Simplified double peak spectral model for ocean waves

Knut Torsethaugen
SINTEF Fisheries and Aquaculture, Trondheim Norway.
Sverre Haver
STATOIL, Stavanger Norway

ABSTRACT
A simplified version of the double peak spectral model for ocean waves (Torsethaugen, 1993, 1994, 1996) is described. For the locally fully developed sea, the spectral peak period $T_{pf}$ is a function of the significant wave height, $H_s$, $T_{pf}=a_f*H_s^{1/3}$ with $a_f$ slightly dependent on fetch. This classifies sea states as wind sea $T_p<=T_{pf}$ and swell $T_p>T_{pf}$ each consisting of two wave systems. Empirical parameters given by significant wave height and spectral peak period define the spectral form parameters and energy distribution between the two wave systems in the model. The adequacy of the model is indicated by comparing the model with measured wave spectra from the Norwegian Continental shelf.

KEY WORDS: Ocean waves; wave spectra; double peak; wind sea; swell

INTRODUCTION
For a number of years, the Torsethaugen double peak spectral model has frequently been used for design purposes at the Norwegian Continental Shelf, both in connection with numerical analyses and model tests. The original Torsethaugen model was established by fitting two JONSWAP shaped models to average measured spectra from the Norwegian Continental Shelf. The observed sea states were grouped with respect to significant wave height, $H_s$, and spectral peak period, $T_p$. This means that average spectra were established from a broad range of combinations of $H_s$ and $T_p$. Regarding the adequacy of the original fit to the average spectra, reference is made to Torsethaugen (1993, 1994, 1996).

Over the years, users of the spectral model have often raised the question if the model can be simplified without a significant loss of validity. The final use of the model has always been rather simple, since the spectrum is defined as the total significant wave height and the period of the dominating spectral peak are given. The complexity lies in the number of parameters involved in parameterize the spectral model in merely two parameters.

The purpose of the present study is to
- reduce the number of free parameters, requiring that the simplified model should represent a fit to the average spectra of an accuracy being comparable to the original Torsethaugen model.
- analyze the model in order to find a better understanding of some of the physical and site dependency for the empirical parameters.

The adequacy of the simplified model is indicated by comparing it with a subset of available average spectra. In this connection, the model is also compared to wave spectra measured by the Miros radar. This is of special interest since the original model is fitted to average spectra obtained from buoy measurements. The simplified model is also compared to the Ochi-Hubble double peak spectral model since the latter is more frequently adopted outside the Norwegian Continental Shelf.

As mentioned above, the Torsethaugen model is fitted to average spectra established for various combinations of $H_s$ and $T_p$. When utilizing a model aiming towards describing the expected spectral shape of a sea state characterized by merely two parameters, one has to realize that on a case by case comparison a considerable scatter around the expected shape is observed. In particular this is the case for sea states being of a combined nature. The scatter is most clearly pronounced for the low frequency part of the spectrum where the swell components are located. The importance of the scatter for structural response predictions depends very much on the transfer function of the response problem under consideration. If the response is sensitive to the swell period band, one should ensure some robustness against the effect of the scatter around the average spectral shape. By utilizing an average spectral shape a source of randomness is neglected, this should be kept in mind when executing a structural reliability analysis.

A possible generalization of the spectral modeling would be to suggest a family of spectral shapes for each sea state class, where each member of the family is given a certain probability of occurrence.
It is also to be noted that a spectral model assumed to represent the expected spectral shape for a given combination of $H_I$ and $T_p$ in principle is merely validated for sea states covered by the observations. When utilizing such a model well beyond the range of experienced sea states, as is the case when the model is adopted for 100- and 10000-year sea states, the accuracy of the model is of course associated with uncertainties due to the extrapolation beyond the fitting region. For sea states of such a severity as indicated above, uncertainties will be present whether one chooses a single peak spectral model or a double peak model. A double peak model is in qualitative agreement with experienced real sea wave spectra, and we expect them to correspond to a broader range of applicability than single peak spectral models.

The present spectral model is limited to account for two distinct sea wave systems where one is caused by local wind. In certain areas of the world, it is claimed that one typically experience triple peak wave spectra or more than one swell peak. If this is the case for the design sea state regime, a better way is probably to establish the above mentioned families of spectra for various combinations of $H_I$ and $T_p$. Such verification is left for possible future work.

