SIMULATION OF TRANSMISSION LINE TRANSIENTS USING VECTOR FITTING AND MODAL DECOMPOSITION

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Abstract – This paper introduces a fast and robust method for rational fitting of frequency domain responses, well suited for both scalar and vector transfer functions. Application of the new method results in increased computational efficiency for transmission line models using modal decomposition with frequency dependent transformation matrices. This is due to the fact that the method allows the fitted elements of each eigenvector to share the same set of poles, and that accurate fitting can be achieved with a relatively low number of poles.

1 INTRODUCTION

Accurate calculation of electromagnetic transients in power systems requires the frequency dependent effects of transmission lines to be taken into account. Frequency dependent line models can in principle easily be formulated in the time domain via numerical convolutions [1], but the resulting model is computationally inefficient. The efficiency is greatly improved if the impulse responses for the line are fitted using rational functions in the frequency domain [2-3], *z*-domain [4-5], or approximated directly in the time domain [6-7], as this leads to a recursive formulation of the time domain convolution integrals. Further savings can be achieved by introducing modal decomposition [8], as the transformation matrix is almost constant for many overhead lines of practical interest. In such cases, the number of scalar convolutions to be carried out is reduced from $4n^2$ to $4n$, where n is the number of conductors. However, in case of cable systems, multi-circuit overhead lines and strongly asymmetric overhead lines, the transformation matrix may be strongly frequency dependent. Taking this frequency dependence into account requires the handling of a frequency dependent transformation matrix [9]. This increases the number of convolutions to $4n^2+4n$, making the modal domain approach slightly more time consuming than the phase domain approach.

The propagation function H may in some cases of cable systems contain widely different time delays for the individual modes. In such cases it is very difficult to fit H with rational functions directly in the phase domain, as a very high order fit may be needed. However, this problem is easily overcome with the modal domain models, because there is only a single time delay associated with each mode of propagation. The modal time delays can therefore readily be eliminated from the modal components of H [10], yielding smooth functions which can be fitted with low order rational polynomials.

In this paper we briefly review the methodology of modal decomposition applied to transmission line modeling. We then introduce a new powerful fitting technique which allows us to fit the transformation matrix column-by-column. This strategy is shown to result in substantial savings in computational time for the time step loop, as compared to traditional element-by-element fitting. The method is also used to fit the responses for modal propagation and modal characteristic admittance. Unlike traditional Marti-fitting [3], the resulting rational function approximation is not restricted to real poles and zeros, which is shown to be advantageous when fitting the transformation matrix and the modal characteristic admittance.

PE-347-PWRD-0-01-1997 A paper recommended and approved by the IEEE Transmission and Distribution Committee of the IEEE Power Engineering Society for publication in the IEEE Transactions on Power Delivery. Manuscript submitted July 26, 1996; made available for printing January 8, 1997.

2 MODAL DECOMPOSITION

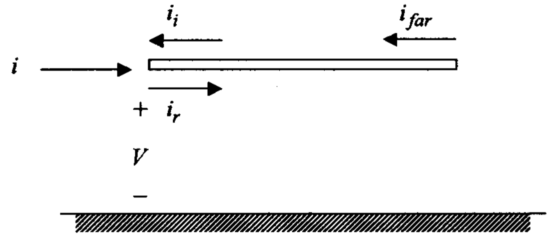

Fig. 1 *Traveling waves at transmission line end*

The frequency domain solution of the traveling wave equation can at each end of a transmission line be expressed [7] by the well known matrix-vector expression

$$Y_c v - i = 2i_i = 2H\, i_{far} \tag{1}$$

where the propagation function H and the characteristic admittance Y_c are given by

$$H = \exp(-\sqrt{YZ}\, l) \tag{2}$$

$$Y_c = Z^{-1}\sqrt{ZY} \tag{3}$$

Z and Y are the series impedance and shunt admittance per unit length of the transmission line. For an *n*-conductor system these are $n \times n$ matrices.

Equation (1) represents n coupled scalar equations, but may be replaced by n uncoupled equations by introducing modal quantities :

$$i = T_I\, i^m \tag{4}$$

$$v = T_V\, v^m \tag{5}$$

Here, T_I and T_V are the right eigenvector matrices of YZ and ZY respectively. Superscript m denotes modal quantities. Substituting (4) and (5) in (1) gives

$$Y_c^m v^m - i^m = 2H^m i_{far}^m \tag{6}$$

The diagonal matrices H^m and Y_c^m are related to their phase domain counterparts by :

$$H = T_I H^m T_I^{-1} \tag{7}$$

$$Y_c = T_I Y_c^m T_I^T \tag{8}$$

where we have used [8] the relation :

$$T_V = T_I^{-T} \tag{9}$$

In practical calculations the modal domain transmission line model is linked to the phase domain host program by the relations

$$v^m = T_V^{-1} v = T_I^T v \tag{10}$$

$$i = T_I\, i^m \tag{11}$$

Thus, the modal calculations have been manipulated to involve only the matrices T_I, T_I^T, Y_c^m, and H^m.

