EQUAL-AREA CRITERION APPLIED ON POWER TRANSFER CORRIDORS

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ABSTRACT

This paper presents a novel adaptation of the equal-area criterion. The adapted criterion provides a new possibility to study the stability criteria of critical power transfer corridors, supporting the specification of the secure power transfer capacity of the interconnected power system.

Furthermore, the authors describe how the adapted equal-area criterion can be employed in the design of System Integrity Protection Schemes to prevent instability and mitigate consequences of extraordinary events. The concept is tested on the benchmark model IEEE Reliability Test System 1996.

KEY WORDS

Equal-Area Criterion, Power System Stability, Wide Area Monitoring, Protection and Control, Security Assessment, Power Transfer Corridor

1. INTRODUCTION

Extraordinary events in the electrical power system refer to disturbances with potentially high societal impact and low probability to occur. As both probability and consequences of extraordinary events are highly unpredictable, there are difficulties to economically justify major power system reinforcements based on their prevention, [1].

Extraordinary events are often characterised by instability phenomena, [2], leading to the triggering of component protections, resulting in a wide-spread interruption or blackout, where the affected region is difficult to anticipate. As the system becomes unstable, only pre-designed automatic remedies, such as System Integrity Protection Schemes (SIPS), are able to prevent the un-controlled disconnection of power system components and splitting of the system.

SIPS are increasingly utilized in power systems worldwide, providing both increased transfer capacity and security [3]. The improved situational awareness provided by Phasor Measurement Units (PMU) and Wide Area Monitoring Systems (WAMS) opens for further improvements of conventional SIPS, providing robustness against unforeseen disturbances. The main purpose of SIPS is to prevent instability and to maintain an interconnected operation of the power system.

Stability phenomena related to extraordinary events are mainly large-disturbance voltage and rotor angle stability, where the latter is often referred to as transient (rotor angle) stability [4]. Frequency instability may also be an issue, typically related to the shortage of spinning reserves in island operation. Although the different stability phenomena are interrelated, the concerns of this paper are limited to aspects of transient rotor angle stability only.

Transient rotor angle stability is often analysed using simplifications, such as single-machine infinite-bus equivalent models, where stability margins are determined using the renowned equal-area criterion of a synchronous machine.

Since stability stipulates that every machine needs to fulfil the equal-area criterion, several studies focus on the identification of critical machines which are likely to lose synchronism with the remaining system [5-13]. In the case with multiple critical machines, it is possible to cluster these into an equivalent one-machine-infinite-bus (OMIB) system, as described in [5]. An extension of the traditional equal-area criterion is suggested in [6, 7], where the transient stability margin and critical clearing time of critical machines are assessed without equivalent models. Techniques for preventive and emergency transient stability control are described in [8-10]. Here, the single-machine-equivalent (SIME) method is utilised, and the emergency control actions are defined on the basis of identifying critical machines, which are tripped in an iteratively manner until the system reaches stable operation. Emergency controls based on online measurements provided by PMUs are suggested in [11-13]. Here, the equal-area criterion is used to identify critical machines and assess the adequacy of emergency control actions.

This paper presents a novel adaptation of the equal-area criterion, providing new possibilities to study the stability criteria of critical power transfer corridors (PTC) and specifying the secure power transfer capacity of the interconnected power system. The authors describe how the adapted equal-area criterion can be employed in the design of adequate mitigating actions of SIPS, to limit the consequences of extraordinary events.
The paper is organized in the following manner: Chapter 2 holds the theoretical background of the equal-area criterion, including a description of the concept to apply the criterion on a PTC. The utilization of the adapted equal-area criterion in SIPS design is described in chapter 3. Chapter 4 holds a case study made on the IEEE Reliability Test System 1996. Discussion and conclusions are included in chapter 5.

