Simplified double peak spectral model for ocean waves

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ABSTRACT

A simplified version of the double peak spectral model for ocean waves (Torsethaugen, 1993, 1994, 1996) is described. For the locally fully developed sea, the spectral peak period T_{pf} is a function of the significant wave height, H_s, $T_{pf}=a_f*H_s^{1/3}$ with a_f slightly dependent on fetch. This classifies sea states as wind sea $T_p<=T_{pf}$ and swell $T_p>T_{pf}$ each consisting of two wave systems. Empirical parameters given by significant wave height and spectral peak period define the spectral form parameters and energy distribution between the two wave systems in the model. The adequacy of the model is indicated by comparing the model with measured wave spectra from the Norwegian Continental shelf.

KEY WORDS: Ocean waves; wave spectra; double peak; wind sea; swell

INTRODUCTION

For a number of years, the Torsethaugen double peak spectral model has frequently been used for design purposes at the Norwegian Continental Shelf, both in connection with numerical analyses and model tests. The original Torsethaugen model was established by fitting two JONSWAP shaped models to average measured spectra from the Norwegian Continental Shelf. The observed sea states were grouped with respect to significant wave height, H_s, and spectral peak period, T_p. This means that average spectra were established from a broad range of combinations of H_s and T_p. Regarding the adequacy of the original fit to the average spectra, reference is made to Torsethaugen (1993, 1994, 1996).

Over the years, users of the spectral model have often raised the question if the model can be simplified without a significant loss of validity. The final use of the model has always been rather simple, since the spectrum is defined as the total significant wave height and the period of the dominating spectral peak are given. The complexity lies in the number of parameters involved in parameterize the spectral model in merely two parameters.

The purpose of the present study is to

- reduce the number of free parameters, requiring that the simplified model should represent a fit to the average spectra of an accuracy being comparable to the original Torsethaugen model.
- analyze the model in order to find a better understanding of some of the physical and site dependency for the empirical parameters.

The adequacy of the simplified model is indicated by comparing it with a subset of available average spectra. In this connection, the model is also compared to wave spectra measured by the Miros radar. This is of special interest since the original model is fitted to average spectra obtained from buoy measurements. The simplified model is also compared to the Ochi-Hubble double peak spectral model since the latter is more frequently adopted outside the Norwegian Continental Shelf.

As mentioned above, the Torsethaugen model is fitted to average spectra established for various combinations of H_s and T_p. When utilizing a model aiming towards describing the expected spectral shape of a sea state characterized by merely two parameters, one has to realize that on a case by case comparison a considerable scatter around the expected shape is observed. In particular this is the case for sea states being of a combined nature. The scatter is most clearly pronounced for the low frequency part of the spectrum where the swell components are located. The importance of the scatter for structural response predictions depends very much on the transfer function of the response problem under consideration. If the response is sensitive to the swell period band, one should ensure some robustness against the effect of the scatter around the average spectral shape. By utilizing an average spectral shape a source of randomness is neglected, this should be kept in mind when executing a structural reliability analysis.

A possible generalization of the spectral modeling would be to suggest a family of spectral shapes for each sea state class, where each member of the family is given a certain probability of occurrence.

It is also to be noted that a spectral model assumed to represent the expected spectral shape for a given combination of H_s and T_p, in principle is merely validated for sea states covered by the observations. When utilizing such a model well beyond the range of experienced sea states, as is the case when the model is adopted for 100- and 10000-year sea states, the accuracy of the model is of course associated with uncertainties due to the extrapolation beyond the fitting region. For sea states of such a severity as indicated above, uncertainties will be present whether one chooses a single peak spectral model or a double peak model. A double peak model is in qualitative agreement with experienced real sea wave spectra, and we expect them to correspond to a broader range of applicability than single peak spectral models.

The present spectral model is limited to account for two distinct sea wave systems where one is caused by local wind. In certain areas of the world, it is claimed that one typically experience triple peak wave spectra or more than one swell peak. If this is the case for the design sea state regime, a better way is probably to establish the above mentioned families of spectra for various combinations of H_s and T_p.

From a design point of view, final conclusions regarding the adequacy of a spectral model require that it is applied for predicting structural response. In this connection it is of interest to compare the results obtained using the model versus: i) Using the observed average spectrum, ii) Using all observed spectra for given combinations of H_s and T_p. Such verification is left for possible future work.

BASIC ELEMENTS IN SPECTRAL WAVE MODELS

Basic sea states

For this model it is assumed that ocean waves at a location can be divided into two main parts

a) Wind sea, i.e. seas generated by the local wind.
b) Swell seas, i.e. waves entering into the location from other areas.

Generally sea states can occur with more than one swell component and be influenced by fetch, breaking waves, wave refraction and reflection, local current and large current circulation systems. These effects are dependent on the location. The spectral models discussed are limited to deepwater waves on the open ocean, ignoring the influence from current and other refraction and diffraction effects.

Basic spectral models

There are two different approaches to spectral modeling.

a) Single sea state models, describing locally generated wind sea states, are based on fundamental understanding of wind wave generation. These models are the basis for all other models and are essential for the understanding of ocean waves.
b) Statistical spectral models that are based on the basic models or a combination of such models. They describe the most probable spectrum or a distribution of wave spectra for a given sea state. These models will be based on a large amount of measured wave spectra.

Basic spectral elements

In the derivation of spectral models two basic elements are involved

a) The high frequency tail.
b) The spectral peakedness.

a) High frequency tail

The behavior of the high frequency part of the spectrum is given by the energy balance for waves generated by the local wind fields.