3 VECTOR FITTING BY OPTIMAL SCALING

In this chapter we show that substantial savings in the time step loop can be obtained if the elements of each eigenvector are fitted using the same set of poles. This is achieved by the introduction of a new fitting methodology—vector fitting by optimal scaling.

3.1 State equation realization

A linear system can in general be represented by the state equation realization (SER)

$$\dot{x}=Ax+Bu \tag{12}$$

$$y=Cx+Du \tag{13}$$

In transient calculations we need SERs for T_I, T_I^T, Y_c^m, and H^m. Note that in the calculations we need only access to the variables for input u and output y, whereas the state variables x are used as "internal" variables only. This allows us to use a diagonal A-matrix, since this corresponds to choosing a particular set of state variables (via a similarity transformation). The selection of a diagonal A-matrix reduces the computational burden of the time domain calculations.

Let $G(s)$ be a matrix transfer function of dimension $n \times n$. G is related to its SER by the expression

$$G(s)=C(sI-A)^{-1}B+D \tag{14}$$

The elements of the diagonal A-matrix are seen to be the poles of G. In the case of T_I, Y_c^m and H^m, the transfer matrices have an infinite number of poles. Hence, our task is to find a finite order approximation.

Assume that we fit the transformation matrix $T_I(s)$ element-by-element by a SER of order N. This implies that each element is represented by a SER of dimension N. For an n-conductor system the total number of poles for each column of T_I equals

$$N_{tot}=nN \tag{15}$$

We can, however, do better than this. By suitable frequency dependent normalization the elements of T_I become smooth functions of frequency. This makes it possible to choose the same poles for all elements of a column of T_I. (See Appendix A for additional justification). Thus, the total number of poles for a single column then becomes equal to

$$N_{tot}=N \tag{16}$$

A detailed analysis (Appendix B) shows that this will *increase the computational efficiency for T_I in the time step loop by approximately a factor of 3*. It should be noted that an additional requirement of all eigenvectors sharing the same poles would not lead to further savings. This is due to the fact that the SER for each vector has to be implemented separately. Similarly, nothing is achieved by requiring the elements of Y_c^m and H^m (diagonal) to have the same poles.

3.2 Optimal scaling

In this section we introduce a new, general fitting methodology which allows the columns of a matrix to be fitted with the same set of poles. The method is explained for the transformation matrix T_I, but will also be used for fitting the diagonal elements of Y_c^m and H^m.

As in [9] we normalize the eigenvectors by fixing one element to unity, giving a transformation matrix T_I^0. Artificial eigenvector switchovers are avoided by means of a switching-back procedure, similarly as described in [11].

In the following we introduce an intermediate scaling (smoothing) of each column t_I^0 with the purpose of obtaining poles suitable for the fitting of the original (unscaled) vector t_I^0. We use for smoothing a scalar (complex) scaling function $\sigma(s)$ so that the scaled column $t_I=\sigma t_I^0$ can be fitted with a set of prescribed stable poles given in the diagonal matrix $\tilde{A}$. In addition, we append a unity to the original vector:

$$\sigma\begin{bmatrix} t_I^0 \\ 1 \end{bmatrix}=\tilde{C}(sI-\tilde{A})^{-1}\tilde{b}+\tilde{d} \tag{17}$$

In order to prevent ambiguity of the solution we impose the condition

$$\tilde{d}=\begin{bmatrix} t_I^0(s=\infty) \\ 1 \end{bmatrix} \tag{18}$$

which implies that σ approaches unity at high frequencies. The elements of the column vector $\tilde{b}$ are arbitrarily set to unity. Equation (17) is manipulated into an overdetermined set of linear equations, one equation for each frequency point. This yields the sparse system

$$Fx=g \tag{19}$$

where the solution vector x contains the elements of $\tilde{C}$ as well as of $\sigma(s)$. Equation (17) is split into its real and imaginary part when assembling (19), thereby ensuring $\tilde{C}$ to have real elements only.

Our experience is that specifying the poles in $\tilde{A}$ to be real and logarithmically distributed gives a very accurate fit for t_I. This choice is somewhat arbitrary and is based on the requirement of adequate covering of the whole frequency range considered (see Appendix A).