2. THEORETICAL BACKGROUND

2.1 Equal-Area Criterion of a single machine

The equal-area criterion of a single synchronous machine, in a multi-machine system exposed to a disturbance, can be formulated as:

\[ A_{acc} - A_{dec} \leq 0 \]  (1)

where \( A_{acc} \) and \( A_{dec} \) are the accelerating and decelerating areas as depicted in the synchronous machine power-angle characteristics illustrated in Fig 1. Equality occurs if the maximum rotor angle, \( \delta_M \), coincides with the post-fault unstable equilibrium angle, \( \delta_U \), i.e. for the machine to remain stable, the following criteria needs to be fulfilled:

\[ \delta_M \leq \delta_U \]  (2)

From in Fig 1, it is clear that the accelerating and decelerating areas can be calculated as:

\[ A_{acc} = \int_{\delta_S}^{\delta_{CT}} (P_M - P_f(\delta)) \, d\delta \]  (3)

\[ A_{dec} = \int_{\delta_{CT}}^{\delta_M} (P_E(\delta) - P_M) \, d\delta \]  (4)

where \( \delta_S \) and \( \delta_{CT} \) are the rotor angles at the pre-fault steady-state equilibrium and at the time of fault clearing, respectively. \( P_M \) is the mechanical power of the turbine (assumed constant), and \( P_f(\delta) \) and \( P_E(\delta) \) are the underfault and post-fault electrical power of the machine, respectively.

The rotor angle \( \delta \) needs to be related to a reference, and often a centre of angle reference is used, [5], but theoretically any angle reference can be used.

2.2 Equal-Area Criterion of a Power Transfer Corridor

Fig 2 shows a power system, consisting of a sub-system and a main system, interconnected via a single power transfer corridor (PTC). In this system, it is possible that all synchronous machines inside the sub-system can be identified as critical for certain contingencies. This is exemplified by Fig 3, where all generators in the sub-system accelerate relative the main system after a critical contingency.

Clustering all critical machines into an equivalent model, as described in [5], the entire sub-system can be assessed against the main system. This implies that, at steady state, the equivalent mechanical power of the sub-system equals the power flow of the PTC. Together with the angle difference between the equivalents of the sub- and main systems, the equal-area criterion of the PTC can be assessed.

If the PTC consists of only a single tie-line, the rotor angle reference can be selected so that \( \delta \) corresponds to the voltage angle difference over the PTC. Thus, the power-angle characteristics of the sub-system and main
system equivalents correspond to the power flow and angle over the PTC. The equal-area criterion of the PTC can then be described by equations (1)-(4).

The loss of a line or generator in close vicinity of a PTC may prove to be especially critical: significantly decreasing the power-angle characteristics of the PTC, resulting in an equivalent mechanical power which exceeds the critical loading level of the post-fault system. This scenario is exemplified by Fig 4, where a SIPS is suggested to decrease the equivalent mechanical power of the post-fault system to a new stable operation point.

Fig 4. Simplified power-angle characteristics of a PTC, describing pre-fault, $P_{0E}(\delta)$, under-fault, $P_i(\delta)$, and post-fault, $P_{1E}(\delta)$, characteristics. $P_{OM}$ and $\delta_{0S}$ represent the pre-fault steady-state operating point. $\delta_{SIPS}$ represent the angle difference at the instant when the SIPS is activated, with $P_{1M}$ and $\delta_{1S}$ as the post-SIPS stable equilibrium point and $\delta_U$ as the corresponding unstable equilibrium.

At the pre-fault state, the steady-state operation point is characterized by the PTC power flow, $P_{0E}(\delta)$, the PTC equivalent mechanical power of the sub-system, $P_{OM}$, and the voltage angle between the equivalents of the sub- and main systems, $\delta_{0S}$. A critical contingency moves the system to an unstable state, since the pre-fault mechanical power is higher than the critical level of the post-fault power-angle characteristics, $\max\{P_{1E}(\delta)\}$. At $\delta = \delta_{SIPS}$, the mitigating action of a SIPS system is assumed to decrease the mechanical power to a potentially stable post-fault level, $P_{1M}$.