An important concept in wind wave generation is the so-called equilibrium range. When the wind blows for a long time or over a long fetch, the wave energy for a given frequency reaches an upper limit, the equilibrium range. This means that the energy input from the wind is balanced by energy loss to other frequencies and by wave breaking. Since Phillips (1958) introduced the equilibrium range, a discussion has been going on how to define the high frequency range. According to Philips, the equilibrium range is found by dimension analysis to be:

$$S(f) \approx \alpha (2\pi)^{-5} g^2 f^{-5} \qquad (1)$$

where α was an absolute constant (7.4×10^{-4}) and g the gravity constant.

This equilibrium range was used by Pierson and Moskowitz (1964) when they developed the PM-spectrum for fully developed wind sea. Hasselman et al. (1973) also adopted this formulation in the JONSWAP spectrum for growing sea states under fetch limited conditions. Based on measured spectra, (Toba, 1972), theoretical studies of wave-wave interactions and study of energy input mechanisms from wind and wave breaking, Phillips (1984) found that the equilibrium range for high frequencies rather goes as:

$$S(f) \approx u^* g (2\pi)^{-4} f^{-4} \qquad (2)$$

where u* is the friction wind velocity. This value of the exponent is commonly accepted as the (theoretical) value for the high frequency tail (equilibrium range) for wind generated sea today. For measured spectra this exponent seems to vary between 4 and 5. Values even outside this range are found in some few cases. This can be an effect of the measuring technique, bottom effects, effects of ocean currents, or other effects.

b) Spectral peakedness

In the simplest form for a wave spectrum the energy follows the equilibrium range up to the spectral peak and then drops to zero. A wave spectral model developed by Bretschneider (1958) was formulated as:

$$S(f) = \alpha g^2 (2\pi)^{-5} f^{-5} e^{(-0.675(1/T_m f)^4)} \qquad (3)$$

The wave period T_m is given by $T_m = T_p/1.17$

For high frequencies, Eq. (3) is in agreement with Phillips first version of the equilibrium range and confirms the value of α found from measurements. Later Pierson and Moskowitz came up with the spectral form:

$$S(f) = \alpha g^2 (2\pi)^{-5} f^{-5} e^{(-\beta (f_0/f)^4)} \qquad (4)$$

where $\alpha = 8.10 \times 10^{-3}$, $\beta = 0.74$ and $f_0 = g/2\pi U$ (U= wind speed 19.5 m above sea level). Here the wave period $T_0 = 1/f_0$ is given by $T_0 = T_p/1.14$

The original PM-spectrum represents the spectrum for the fully developed wind sea (infinite fetch) and has a high frequency tail that falls off as f^{-5} . The spectral peakedness is defined by the exp ($-\beta(f_0/f)^4$) factor.

As part of the Joint North Sea Wave Project (JONSWAP) Hasselmann et. al. (1973) developed the JONSWAP spectrum where a peak enhancement factor γ was introduced to represent fetch limited wind sea:

$$S(f)=\alpha g^2(2\pi)^{-4} f^{-5} e^{(-1.25(f_p/f)^4)} * \gamma^{\exp\{(-(f-f_p)^2/(2\sigma^2 f_p^2))\}} \quad (5)$$

where $\sigma=\sigma_a$ for $f<f_p$ and $\sigma=\sigma_b$ for $f>f_p$, f_p represents the spectral peak frequency.

The JONSWAP spectrum is a PM-spectrum multiplied with a peak enhancement function $\gamma(f_p)$. The JONSWAP spectrum contains 5 parameters f_p,α,γ, σ_a and σ_b and the most probable values for some of these read: $\gamma=3.3$, $\sigma_a=0.07$ and $\sigma_b=0.09$

The values for α and f_p depend on wind speed, fetch and duration.

The Γ-spectrum

The generalized Γ spectrum is defined as:

$$S(f)=E\,S_n(f_n) \quad (6)$$

where:

$S(f)$ is wave energy density (m²s)

f is wave frequency (Hz)

$E=m_0/f_p=(1/16)H_s^2/f_p$ (m²s)

$m_0=\int_0^\infty S(f)\,df$

f_p is spectral peak frequency (Hz)

$H_s=4\sqrt{m_0}$ is significant wave height (m)

$S_n(f_n)$ is non dimensional spectral density

$f_n=f/f_p$

The non dimensional spectral density can be written:

$$S_n(f_n)=G_0 A_\gamma \Gamma_s(f_n;N,M)\gamma_F(f_n;\gamma,\sigma) \quad (7)$$

where:

$$\Gamma_s(f_n;N,M)=f_n^{-N} e^{-(N/M)f_n^{-M}} \quad (8)$$

$$\gamma_F(f_n;\gamma,\sigma)=\gamma^{\exp[-(1/(2\sigma^2))(f_n-1)^2]} \quad (9)$$

Eq. (8) is the Pierson-Moskowitz form of the wave spectrum, while Eq. (9) represents the JONSWAP peak enhancement function. The parameter $\sigma=0.07$ for $f_n<1$ and $\sigma=0.09$ for $f_n>1$ are usually adopted.

The normalizing factor related to be the P-M form is

$$G_0=\{(1/M)(N/M)^{-(N-1)/M}\Gamma((N-1)/M)\}^{-1} \quad (10)$$

where $\Gamma()$ is the gamma function. N represents the frequency exponent for the high frequency range of the spectrum, while M is related to the spectral width. For M=N=4, which is used herein, $G_0=3.26$.

Regression analyses, Torsethaugen (1993), show that the normalizing factor, A_γ, for N=4 can be approximated as:

$$A_\gamma=(1+1.1(\ln\gamma)^{1.19})/\gamma \quad (11)$$

Double peak wave spectra.

