Hydroelastic Modelling of Net Structures Exposed to Waves and Current

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ABSTRACT: A dynamic model for 3D net structures exposed to waves and current is proposed. The model is based on a super-element formulation where the net structure is divided into four-sided super-elements interconnected in each corner node. The hydrodynamic and structural forces are calculated in each node where the equation of motion are solved. The hydrodynamic forces on the net is calculated using formulas based on empirical studies of net panels in steady current, and are assumed to be drag dominated. The structural forces are calculated by assuming that each element consists of six nonlinear springs, interconnecting each node to the other three. The model is partially validated by comparing it with experimental measurements.

Introduction

The fish farming and aquaculture industry are expanding and the demand for suitable locations for fish farms is increasing. In the future more of the fish farms will be located offshore, as the number of suitable near-shore locations are limited. A move towards the use of offshore locations are also motivated from environmental and aesthetic aspects of the industry, and future fish-farms will most likely be in the form of large scale offshore installations, rather than small near-shore farms.

This calls for new technological challenges, as fish farms are being installed at locations that are more exposed to waves, wind and current. In the design process and before the installation of such structures it is necessary to assess the behaviour of the structure when it is exposed to the environmental forces typical of the location. In the design phase of future aquacultural installations numerical models which can predict the behaviour of structures in different seastates will be important. Assessments of dynamic wave loads is also required in the new Norwegian design rules (Norsk Standard, 2003) for marine fish farms.

The fish farming industry has traditionally been "low-tech", and the method for making a structure more resistant to sea loads is to build it heavier and stronger. An alternative strategy is to make the structure "intelligent", enabling the structure to adapt to the surroundings in such a way that the forces and motions are kept within certain limits. This can be achieved by designing the structure with passive (for instance by introducing flexible elements) or active control elements (for instance thrusters). To achieve this the control system needs a model of the physics involved, and a dynamic model of the behaviour of the structures is required.

Flexible net structures are a major part of marine fish farm, and such net structures are hydroelastic. This means that the force on the structure, and the structures deformation are highly dependent. In contrast to large floating oil installations which can be assumed to be rigid, the fish farms are complex flexible structures with an infinite number degrees of freedom. These circumstances makes it necessary to develop new numerical tools for simulating

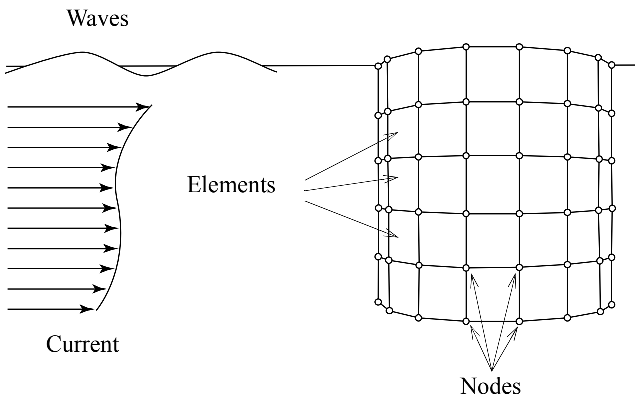

Figure 1. Model overview

the behaviour of such structures, and the methods used for ordinary floating bodies have limited applicability.

Model Description

The model proposed in this paper is a fully 3D dynamic model where the equations of motion are solved in the time domain. The present model is based on a model of a single net panel (Lader and Fredheim, 2001 and Lader et. al., 2001), and is a direct extension of this work. A tentative description of the model is also given in Lader et al. (2003).

Overview

The net structure is modelled by using elements and nodes (Fig. 1). The elements are four-sided with a node in each corner. Hydrodynamic and structural forces are calculated for each element based on the wave particle velocity, current velocity, structural velocity, and node positions. A mass (inertia) and a weight (gravity) are associated with each node, and the nodes can be either free, fixed or have their motion prescribed. This enables, for example, selected nodes to be fixed or connected to other structures which dominates the motion. Movement of the free nodes are not restricted, and the node points are allowed full 3D movement. The forces are collected in each node, where the equation of motion ($F = ma$) is evaluated, yielding the acceleration. The position and the velocity of the nodes are then found by time integrating the acceleration.

