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THE ROLE OF FROUDE NUMBER IN MODELS OF BAROCLINIC COASTAL CURRENTS

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INTRODUCTION

The laboratory results of Vinger & McClimans (1980) implied that the densimetric Froude Number of a baroclinic coastal current $F = u/c_i$, where u is the longshore current velocity and c_i the celerity of interfacial Kelvin waves, controls the dynamics of flow development. In particular, for F<1, large meanders develop into large cyclonic and anticyclonic eddy pairs while for F>1 the flow follows the coast allowing only for the growth of cyclonic eddies along the seaward front (Fig 1). A more systematic investigation (McClimans & Green, 1982) confirmed the first results. These tests were run with a source intended to simulate the situation of the Skagerrak outflow along the west coast of Norway. Both insitu current measurements (Lønseth, et al, 1983) and satellite thermal imagery (McClimans & Nilsen, 1982) imply that F<1 most of the time for the Norwegian Coastal Current (NCC) (Fig 2).

The purpose of this article is to highlight the role of F for equilibrium width and wave growth characteristics, using variants of the 2-layer Margules (geostrophic) flow (Fig 3). We leave open the question as to which factors may control F in the Skagerrak and what range of values it may attain, hoping that the readers of Ocean Modelling will be interested in pursuing the issue.

WIDTH OF THE NCC

What is the relation between the width of a baroclinic coastal current B and the Rossby deformation radius $r_o = c_i/f$, for which f is the Coriolis parameter? The results of Griffiths & Linden (1981) which imply $B/r_o \sim 3$ have been used by James & McClimans (1983) to produce a good numerical simulation of the laboratory test results for F = 0.6. Both agree favorably with available observations from the NCC.

The geostrophic balance of the Margules current with a seaward front at y=B is (see Fig 3)

$$g'h_1 = Bfu \qquad (1)$$

where $g' = g(\rho_2-\rho_1)/\rho_2$ is the reduced gravity, g is the acceleration of gravity, ρ_i are the densities of layers i and h_1 is the thickness of the Coastal Current at the coast. It will be assumed that (1) holds as $h_1 \rightarrow h_2$, which appears to be the limiting case for the NCC.

For the above situation, $c_i = (g'h_1h_2/(h_1+h_2))^{\frac{1}{2}}$ and (1) can be rewritten

$$\frac{B}{r_o} = \frac{h_1 + h_2}{h_2 F} \qquad (2)$$

emphasizing the fundamental roles of r_o and F for B in this simple hydraulic model, which appears to be a good approximation of the NCC.

The earlier mentioned limiting case $h_1 \to h_2$ leads to $B/r_o = 2/F$ for the NCC. Thus, if $B/r_o \sim 3$, the densimetric Froude number for the NCC is 2/3. Indeed, the type of instabilities produced in the laboratory by Vinger & McClimans (1980) for F = 0.6 are quite similar to those observed in nature (Johannessen & Mork, 1979; McClimans & Nilsen, 1982). Compare Fig 1a with Fig 2.

RELATION BETWEEN DENSIMETRIC FROUDE NUMBERS

Several theoretical and experimental results in the literature are presented in terms of the rotational densimetric Froude Number $F_1 = f^2B^2/g'h_1$. Using (1), (2) and $f = c_i/r_o$ we obtain

$$F_1 = (h_1 + h_2)/h_2 F^2 = \left(\frac{B}{r_o}\right)^2 \frac{h_2}{(h_1 + h_2)} = \frac{B}{r_o F} \qquad (3)$$

WAVES IN THE NCC

A dependency of the width and stability of the NCC on F has been suggested. The wavelength λ of large disturbances of the type seen in Fig 1 are shown in Fig 4 for several baroclinic coastal current situations (McClimans & Green, 1982). For many of the situations, $h_2/(h_1 + h_2) = O(1)$, thus, the absissa shows not only the range of B/r_o, but also the range in F.

The data include a case of supercritical flow (F>1) for Lake Superior (Green, 1983), probably a result of upstream accelerations. Remote thermal images of this coastal current reveal instabilities like those for F>1 in Fig 1.

With the exception of the results of Stern (1980), which were probably affected by surface tension, the wavelengths follow a range $5<\lambda/r_o<15$. As far as we know only the experiments of Vinger & McClimans (1980) and McClimans & Green (1982) have used F as a control variable. The observed phase speeds c of the waves were, on the average, about 15% of c_i. Both phase speed and growth rate depend on F.