In the scenario described by Fig 4, equations (3) and (4) require the following modifications:

$$A_{acc} = \int_{\delta_{0S}}^{\delta_{CT}} (P_{OM} - P_i(\delta)) \, d\delta$$

$$+ \int_{\delta_{CT}}^{\delta_{SIPS}} (P_{OM} - P_{1E}(\delta)) \, d\delta$$

$$A_{dec} = \int_{\delta_{SIPS}}^{\delta_{M}} (P_{1E}(\delta) - P_{1M}) \, d\delta$$

The equal-area criterion of a PTC can thus be further utilised in contingency analysis, where assessing the stability margins of a critical PTC can provide an improved overview of the system operation. In the case when assessing the system response after multiple subsequent contingencies, the PTC stability margins can be used to identify the system’s vulnerability of extraordinary events.

3. EQUAL-AREA CRITERION APPLIED ON SIPS DESIGN

The main purpose of the SIPS is to regain a stable steady-state operation of the power system. A sufficient level of mitigating action is needed in order for the system to maintain its stable operation in accordance with the equal-area criterion. The SIPS system introduced in the previous section could be based on, for example, load-shedding or generation-rejection. In the following, the suggested design procedure of a generation-rejection scheme is described.

The SIPS design, having the goal to identify suitable generators to achieve sufficient stabilizing performance, is proposed to be done by:
1. Identification of critical contingencies
2. Equal-area criterion assessment of critical contingencies
3. Selection of suitable generators to participate in SIPS action

This procedure is described in the following subsections and tested in the power system analysis described in chapter 4.

3.1 Identification of critical contingencies

The critical contingencies referred to here, are the contingencies leading to rotor-angle instability in the sense that all machines within a specific sub-system are identified as critical.

Critical contingencies can be identified through a standard contingency analysis, assessing the consequences of e.g. all $N-1$ contingencies. The contingency analysis should be based on dynamic simulations, rather than steady-state power flow calculations, since the transient stability of the system is to be assessed.

3.2 PTC equal-area assessment of critical contingencies

For each identified critical contingency, the SIPS activation instant and corresponding angle needs to be assessed. In this way, the size of the accelerating area can be calculated, which defines the minimum size of the decelerating area that fulfills the equal-area criterion. Thus, the minimum level of mitigating actions necessary to maintain stable operation can be identified.
3.3 Selection of suitable generators to participate in SIPS action

There are two criteria in the selection procedure of generators that have to be addressed: firstly, the generators should have a power production level corresponding at least to the minimum level of mitigating actions, secondly, the impact on the sub-system and on the PTC characteristics should be limited in order for the equal-area criterion to be utilised.

A suitable set of generators needs to be selected among the critical machines as a basis to perform desired SIPS actions, in order to assess the reliability of the new steady-state scenario.

4. CASE STUDY

4.1 System model

The study is performed on the IEEE Reliability Test System 1996, which is a benchmark model for reliability assessment studies.

The IEEE Reliability Test System 1996, defined in [14], consists of 73 buses in three sub-systems, area A, B, and C, as shown in Fig 5. Each area has approximately 3.4GW of installed production and a peak load of 2.8GW. The areas are interconnected by five tie-lines, where the A-C and B-C tie-lines form a power transfer corridor between area C and the rest of the system, referred to as PTC.

The studied scenario is a low load scenario, with total demand approximately 50% of the system peak demand. The power exchange between areas is listed in Table I, with area A as a transit region, and areas B and C as import and export regions, respectively.

The loads in the system are represented by steady-state and dynamic load models, based on a composite of constant power, constant current and constant admittance, as defined by equations (7)-(10):

\[ P_0 = P_N \left( \frac{U_0}{U_N} \right)^2 \]  
\[ P_d = P_0 \frac{U_d}{U_0} \left( 0.4 + 0.6 \frac{U_d}{U_0} \right) \]  
\[ Q_0 = Q_N \]  
\[ Q_d = Q_0 \left( \frac{U_d}{U_0} \right)^2 \]

where the sub-indices \( N, 0, \) and \( d \) represent nominal, steady-state, and dynamic values, respectively. \( P \) and \( Q \) refer to the active and reactive power of the load, with \( U \) as the bus voltage.

