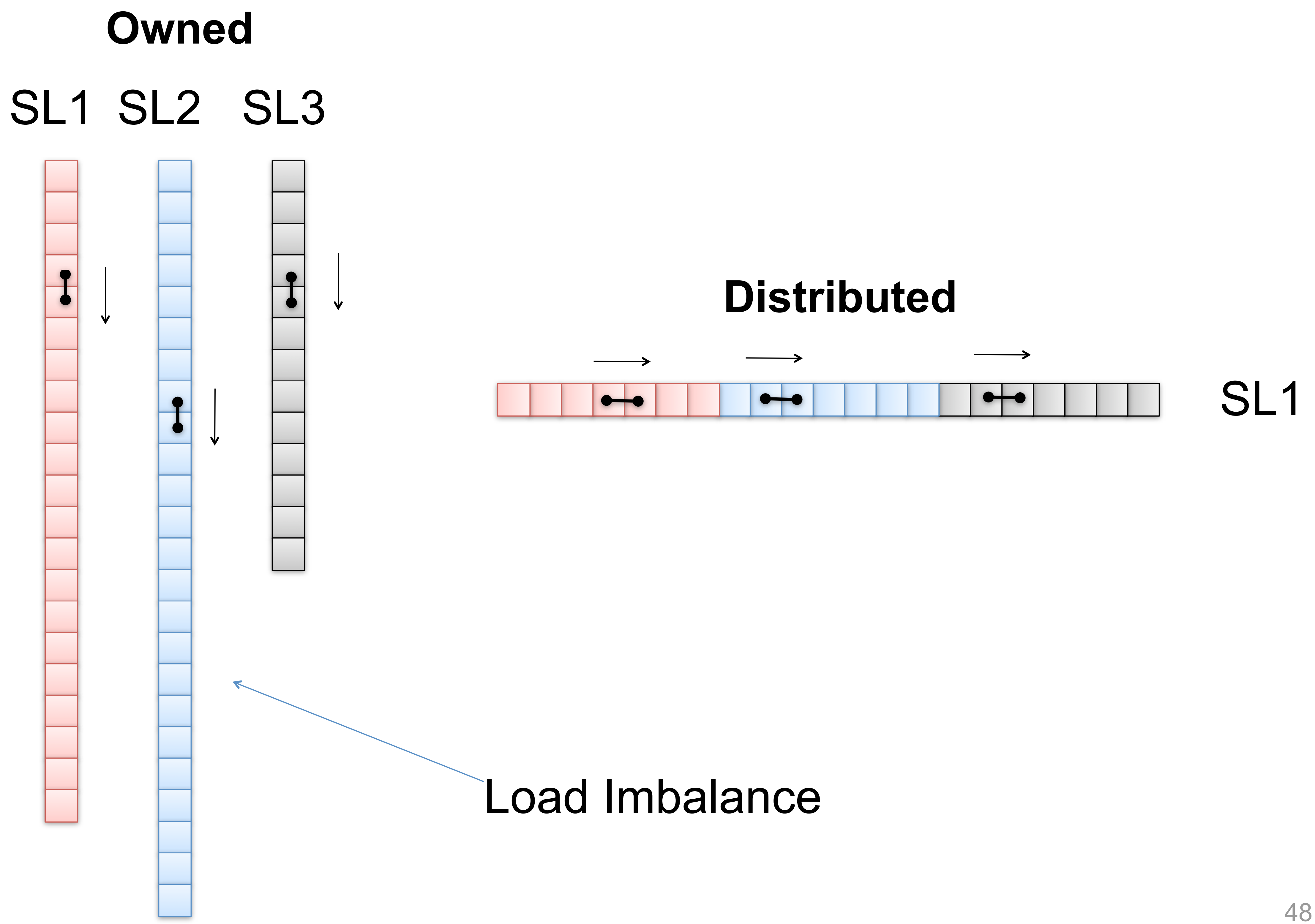
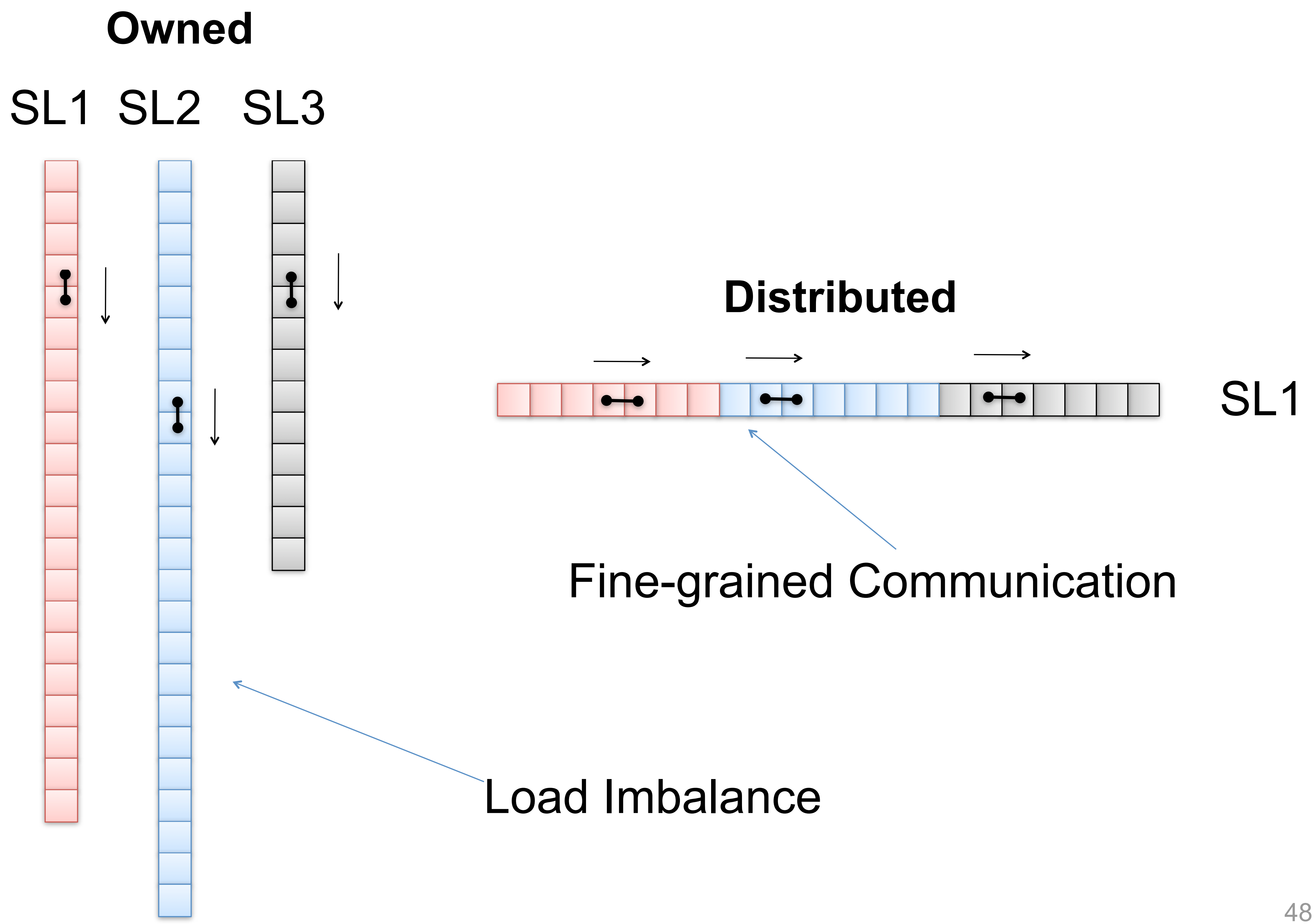
Parallel 1-D Solves, illustration