**BASIC ELEMENTS IN SPECTRAL WAVE MODELS**

### Basic sea states

For this model it is assumed that ocean waves at a location can be divided into two main parts

- a) Wind sea, i.e. seas generated by the local wind.
- b) Swell seas, i.e. waves entering into the location from other areas.

Generally sea states can occur with more than one swell component and be influenced by fetch, breaking waves, wave refraction and reflection, local current and large current circulation systems. These effects are dependent on the location. The spectral models discussed are limited to deepwater waves on the open ocean, ignoring the influence from current and other refraction and diffraction effects.

### Basic spectral models

There are two different approaches to spectral modeling.

- a) Single sea state models, describing locally generated wind sea states, are based on fundamental understanding of wind wave generation. These models are the basis for all other models and are essential for the understanding of ocean waves.
- b) Statistical spectral models that are based on the basic models or a combination of such models. They describe the most probable spectrum or a distribution of wave spectra for a given sea state. These models will be based on a large amount of measured wave spectra.

### Basic spectral elements

In the derivation of spectral models two basic elements are involved

- a) The high frequency tail.
- b) The spectral peakedness.

#### a) High frequency tail

The behavior of the high frequency part of the spectrum is given by the energy balance for waves generated by the local wind fields.

An important concept in wind wave generation is the so-called equilibrium range. When the wind blows for a long time or over a long fetch, the wave energy for a given frequency reaches an upper limit, the equilibrium range. This means that the energy input from the wind is balanced by energy loss to other frequencies and by wave breaking. Since Phillips (1958) introduced the equilibrium range, a discussion has been going on how to define the high frequency range. According to Phillips, the equilibrium range is found by dimension analysis to be:

$$ S(f) = \alpha (2\pi)^2 g^2 f^{-5} $$

where $\alpha$ was an absolute constant ($7.4 \times 10^3$) and $g$ the gravity constant.

This equilibrium range was used by Pierson and Moskowitz (1964) when they developed the PM-spectrum for fully developed wind sea. Hasselman et al. (1973) also adopted this formulation in the JONSWAP spectrum for growing sea states under fetch limited conditions. Based on measured spectra, (Toba, 1972), theoretical studies of wave-wave interactions and study of energy input mechanisms from wind and wave breaking, Phillips (1984) found that the equilibrium range for high frequencies rather goes as:

$$ S(f) \approx u^* g (2\pi)^4 f^{-4} $$

where $u^*$ is the friction wind velocity. This value of the exponent is commonly accepted as the (theoretical) value for the high frequency tail (equilibrium range) for wind generated sea today. For measured spectra this exponent seems to vary between 4 and 5. Values even outside this range are found in some few cases. This can be an effect of the measuring technique, bottom effects, effects of ocean currents, or other effects.

#### b) Spectral peakedness

In the simplest form for a wave spectrum the energy follows the equilibrium range up to the spectral peak and then drops to zero. A wave spectral model developed by Bretschneider (1958) was formulated as:

$$ S(f) = \alpha g^2 (2\pi)^2 f^{-5} e^{-\left(\frac{f}{f_0}\right)^2} $$

The wave period $T_m$ is given by $T_m = T_p/1.17$

For high frequencies, Eq. (3) is in agreement with Phillips first version of the equilibrium range and confirms the value of $\alpha$ found from measurements. Later Pierson and Moskowitz came up with the spectral form:

$$ S(f) = \alpha g^3 (2\pi)^5 f^{-8} e^{-\left(\frac{f}{f_0}\right)^2} $$

where $\alpha = 8.10 \times 10^3$, $\beta = 0.74$ and $f_0 = g/2\pi U$ (U= wind speed 19.5 m above sea level). Here the wave period $T_0 = 1/f_0$ is given by $T_0 = T_p/1.14$

The original PM-spectrum represents the spectrum for the fully developed wind sea (infinite fetch) and has a high frequency tail that falls off as $f^5$. The spectral peakedness is defined by the exp (-$\beta(f/f_0)^2$) factor.