In order to get a SER for the original vector t_I^0 we partition the SER in (17) into two parts, h and h_1, where h_1 is the last element in the SER:

$$\sigma\begin{bmatrix} t_I^0 \\ 1 \end{bmatrix}=\tilde{C}(sI-\tilde{A})^{-1}\tilde{b}+\tilde{d}=\begin{bmatrix} h(s) \\ h_1(s) \end{bmatrix} \tag{20}$$

It is seen from (20) that h_1 represents a SER for σ. Because the poles of h_1 are identical to those of h, it follows that the poles of t_I^0 become equal to the zeros of h_1. The zeros of h_1 are calculated directly from its SER, as shown in Appendix C.

We next calculate the SER of t_I^0 by means of (21), with the zeros of h_1 used as elements (poles) for A:

$$t_I^0=C(sI-A)^{-1}b+d \tag{21}$$

As in (17), $d=t_I^0(s=\infty)$, and b is a column vector of ones. Equation (21) is manipulated into an overdetermined linear set of equations, similarly to (19). However, the solution vector x now contains only the elements of C as no scaling is involved.

It should be noted that the new poles used in (21) are both real poles and complex conjugate pairs. This is also the case for the zeros for the transfer matrix corresponding to the SER of t_I^0.

The fitting technique by optimal scaling is equally well suited for fitting scalar functions as vectors. In (17) the vector t_I^0 is then replaced by the scalar function. We use this technique for finding the rational function approximation for the elements of the diagonal matrices H^m and Y_c^m.

In case of T_I^T, the SER is calculated from the SER of T_I using formulae shown in Appendix D.

3.3 Stability

Because the "new" poles used in (21) are zeros that were calculated without any restrictions, they may in principle turn out to be unstable. Our experience is that when the function to be fitted represents a stable system, then the technique by optimal scaling also results in a stable fit. The reason for this is simply that if the rational function approximation contained an unstable pole, then the resulting fit would be poor. An exception is when the unstable pole is canceled by an unstable zero. This problem is, however, easily overcome by deleting the unstable poles in (21) prior to determination of C.

3.4 Removal of time delays from the modal propagation functions

Many years experience with Marti-fitting has shown that the scalar transfer function H_k^m can be accurately approximated by a minimum phase shift function plus a time delay :

$$H_k^m(\omega)\approx h_{\min}(\omega)\exp(-j\omega\tau) \tag{22}$$

In the case of Marti-fitting, only the magnitude function was considered in the fitting process. This allowed the time delay to be calculated by comparing the phase angle of the propagation function with that of the rational approximation. However, in case of fitting by optimal scaling, both the real and imaginary part of H_k^m are used in the fitting process. This makes it necessary to remove the time delay *before* the fitting is carried out, which is done by multiplying H_k^m with the factor $\exp(j\omega\tau)$. As shown in Appendix E, the time delay for each mode can be calculated by the expression

$$\tau=\frac{l}{v(\Omega)}+\frac{\angle h_{\min}(\Omega)}{\Omega} \tag{23}$$

where Ω is the highest frequency point of interest, v is the modal velocity and l the length of the transmission line. Because accurate representation of the "toe portion" of $H_k^m(t)$ would require a very high order fitting, we choose Ω to be the frequency point where $|H_k^m(\omega)|=0.1$, as illustrated in figure 2. This gives an accurate fit for $\omega<\Omega$, and a reasonable representation for $\omega>\Omega$. We ensure that the rational approximation approaches zero as $\omega\rightarrow\infty$ by specifying the D-matrix of the SER to be zero when doing the fitting.

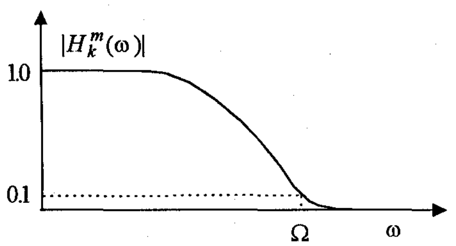

Fig. 2 *Upper frequency limit* Ω

$\angle h_{\min}(\Omega)$ in (23) is calculated by a formula derived by Bode [14], which relates the phase angle for a minimum phase shift function to its magnitude function. The formula is shown in (24), in a slightly rewritten form :

$$\angle h(j\omega_1)=\frac{\pi}{2}\left.\frac{d(\ln|h(j\omega)|)}{d(\ln\omega)}\right|_{\omega=\omega_1}+\Delta(u) \tag{24}$$

where

$$\Delta(u)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\left(\left|\frac{d(\ln(|h(j\omega)|)}{du}\right|-\left|\frac{d(\ln(|h(j\omega)|)}{du}\right|_{u=0}\right)\ln(\coth\frac{|u|}{2})du \tag{25}$$

and

$$u=\ln\frac{\omega}{\omega_1} \tag{26}$$

The first term alone gives a good estimate for the phase angle. However, an improved result is achieved by including the second term $\Delta(u)$. We limit the integration in (25) to the interval $\omega\in[0.1\Omega,10\Omega]$. This can be done since the shape of the magnitude function more than one decade of frequency away has only a very small effect on the phase angle at $\omega=\Omega$. The derivatives in (24)-(26) are evaluated by simple numerical differentiation, as is also the case for the integration. We typically use 10 frequency points in the integration, which results in the evaluation of $\angle h_{\min}(\Omega)$ to be very fast.