Low frequency waves can propagate faster than the generating wind field and reach areas not influenced by this wind field or at least before the area is influenced directly by it. This swell component will add to the locally generated wind sea and create double (or multiple) peak spectra. The spectra for the various sea systems will usually correspond to different peak frequencies and different directions of propagation. Sea wave spectra can be rather complicated and be a result of several swell systems in addition to local generated waves. Some spectral models have been developed to give a realistic approach for double peak cases.

a) Ochi and Hubble

Ochi and Hubble (1976) formulated a double peak spectral model where the resulting spectrum was modeled as a sum of two Γ-distributions:

$$S(f)=\sum_{j=1}^{2}\frac{H_{sj}^2 T_{pj}}{4\Gamma(\lambda_j)}\frac{(\lambda_j+0.25)^{\lambda_j}}{(T_{pj}f)^{(4\lambda_j+1)}}\exp\left\{-\frac{(\lambda_j+0.25)}{(T_{pj}f)^4}\right\} \quad (12)$$

where Γ() is the gamma function.

This spectrum has 3 parameters for each wave system, significant wave height, spectral peak period and the shape factor λ. The following recommendations were made for the shape factors for the swell sea system, j=1, and the wind sea system, j=2:

$\lambda_1=\lambda_{10}$

$\lambda_2=\lambda_{20}e^{\lambda_{21}H_s}$ (H_s in m)

Comparison with measured spectra gave $\lambda_{10}=3.0$, $\lambda_{20}=1.54$ and $\lambda_{21}=-0.062$ (m^{-1}) as the most probable values. For the most probable spectrum the swell system the high frequency tail goes as $f^{-11.9}$, while for the wind sea system, the high frequency exponent varies with H_s. It equals 8 for low values and reduces to 3 for high values.

The variation (within 95% confidence) in the spectrum is expressed by 10 parameters sets for λ_{10}, λ_{20} and λ_{21}. For one value of H_s a set of parameters can be found that gives a spectrum close to the wanted peak period. The Occhi-Huble spectral model is in this respect a one-parameter model (H_s) giving a set of probable spectral forms. Examples are given below.

b) The Torsethaugen spectrum

A double peak spectral model was developed based on measured spectra for Norwegian waters (Haltenbanken and Statfjord). The input to the model is significant wave height, H_s, for total sea and spectral peak period, T_p, for the primary (highest) peak. The model parameters are found by fitting a Γ- spectral form to the average measured spectra for given classes of sea states. Each sea system is defined by five parameters H_s, T_p, γ, N and M. They are all parameterized in terms H_s and T_p by means of regression analyses and curve fitting. This includes also the ratio of the significant wave height between the two sea systems, swell and wind sea.

The locally fully developed sea concept is used to divide the T_p-H_s space in two different types.

a) Wind dominated sea: $T_p<T_{pf}$
b) Swell dominated sea $T_p>T_{pf}$.

T_{pf} is the spectral peak period for fully developed sea at the actual location. This is determined by the maximum fetch given by the topography or the typical extent for low pressures in the area. It gives a relation between the maximum wind wave energy (or H_s) and the corresponding spectral peak period for the actual location. (see Eq. (13) below)

For type a), the sea states have a significant wave height that is higher than the value corresponding to locally fully developed sea with the given spectral peak period. This means that a secondary system (swell) containing the extra energy must be present.

For type b), the situation is more complicated and includes at least two different situations:
1) Dominating swell in addition to local wind sea.
2) Decaying wind seas. If the local wind is decaying, the waves will loose energy for high frequencies. This swell ("old" wind sea) can represent the highest spectral peak if no new local wind seas are generated with sufficient spectral peak density

For type a), wind dominated sea; the secondary system represents an average of swell sea for the area. For locations not particularly exposed to swell, the probability to find sea states with this combination of H_s, T_p should be low. For type b), swell dominated sea, the secondary sea system represents local wind sea.

The double peak spectral model gives a parametric description for four types of peaks.

a) Primary wave system generated by the local wind field
b) Primary wave system dominated by swell
c) Secondary wind sea system
d) Secondary swell system
The average wave spectra is found as a sum of a primary and a secondary wave system each given by the Γ- spectrum.

FORMULATION OF THE SIMPLIFIED DOUBLE PEAK SPECTRUM

The original formulation of the double peak wave spectrum contained many empirical parameters. Some of these parameters have only effect for low sea states and are of marginal importance for design. The present study describes a simplified version.

Fully developed sea.
The distinction between wind dominated and swell dominated sea states is found to be defined by the fully developed sea for the location where T_p is given by:

$$T_{pf} = a_f H_s^{1/3} \tag{13}$$

If $T_p \leq T_{pf}$, the primary spectral peak corresponds to the local wind system. If $T_p > T_{pf}$, the primary spectral peak corresponds to the swell system.

Eq. (13) is derived as follows:
From the JONSWAP experiment:

$$H = k_1 F^{1/2} \tag{14}$$

$$T = k_2 F^{1/3} \tag{15}$$

H, T and F are dimensionless values for H_s, T_p and fetch F_e defined by:

$$H = gH_s / u^2 \tag{16}$$

$$T = gT_p / u \tag{17}$$

$$F = gF_e / u^2 \tag{18}$$

where u is a wind velocity. The JONSWAP experiment, Hasselmann et al. (1973), used u= u_{10}, where u_{10} is the wind speed 10m above sea level and found that $k_1 = 1.6 * 10^{-3}$ and $k_2 = 0.286$

From these expressions the following can be derived:

$$T / H^{1/3} = k_3 F^{1/6} \; ; \; k_3 = k_2 / k_1^{1/3} \tag{19}$$

Introducing the definitions of the normalized quantities, one obtains:

$$T_{pf} = k_f F_e^{1/6} H_s^{1/3} \tag{21}$$

where T_{pf} corresponds to the peak period for the locally fully developed sea, and $k_f = k_3 * g^{-1/2} = 0.78$ ($sm^{-1/2}$).
This gives

$$a_f = k_f F_e^{1/6} \tag{22}$$

The factor a_f is seen to be slightly dependent on fetch. It has the value:
$a_f = 6.6$ ($sm^{-1/3}$) for F_e= 370 km and
$a_f = 5.3$ ($sm^{-1/3}$) for F_e= 100 km.