Hydrodynamic Forces

The hydrodynamic forces are divided into drag ($\boldsymbol{F}_D$)[1] and lift ($\boldsymbol{F}_L$) force, and are calculated in each individual node. The drag force is parallel to the velocity direction ($\boldsymbol{F}_D \parallel \boldsymbol{U}$), and the lift force is perpendicular to the flow direction ($\boldsymbol{F}_L \perp \boldsymbol{U}$). The drag and lift force are calculated from the general expression:

$$\boldsymbol{F}_D = \frac{1}{2}\rho C_D A |\boldsymbol{U}|^2 \boldsymbol{n}_D \tag{1}$$

$$\boldsymbol{F}_L = \frac{1}{2}\rho C_L A |\boldsymbol{U}|^2 \boldsymbol{n}_L \tag{2}$$

where ρ is the mass density of water, C_D and C_L are the non-dimensional drag and lift coefficients respectively, A is a characteristic area and $\boldsymbol{U}$ is the relative velocity between the fluid and the structure. $\boldsymbol{n}_D$ and $\boldsymbol{n}_L$ are the unit vectors in the direction of the drag and lift force respectively.

The hydrodynamic forces in each node are calculated by dividing the area around each node into eight triangles as shown in Figure 2a. Every triangle has one corner in the node point ($\boldsymbol{x}_1$ in Fig. 2b), one in an element centre ($\boldsymbol{x}_3$) and one in the mid-point between the node and an adjacent node ($\boldsymbol{x}_2$). $\boldsymbol{F}_D$ and $\boldsymbol{F}_L$ are calculated on each triangle, and then summed up to give the total hydrodynamic force in the node.

The velocity $\boldsymbol{U}$ is the relative velocity between the fluid and the net:

$$\boldsymbol{U} = \boldsymbol{U}_w + \boldsymbol{U}_c - \boldsymbol{U}_s \tag{3}$$

where $\boldsymbol{U}_w$ is the wave particle velocity, $\boldsymbol{U}_c$ is the current velocity and $\boldsymbol{U}_s$ is the structural velocity (the velocity of the node). The wave particle velocity is calculated using linear, regular wave theory.