4 TIME DOMAIN IMPLEMENTATION

The transmission line model as defined by the traveling wave equation (1) and (6) is transformed from the frequency domain into the time domain by application of the convolution theorem. We start by noting that a convolution between an arbitrary matrix G and an input vector u may be expressed :

$$y=G*u=G_0u+h \tag{27}$$

where $*$ denotes the convolution operator. G_0u is an instantaneous term and h a history term. Considering one end of the line, the modal domain traveling wave equation (6) becomes in the time domain

$$Y_c^m*v^m-i^m=2H^m*i_{far}^m \tag{28}$$

Applying (27) to (28) gives

$$(Y_{c0}^m v^m+h_1)-i^m=2h_2 \tag{29}$$

(H_0^m is equal to 0 due to the time delay between the line ends.) The modal domain currents and voltages in (29) are transformed into the phase domain using (10), (11), and (27) :

$$v^m=T_{I0}^T v+h_3\ ,\ i=T_{I0}i^m+h_4 \tag{30}$$

Substituting (30) in (29) and premultiplying with T_{I0} gives the final result :

$$(T_{I0}Y_{c0}^m T_{I0}^T)v-i=T_{I0}[2h_2-h_1-Y_{c0}^m h_3]-h_4 \tag{31}$$

which is conveniently expressed by the Norton equivalent in figure. 3. This representation is similar to the one used in [9].

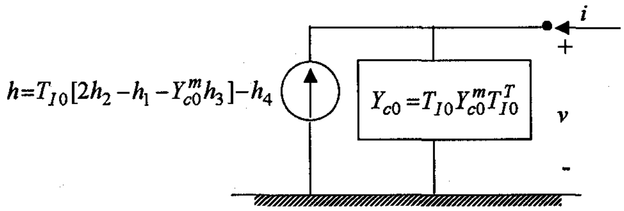

Fig. 3 *Network representation of line end*

The history terms h_1, h_2, h_3, h_4 and the instantaneous terms Y_{c0}^m, T_{I0} and T_{I0}^T are calculated from the respective SERs assuming trapezoidal integration.

5 CALCULATED RESULTS FOR CABLE SYSTEM

In the following we show results for the 66 kV cable system in figure 4. The cable series impedance Z is calculated using simplified formulae given in [12].

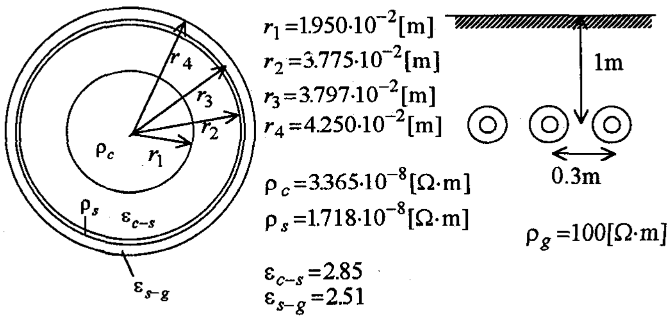

Fig. 4 *Cable system data*

5.1 Rational function approximation

The elements of T_I, Y_c^m and H^m were fitted using the optimal scaling technique in the frequency range 0.1Hz–1MHz, with 5 frequency points per decade of frequency. The cable length was 10km.

Figure 5 shows the magnitude of the six elements of the third column of T_I , and the magnitude of the complex deviation (fitting error). The fit is seen to be very accurate.

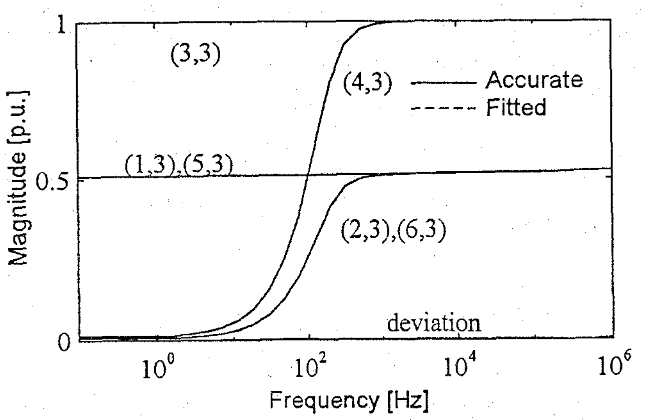

Fig. 5. *Elements of third column of T_I (5th order approximation)*

As "starting" poles in (17) we used 5 real poles, logarithmically distributed between 0.1Hz and 1MHz. The calculated "new" set of poles used in (21) is shown in table 1. It is seen that the fitting technique resulted in a complex conjugate pair, in addition to three real poles.