As part of the Joint North Sea Wave Project (JONSWAP) Hasselmann et. al. (1973) developed the JONSWAP spectrum where a peak enhancement factor $\gamma$ was introduced to represent fetch limited wind sea:
\[ S(f) = \alpha g^2 (2\pi)^{-4} f^{-5} e^{(-1.25(f_p / f)^{3})} \]
\[ \times \exp\left\{ -(1-f/f_p)^2/(2\sigma^2 f^2) \right\} \]
where \( \sigma = \sigma_f \) for \( f < f_p \) and \( \sigma = \sigma_o \) for \( f > f_p \), \( f_p \) represents the spectral peak frequency.

The JONSWAP spectrum is a PM-spectrum multiplied with a peak enhancement function \( \gamma(f) \). The JONSWAP spectrum contains 5 parameters \( f_p, \alpha, \gamma, \sigma_f \) and \( \sigma_o \) and the most probable values for some of these read: \( \gamma = 3.3, \sigma_f = 0.07 \) and \( \sigma_o = 0.09 \). The values for \( \alpha \) and \( f_p \) depend on wind speed, fetch and duration.

The \( \Gamma \)-spectrum

The generalized \( \Gamma \) spectrum is defined as:
\[ S(f) = E S_n(f_n) \]
where:
\[ S(f) \] is wave energy density (m^2/s)
\[ f \] is wave frequency (Hz)
\[ E = m_0 / f_p = (1/16) H_s^2 / f_p \] (m^2/s)
\[ f_p \] is spectral peak frequency (Hz)
\[ H_s \] is significant wave height (m)
\[ S_n(f_n) \] is non dimensional spectral density
\[ f_n = f / f_p \]

The non dimensional spectral density can be written:
\[ S_n(f_n) = G_\alpha A \gamma \Gamma (f_n; N, M) \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \g
For type b), the situation is more complicated and includes at least two different situations:

1) Dominating swell in addition to local wind sea.
2) Decaying wind seas. If the local wind is decaying, the waves will lose energy for high frequencies. This swell ("old" wind sea) can represent the highest spectral peak if no new local wind seas are generated with sufficient spectral peak density.

For type a), wind dominated sea; the secondary system represents an average of swell sea for the area. For locations not particularly exposed to swell, the probability to find sea states with this combination of $H_{s}, T_p$ should be low. For type b), swell dominated sea, the secondary sea system represents local wind sea.

The double peak spectral model gives a parametric description for four types of peaks.

a) Primary wave system generated by the local wind field
b) Primary wave system dominated by swell
c) Secondary wind sea system
d) Secondary swell system

The average wave spectra is found as a sum of a primary and a secondary wave system each given by the $\Gamma$- spectrum.

**FORMULATION OF THE SIMPLIFIED DOUBLE PEAK SPECTRUM**

The original formulation of the double peak wave spectrum contained many empirical parameters. Some of these parameters have only effect for low sea states and are of marginal importance for design. The present study describes a simplified version.

**Fully developed sea.**

The distinction between wind dominated and swell dominated sea states is found to be defined by the fully developed sea for the location where $T_p$ is given by:

$$ T_p = a_T H_s^{1/3} \quad (13) $$

If $T_p \leq T_p$, the primary spectral peak corresponds to the local wind system. If $T_p > T_p$, the primary spectral peak corresponds to the swell system.

Eq. (13) is derived as follows:

From the JONSWAP experiment:

$$ H = k_F F_1/2 \quad (14) $$

$$ T = k_F F_1/3 \quad (15) $$

$H, T$ and $F$ are dimensionless values for $H_{s}, T_p$ and fetch $F_p$ defined by:

$$ H = g H_s / u^2 \quad (16) $$

$$ T = g T_p / u \quad (17) $$

$$ F = g F_p / u^2 \quad (18) $$

where $u$ is a wind velocity. The JONSWAP experiment, Hasselmann et al. (1973), used $u = u_{10}$, where $u_{10}$ is the wind speed 10m above sea level and found that $k_1 = 1.6 \times 10^{-7}$ and $k_2 = 0.286$

From these expressions the following can be derived:

$$ T / H^{1/3} = k_3 F_1/6 ; \quad k_3 = k_2 / k_1^{1/3} \quad (19) $$

Introducing the definitions of the normalized quantities, one obtains:

$$ T_p = k_p F_1/6 H_1/3 \quad (21) $$

where $T_p$ corresponds to the peak period for the locally fully developed sea, and $k_p = k_{1/2} g^{-1/2} = 0.78$ (sm$^{-1/2}$).