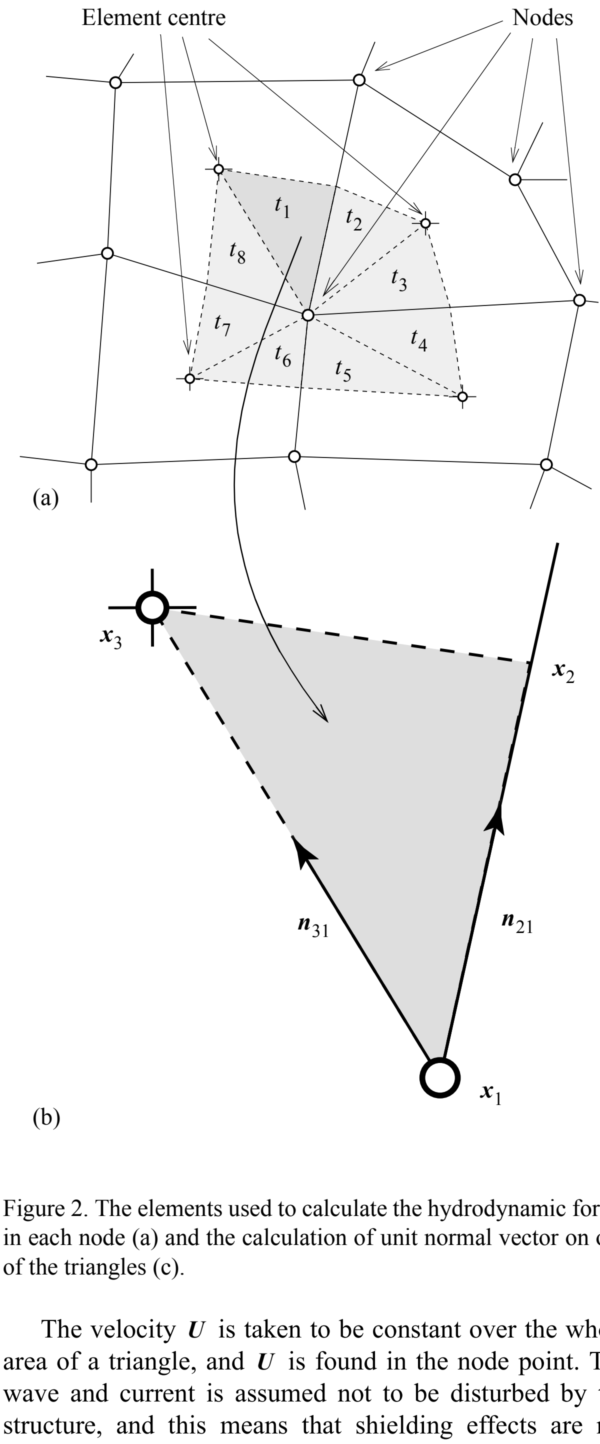

Figure 2. The elements used to calculate the hydrodynamic forces in each node (a) and the calculation of unit normal vector on one of the triangles (c).

The velocity $\boldsymbol{U}$ is taken to be constant over the whole area of a triangle, and $\boldsymbol{U}$ is found in the node point. The wave and current is assumed not to be disturbed by the structure, and this means that shielding effects are not taken into account in the model.

1. Bold type faces indicates vectors

The triangle normal unit vector ($\boldsymbol{n}_{tn}$), which is used to calculate the lift unit vector ($\boldsymbol{n}_L$), is found in each individual triangle by taking the cross product of two unit vectors ($\boldsymbol{n}_{21}$ and $\boldsymbol{n}_{31}$) given by (Fig. 2b):

$$\boldsymbol{n}_{21} = \frac{\boldsymbol{x}_2 - \boldsymbol{x}_1}{|\boldsymbol{x}_2 - \boldsymbol{x}_1|} \tag{4}$$

$$\boldsymbol{n}_{31} = \frac{\boldsymbol{x}_3 - \boldsymbol{x}_1}{|\boldsymbol{x}_3 - \boldsymbol{x}_1|} \tag{5}$$

and the element normal unit vector are then given by:

$$\boldsymbol{n}_{tn} = \frac{\boldsymbol{n}_{21} \times \boldsymbol{n}_{31}}{|\boldsymbol{n}_{21} \times \boldsymbol{n}_{31}|} \tag{6}$$

The direction of $\boldsymbol{n}_{tn}$ is by definition in the positive flow direction.

The unit vectors for drag and lift are then given by:

$$\boldsymbol{n}_D = \frac{\boldsymbol{U}}{|\boldsymbol{U}|} \tag{7}$$

$$\boldsymbol{n}_L = \frac{(\boldsymbol{U} \times \boldsymbol{n}_{tn}) \times \boldsymbol{U}}{|(\boldsymbol{U} \times \boldsymbol{n}_{tn}) \times \boldsymbol{U}|} \tag{8}$$

The drag and lift coefficients (C_D and C_L) are calculated using formulas found by Aarsnes et al. (1990). These formulas are based on both theoretical work and comprehensive model tests (Rudi et al., 1988). C_D and C_L are given by:

$$C_D = 0.04 + (-0.04 + S - 1.24S^2 + 13.7S^3)\cos(\alpha) \tag{9}$$

$$C_L = (0.57S - 3.54S^2 + 10.1S^3)\sin(2\alpha) \tag{10}$$

where S is the solidity of the net, which is the ratio between the solid area of the net (A_s) and the total area enclosed by the net (A): $S = A_s/A$. α is the angle of attack, and is defined to be the angle between the direction of $\boldsymbol{U}$ and the normal vector of the triangle ($\boldsymbol{n}_{tn}$). The formulas should not be used for S higher than 0.35.

The model tests on witch the formulas are based were done with Reynolds numbers in the range 1400 to 1800, and the Reynolds number dependency of the drag and lift forces are thus not included in the formulas. Also note that these formulas are strictly valid for stationary flow only.

The triangle area A is not constant, but varies with the deformation of the elements. Consequently the solidity S also varies accordingly since the solid area of the net elements (A_s) is constant. The solidity used to calculate the hydrodynamical forces on each net elements must therefore be corrected for on each individual triangle based on the instantaneous area of that element:

$$S = S_0 \frac{A_0}{A} \tag{11}$$

where S and A is the instantaneous solidity and area of the triangle respectively, and S_0 and A_0 is the solidity and area of the triangle when it is undeformed.