Table 1 *Poles of rational function approximation*

-1.7714E6	-2.6904E3	-944.96±j473.82	-2.0957

Figure 6 shows the phase angles corresponding to the magnitudes in figure 5. The angles of the rational function approximation are seen to be in close agreement with the original angles. We also note that the phase angles of elements (2,3), (6,3) and (4,3) have a net phase shift of –180°, whereas the corresponding magnitude functions (figure 5) are constant at both very low and very high frequencies. This implies that these elements contain a zero in the *right* half plane. This shows that in case of cable systems, one cannot assume all the zeros to lie in the left half plane, as is traditionally done in Marti-fitting [3].

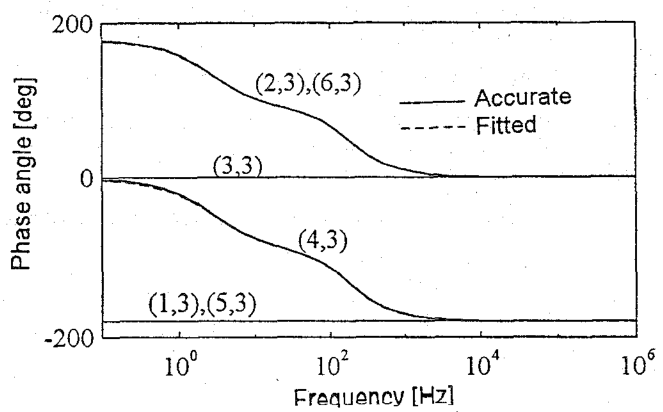

Fig. 6 *Phase angle of elements of third column of T_I*

Figure 7 shows the fitted elements of the modal characteristic admittance matrix, Y_c^m . Each element was fitted separately, using 7 real poles as "starting" poles. These were logarithmically distributed between 0.1Hz and 1MHz. Also is shown the magnitude of the complex deviation. Again, the fit is very accurate.

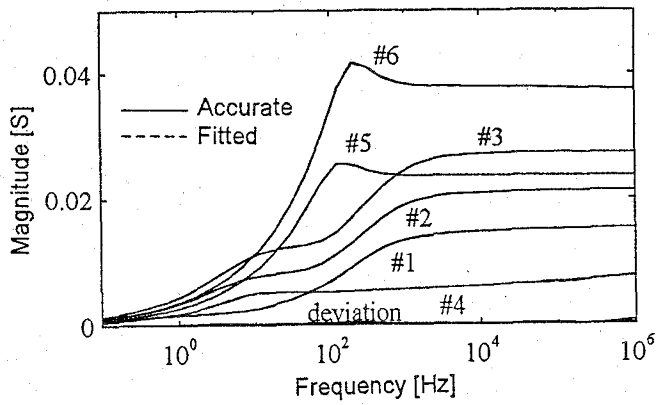

Fig. 7 *Modal characteristic admittance (7th order approximation)*

Figure 8 shows the fitted elements of the modal propagation, H^m , after removing time delays by (23). The elements were fitted separately, using 8 real poles as "starting poles", logarithmically distributed between 0.1Hz and 1MHz. The fit is seen to be quite good, perhaps with the exception of the toe portion of element #6.

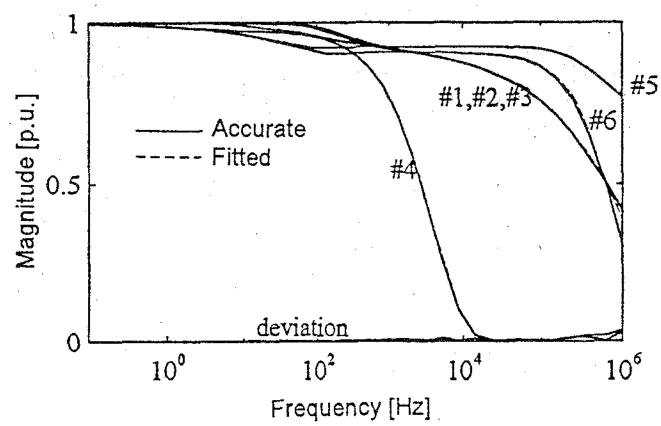

Fig. 8 *Modal propagation (8th order approximation)*

5.2 Time domain simulation

The accurate representation of the frequency dependence of the transformation matrix is particularly important when calculating induced currents [13] and induced voltages [7] in cable sheaths. In what follows we show calculated results for the cable system of figure 4, (cable length=10km). The responses were fitted in the interval 1Hz–1MHz using 5–6 poles for each column of T_I, 7 poles for each element of Y_c^m and 8 poles for each element of H^m. The SER for T_I^T was calculated directly from the SER of T_I, as previously mentioned.