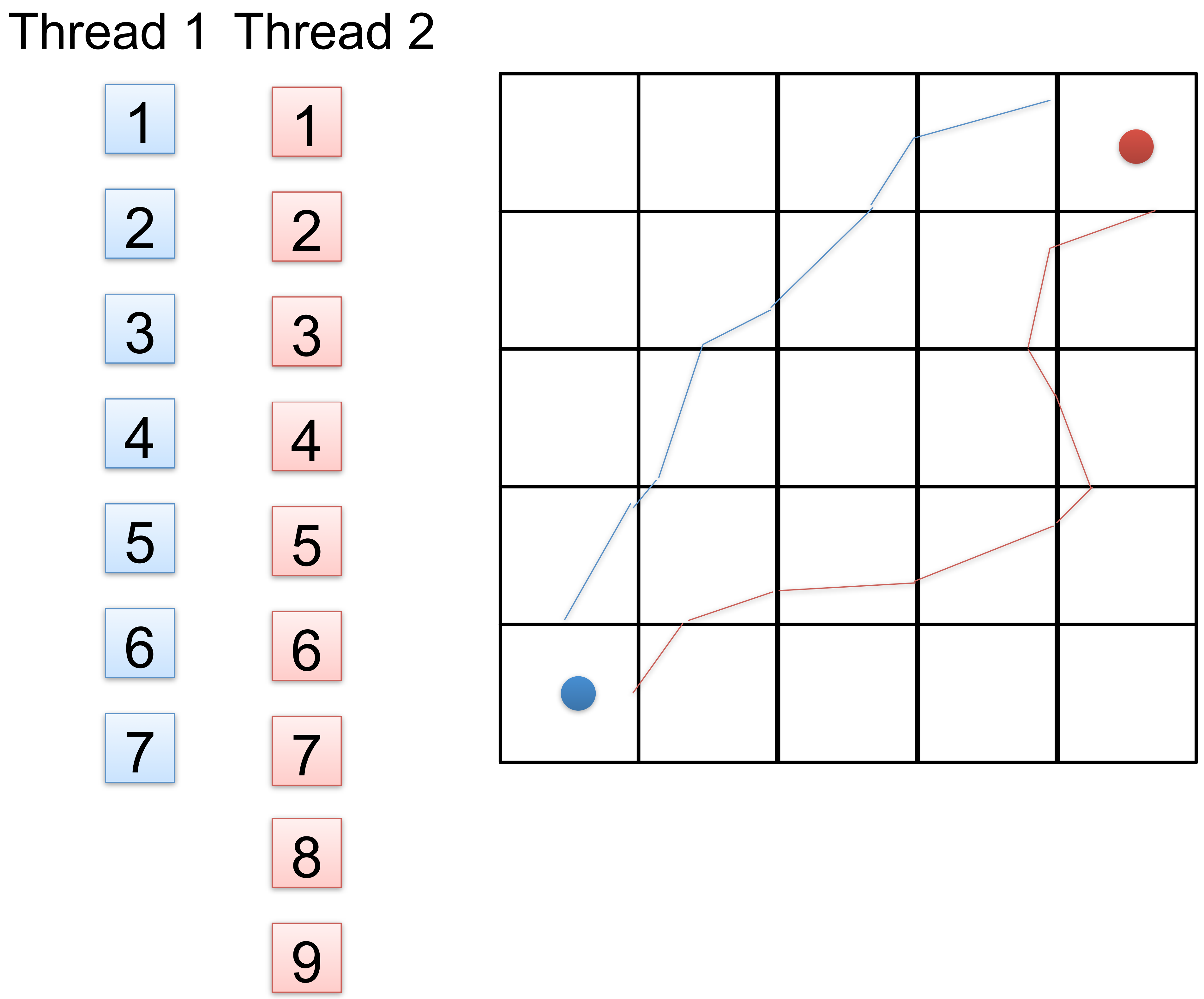
Parallel 1-D Solves, illustration



Parallel 1-D Solves, illustration

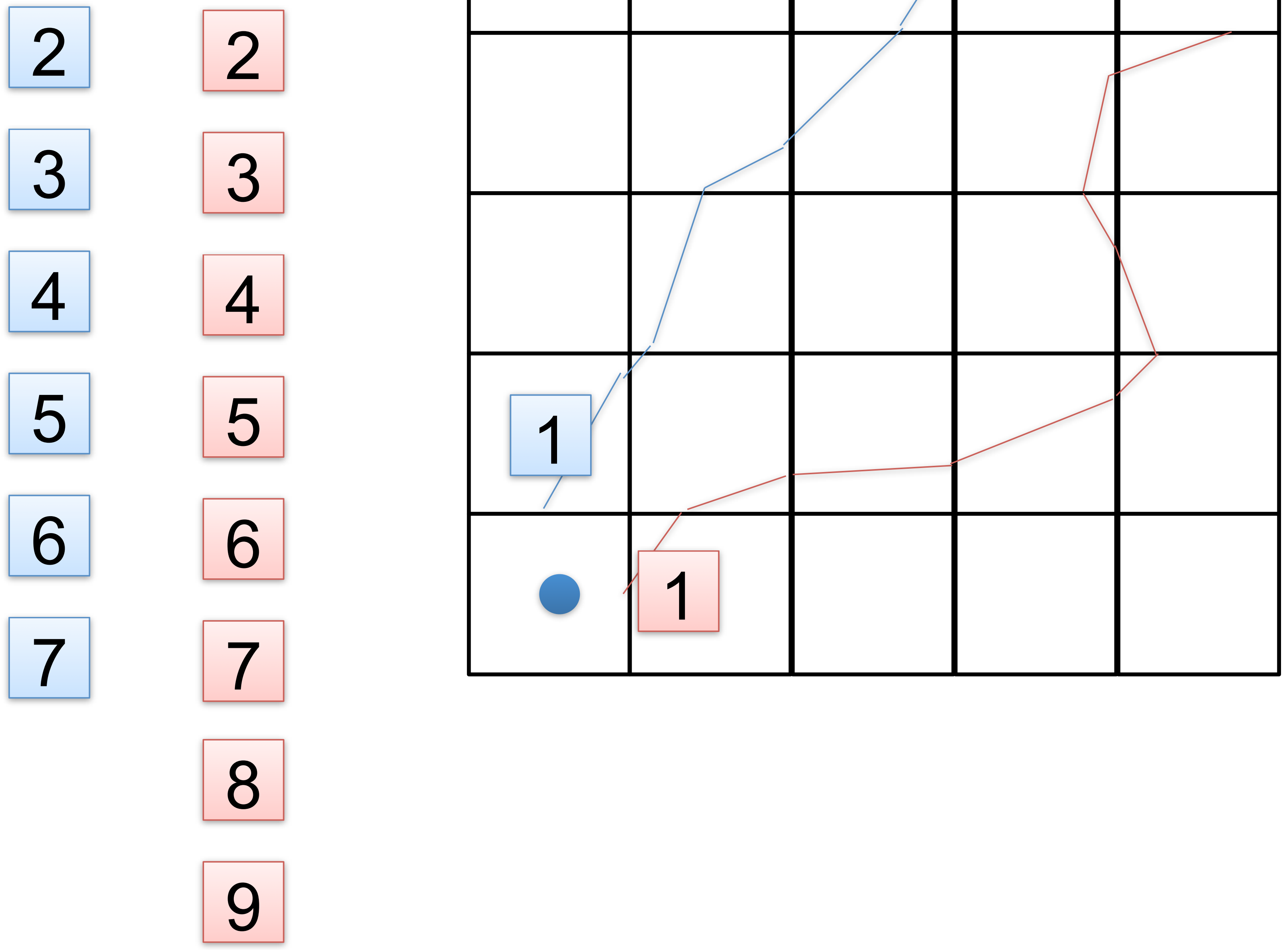


Parallel Mapping, illustration



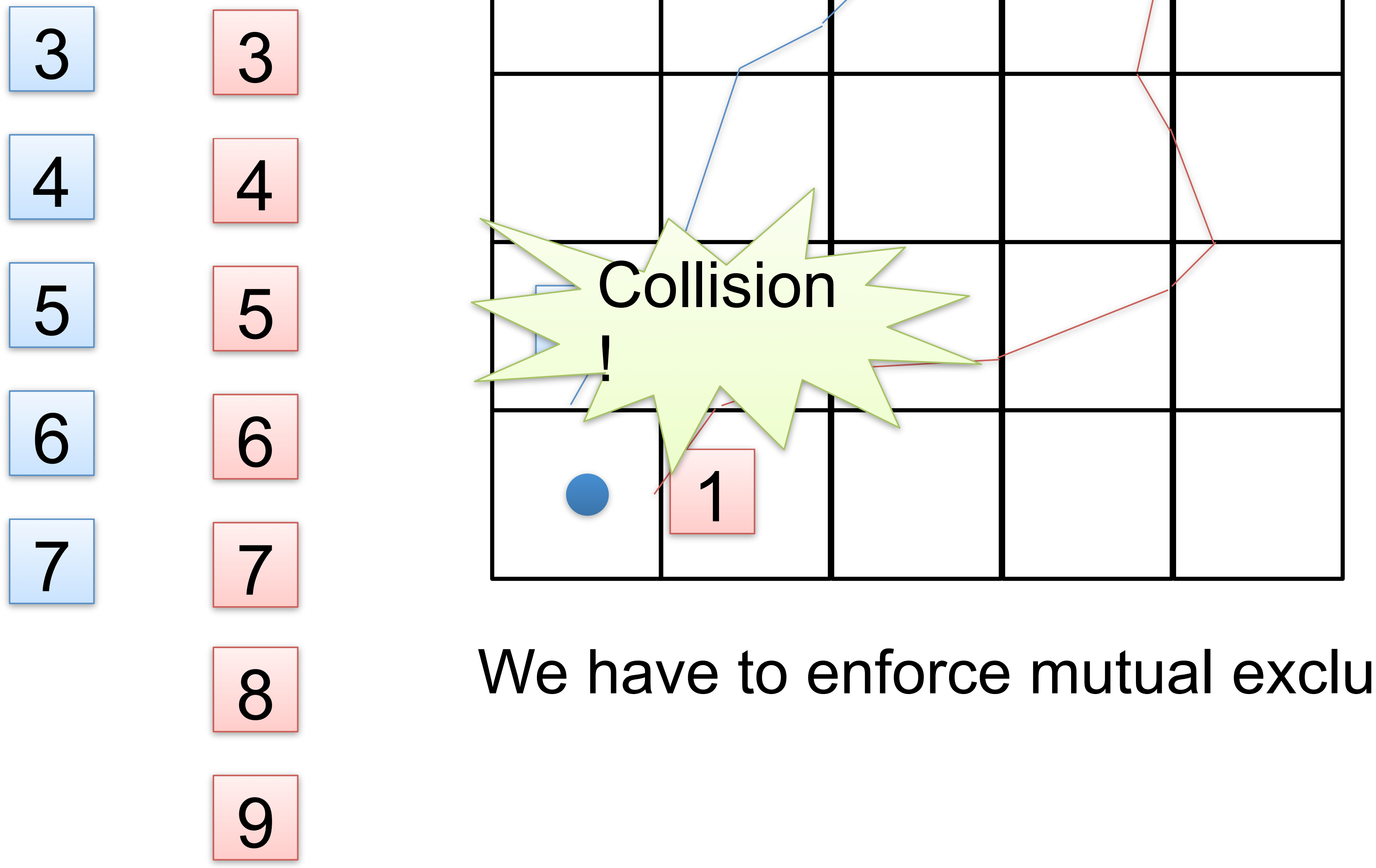
Parallel Mapping, illustration

Thread 1 Thread 2



Parallel Mapping, illustration

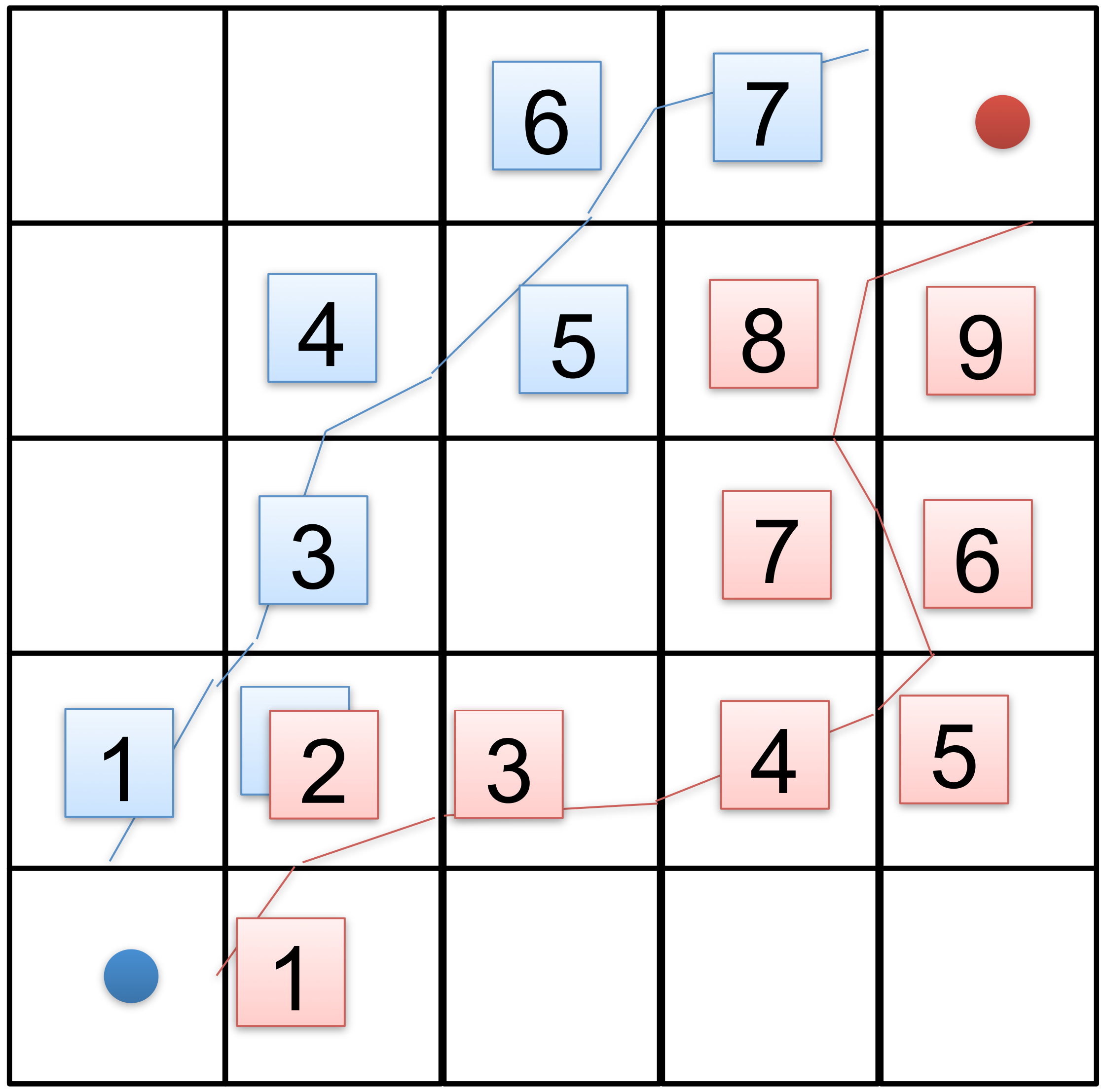
Thread 1 Thread 2



We have to enforce mutual exclusion

Parallel Mapping, illustration