Structural Forces

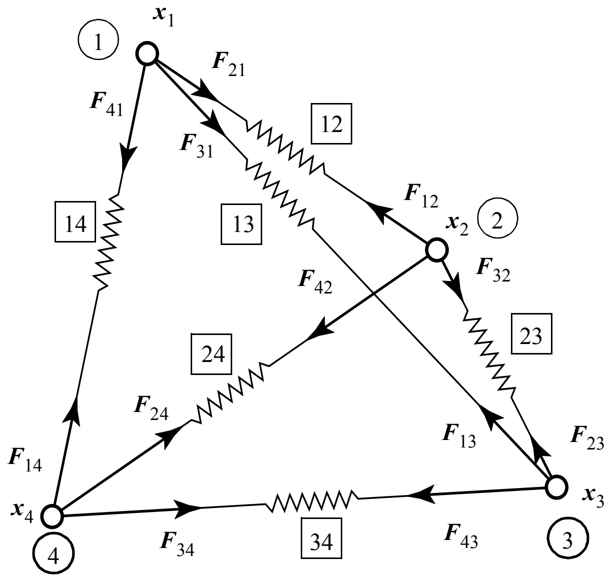

Figure 3. The structural model of the element. Node numbers are indicated in circles, and spring numbers in squares.

The elements are modelled structurally as shown in Fig. 3. Each node is connected with the other nodes through a non-linear spring. Each spring is assumed to have the following force - elongation relationship:

$$\begin{aligned} F_s &= C_1\varepsilon + C_2\varepsilon^2 & \varepsilon > 0 \\ F_s &= 0 & \varepsilon \le 0 \end{aligned} \tag{12}$$

where F_s [N] is the structural force, and ε [-] is the global elongation. The global elongation is given by $\varepsilon = (l - l_0)/l_0$ where l_0 is the undeformed length and l is the deformed length of the panel. C_2 and C_1 are constants describing the force/elongation characteristics of each spring. In each four sided element there are six springs, yielding twelve force contributions, three in each node. The main reason for introducing the diagonal springs is to be able to model both square and diamond oriented meshes without having to re-grid the net. The force in the spring between node n and node m ($\boldsymbol{F}_{nm}$) are calculated from the following equations:

$$\boldsymbol{F}_{nm} = F_{nm}\boldsymbol{n}_{nm} \tag{13}$$

here F_{nm} is the force magnitude, and $\boldsymbol{n}_{nm}$ is the unit directional vector for the force. The force magnitude is calculated from:

$$\begin{aligned} F_{nm} &= C_{1nm}\varepsilon_{nm} + C_{2nm}\varepsilon_{nm}^2 & \varepsilon_{nm} > 0 \\ F_{nm} &= 0 & \varepsilon_{nm} \le 0 \end{aligned} \tag{14}$$

where the elongation ε_{nm} is given by:

$$\varepsilon_{nm} = \frac{l_{nm} - l_{0nm}}{l_{0nm}} = \frac{|\boldsymbol{x}_n - \boldsymbol{x}_m| - |\boldsymbol{x}_{0n} - \boldsymbol{x}_{0m}|}{|\boldsymbol{x}_{0n} - \boldsymbol{x}_{0m}|} \tag{15}$$

where $\boldsymbol{x}_{0n}$ and $\boldsymbol{x}_{0m}$ is the position of the nodes when the element is undeformed, and no internal forces are acting in the element. $\boldsymbol{x}_n$ and $\boldsymbol{x}_m$ are the instantaneous position of the nodes as indicated previously (Fig. 3). The unit directional vector is given by:

$$\boldsymbol{n}_{nm} = \frac{\boldsymbol{x}_n - \boldsymbol{x}_m}{|\boldsymbol{x}_n - \boldsymbol{x}_m|} \tag{16}$$