Peak enhancement factor γ.
According to Mitsuyasu (1980), the peak enhancement factor

$$\gamma = k_4 F^{-1/7} \tag{23}$$

Introducing the average wave steepness:

$$s = (2\pi / g) H_s / T_p^2 = 2\pi H / T^2 = 2\pi (k_1 / k_2{}^2) F^{-1/6} \tag{24}$$

one may express γ as:

$$\gamma = k_g s^{6/7} \; ; \quad k_g = k_4 (k_2{}^2 / 2\pi k_1)^{6/7} \tag{25}$$

The value of k_4 is given by Mitsuyasu (1980) as $k_4 = 7.0$ resulting in k_g= 42.2 with values of k_1 and k_2 taken from the JONSWAP experiment (Eqs. (14) and (15)). The value of k_g will probably depend on the location (fetch and wind speed). It will increase in fetch limited areas reflecting more peaked spectra.
In the original model, $k_g = 35(1+3.5e^{-Hs})$ (H_s in m) was used based on best fit to data from the Statfjord area. For H_s> 5 m this is a constant = 35. This value is used in the simplified spectral model.

Wave period scale factors
For each of the sea systems a non-dimensional period scale is introduced by using a lower and an upper value for T_p. The lower period limit depends on H_s and is related to wave breaking. The parameter used in the original version was found by best fit to data and corresponds to a steepness, s = 0.16. This is slightly above the breaking criteria for regular waves, s= 1/7. For the upper period limit a constant value, representing an absolute upper value for the spectral peak period, is used.

The two limits are set to:
Lower limit

$$T_l = a_e H_s^{1/2} \tag{26}$$

Upper limit

$$T_u = a_u \tag{27}$$

Non-dimensional scales for the spectral peak period are defined as:
Wind sea

$$\varepsilon_l = (T_{pf} - T_p)/(T_{pf} - T_l) \tag{28}$$

Swell

$$\varepsilon_u = (T_p - T_{pf})/(T_u - T_{pf}) \tag{29}$$

Comment: For values of T_P below T_l or above T_u, ε- values should be set equal to 1.

High frequency tail and spectral form
The high frequency tail of the wind spectrum goes like f^{-N}. Measured spectra often show a value for N varying from 4 to 5. In the original version, Torsethaugen (1993), the following expression for N was used:

$$N = k_0 H_s^{1/2} + k_{00} \tag{30}$$

where k_0= 0.5 ($m^{-1/2}$) and k_{00}=3.2.

For most practical applications, the exponent of the high frequency tail is not important. It may have a slight effect on the results for a lightly damped structural system with a natural frequency in the high frequency range. In view of this, the simplified model is using N=4 for all sea states. This will be conservative in case of a lightly damped structural system.
For the spectral width parameter, M, in the Γ-spectrum, 4 is used for all sea states.

Spectral parameters for
a) Wind dominated sea $T_p <= T_{pf}$

1)Primary peak

i) Significant wave height,

$$H_{w1} = R_w H_s \; ; \quad R_w = (1-a_{10})e^{-(\varepsilon_l / a_1)^2} + a_{10} \tag{31}$$

ii) Spectral peak period

$$T_{pw1} = T_p \tag{32}$$

iii) Peak enhancement factor,

$$\gamma = k_g s_p^{6/7}; \quad s_p = (2\pi / g) H_{w1} / T_{pw1}^{2} \tag{33}$$

2)Secondary peak

i) Significant wave height

$$H_{w2} = (1 - R_w^2)^{1/2} H_s \tag{34}$$

ii) Spectral peak period

$$T_{pw2} = T_{pf} + b_1 \tag{35}$$

iii) Peak enhancement factor

$$\gamma = 1 \tag{36}$$

b) Swell dominated sea $T_p > T_{pf}$

1)Primary peak

i) Significant wave height

$$H_{s1} = R_s H_s; \quad R_s = (1-a_{20})e^{-(\varepsilon_u / a_2)^2} + a_{20} \tag{37}$$

ii) Spectral peak period

$$T_{ps1} = T_p \tag{38}$$

iii) Peak enhancement component

$$\gamma = \gamma_f (1 + a_3 \varepsilon_u); \quad \gamma_f = k_g s_f^{6/7}, s_f = (2\pi / g) H_s / T_{pf}^2 \tag{39}$$

2)Secondary peak

i) Significant wave height

$$H_{s2} = (1 - R_s^2)^{1/2} H_s \tag{40}$$

ii) Spectral peak period

$$T_{ps2} = a_f H_{s2}^{1/3} \tag{41}$$

iii) Peak enhancement factor

$$\gamma = 1 \tag{42}$$

The numerical values for all parameters are given in Table 1.