Thread 1 Thread 2



We have to enforce mutual exclusion

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Hamster

- Single-phase tracer flow
- Uniformly refined CCAR grid
- Streamlines are implemented in a way comparable to most industrial codes (3DSL, FrontSim)
  - SPU upwinding scheme for 1-D solves
  - Simple averaging scheme for mapping
  - Dual trace for storing segments
  - 2-point flux approximation (7-point stencil)
  - GMRES solver preconditioned using BoomerAMG from the HYPRE package
  - Tracing using Pollock's method

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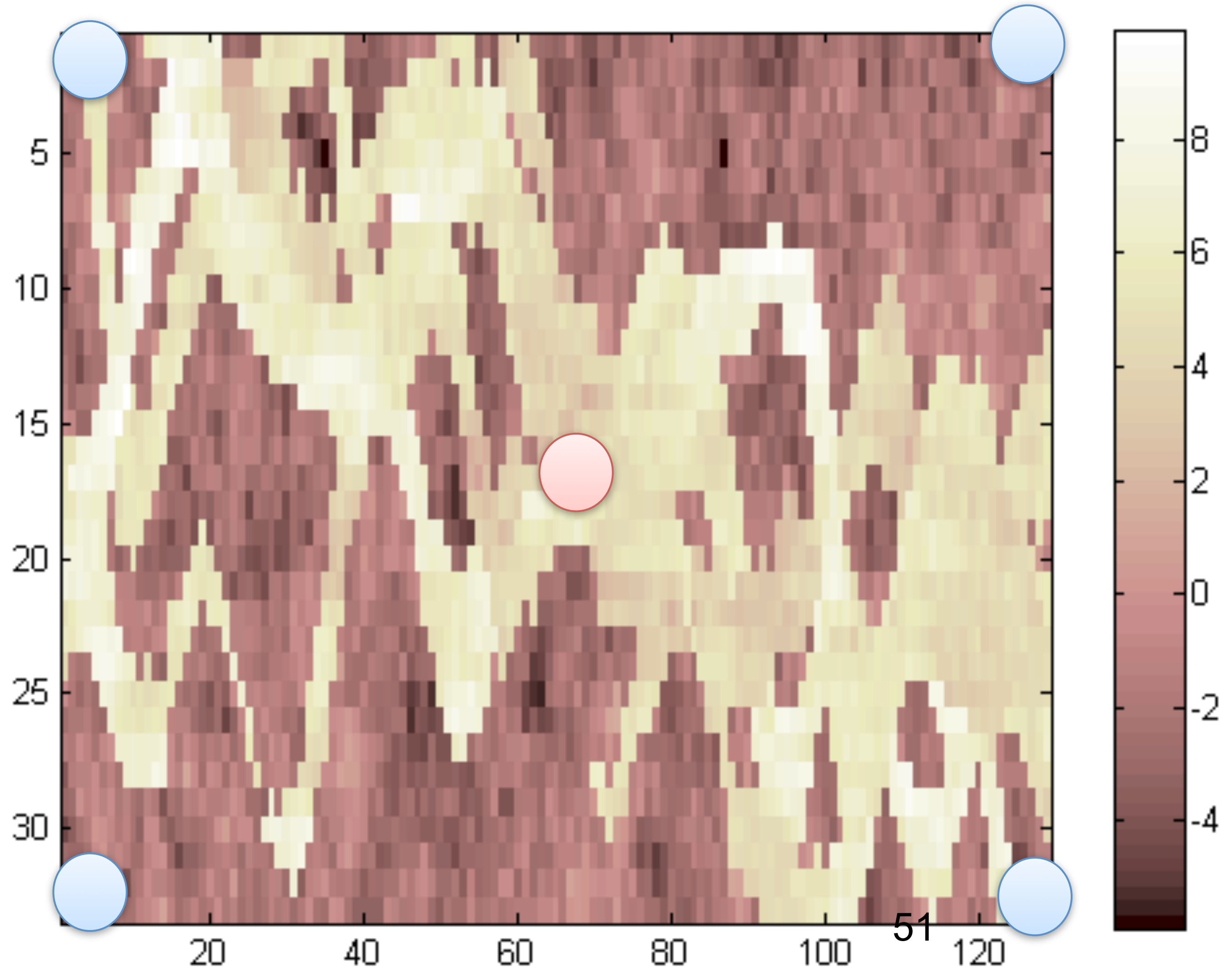
Test Case #1, SPE10

🔵 Production well at constant BHP of 4000 psi

🔴 Injection well at constant BHP of 10000 psi

- We cut out a domain of size 32x128x32 from the bottom 32 layers of SPE10
  - 131,072 cells
- One global time step of 2000 days (~5 years)
- 10,096 streamlines