Equation of Motion

The structural and hydrodynamical forces are calculated in each element, and the forces are then collected in each node, where the equation of motion is evaluated. For one node surrounded by the four elements e_1 to e_4 (Fig. 4), and the eight triangles t_1 to t_8 (Fig. 2a) the total structural and hydrodynamical forces in the node is given by:

$$\begin{aligned}\boldsymbol{F}_{structural} = {} & \boldsymbol{F}_{13}^{e_1} + \boldsymbol{F}_{23}^{e_1} + \boldsymbol{F}_{43}^{e_1} + \boldsymbol{F}_{14}^{e_2} + \boldsymbol{F}_{24}^{e_2} + \boldsymbol{F}_{34}^{e_2} \\ & + \boldsymbol{F}_{21}^{e_3} + \boldsymbol{F}_{31}^{e_3} + \boldsymbol{F}_{41}^{e_3} + \boldsymbol{F}_{12}^{e_4} + \boldsymbol{F}_{32}^{e_4} + \boldsymbol{F}_{42}^{e_4}\end{aligned} \tag{17}$$

$$\boldsymbol{F}_{hydrodynamic} = \sum_{n=1}^{8} \boldsymbol{F}_L^{t_n} + \boldsymbol{F}_D^{t_n} \tag{18}$$

where $\boldsymbol{F}_L^{t_n}$ and $\boldsymbol{F}_D^{t_n}$ is the lift and drag force respectively of triangle n.

The equation of motion now becomes:

$$\boldsymbol{F}_{structural} + \boldsymbol{F}_{hydrodynamic} + wg\boldsymbol{n}_g = m\boldsymbol{a} \tag{19}$$

where w and m is the weight and mass respectively associated with the node, and $\boldsymbol{n}_g$ is the gravity unit vector equal $[0, 0, -1]$.

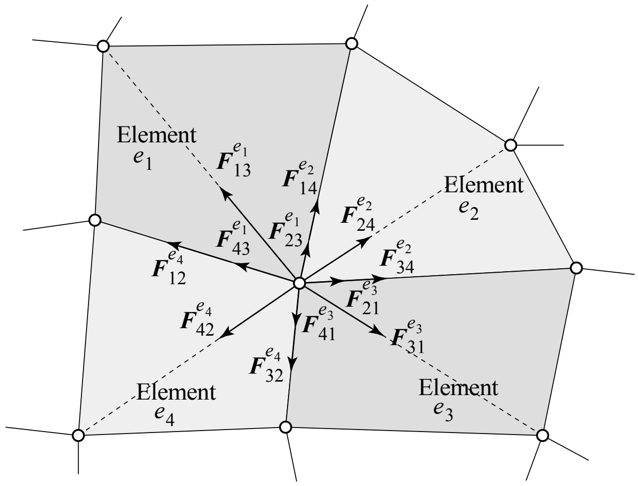

Figure 4. Structural forces in one node.

Time Integration

From the equation of motion the acceleration of each element is found. To calculate the movement of the node, the acceleration is integrated twice:

$$\begin{aligned}\boldsymbol{u} &= \int \boldsymbol{a}dt \\ \boldsymbol{x} &= \int \boldsymbol{u}dt = \iint \boldsymbol{a}dtdt\end{aligned} \tag{20}$$

The numerical integration of Equation 20 is performed by using the `ode15s` routine implemented in the Matlab (v 6.1) software package (Shampine and Reichelt, 1997). `ode15s` is a variable order solver based on the numerical differentiation formulas (NDFs), and this method is chosen since the problem has a stiff characteristic due to the high structural eigenfrequency, much higher than the common wave frequency.

