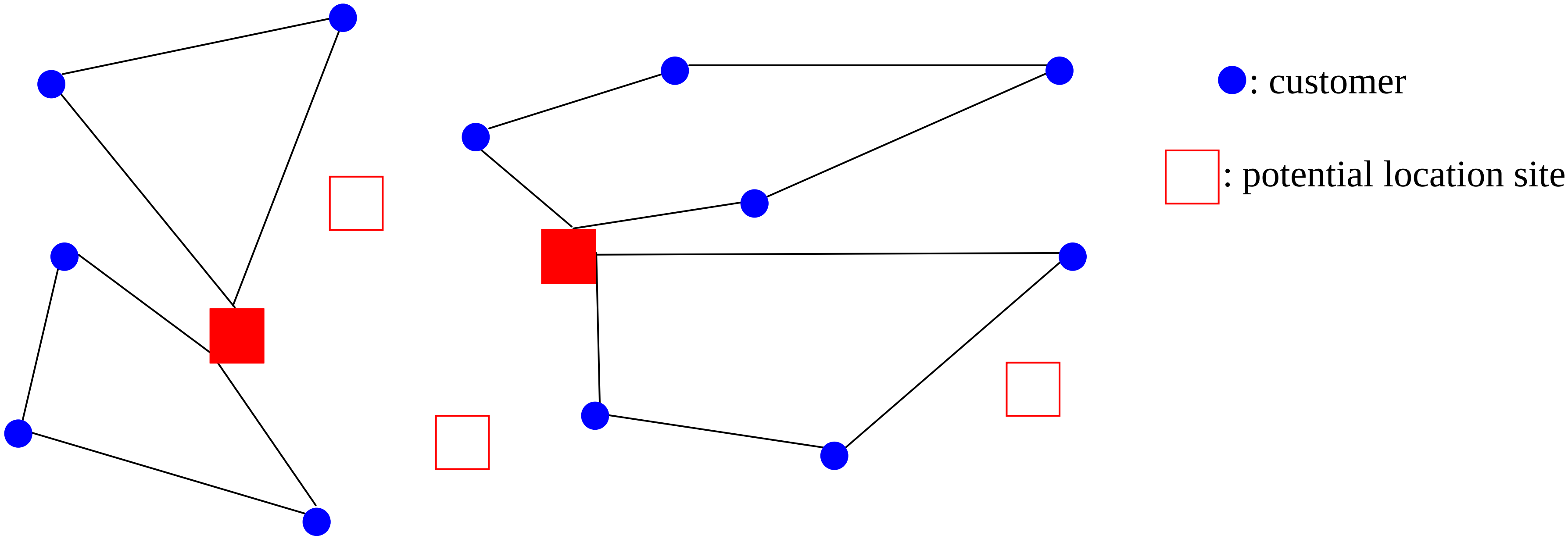
NEW HYBRID HEURISTICS FOR A LOCATION-ROUTING PROBLEM

Célia BOULANGER[1]

Frédéric SEMET[1]

[1]LAMIH-ROI, UMR CNRS, Université de Valenciennes, France

The capacitated location-routing problem (CLRP)



Determine simultaneously a set of depot locations and a set of collection routes to serve a set of customers

in order to

- Minimize the total cost : fixed cost + transportation cost
- Satisfy capacity requirements

Litterature review

- Early papers:
  Chan and Hearn (1977) :
  Laporte and Nobert (1981)

- CLRP :
  Wu, Low and Bai (2002)
  Prins, Prodhon, Wolfler-Calvo (2006, 2007)

- Survey :
  Nagy, Salhi (2007)

A four-index formulation of the CLRP

Notation :

Define on a graph : $G = (V \cup W, E)$ where

V: potential location sites for the depots

W: set of customers

$C = (c_{ij})$: cost matrix

d_i : demand of customer i

Q : vehicle capacity

$\overline{Q}$: depot capacity

At most m depots are opened

Variables :

$$x_{ij}^{kf} = \begin{cases} 1 \text{ if vehicle } k \text{ based at depot } f \text{ travels directly from } i \text{ to } j \\ 0 \text{ otherwise} \end{cases}$$

$$y_f = \begin{cases} 1 \text{ if a depot is located in } f \\ 0 \text{ otherwise} \end{cases}$$

Minimize $$\sum_f C_f y_f + \sum_i \sum_j \sum_k \sum_f c_{ij}^{kf} x_{ij}^{kf}$$

subject to

$$\sum_j \sum_k \sum_f x_{ij}^{kf} = 1 \qquad \forall\, i \in W$$

$$\sum_i x_{i\ell}^{kf} - \sum_j x_{\ell j}^{kf} = 0 \qquad \forall\, \ell \in W;\ \forall\, k, f$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij}^{kf} \le |S| - V(S) \qquad S \subset W,\ 2 \le |S| \le (n-2);\ \forall\, k, f$$

$$\sum_i \sum_f x_{fi}^{kf} \le 1 \qquad \forall\, k$$

$$\sum_k \sum_i d_i \sum_j x_{ij}^{kf} \le \overline{Q}\, y_f \qquad \forall\, f$$

$$\sum_f y_f \le m$$

$$x_{ij}^{kf} = 0 \text{ or } 1,\ y_f = 0 \text{ or } 1$$

If we fix the location variables y_f :