This gives

$$ a_T = k_p F_1/6 \quad (22) $$

The factor $a_T$ is seen to be slightly dependent on fetch. It has the value:

- $a_T = 6.6$ (sm$^{-1/2}$) for $F_p = 370$ km and
- $a_T = 5.3$ (sm$^{-1/2}$) for $F_p = 100$ km.

**Peak enhancement factor $\gamma$.**

According to Mitsuyasu (1980), the peak enhancement factor

$$ \gamma = k_p F_1^{-1/7} \quad (23) $$

Introducing the average wave steepness:

$$ s = (2\pi / g) H_s / T_p^2 = \pi H_s / T_p^2 = 2\pi (k_1 / k_2) F_1^{-1/6} \quad (24) $$

one may express $\gamma$ as:

$$ \gamma = k_p s^{-6/7} ; \quad k_p = k_4 (k_2^2 / 2\pi k_1)^{6/7} \quad (25) $$

The value of $k_4$ is given by Mitsuyasu (1980) as $k_4 = 7.0$ resulting in $\gamma = 42.2$ with values of $k_1$ and $k_2$ taken from the JONSWAP experiment (Eqs. (14) and (15)). The value of $k_4$ will probably depend on the location (fetch and wind speed). It will increase in fetch limited areas reflecting more peaked spectra.

In the original model, $k_p = 35 (1+3.5 e^{150})$ ($H_s$ in m) was used based on best fit to data from the Statfjord area. For $H_s > 5$ m this is a constant = 35. This value is used in the simplified spectral model.

**Wave period scale factors**

For each of the sea systems a non-dimensional period scale is introduced by using a lower and an upper value for $T_p$. The lower period limit depends on $H_s$ and is related to wave breaking. The parameter used in the original version was found by best fit to data and corresponds to a steepness, $s = 0.16$. This is slightly above the breaking criteria for regular waves, $s = 1/7$. For the upper period limit a constant value, representing an absolute upper value for the spectral peak period, is used.

The two limits are set to:

Lower limit

$$ T_l = a_T H_s^{1/2} \quad (26) $$

Upper limit

$$ T_u = a_T \quad (27) $$

Non-dimensional scales for the spectral peak period are defined as:

Wind sea

$$ \xi = (T_p - T_l) / (T_u - T_l) \quad (28) $$

Swell

$$ \xi = (T_p - T_u) / (T_u - T_l) \quad (29) $$

Comment: For values of $T_p$ below $T_l$ or above $T_u$ $\xi$ values should be set equal to 1.

**High frequency tail and spectral form**

The high frequency tail of the wind spectrum goes like $f^{-N}$.

Measured spectra often show a value for $N$ varying from 4 to 5. In the original version, Torsethaugen (1993), the following expression for $N$ was used:

$$ N = k_p H_s^{1/2} + k_0 g \quad (30) $$

where $k_0 = 0.5$ (m$^{-2/3}$) and $k_0 g = 3.2$. 

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For most practical applications, the exponent of the high frequency tail is not important. It may have a slight effect on the results for a lightly damped structural system with a natural frequency in the high frequency range. In view of this, the simplified model is using N=4 for all sea states. This will be conservative in case of a lightly damped structural system.

For the spectral width parameter, M, in the $\Gamma$-spectrum, 4 is used for all sea states.

### Spectral parameters for

#### a) Wind dominated sea $T_p \leq T_{pf}$

1) **Primary peak**
   i) Significant wave height,
   \[ H_{w1} = R_w H_s, \quad R_w = (1 - a_{10}) e^{-(\epsilon_1 / a_{10})^2} + a_{10} \]  
   (31)
   ii) Spectral peak period
   \[ T_{p1} = T_p \]  
   (32)
   iii) Peak enhancement factor,
   \[ \gamma = k_{s} s_p^{6/7}; \quad s_p = (2\pi / g) H_{w1} / T_{p1}^2 \]  
   (33)

2) **Secondary peak**
   i) Significant wave height
   \[ H_{w2} = (1 - R_w^2)^{1/2} H_s \]  
   (34)
   ii) Spectral peak period
   \[ T_{p2} = T_{pf} + b_1 \]  
   (35)
   iii) Peak enhancement factor
   \[ \gamma = 1 \]  
   (36)