In this study, the optional DC-link is excluded, synchronous condensers are exchanged with SVCs, and the dynamic models suggested in [15] are used to represent the synchronous generator and turbine systems.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>INTER-AREA POWER EXCHANGE OF THE STUDIED OPERATING SCENARIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area A→B Power flow (MW)</td>
<td>220</td>
</tr>
<tr>
<td>Area C→A Power flow (MW)</td>
<td>240</td>
</tr>
<tr>
<td>Area C→B Power flow (MW)</td>
<td>420</td>
</tr>
<tr>
<td>Area A Power exchange (MW)</td>
<td>15</td>
</tr>
<tr>
<td>Area B Power exchange (MW)</td>
<td>-640</td>
</tr>
<tr>
<td>Area C Power exchange (MW)</td>
<td>655</td>
</tr>
</tbody>
</table>

4.2 Identification of critical contingencies

Critical contingencies are identified through an \( N-1 \) contingency analysis, including 3-phase short-circuit faults on transmission lines, transformers, and generators,
with 100ms duration, followed by the disconnection of the affected unit. The results show that the trip of the A-C tie-line leads to instability, as shown in Fig 6.

Further analysis of the critical contingency, shows that all machines in area C accelerate out of synchronism relative the rest of the system, as described by the generator terminal voltage angles shown in Fig 3. Hence, for this contingency, the machines in area C are considered critical and can be clustered into a single-machine equivalent to analyze the transient stability of the system.

**4.3 PTC equal-area assessment of critical contingencies**

The equal-area assessment of the PTC for the identified critical contingency is done in three steps:

1. Identifying the instant of SIPS activation
2. Assessing the accelerating area of the PTC before SIPS activation
3. Assessing the minimum necessary level of rejected generation to fulfill the equal-area criterion

**4.3.1 Identifying the instant of SIPS activation**

A generation-rejection SIPS is considered to be utilized to prevent instability if the A-C tie-line is tripped. Measurement data from a WAMS are used as input to the SIPS, where $\delta_{BC}$ the voltage angle difference over PTC$_C$ shown in Fig 6, is utilised as activation signal.

The total inherent time delay of the SIPS, from PMU measurement to execution of mitigating action, is assumed to be no longer than 100ms. This seems realistically achievable, based on actual measurements of a PMU based Wide Area Power Oscillation Controller as well as the delays of a Wide Area Monitoring and Control System presented in [16].

By studying the results of the $N-1$ contingency analysis, presented in Fig 6, appropriate trigger levels for arming and activation of the SIPS are identified:

- $\delta_{arming}$: $\delta_{BC} \geq 40^\circ$
- $\delta_{activation}$: $\delta_{BC} \geq 50^\circ$

To prevent unwanted SIPS action during switching events, the internal time delay between arming and earliest activation is set to 200ms.

The specified SIPS trigger levels and delays are displayed in Fig 6, together with the resulting activation angle of the SIPS and corresponding instant:

- $\delta_{SIPS}$: $\delta_{BC} \leq 67^\circ$
- $t_{SIPS}$: $t_0 + 1.0$s

where $t_0$ is the instant of the occurrence of the fault.

**4.3.2 Assessing the accelerating area of the PTC before SIPS activation**

The power-angle characteristics of the PTC is shown in Fig 8, together with the identified SIPS activation angle, $\delta_{SIPS}$, for the critical contingency (the trip of the A-C tie-line).