Minimize $$\sum_i \sum_j \sum_k \sum_f c_{ij}^{kf} x_{ij}^{kf}$$

subject to

$$\sum_j \sum_k \sum_f x_{ij}^{kf} = 1 \qquad \forall\, i \in W$$

$$\sum_i x_{i\ell}^{kf} - \sum_j x_{\ell j}^{kf} = 0 \qquad \forall\, \ell \in W;\ \forall\, k, f$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij}^{kf} \le |S| - V(S) \qquad S \subset W,\ 2 \le |S| \le (n-2);\ \forall\, k, f$$

$$\sum_i \sum_f x_{fi}^{kf} \le 1 \qquad \forall\, k$$

$$\sum_k \sum_i d_i \sum_j x_{ij}^{kf} \le \overline{Q} \qquad \forall\, f$$

$$x_{ij}^{kf} = 0 \text{ or } 1$$

$\rightarrow$ MDVRP

If we fix the customers served by vehicle k :

$$z_{kf} = \begin{cases} 1 \text{ if the route of vehicle } k \text{ is rooted at depot } f \\ 0 \text{ otherwise} \end{cases}$$

$$\text{Minimize} \quad \sum_f C_f y_f + \sum_k \sum_f \bar{c}_{kf} z_{kf}$$

subject to

$$\sum_{k \in R(i)} \sum_f z_{kf} = 1 \qquad \forall\, i \in W$$

$$\sum_f z_{kf} \le 1 \qquad \forall\, k$$

$$\sum_k D_k z_{kf} \le \bar{Q}\, y_f \qquad \forall\, f$$

$$\sum_f y_f \le m$$

$$z_{kf} = 0 \text{ or } 1 \;,\; y_f = 0 \text{ or } 1$$

If we restrict the set of routes s.t.
each customer is served by one and only one vehicle :

$$\text{Minimize} \quad \sum_f C_f y_f + \sum_k \sum_f \bar{c}_{kf} z_{kf}$$

subject to

$$\sum_f z_{kf} = 1 \qquad \forall k$$

$$\sum_k D_k z_{kf} \le \bar{Q} \, y_f \qquad \forall f$$

$$\sum_f y_f \le m$$

$$z_{kf} = 0 \text{ or } 1 \,,\ y_f = 0 \text{ or } 1$$

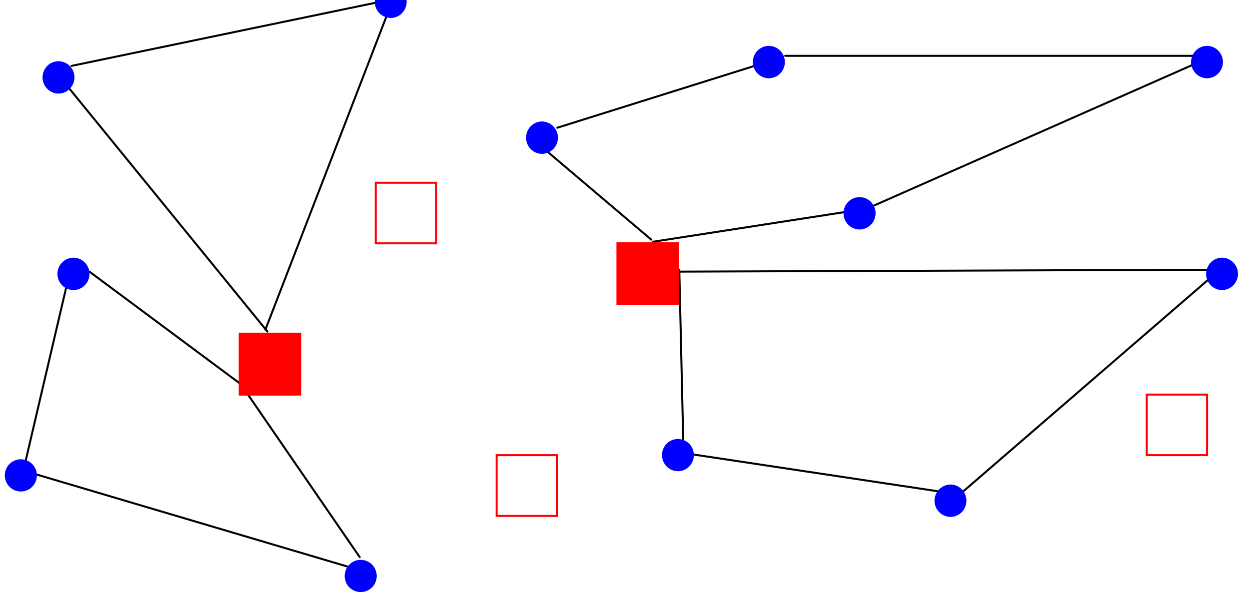
$\rightarrow$ CPLP

A generic heuristic approach for the CLRP

Phase I Solve a MDVRP $\rightarrow$ a set of collection routes

Phase II Solve a CPLP $\rightarrow$ a set of opened depots

Phase I : Routing phase



Adapted version of TS algorithm (Cordeau, Laporte, Gendreau)