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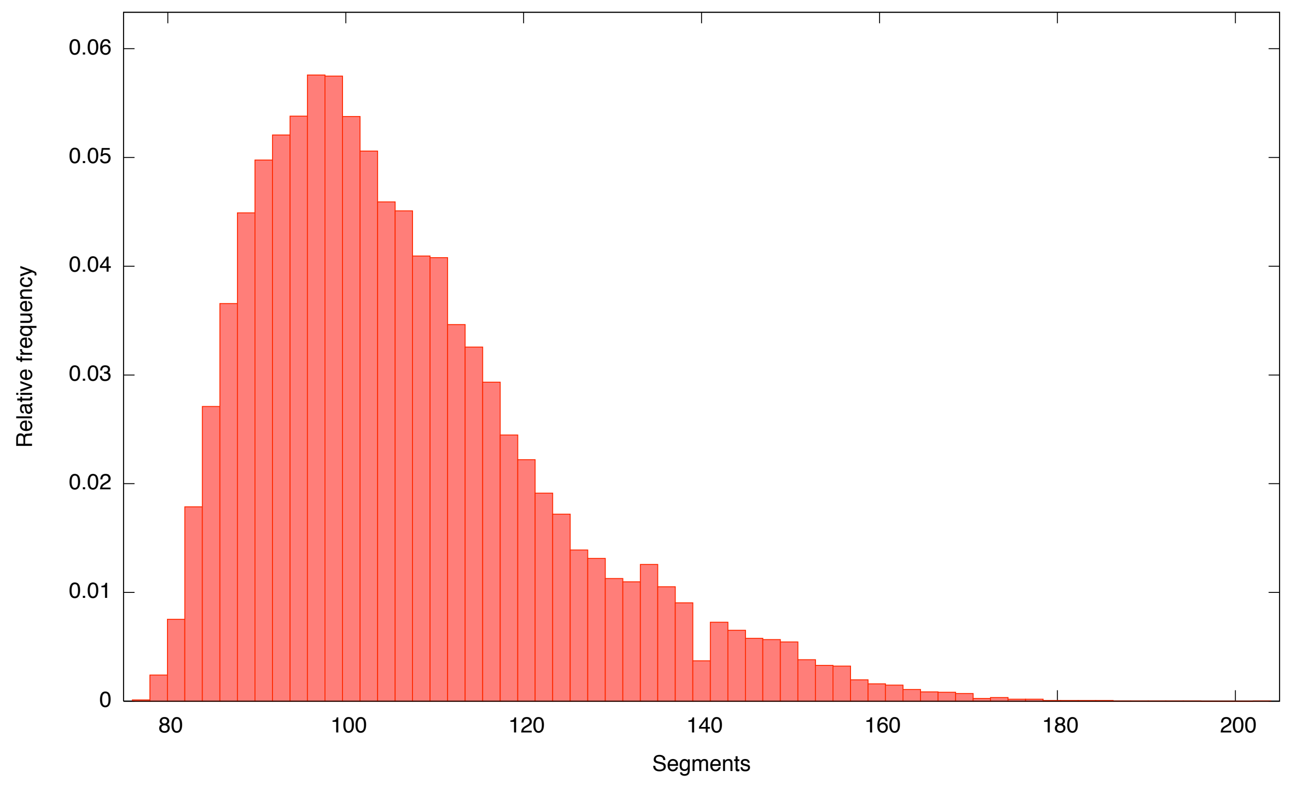
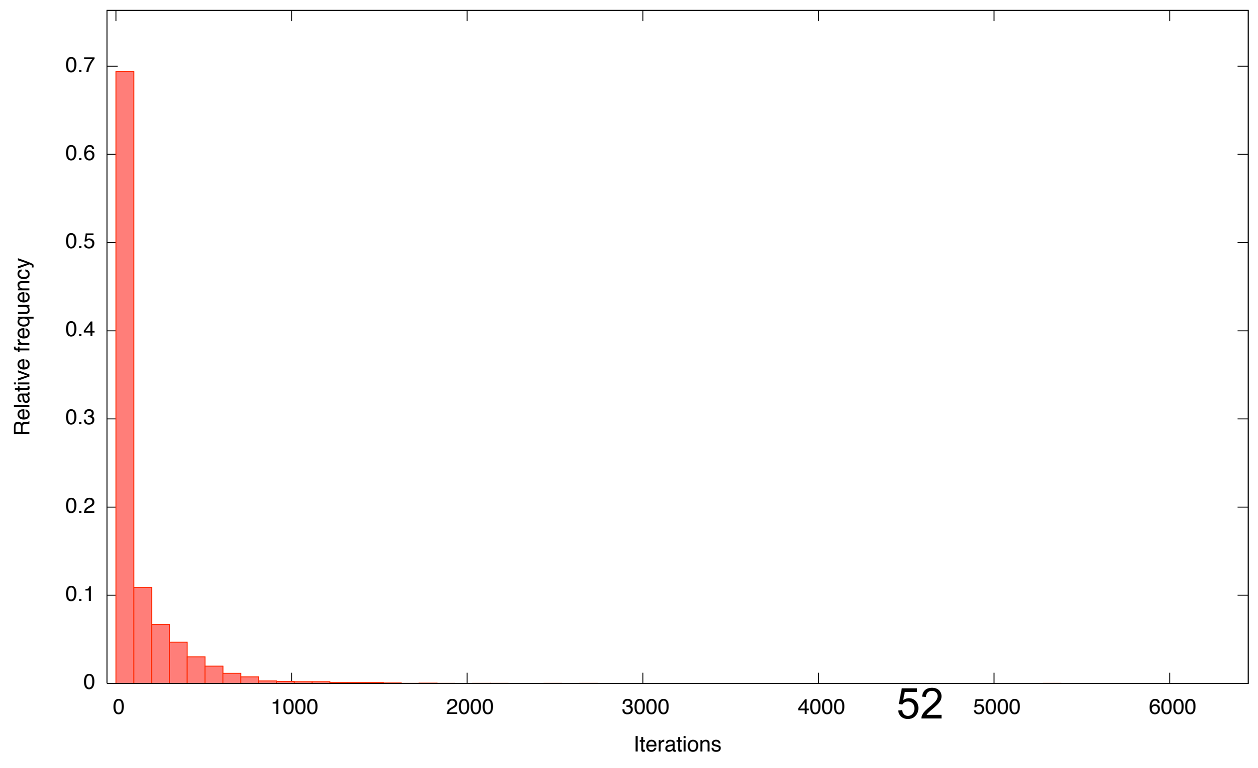
Streamline Statistics, SPE10

	Min	Max	Median	Average	Std.Dev
Number of Segments	77	203	103	107	106
Number of Iterations	1	6296	19	119	244





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Performance on 4 way dual-core Opteron

Threads	Execution Time (s)	Speedup
1	12.01	1.00
2	6.64	1.82
4	3.74	3.23
8	2.21	5.70