STABILITY OF THE NCC

Several investigators have considered the stability of baroclinic coastal currents. The general view is a baroclinically driven flow developing barotropic instability by virtue of horizontal shear. The degree to which the instability gains energy from the buoyancy flux depends on the depth ratio h_1/h_2 and F. Perturbation stream functions of the form $\psi=\phi(y)\exp(ik(x-(c+ic_y)t))$ in which k is a wave number, c is the phase speed and c_y is the growth rate, are integrated in time t.

Hart (1974) imposed a horizontal shear within an upper buoyant layer of a two-layer flow and chose a sine series for ϕ. The most rapidly growing wave had an angle of growth from the coast $\alpha = \tan^{-1}(c_y/c)=28^o$. F was not given, but Hart's result c/u=0.15 implies F≃1. Mysak (1977) solved for constant speeds in each layer and obtained much smaller wave growths.

Jones (1977), Stern (1980) and Mork (1980) investigated baroclinic coastal currents with seaward fronts. Only Jones computed the growth rate, using a cosine series for ϕ and a rigid lid. He obtained an angle $\alpha = 20^o$ for $h_1/h_2 \cong 0.2$ and $F \cong 0.6$. α increased with increasing h_1/h_2 and decreasing F.

Killworth (1983), using a two-layer front model obtained large wave growths for $h_1/h_2=0(1)$ and $F \sim 0$, although the direct application to the NCC is not apparent.

In the following, the results of a simple modified version of Mork (1980) with a hint of Hart (1974) will be compared with the laboratory growth rates of McClimans & Green (1982). The model (Mork, 1985) is shown in Fig 5. The y-dependency of ϕ in this reduced gravity model is resolved with Bessel functions in the wedge region, coupled with an exponential decrease seaward of the front. The coupling conditions are developed at the free-moving velocity front (dashed line in Fig 5) where the maximum lateral excursions are found.

Instability conditions in this model are determined by F, λ and the depth ratio h_o/h_1, where h_o is the thickness of the outside surface layer. The wall effect on the frontal wave is shown to be negligible for this barotropic instability when the depth ratio $h_o/h_1 \ll 1$. When $h_o \to 0$ the reduced gravity model approaches a neutral state.

The results of the theory for the most unstable waves are compared with laboratory observations of growth rates in Fig. 6. Both emphasize the importance of F in modeling baroclinic coastal current dynamics. Detailed calculations reveal a sensitivity to λ. The variations of α with λ and h_o/h_1 are to be found in Mork (1985).

DISCUSSION

Experience with the NCC north of the Lista promentory in Southern Norway indicates that the initial condition is regulated somewhat by topography. This is a convenient model condition. In fact, results of Stigebrandt's (1984) model of the daily volume flux from the Skagerrak during 1982, using constant $F \sim 1$, were recently applied to a laboratory model of the northern North Sea with a reasonably good simulation of the measured coastal current dynamics from that year. Our efforts are now directed toward greater detail of the influence of local forcing on the time development of the instabilities. However, there are still unresolved issues on the nature of the control conditions to the south of the area of interest.

The Froude Number dependency of the width of the coastal current is implicit in modeling and analyzing coastal current dynamics. Given the source density and flux, and therefore a measure for h_1, it remains to find a deterministic relation for F (or u) for computing B and the stability characteristics of the flow. In the absence of apriori knowledge of the total energy of the upstream flow, or its equivalent, it is tempting to fix the initial densimetric Froude Number F_0, as has been done in the earlier mentioned laboratory studies. Under certain circumstances (e.g. topographic control) this may be justified. The question is what regulates F_0 and what are its normal ranges?