Assuming a constant mechanical power of the system, the accelerating area before the SIPS activation, as shown in Fig 8-I, is then approximated to:

$$ A_{acc} \approx 5000\text{MW} $$

The mechanical power of the turbines is, however, not constant but depending on the response of the governor controllers. Assuming that the response of each machine can be approximated by its speed-droop, then the response of the system can be approximated by a piece-wise linear speed-droop, $R$. The mechanical power of the system, as a function of the frequency change, can thus be approximated as:

$$ P_m(\Delta f) = (1 - \frac{\Delta f}{R}) \times P_{OM} $$

where $\Delta f$ is the per unit change in frequency and $P_{OM}$ is the mechanical power at the pre-fault instant.

**Fig 7. Area C frequency response for the critical contingency.**

The speed-droop of the system can be assessed during operation, studying the frequency response of a known disturbance, e.g. the trip of a generator, as:

$$ R = -\frac{\Delta f}{\Delta P_g} \times P_G $$

where $\Delta P_g$ is the production change and $P_G$ is the total
production of the system. In the studied scenario, the speed-droop of the system is approximated to:

\[ R = 4.3\% \]

The frequency in area C, measured at the B-C tie-line, is shown in Fig 7.

From this frequency measurement and the calculated speed-droop of the system, the approximate equivalent mechanical power of the sub-system, \( P_M(\Delta \omega) \), is derived from equation (11), and illustrated in Fig 8-II.

The PTCC accelerating area can then be approximated to:

\[ A_{acc} P_M(\Delta \omega) = 3000\text{MW} \]

This area is drastically smaller than the area calculated using constant mechanical power. This implies that the impact of simplified assumptions is large, thus to design appropriate SIPS solutions sufficient details are needed to be considered.

### 4.3.3 Assessing the minimum level of rejected generation to fulfil the equal-area criterion

Assuming that the SIPS action affect the frequency in area C can be approximated by a linear decay, with nominal frequency reached at the maximum angle, \( \delta_M \), the decelerating area can be assessed as shown in Fig 8-III. The minimum SIPS action that fulfil the equal-area criterion is then approximated to:

\[ P_{SIPS} \geq 206\text{MW} \]

The resulting maximum angle equals:

\[ \delta_M = 99^\circ \]

### 4.4 Selection of suitable generators to participate in SIPS action

Table II lists all generators in operation in area C.

<table>
<thead>
<tr>
<th>Bus number and generator ID</th>
<th>Pre-fault production (MW)</th>
<th>Selected SIPS solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>302 G1</td>
<td>10</td>
<td>B3</td>
</tr>
<tr>
<td>302 G2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>313 G1</td>
<td>197</td>
<td>B3</td>
</tr>
<tr>
<td>313 G2</td>
<td>197</td>
<td>C1</td>
</tr>
<tr>
<td>313 G3</td>
<td>197</td>
<td></td>
</tr>
<tr>
<td>315 G1-5</td>
<td>5x12</td>
<td></td>
</tr>
<tr>
<td>315 G6</td>
<td>155</td>
<td>C2</td>
</tr>
<tr>
<td>316 G1</td>
<td>114</td>
<td>C3</td>
</tr>
<tr>
<td>321 G1</td>
<td>400</td>
<td>A1</td>
</tr>
<tr>
<td>322 G1-6</td>
<td>6x25</td>
<td></td>
</tr>
<tr>
<td>323 G1</td>
<td>155</td>
<td>B1</td>
</tr>
<tr>
<td>323 G2</td>
<td>155</td>
<td></td>
</tr>
<tr>
<td>323 G3</td>
<td>350</td>
<td>A2</td>
</tr>
</tbody>
</table>

If basing the SIPS solution on the rejection of a single generator, only the machines on bus 321 and 323 (G3) have sufficient production, i.e. \( P_G \geq P_{SIPS} \). These machines are selected to represent the solutions SIPS\(_{A1}\) and SIPS\(_{A2}\), respectively.

Various generator selections are possible for SIPS solutions based on tripping several generators. Here three solutions have been selected: SIPS\(_{B1-B3}\).

Solutions SIPS\(_{C1-C3}\) are based on single machines with production less than the identified minimum \( P_{SIPS} \) level.