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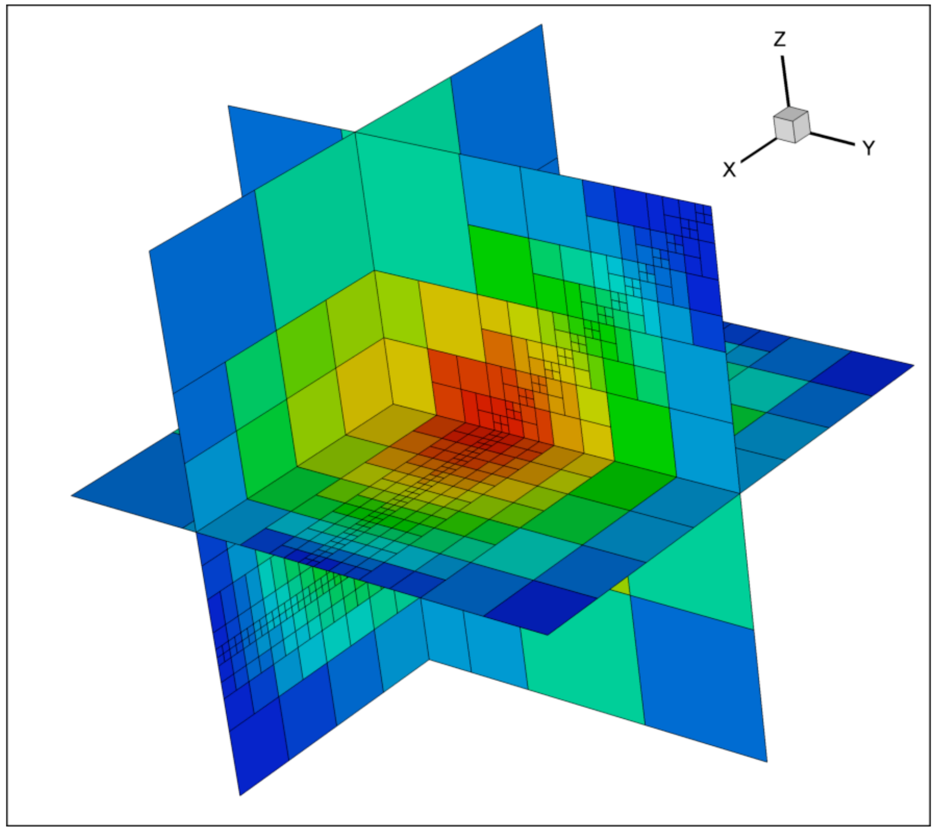
Scalability Problems

- Did not take NUMA into account
  - Linux does not yet support page migration
- Cell and face ordering on unstructured grids
  - Every cache line should be used
  - Communication
  - False sharing



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Performance on Sun Niagara 2 (UltraSPARC-T2)

Threads	Execution Time (s)	Speedup
1	84.40	1.00
2	42.44	1.99
4	21.36	3.95
8	10.91	7.73
16	6.27	13.47
32	3.81	22.15
64	157.88	0.53

Manual (Optimal) Load Balancing

- In the static case, we can improve load balance by assigning a **weight** representing the compute time
- We then formulate a discrete optimization problem using these weights
- Possible parameters for load balancing weights
  1. Rock properties (segment counts)
  2. Total time of flight (iterations)
  3. Well placement (segment count)
  4. Size of multi-phase region (flash calculations)
  5. Cache misses and communication costs (hardware)

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Load Balanced Algorithm

Algorithm 2 Load Balanced Streamline Simulation

```
Require: $T_{end} \leftarrow$ simulation end time
Require: $dt \leftarrow$ global timestep
Require: $T \leftarrow dt$
 1: repeat
 2:    Solve pressure equation
 3:    Calculate velocity field
 4:    repeat
 5:       Select launch points for streamlines and build bundle
 6:       Assign launch points in bundle to threads
 7:       for all streamlines in bundle do
 8:          1:st trace, count segments
 9:          2:nd trace, pick up pressure grid data, store the segments
10:       end for
11:       Load balance bundle
12:       for all streamlines in bundle do
13:          Build streamline grids from segment and pressure grid data
14:          Solve the corresponding 1-D transport equations
15:       end for
16:       Map new values of the transport variables to the pressure grid
17:    until Domain is sufficiently covered
18:    $T \leftarrow T + dt$
19: until $T \geq T_{end}$
```

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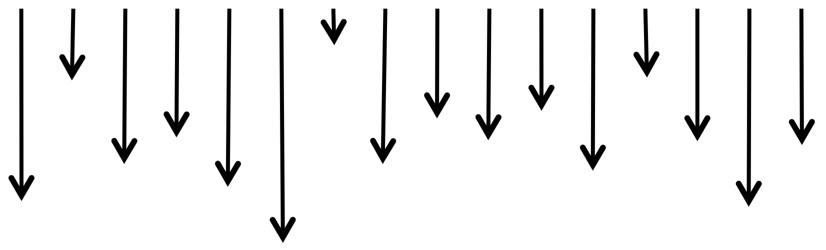
Reassignment problem, 16 streamlines

Tracing



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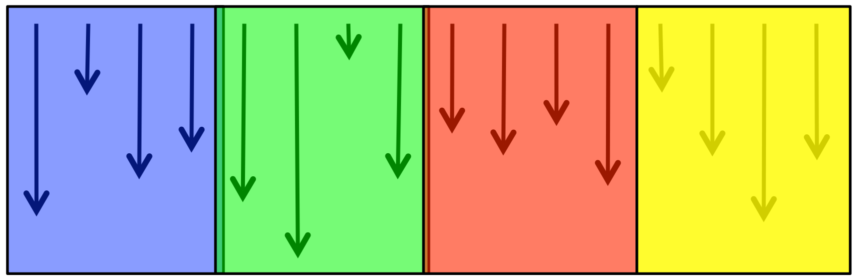
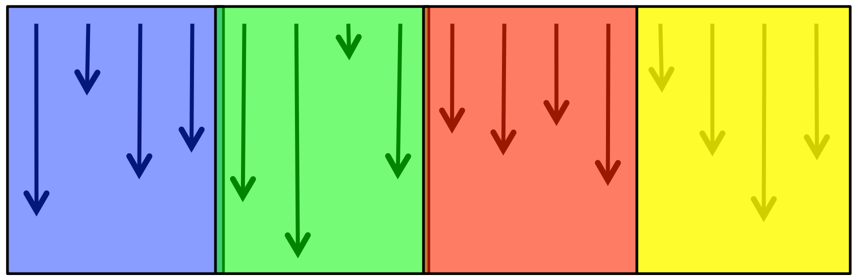
Reassignment problem, 16 streamlines