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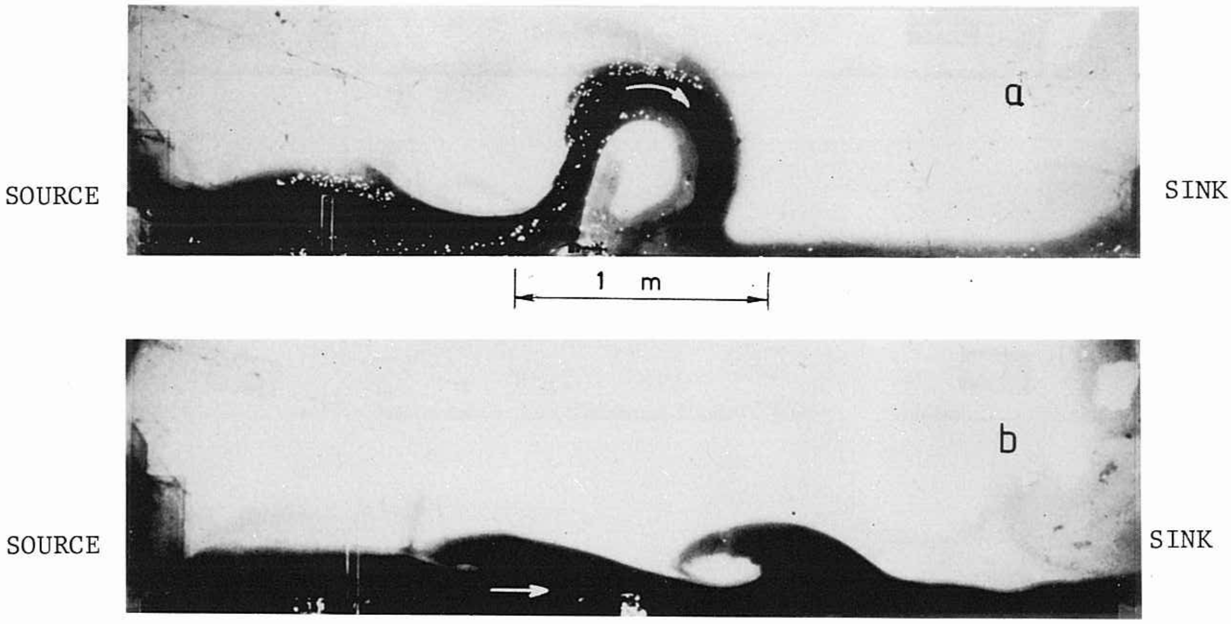

Figure 1. Plan view of coastal current instabilities (a) F=0.6 (b) F=1.2. (Results from Vinger and McClimans, 1980.)

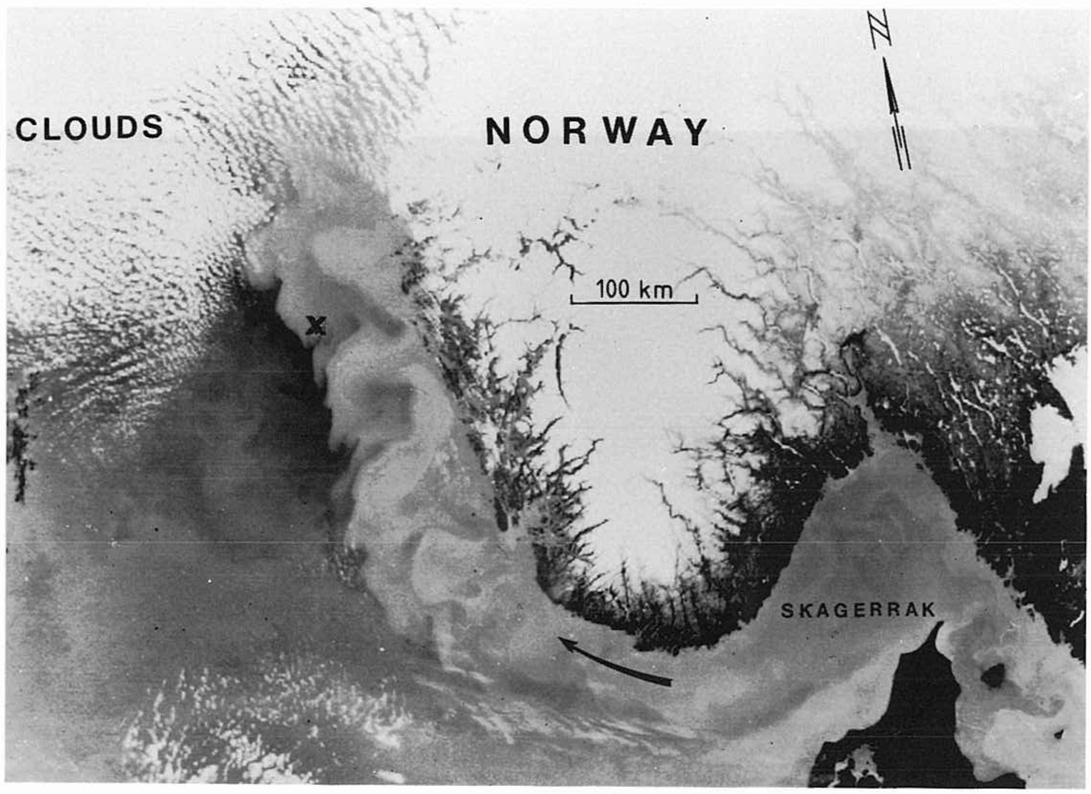

Figure 2. A thermal image of the coastal current in the Skagerrak and along the west coast of Norway. The outflow is on the order of 1 Sverdrup.