Table 1.Empirical parameter values for simplified model

Parameter	Value	Used in formulae
a_f	6.6 $sm^{-1/3}$	13,41
a_e	2.0 $sm^{-1/2}$	26
a_u	25 s	27
a_{10}	0.7	31
a_1	0.5	31
k_g	35.0	33,39
b_1	2.0 s	35
a_{20}	0.6	37
a_2	0.3	37
a_3	6	39

Resulting spectral formula
The resulting spectra can be formulated as:

$$S(f_n) = \sum_{j=1}^{2} E_j S_{jn}(f_{jn}) \tag{43}$$

j = 1 primary sea system, j = 2 secondary sea system.

$$E_1 = (1/16) H_1^2 T_{p1} \text{ and } E_2 = (1/16) H_2^2 T_{p2} \tag{44}$$

$$S_{1n}(f_{1n}) = G_0 A_\gamma f_{1n}^{-4} e^{-f_{1n}^{-4}} \gamma^{(\exp -(1/2\sigma^2)(f_{1n}-1)^2)} \tag{45}$$

$$S_{2n}(f_{2n}) = G_0 f_{2n}^{-4} e^{-f_{2n}^{-4}} \tag{46}$$

$f_{1n}=f*T_{p1}$, $f_{2n}=f*T_{p2}$, $G_0 = 3.26$, $A_\gamma = (1+1.1 [\ln\gamma]^{1.19})/\gamma$
and $\sigma = 0.07$ for $f_n < 1$ and $\sigma = 0.09$ for $f_n > 1$

For wind sea dominated cases i.e. $T_p<T_{pf}$ we have:
$H_1=H_{w1}$ and $H_2 = H_{w2}$ see Eqs. (31) and (34)
$T_{p1}=T_{pw1}$ and $T_{p2}=T_{pw2}$ see Eqs. (32) and (35)
γ given by Eqs. (33) and (36).

For swell sea dominated cases i.e. $T_P>T_{pf}$ we have
$H_1=H_{s1}$ and $H_2 = H_{s2}$ see Eqs. (37) and (40)
$T_{p1}=T_{ps1}$ and $T_{p2}=T_{ps2}$ see Eqs. (38) and (41)
γ given by Eqs. (39) and (42)

Parameter variations
The total wave energy is divided between the two sea systems according to the parameter shown in Figures 1 and 2 for wind dominated sea and swell dominated sea respectively. Curves are given for different values of the spectral peak period. The energy in the primary peak reaches a constant level for spectral peak periods far from the one peak sea state.

The variation in the peak enhancement factor for the primary peak is shown in Figures 3 and 4 for wind sea and swell, respectively. For the secondary peak the peak enhancement factor is set equal to 1 for all sea states. Curves are given for different values of the spectral peak period.

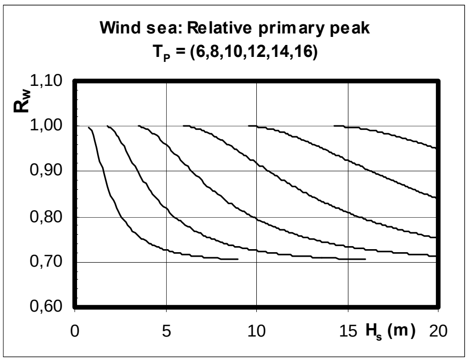

Figure 1. Ratio wind sea significant wave height to total significant wave height for wind dominated sea $T_p \le T_{pf}$. Curves are given for T_p values from 6s to 16 s from left to right.

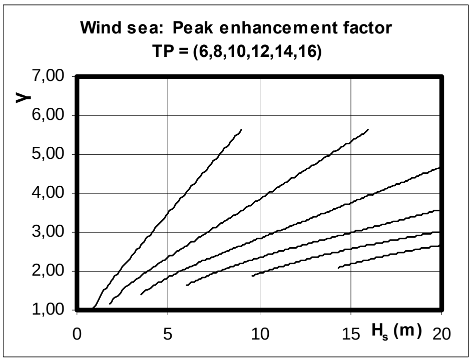

Figure 2. Variations in the peak enhancement factor γ for wind dominated sea $T_p \le T_{pf}$. Curves are given for T_p values from 6s to 16 s from left to right.

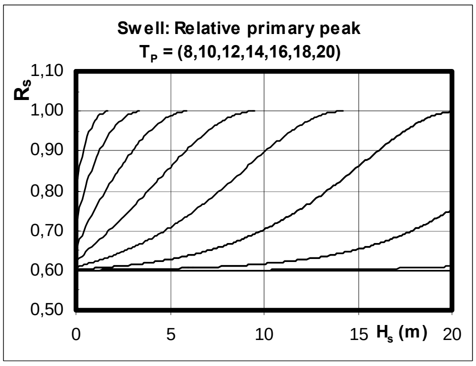

Figure 3. Ratio swell significant wave height to total significant wave height for wind dominated sea $T_p > T_{pf}$. Curves are given for T_p values from 8s to 20 s from left to right.

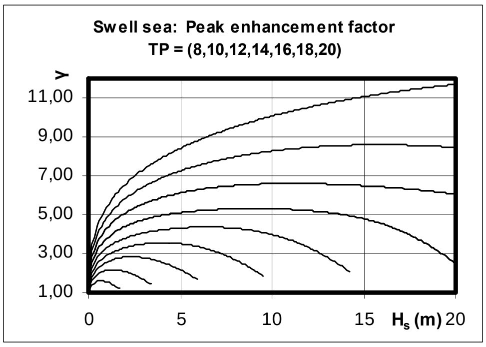

Figure 4. Ratio swell significant wave height to total significant wave height for swell dominated sea $T_p > T_{pf}$. Curves are given for T_p values from 8s to 20 s from bottom left to top.

COMPARISON WITH MEASURED WAVE SPECTRA

The following comparisons between the simplified model and measured data are presented.

a) Average measured wave spectra for Statfjord
b) Spectral moments represented by wave period parameter T_{m02} for Halten area.
c) Average spectra measured by the MIROS radar at Gullfaks C

Statfjord.