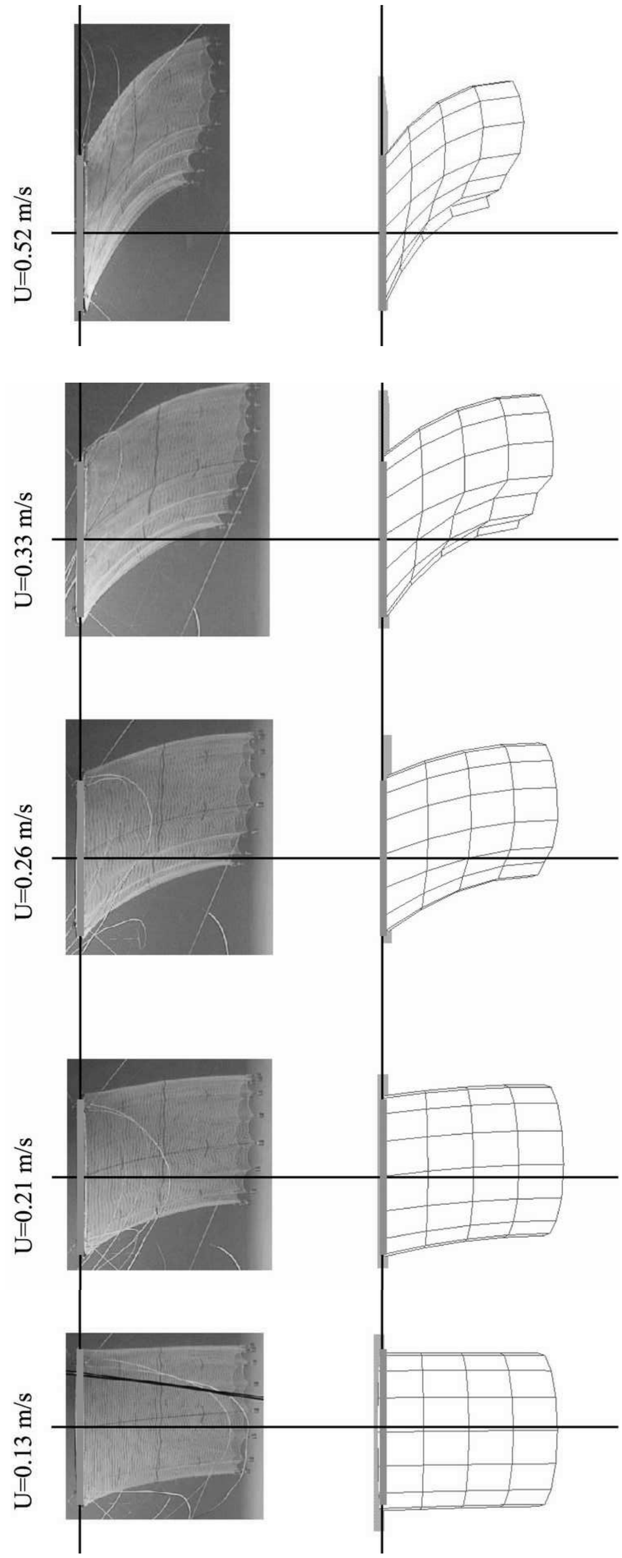

Figure 5. Deformation of net cylinder exposed to different current velocities. Comparison between flume tank experiments (top) and numerical calculations (bottom).

Comments on the Solution Strategy

The method of solution used in this model is based on a direct explicit approach. The highly hydroelastic and non conservative nature of the problem exhibit difficulties with an implicit approach. An implicit approach will be advantageous with respect to usable time step, numerical stability bounds and probably total simulation efficiency. Stiffness matrix inversion for each time step will also be required due to large non-linear deformations of the net structure. The possibility of slack events in the net structure is high and may cause singularities in the stiffness matrix inversion. Thus, the gain in improved efficiency and stability with reduced time step requirements is lost. The cost of an explicit method is however increased simulation time due to the routine `ode15s` automatically selection of very small time steps.

Experimental validation

The numerical model has been validated by comparing numerical simulations with physical experiments. The behaviour of a cylindrical net structure exposed to both accelerating and steady current was simulated by the numerical model, and physical experiments were carried out at *the North Sea Centre Flume Tank* in Hirtshals, Denmark.

A comparison of the deformation of the cylindrical net structure in the numerical model, and the physical experiments are given in Figure 5. This comparison indicates that the agreement between model and experiments are satisfactory for velocities above 0.21 m/s, and that the model under-predicts the deformation at low velocities. This is also consistent with the force measurements conducted in the tests, and can be partly explained by the Reynolds number dependency of the hydrodynamic loads, which are not included in the present model. More details of the validation are given in Lader et al. (2003).

Summary

A dynamic model of a 3D net structure is proposed and implemented using Matlab. The model uses a direct explicit approach to the problem. This is due to the strong hydroelastic effects in the problem as the structure can experience large deformations due to the exposure to waves and current. The model is partially validated against measurements of a cylindrical net structure in accelerating and steady current.

The model is relevant for use in the design of marine fish farms, as new design rules require assessment of dynamic waveinduced loads on such structures. The move towards utilisation of locations with increasing exposure to waves and current makes it important to have predictable numerical tools in the design process, and the possible introduction of automatic control elements in aquaculture structures also makes it necessary to develop such numerical models.

Acknowledgement

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