#### b) Swell dominated sea $T_p > T_{pf}$

1) **Primary peak**
   i) Significant wave height
   \[ H_{s1} = R_s H_s, \quad R_s = (1 - a_{20}) e^{-(\epsilon_2 / a_{20})^2} + a_{20} \]  
   (37)
   ii) Spectral peak period
   \[ T_{p1} = T_p \]  
   (38)
   iii) Peak enhancement component
   \[ \gamma = \gamma_f (1 + a_s e_u); \quad \gamma_f = k_{s} s_f^{6/7}, s_f = (2\pi / g) H_s / T_{pf}^2 \]  
   (39)

2) **Secondary peak**
   i) Significant wave height
   \[ H_{s2} = (1 - R_s^2)^{1/2} H_s \]  
   (40)
   ii) Spectral peak period
   \[ T_{p2} = a_s H_{s2}^{1/3} \]  
   (41)
   iii) Peak enhancement factor
   \[ \gamma = 1 \]  
   (42)

The numerical values for all parameters are given in Table 1.

### Table 1. Empirical parameter values for simplified model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Used in formulae</th>
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<td>$\epsilon_1$</td>
<td>6.6 sm$^{-0.3}$</td>
<td>13,41</td>
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<tr>
<td>$\epsilon_2$</td>
<td>2.0 sm$^{-0.2}$</td>
<td>26</td>
</tr>
<tr>
<td>$a_{10}$</td>
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<tr>
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<td>37</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.6</td>
<td>37</td>
</tr>
</tbody>
</table>

### Resulting spectral formula

The resulting spectra can be formulated as:

\[ S(f_j) = \sum_{j=1}^{2} E_j S_p(f_j, \gamma) \]  
(43)

\[ j = 1 \text{ primary sea system, } j = 2 \text{ secondary sea system.} \]

\[ E_1 = (1/16)H_r^2 T_{p1} \text{ and } E_2 = (1/16)H_r^2 T_{p2} \]  
(44)

\[ S_{1n}(f_j) = G_0 f_j^{-4} e^{-f_j^{-4} \gamma (\exp(-1/2 \sigma_j^2 f_j^{-4} - 1))} \]  
(45)

\[ S_{2n}(f_j) = G_0 f_j^{-4} e^{-f_j^{-4} \gamma} \]  
(46)

For wind sea dominated cases i.e. $T_p \leq T_{pf}$ we have:

\[ H_1 = H_{w1} \text{ and } H_2 = H_{w2} \text{ see Eqs. (31) and (34)} \]

For swell sea dominated cases i.e. $T_p > T_{pf}$ we have:

\[ H_1 = H_{s1} \text{ and } H_2 = H_{s2} \text{ see Eqs. (32) and (35)} \]

$\gamma$ given by Eqs. (33) and (36).

### Parameter variations

The total wave energy is divided between the two sea systems according to the parameter shown in Figures 1 and 2 for wind dominated sea and swell dominated sea respectively. Curves are given for different values of the spectral peak period. The energy in the primary peak reaches a constant level for spectral peak periods far from the one peak sea state.

The variation in the peak enhancement factor for the primary peak is shown in Figures 3 and 4 for wind sea and swell, respectively. For the secondary peak the peak enhancement factor is set equal to 1 for all sea states. Curves are given for different values of the spectral peak period.
### COMPARISON WITH MEASURED WAVE SPECTRA

The following comparisons between the simplified model and measured data are presented.

a) Average measured wave spectra for Statfjord
b) Spectral moments represented by wave period parameter $T_{m02}$ for Halten area.

c) Average spectra measured by the MIROS radar at Gullfaks C

### Statfjord.

The spectral model is compared to measured spectra for Statfjord. The points represent average measured spectra and the solid line show the simplified model spectrum.

The examples below show that the simplified model fits very well to the spectral peak, but minor deviations in the high frequency tail will be found for significant wave height above 5 m.

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**Figure 1**: Ratio wind sea significant wave height to total significant wave height for wind dominated sea $T_p = T_{pf}$. Curves are given for $T_p$ values from 6s to 16 s from left to right.