The spectral model is compared to measured spectra for Statfjord. The points represent average measured spectra and the solid line show the simplified model spectrum

The examples below show that the simplified model fits very well to the spectral peak, but minor deviations in the high frequency tail will be found for significant wave height above 5 m.

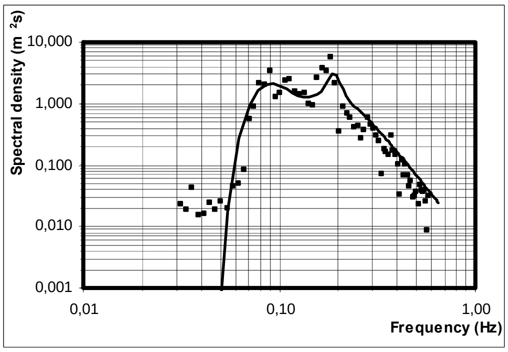

Figure 5: Comparison of wave spectra for Statfjord.
Solid line: Simplified model. Dots: Measured data.
Sea state: H_s=2.48m, T_p=5.3s. No. of spec.= 6

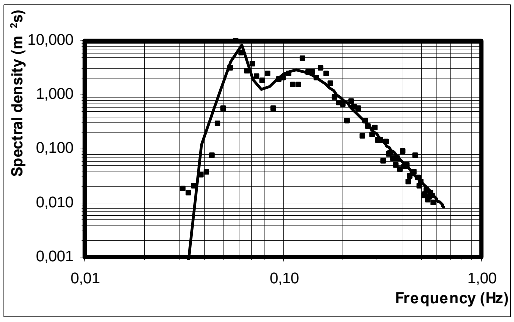

Figure 6: As for Figure 5
Sea state: H_s=2.59m, T_p=16.8s. No. of spec= 14

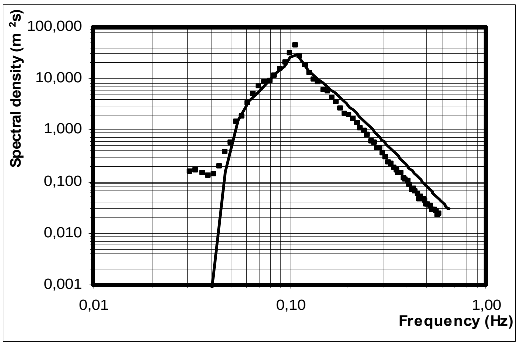

Figure 7: As for Figure 5
Sea state: H_s=5.24m, T_p=9.4s. No. of spec.= 279

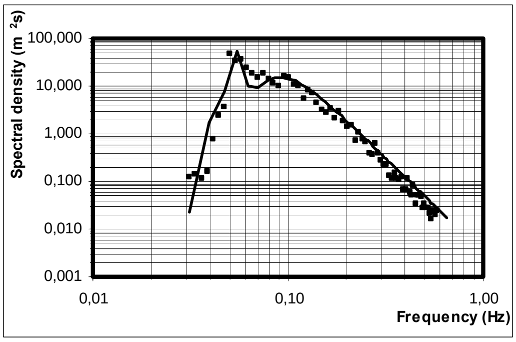

Figure 8: As for Figure 5
Sea state: H_s=5.28m, T_p=18.5s. No. of spec.= 14

Halten

The average wave period parameters $T_{02} = (m_0/m_2)^{1/2}$ given by the spectral moments, m_0 and m_2 are computed from data for Halten for one year. Figure 9 show one example of how T_{02} varies with T_p for sea state with significant wave height 2.5-3 m. We see that there are only minor differences for high values of T_p between the two versions of the double peak model and that the model represents the measurements quit well. The same tendency is found for all sea states. The deviation between the two models for high values of T_p is caused by the difference in the high frequency exponent.

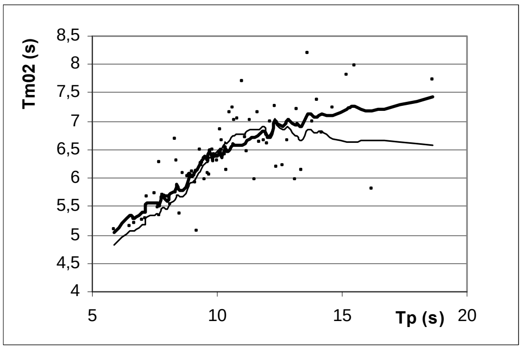

Figure 9: Comparison of wave period parameter, T_{02} for Halten data. Sea state: H_s=2.75m Dots: Measurements, Solid line: Model (thick line: simplified, thin line original).

Radar data from Gullfaks C.

The simplified model spectra are compared to wave spectra from the Gullfaks C platform. Wave spectra are measured by the MIROS wave radar for the period 1998 to February 2002 for every 20 min. The total number of spectra is 105.403. The raw spectra are analyzed to find the significant wave height and the spectral peak period. The average spectra are found for each class of sea state where a class is defined for intervals of H_s equal 0.5 meters (1 meter above 5 meter) and intervals for T_p equal to 1 sec.

All spectra show an energy level for low frequencies not seen for buoy data. For low sea states the measured peaks are smoother than the corresponding model peak, and also smother than corresponding peaks in spectra from buoys. For all other sea states the model represents the measured average spectra quit good. Examples are shown in figures 10-13 below.