5.2.1 Short circuit response

In this test a 1 p.u. step voltage was applied to the core of the leftmost cable (sending end), while the other cores and sheaths were grounded at this end. At the opposite end (receiving end) all cores and sheaths were grounded.

Figure 9 shows the calculated current flowing in the core and sheath of the energized cable at the sending end (solid line). In the same figure are also shown the exact values as calculated by a Fourier transform (dotted line). The currents by the simulation model are seen to be on top of those by the Fourier method, implying both that the rational function approximation is very good, and that the model has been correctly implemented in the time step loop.

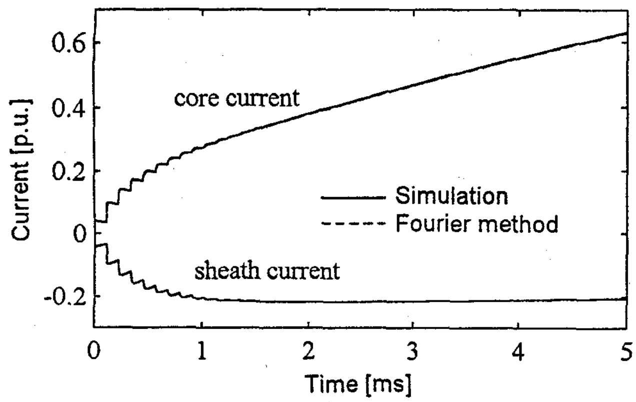

Fig. 9 *Short circuit current in core and sheath of energized cable (step response)*

5.2.2 Open circuit response

In this test a 1 p.u. step voltage was applied to the core of the leftmost cable (sending end), while the other cores and sheaths were grounded at this end. At the opposite end (receiving end), all cores and sheaths were open circuited. Figure 10 shows the calculated receiving end sheath voltage of the energized cable. The calculated voltage is seen to be in good agreement with the theoretically accurate solution by the Fourier method.

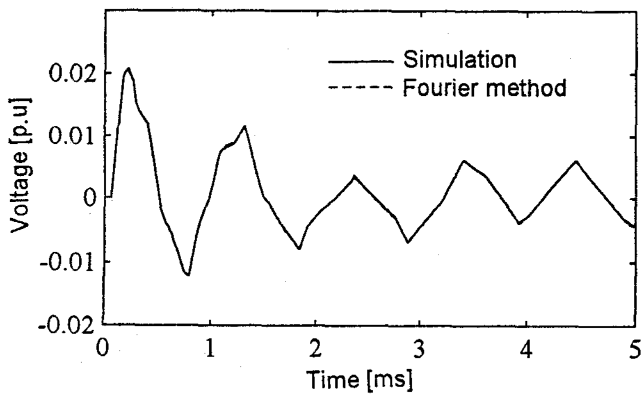

Fig. 10 *Open circuit voltage on sheath of energized cable (step response)*

5.3 Application to other cable geometries

In this study we have only considered systems of single core coaxial cables. It appears that in these cases the transformation matrix and the modal quantities for characteristic admittance and propagation are in general smooth functions that can easily be fitted with rational functions. An interesting observation is that in the case of cables with low system voltage (e.g. 12, 24 kV), there may appear "peaks" in the transformation matrix and in particular in the modal characteristic admittance. This appears to be a result of that the thickness of the sheath-ground insulation is no longer much smaller than that of the main insulation, so that the capacitances of these two insulations become comparable.

As an example, consider the single core coaxial cable in figure 11, which has a thickness of 2.2 mm for the sheath-ground insulation.