**Figure 2**: Variations in the peak enhancement factor $\gamma$ for wind dominated sea $T_p = T_{pf}$. Curves are given for $T_p$ values from 6s to 16 s from left to right.

**Figure 3**: Ratio swell significant wave height to total significant wave height for wind dominated sea $T_p > T_{pf}$. Curves are given for $T_p$ values from 8s to 20 s from left to right.

**Figure 4**: Ratio swell significant wave height to total significant wave height for swell dominated sea $T_p > T_{pf}$. Curves are given for $T_p$ values from 8s to 20 s from bottom left to top.

**Figure 5**: Comparison of wave spectra for Statfjord. Solid line: Simplified model. Dots: Measured data. Sea state: $H_s = 2.48 m$, $T_p = 5.3 s$. No. of spec. = 6

**Figure 6**: As for Figure 5

Sea state: $H_s = 2.59 m$, $T_p = 16.8 s$. No. of spec. = 14

**Figure 7**: As for Figure 5

Sea state: $H_s = 5.24 m$, $T_p = 9.4 s$. No. of spec. = 279
Halten

The average wave period parameters $T_{o2} = (m_2/m_0)^{1/2}$ given by the spectral moments, $m_0$ and $m_2$ are computed from data for Halten for one year. Figure 9 show one example of how $T_{o2}$ varies with $T_p$ for sea state with significant wave height 2.5-3 m. We see that there are only minor differences for high values of $T_p$ between the two versions of the double peak model and that the model represents the measurements quit well. The same tendency is found for all sea states. The deviation between the two models for high values of $T_p$ is caused by the difference in the high frequency exponent.

Figure 9: Comparison of wave period parameter, $T_{o2}$ for Halten data. Sea state: $H_s=2.75m$ Dots: Measurements, Solid line: Model (thick line: simplified, thin line original).

Radar data from Gullfaks C.

The simplified model spectra are compared to wave spectra from the Gullfaks C platform. Wave spectra are measured by the MIROS wave radar for the period 1998 to February 2002 for every 20 min. The total number of spectra is 105,403. The raw spectra are analyzed to find the significant wave height and the spectral peak period. The average spectra are found for each class of sea state where a class is defined for intervals of $H_s$ equal 0.5 meters (1 meter above 5 meter) and intervals for $T_p$ equal to 1 sec. All spectra show an energy level for low frequencies not seen for buoy data. For low sea states the measured peaks are smoother than the corresponding model peak, and also smoother than corresponding peaks in spectra from buoys. For all other sea states the model represents the measured average spectra quit good. Examples are shown in figures 10-13 below.

Figure 10: Comparison of wave spectra for Gullfaks. Solid line: Simplified model. Dots: Measured data. Sea state: $H_s=1.20m$, $T_p=4.6s$. No. of spec. = 686

Figure 11: Comparison of wave spectra for Gullfaks. Solid line: Simplified model. Dots: Measured data. Sea state: $H_s=1.30m$, $T_p=15.5s$. No. of spec. = 66

Figure 12: Comparison of wave spectra for Gullfaks. Solid line: Simplified model. Dots: Measured data. Sea state: $H_s=8.30m$, $T_p=10.7s$. No. of spec. =17

Figure 13: Comparison of wave spectra for Gullfaks. Solid line: Simplified model. Dots: Measured data. Sea state: $H_s=8.30m$, $T_p=15.5s$. No. of spec. = 29
COMPARISON WITH OTHER MODELS

The one peak case
If we set \( T_p = T_{pe} \) (corresponding to locally fully developed sea) the model will represent a single peak sea state. For this single peak case with long fetch (370 km) the variation in \( \gamma \) is given by:

\[
\gamma = 35s^{\frac{6}{7}}p^{\frac{2}{7}} = 0.94 H_s^{\frac{2}{7}}
\]

This means that the \( \gamma \)-value increases from 1 up to 2.2 for \( H_s = 16 \) m. For fetch 100 km the value is \( \gamma = 1.37 H_s^{\frac{2}{7}} \) giving \( \gamma = 3.0 \) for \( H_s = 16 \) m. This \( \gamma \)-value is close to the JONSWAP value \( \gamma = 3.3 \).

The model thus gives single peak spectra for locally wind sea with reasonable \( \gamma \)-values.