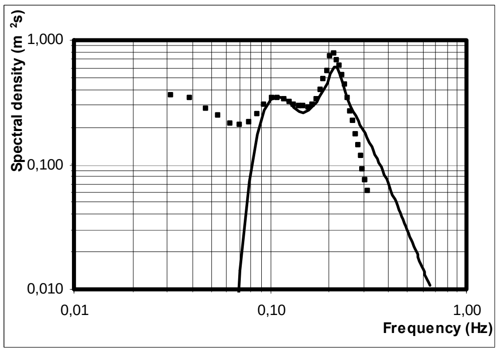

Figure 10: Comparison of wave spectra for Gullfaks.
Solid line: Simplified model. Dots: Measured data.
Sea state: H_s=1.20m, T_p=4.6s. No. of spec. = 686

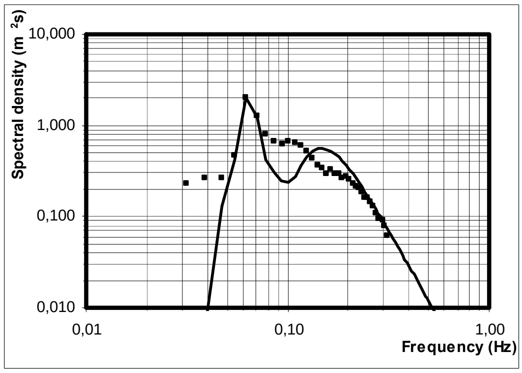

Figure 11: Comparison of wave spectra for Gullfaks.
Solid line: Simplified model. Dots: Measured data.
Sea state: H_s=1.30m, T_p=15.5s. No. of spec. = 66

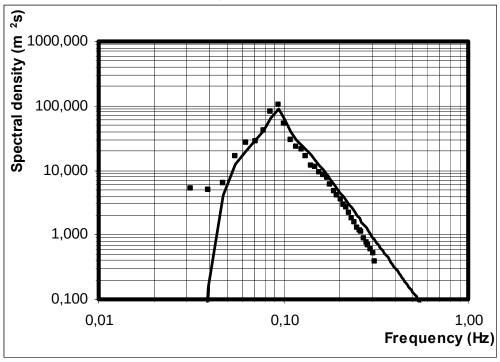

Figure 12: Comparison of wave spectra for Gullfaks.
Solid line: Simplified model. Dots: Measured data.
Sea state: H_s=8.30m, T_p=10.7s. No. of spec. =17

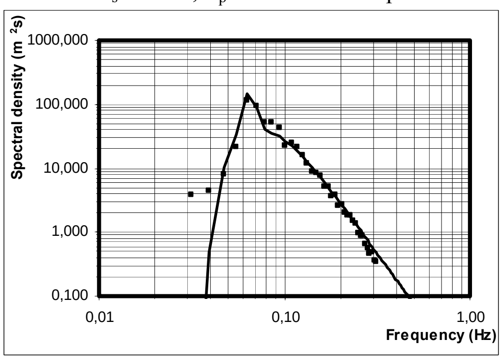

Figure 13: Comparison of wave spectra for Gullfaks.
Solid line: Simplified model. Dots: Measured data.
Sea state: H_s=8.30m, T_p=15.5s. No. of spec. = 29

COMPARISON WITH OTHER MODELS

The one peak case

If we set $T_P=T_{pf}$ (corresponding to locally fully developed sea) the model will represents a single peak sea state. For this single peak case with long fetch (370 km) the variation in γ is given by:

$\gamma = 35 s_p^{6/7} = 0.94\ H_s^{2/7}$ (γ is not allowed to be below 1)

This means that the γ-value increases from 1 up to 2.2 for H_s = 16 m. For fetch 100 km the value is $\gamma = 1.37\ H_s^{2/7}$ giving γ = 3.0 for H_s = 16 m. This γ-value is close to the JONSWAP value γ = 3.3. The model thus gives single peak spectra for locally wind sea with reasonable γ –values.

The Ochi-Hubble spectral model

The Ochi-Hubble spectral model (Ochi-Hubble, 1976) (see also Eq. (12)) gives the most probable spectrum and a class of spectra within a 95% confidence limit for a given H_s. Figures 14 to 17 show some examples of the simplified double peak spectral model and the Ochi-Hubble model compared to measured spectra for Statfjord. Figure 14 shows a comparison for the most probable sea state according to Ochi-Hubble. In Figures 15 the parameters for the Ochi-Hubble spectra is modified to give the same primary peak period as the measured spectrum. Figures 16 and 17 shows to additional spectra for the same value of Hs. In both cases the parameters used for the Ochi-Hubble spectrum gives a spectral peak period near the measured spectra. The figures show a reasonable fit between the two models except for the wind sea dominated sea state showed in figure 16. For this case the Ochi-Hubble model give a high swell peak not found in the data.

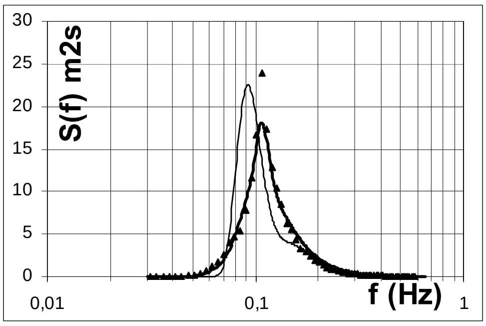

Figure 14: Comparison of the Ochi-Hubble spectrum (thin line) and present model (solid line) for H_s = 4.2m and T_p = 9.3 s. Dots represents average measured spectra from Statfjord