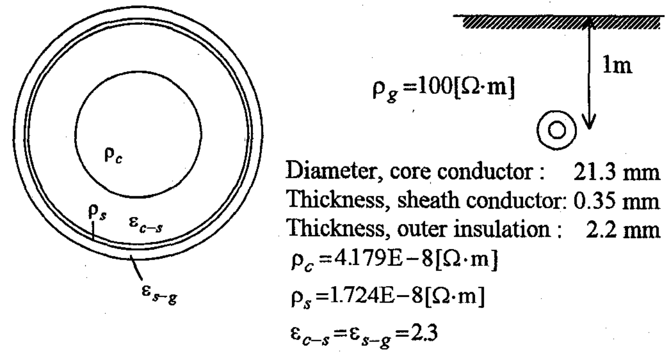

Fig. 11 *Single core coaxial cable.*

The characteristic admittance for the coaxial mode is shown in figure 12 as the thickness of the main isulation is varied between 3 mm and 7 mm. We note that a sharp peak occurs as we decrease the insulation thickness. It should be noted that such peaks are accurately reproduced using our fitting method, as complex poles/zeros are automatically introduced.

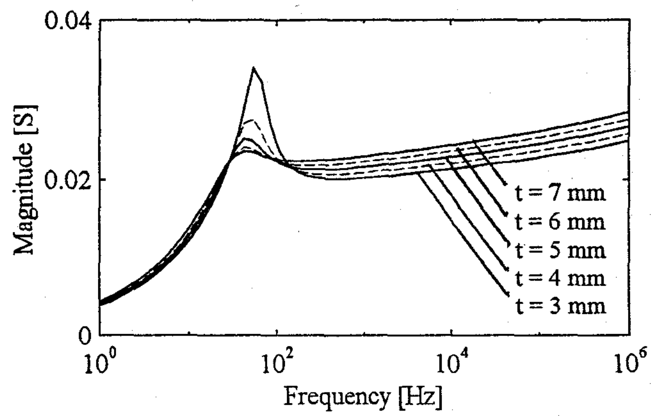

Fig. 12 *Influence of insulation thickness on Y_c^m (coaxial mode)*

6 APPLICATION TO OVERHEAD LINES

The modal domain approach described in the paper has also been applied to a number of different overhead lines, including both single and multi-circuit lines. It turned out that the success obtained with cables could not be automatically achieved with overhead lines. Our fitting technique has no difficulties in fitting the transformation matrix and the modal quantities, but the resulting fit will in general give unstable poles in T_I and Y_c^m. This problem is due to the fact that unstable poles are needed in order to achieve an accurate fit.

As an example, consider the 132kV untransposed single circuit line in figure 13.

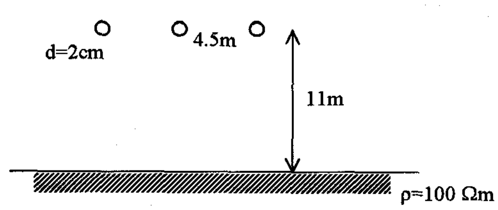

Fig. 13 *Single circuit overhead line*

Figure 14 shows the column of the transformation matrix corresponding to the ground mode (magnitude functions). Two of the elements are equal to unity, but one element (2,1) is frequency dependent, particularly between 1Hz and 100Hz.

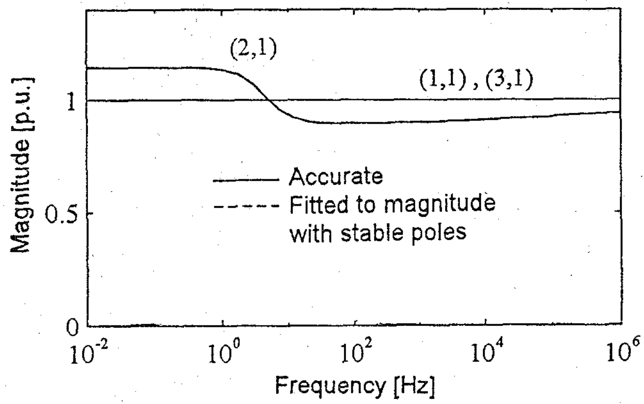

Fig. 14 *First column of T_I (magnitude functions)*

The phase angle for element (2,1) is shown separately in figure 15. Also is shown the phase angle that results when fitting this element to magnitude only with stable poles and zeros. We note that in this frequency interval the fitted response has a negative phase shift whereas the original response has a positive phase shift. In this case it is *not* possible to obtain an accurate fit using stable poles only.