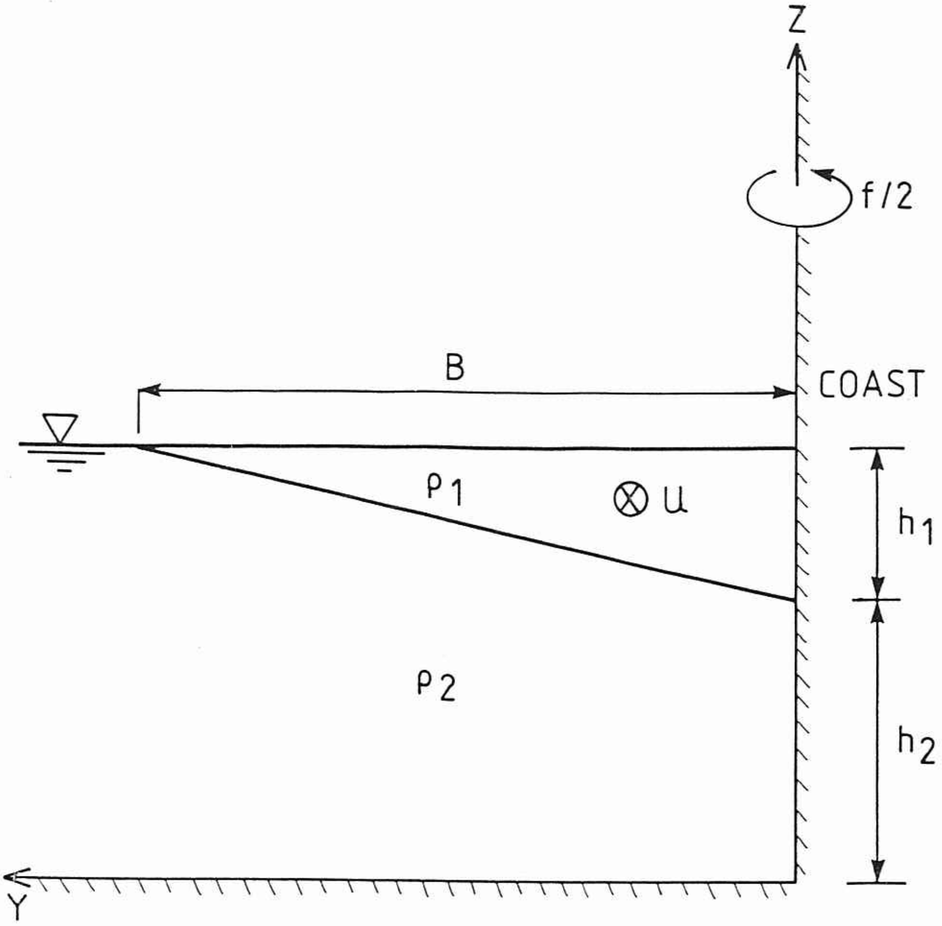

Figure 3. (McClimans et al.) Cross section of the Margules coastal current with appropriate nomenclature.