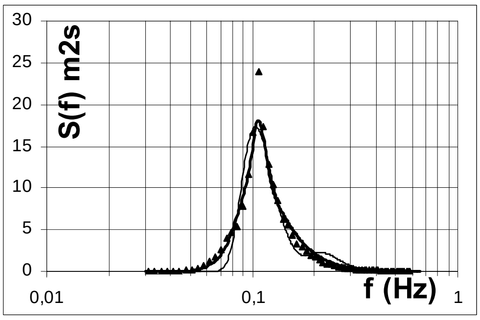

Figure 15: Comparison of the Ochi-Hubble spectrum (thin line) given the same primary peak as measured spectrum and simplified model (solid line) for H_s = 4.2 m and T_p = 9.3 s Dots represents average measured spectra from Statfjord

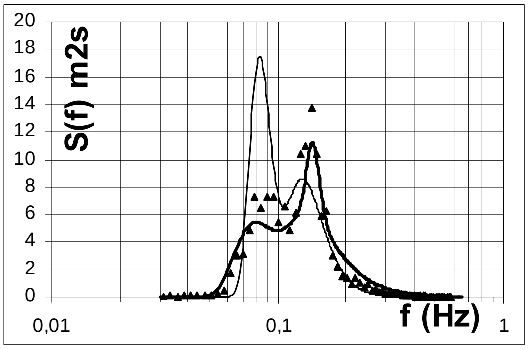

Figure 16: Comparison of the Ochi-Hubble spectrum (thin line) given the same primary peak as measured spectrum and simplified model (solid line) for H_s = 4.0 m and T_p = 7.0 s Dots represents average measured spectra from Statfjord

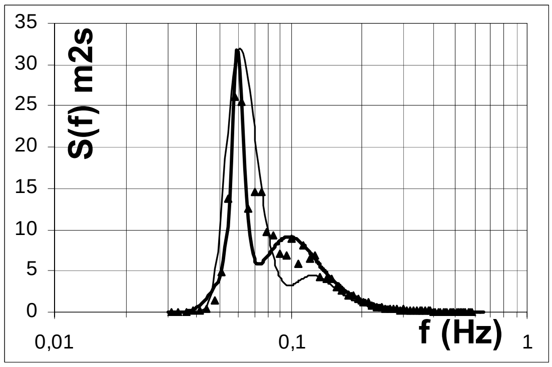

Figure 17: Comparison of the Ochi-Hubble spectrum (thin line) given the same primary peak as measured spectrum and simplified model (solid line) for H_s = 4.3 m and T_p = 16.8 s Dots represents average measured spectra from Statfjord

The original model.

The differences between the original and the simplified model are a) constant exponent for high frequencies (N=-4) in the simplified model, b) simplified dependency of H_s for the peak enhancement factor γ for the primary peak and c) simplified formulation of secondary wind peak. Figure 18 and 19 show the results for two sea states.

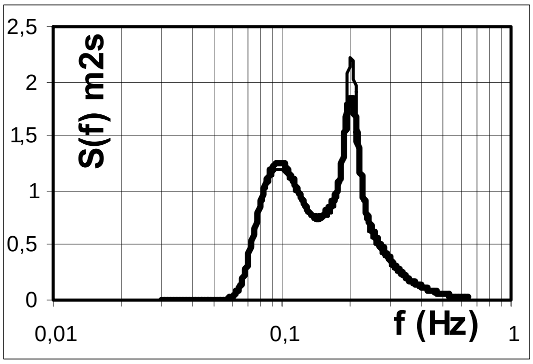

Figure 18: Comparison of the original model (thin line) and present model (solid line) for H_s = 2m and T_p = 5s

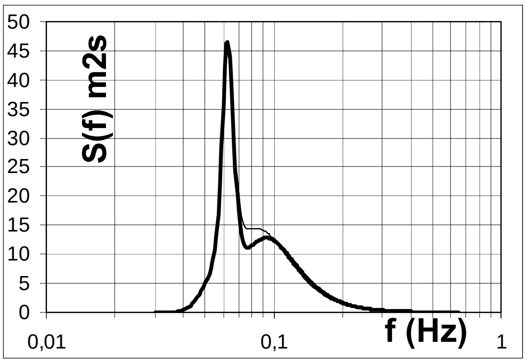

Figure 19: Comparison of the original model (thin line) and present model for H_s = 5m and T_p = 16s

CONCLUSIONS

The findings in this study can be summarized as follows:
A simplified version of the double peak spectral model has been described. The model represents wave conditions in open ocean areas where the waves are dominated by local wind sea, but also exposed to swell. Sea states are classified into two types dependent of the origin of the highest spectral peak. The two types are separated by the relation $T_f=a_f H_s^{1/3}$ into wind sea for $T_p<T_{pf}$ and swell for $T_p>T_{pf}$. The value of a_f is slightly dependent on the maximum fetch for the actual location. The sea state corresponding to $T_p = T_{pf}$ is called the locally fully developed sea and is a single peak sea state. A dimensionless period scale factor defines how far the actual sea state is from the single peak situation. This parameter depends on H_s and T_p and is used to divide the energy between the two spectral peaks. The spectral form for both peaks are given by the Γ spectrum and all parameters is found empirically from the total H_s and primary spectral peak T_P

The simplified model spectra are compared to measured spectra from the Norwegian Continental Shelf, the Ochi-Hubble spectral model and the original double peaked model. The simplified double peak spectral model is for practical use identical to the original model and fits also well to measured data. The deviation can be summarized as follows: For moderate and high sea states the high frequency tail of measured wave spectra seems to fall off faster than the -4 exponent used in the simplified model. For low sea states the measured secondary wind sea peak is slightly broader then found by the model.

Comparison with the Ochi-Hubble spectral model shows some deviations especially for wind dominated seas. The high frequency tail in the Ochi-Hubble model falls off faster than -4. For the examples corresponding to most probable wave situation at Statfjord the two models give similar results. For other sea states deviations are seen, but no comprehensive comparison is done.

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