Phase II : Location phase

Capacitated Plant Location Problem

Minimize $$\sum_{f} C_f y_f + \sum_{k} \sum_{f} \bar{c}_{kf} z_{kf}$$

subject to

$$\sum_{f} z_{kf} = 1 \qquad \forall\, k$$

$$\sum_{k} D_k z_{kf} \le \overline{Q}\, y_f \qquad \forall\, f$$

$$\sum_{f} y_f \le m$$

$$z_{kf} = 0 \text{ or } 1 \;,\; y_f = 0 \text{ or } 1$$

Number of vehicles used set in the solution of the MDVRP

A basic heuristic approach for the CLRP

Phase 0 Feasible solution

Phase I *Routing phase*

Solve the MDVRP using a TS algorithm

Phase II *Location phase*

Solve the CPLP using an exact algorithm

If the set of opened depots differs, Go to Phase I

LRGTS for the CLRP (Prins et al.)

Phase 0 Feasible solution

Phase I *Location phase*

If solution not improved

Create routes based on edge frequencies

Solve the CPLP using a Lagrangean heuristic

Phase II *Routing phase*

Solve the MDVRP using GTS and LS

Go to Phase I for Max Iteration

Computational Results

30 classical instances for the mono-objective CLRP

		Basic algorithm		LRGTS	
Customers	Facilities	Dev.	CPU*	Dev.	CPU **
20	5	0.28	0	0.16	0
50	5	2.07	1	1.53	1
100	5	2.90	3	1.96	5
100	10	6.17	7	2.58	13
200	10	2.41	50	1.81	63

* : PIV 2.7 GHz　　** : PIV 2.4 GHz

An improved heuristic approach for the CLRP

Phase 0 Feasible solution

Phase I *Routing phase*

Solve the MDVRP using a TS algorithm

Phase II *Location phase*

Solve the CPLP using an exact algorithm

If the set of facilities differs, Go to Phase I

else if $\exists$ a feasible solution for the CPLP and $m > 1$,
$m = m - 1$ and go to Phase II

An improved heuristic approach for the CLRP

Phase 0 Feasible solution

Phase I *Routing phase*

Solve the MDVRP using a TS algorithm

Phase II *Location phase*

Solve the CPLP using an exact algorithm

If the set of facilities differs, Go to Phase I

else if ∃ a feasible solution for the CPLP and $m > 1$,

$m = m - 1$ and go to Phase II

else go to Phase III

Phase III *Route selection phase*

Solve the route selection problem

Reinitialize the value of m

go to Phase II

Phase III : Route selection phase

Route Selection Problem

$$a_{ij} = \begin{cases} 1 \text{ if route } j \text{ visited customer } i \\ 0 \text{ otherwise} \end{cases}$$

$$x_j = \begin{cases} 1 \text{ if route } j \text{ is selected} \\ 0 \text{ otherwise} \end{cases}$$

$$\min \sum_i c_i \, x_i$$

s.t.

$$\sum_j a_{ij} \, x_i = 1 \qquad \forall i \in I$$

$$\sum_i x_i \le p$$

$$x_i \in \{0, 1\}$$

Computational Results

30 classical instances for the mono-objective CLRP

		Basic algorithm		Improved algorithm		Improved algorithm +gap		LRGTS	
Customers	Facilities	Dev.	CPU*	Dev.	CPU*	Dev.	CPU*	Dev.	CPU**
20	5	0.28	0	0.00	124	0.00	114	0.16	0
50	5	2.07	1	0.07	240	0.24	205	1.53	1
100	5	2.90	3	0.83	620	0.88	482	1.96	5
100	10	6.17	7	1.17	1529	1.96	1148	2.58	13
200	10	2.41	50	0.37	7789	1.29	5584	1.81	63

* : PIV 2.7 GHz   ** : PIV 2.4 GHz

Improved alg. : 23 new best solutions

Computational Results

30 classical instances for the mono-objective CLRP

		Basic algorithm		Improved algorithm		Improved algorithm +gap		LRGTS	
Customers	Facilities	Dev.	CPU*	Dev.	CPU*	Dev.	CPU*	Dev.	CPU**
20	5	0.28	0	0.00	124	0.00	114	0.16	0
50	5	2.07	1	0.07	240	0.24	205	1.53	1
100	5	2.90	3	0.83	620	0.88	482	1.96	5
100	10	6.17	7	1.17	1529	1.96	1148	2.58	13
200	10	2.41	50	0.37	7789	1.29	5584	1.81	63

* : PIV 2.7 GHz ** : PIV 2.4 GHz

Improved alg. : 23 new best solutions