The Ochi-Hubble spectral model
The Ochi-Hubble spectral model (Ochi-Hubble, 1976) (see also Eq. (12)) gives the most probable spectrum and a class of spectra within a 95% confidence limit for a given \( H_s \). Figures 14 to 17 show some examples of the simplified double peak spectral model and the Ochi-Hubble model compared to measured spectra for Statfjord. Figure 14 shows a comparison for the most probable sea state according to Ochi-Hubble. In Figures 15 the parameters for the Ochi-Hubble spectra is modified to give the same primary peak period as the measured spectrum. Figures 16 and 17 shows to additional spectra for the same value of \( H_s \). In both cases the parameters used for the Ochi-Hubble spectrum gives a spectral peak period near the measured spectra. The figures show a reasonable fit between the two models except for the wind sea dominated sea state showed in figure 16. For this case the Ochi-Hubble model give a high swell peak not found in the data.

![Figure 14](image1.png)
**Figure 14:** Comparison of the Ochi-Hubble spectrum (thin line) and present model (solid line) for \( H_s = 4.2 \) m and \( T_p = 9.3 \) s. Dots represents average measured spectra from Statfjord.

![Figure 15](image2.png)
**Figure 15:** Comparison of the Ochi-Hubble spectrum (thin line) given the same primary peak as measured spectrum and simplified model (solid line) for \( H_s = 4.2 \) m and \( T_p = 9.3 \) s Dots represents average measured spectra from Statfjord.

![Figure 16](image3.png)
**Figure 16:** Comparison of the Ochi-Hubble spectrum (thin line) given the same primary peak as measured spectrum and simplified model (solid line) for \( H_s = 4.0 \) m and \( T_p = 7.0 \) s. Dots represents average measured spectra from Statfjord.

![Figure 17](image4.png)
**Figure 17:** Comparison of the Ochi-Hubble spectrum (thin line) given the same primary peak as measured spectrum and simplified model (solid line) for \( H_s = 4.3 \) m and \( T_p = 16.8 \) s. Dots represents average measured spectra from Statfjord.

The original model.
The differences between the original and the simplified model are:

a) constant exponent for high frequencies (\( N = -4 \)) in the simplified model,

b) simplified dependency of \( H_s \) for the peak enhancement factor \( \gamma \) for the primary peak and

c) simplified formulation of secondary wind peak. Figure 18 and 19 show the results for two sea states.

![Figure 18](image5.png)
**Figure 18:** Comparison of the original model (thin line) and present model (solid line) for \( H_s = 2m \) and \( T_p = 5s \).

![Figure 19](image6.png)
CONCLUSIONS

The findings in this study can be summarized as follows:
A simplified version of the double peak spectral model has been described. The model represents wave conditions in open ocean areas where the waves are dominated by local wind sea, but also exposed to swell. Sea states are classified into two types dependent of the origin of the highest spectral peak. The two types are separated by the relation \( T_p = a \frac{H_s}{1.3} \) into wind sea for \( T_p < T_{pE} \) and swell for \( T_p > T_{pE} \). The value of \( a \) is slightly dependent on the maximum fetch for the actual location. The sea state corresponding to \( T_p = T_{pE} \) is called the locally fully developed sea and is a single peak sea state. A dimensionless period scale factor defines how far the actual sea state is from the single peak situation. This parameter depends on \( H_s \) and \( T_p \) and is used to divide the energy between the two spectral peaks. The spectral form for both peaks are given by the \( \Gamma \) spectrum and all parameters is found empirically from the total \( H_s \) and primary spectral peak \( T_p \).

The simplified model spectra are compared to measured spectra from the Norwegian Continental Shelf, the Ochi-Hubble spectral model and the original double peaked model. The simplified double peak spectral model is for practical use identical to the original model and fits also well to measured data. The deviation can be summarized as follows: For moderate and high sea states the high frequency tail of measured wave spectra seems to fall off faster than the \(-4\) exponent used in the simplified model. For low sea states the measured secondary wind sea peak is slightly broader then found by the model.

Comparison with the Ochi-Hubble spectral model shows some deviations especially for wind dominated seas. The high frequency tail in the Ochi-Hubble model falls off faster than \(-4\). For the examples corresponding to most probable wave situation at Statfjord the two models give similar results. For other sea states deviations are seen, but no comprehensive comparison is done.

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