Tracing

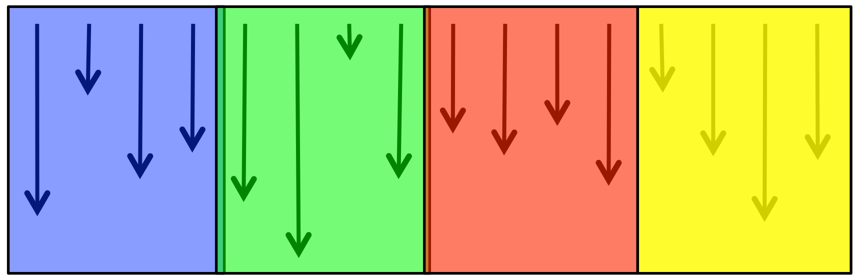


Reassignment problem, 16 streamlines

Tracing

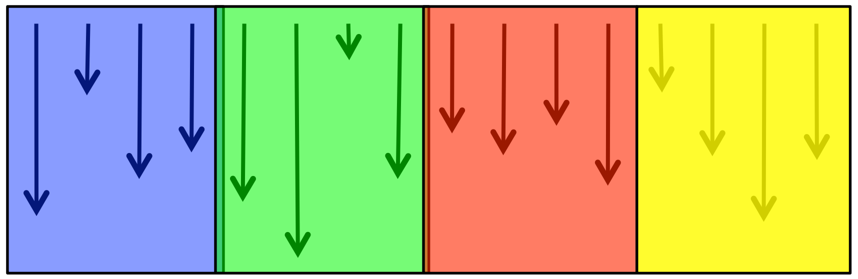


No Load Balancing

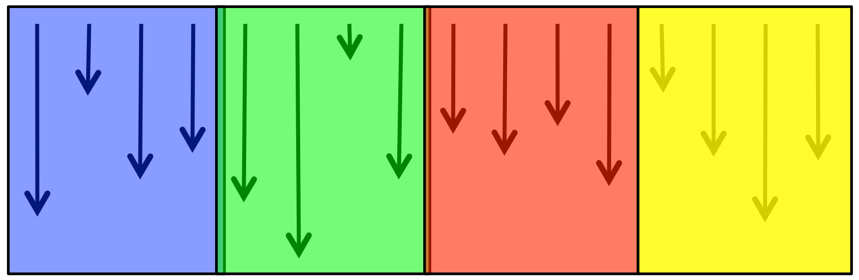


Reassignment problem, 16 streamlines

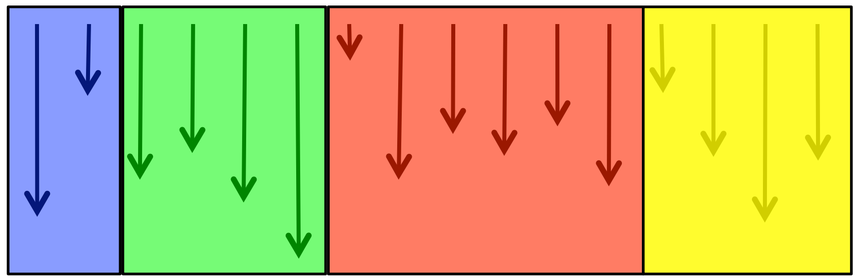
Tracing



No Load Balancing



Load Balancing



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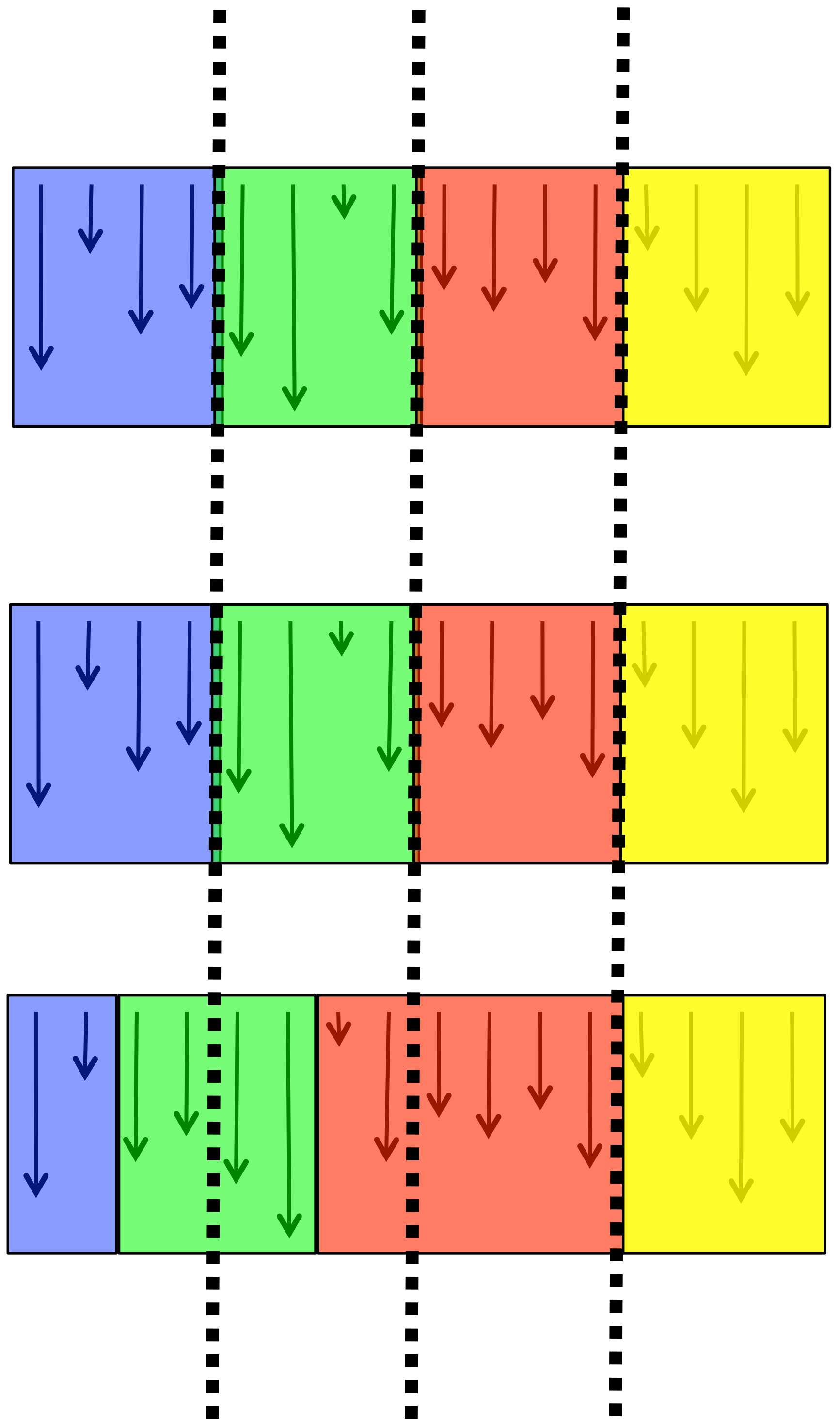
Reassignment problem, 16 streamlines

Tracing

No Load Balancing

Load Balancing



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Percentage of all streamlines moved

	8 Threads	16 Threads
SPE10	28.9%	40.9%
ANOTHER CASE	15.6%	31.0%

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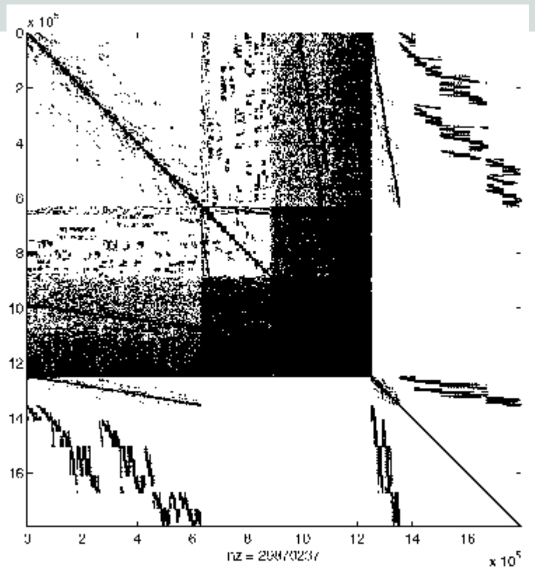
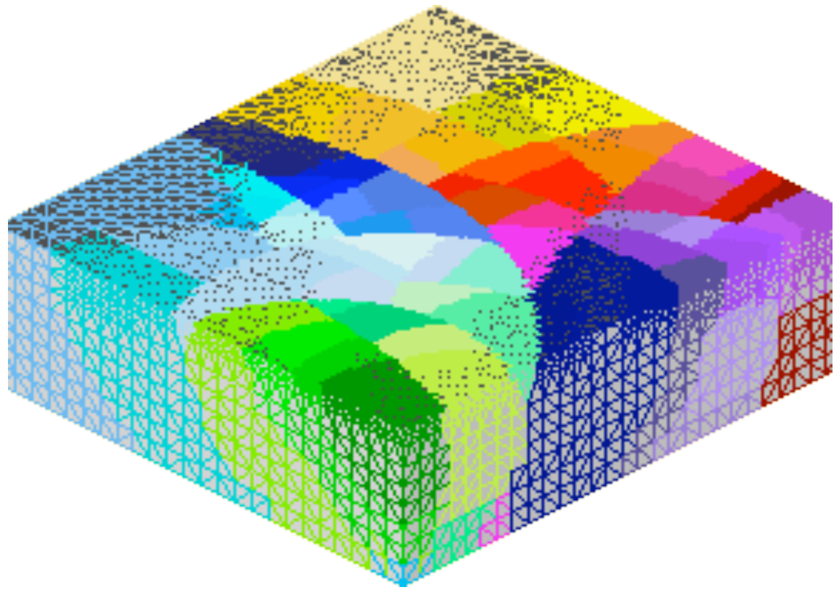
Origins of non-zero structure