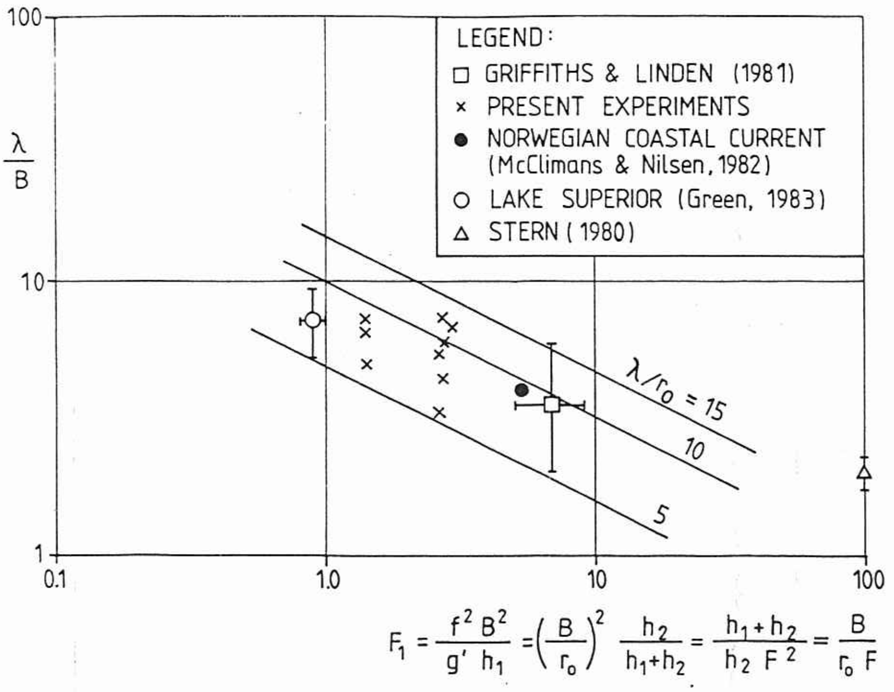

Figure 4. (McClimans et al.) Observed wavelengths of large disturbances in baroclinic coastal currents (McClimans and Green, 1982).

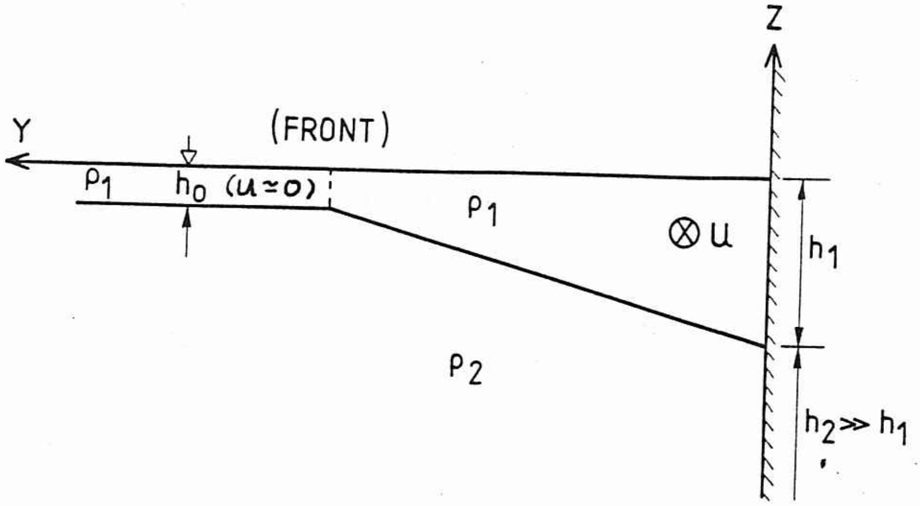

Figure 5. An extended Margules flow model for stability analysis.

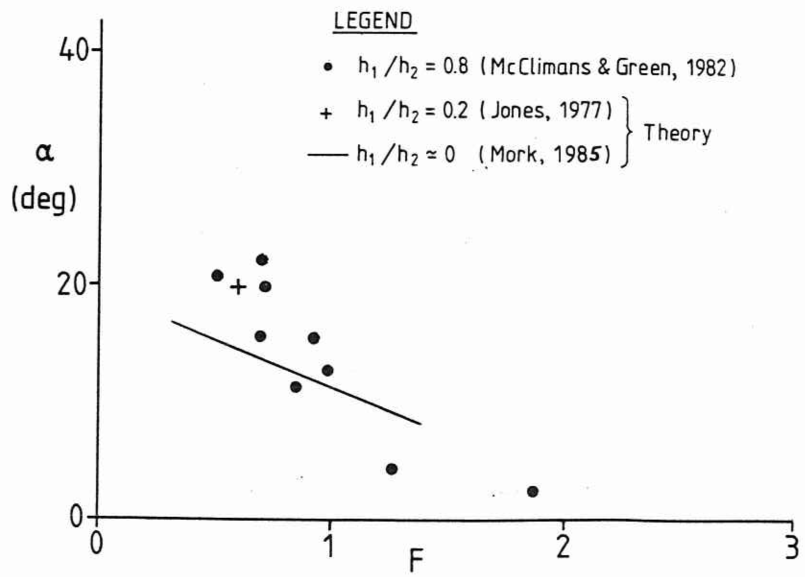

Figure 6. Comparison of theory with the laboratory results of McClimans and Green.

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FIGURE CAPTIONS

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