The selected SIPS solutions are based only on the first criteria defined in section 3.3: the production level of the selected generators. The second criteria relates to the machines’ impact on the PTC and the sub-system, which can be difficult to anticipate. The machines’ reactive power capability and the relative closeness to the PTC determine their influence on the voltage level of the PTC bus. From the single-line diagram in Fig 5, it is noticed that bus 321 (SIPS\(_{A1}\)) is relatively close to the PTC bus, thus this solution might cause voltage instability in the sub-system. This has however not been further investigated in this study.

![Fig 8. Assessment of PTCC accelerating area for the critical contingency, assuming: I) constant mechanical power, II) mechanical power as a function of the frequency deviation. III) Assessment of minimum generation rejection level to fulfill the equal area criterion.](image-url)
4.5 Results from SIPS activation

The response of the selected SIPS solutions are analysed in this section, showing simulation results in Table III.

<table>
<thead>
<tr>
<th>SIPS ID</th>
<th>Rejected power $P_{sips}$ (MW)</th>
<th>Maximum decelerating angle $\delta_M$ (°)</th>
<th>Decelerating area $A_{dec}$ (MW°)</th>
<th>Post-SIPS PTC power transfer $P_{PTC}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>400 (unstable)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>350 76 3000 420</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>310 77 2800 450</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>269 82 3100 470</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>207 83 2000 500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>197 84 1800 500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>155 84 1400 520</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>114 - (unstable)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As anticipated, solution A1 results in an unstable solution. This is due to the location and size of the generator that is rejected in this scheme, leading to a significant voltage drop at the PTC and the system is not able to regain stability. Also solution C3 proved insufficient, which was expected from the insufficient level of rejected power.

Solutions A2, B1, and B2, shown in Fig 9, all respond as expected, with stable solutions and the calculated decelerating areas are approximately equal to the accelerating area. For these solutions, the approximate accelerating and decelerating areas are quite similar. Better approximations are possible to achieve, if considering also the effects of the voltage changes in the sub-system. Furthermore, the participation of each machine in the acceleration of the sub-system should also be considered. Since the accelerating area of the PTC consists of the participation of each machine, the contribution of the rejected machines should be deducted from the total acceleration.

It should be noted that solutions B3, C1, and C2, shown in Fig 10, have decelerating areas considerably smaller than expected. The reason behind this is related to the reactive power capability of the rejected machines. In these three cases, the generators in question were at their under-excitation limit, implying that their disconnection would lead to a voltage rise at the buses in the surrounding area affecting the load of area C. As defined by the dynamic representation of loads, described by equations (8) and (10), a rise in voltage on the load buses lead to an increase in active and reactive load. Thus, the
actions of these three SIPS solutions result in increased load of the sub-system, meaning that the equivalent mechanical power of the sub-system further decreases due to the voltage rise. Hence, in order to properly assess the decelerating areas, also the SIPS affect on the load needs to be considered in the equivalent mechanical power.

5. CONCLUSIONS AND DISCUSSION

The concept of applying the equal-area criterion on critical power transfer corridors appears theoretically feasible. The results from computer simulations show that it is possible to utilise this concept when designing a System Integrity Protection Scheme.

Assumptions regarding the equivalent mechanical power of the sub-system prove to be of high importance for the success of the proposed concept. The speed-droop characteristic and reactive power capabilities of generators, as well as the voltage dependency of loads, have significant impact on the results.

The uncertainties in the approximation of sufficient rejected production, as well as the system impact by the rejected machines, may constitute a challenge during SIPS design in this context. The SIPS functionality and potential voltage and frequency stability problems should be appropriately tested through dynamic analysis.

Utilising data from a Wide Area Monitoring System, the proposed adaptation of the equal-area criterion provide promising applications. In this way, the situational awareness can be enhanced, thus improving the security level of system operation. Furthermore, the proposed concept can be used to increase the efficiency as well as proving the adequacy of existing System Integrity Protection Schemes.

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