- Every row in the matrix corresponds to a set of cells, a small **sub-domain**
- The non-zero pattern can be controlled by reordering of the unknowns (cells)
  - Graph partitioning, Space Filling Curves, Reverse Cuthill-Mckee, ..
- Load balance can be achieved if we replace the simple **N/num_threads** scheme
  - Assumes an efficient a-priori load estimator
  - Area and connectivity of the sub-domains control the amount of communication needed





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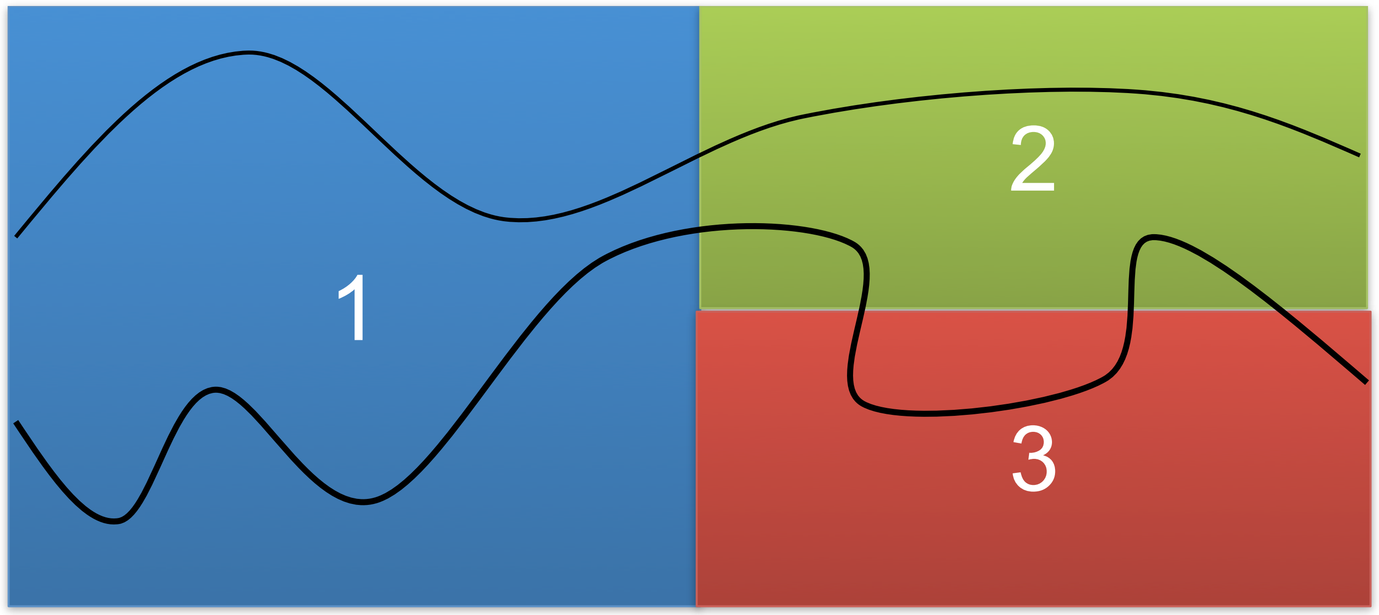
Domains and Streamlines

- Consider a partitioning of the pressure grid into three partitions
  - This is needed for extracting parallelism from the pressure solver
  - Each sub-domain corresponds to set of cells or equivalently, a set of rows of the iteration matrix



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Naive “Parallel” Solution

- Collect the entire velocity field of the sub-domains to a master node
- Trace all the streamlines on the master node
- Copy the streamline grids back to the other nodes
  - Scheduling and load balancing
- Do local transport solve
- Map back

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Analysis

- The entire velocity field must fit in the memory of the master node
  - No chance for giga-cell models
- A node generates lots of communication and the communication will be localized in time (bursty)
  - Very bad for slow interconnects
- A node can only map back streamline data to the sub-domain that it owns
  - More communication needed to pass all the streamlines around in the mapping step
- Probably not very scalable

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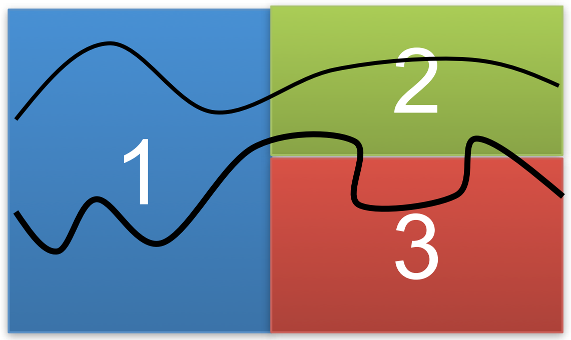
A Pipelined Solution

- For simplicity assume that we trace from left to right
- The segments of an individual streamline may cross all three domains
  - Sub-domains 2 and 3 must wait until the set of streamlines crossing sub-domain 1 have reached the boundary
  - While domains 2 and 3 trace the continuation of this set we can start tracing a new set in sub-domain 1
- In this way we can overlap or **pipeline** the tracing of the streamlines



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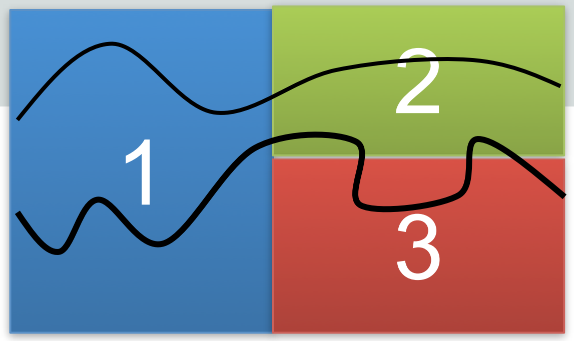
Pipelined Algorithm



- For each processor (sub-domain), repeat:
  1. Wait for start points
  2. Trace all local streamlines
  3. Hand-off exit points to neighbor
- Think of the sub-domains as grid cells in a coarse grid.
- Every sub-domain contains one segment
  - Which consist of smaller segments based on the actual grid cells

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Domain Graph