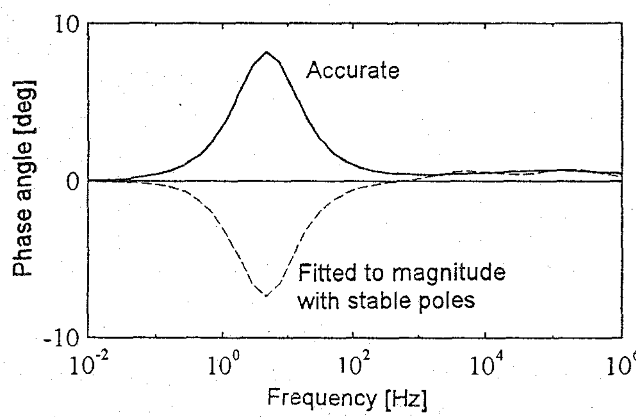

Fig. 15 *Phase angle of element* (2,1) *of T_I*

In principle, we can easily remove the unstable poles from T_I since all elements in each eigenvector have been forced to share the same poles. If p_{unst} is an unstable pole, then multiplication by the scalar function $(s-p_{unst})$ will give a new eigenvector with only stable poles. However, it follows from (8) that this scaling results in the corresponding modal characteristic admittance being scaled by the factor $1/(s-p_{unst})^2$. Forcing T_I^0 to be stable by scaling would therefore move the instabilities over to Y_c^m.

The underlying problem is that the characteristic admittance matrix, which is intrinsically stable in the phase domain, may have an unstable modal decomposition. This decomposition may be expressed as :

$$Y_c = T_I Y_c^m T_I^T = \sum_{k=1}^{n} t_{Ik} Y_{ck}^m t_{Ik}^T = \sum_{k=1}^{n} Y_{ck} \qquad (32)$$

where Y_{ck}^m is element (k,k) of Y_c^m. When the sum in (32) is taken to form a phase domain element, the poles of that element becomes equal to the poles of the contributing terms. The resulting zeros of the phase domain element will, however, be different from the zeros of the contributing terms. This permits unstable poles in Y_{ck} to be canceled by unstable zeros. Thus, some Y_{ck} could be unstable while the resultant Y_c is stable. In case of overhead lines this problem appears to frequently take place as non-perfect cancellations between unstable poles and unstable zeros. This makes it generally impossible to obtain a perfectly accurate fit for the modal components using stable poles only.

Figures 16 and 17 show the magnitude functions and phase angles for the contributions Y_{ck} to element (1,1) of Y_c. The fitting technique was modified so as to fit to magnitude only with stable poles and zeros. We note that although the magnitude functions have been accurately fitted, the phase angles are in error for contributions #1 and #3. However, when all three elements were added together to form the phase domain element $Y_c(1,1)$, the resulting element could be accurately fitted with stable poles.

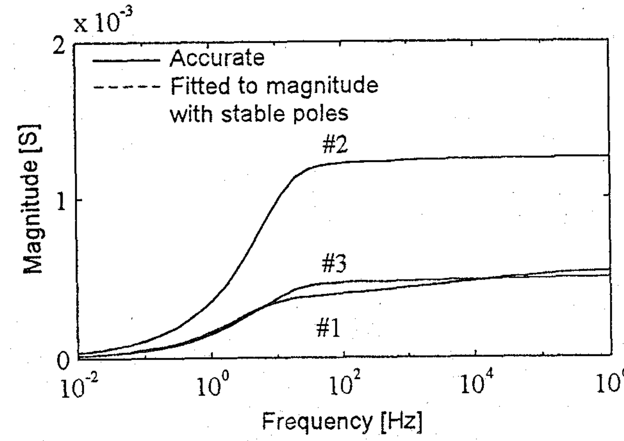

Fig. 16 *Magnitude of modal contributions $Y_{ck}(1,1)$ to $Y_c(1,1)$.*

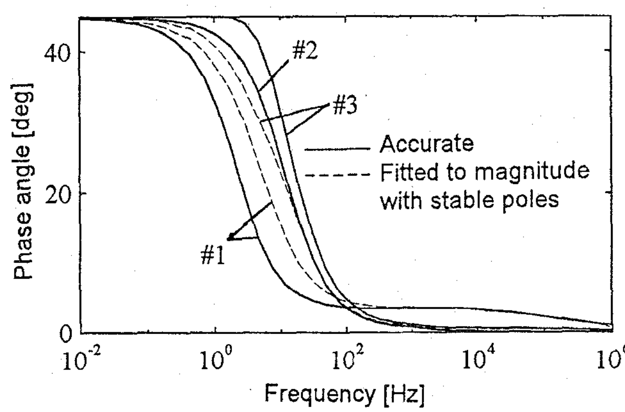

Fig. 17 *Phase angle of modal contributions $Y_{ck}(1,1)$ to $Y_c(1,1)$*

7 OVERVIEW OF RESULTS

The frequency dependence of the modal transformation matrix encountered in transmission line transient calculations can be handled by a modal domain approach involving an additional convolution for the transformation matrix. For an n-conductor system this gives a total number of convolutions equal to $4n^2+4n$, as compared to $4n$ when a constant transformation matrix is used. A significant computational burden obviously lies in the $4n^2$ convolutions for the transformation matrix.

In this paper *we reduce the computational burden* for these convolutions *to approximately one third by using vector fitting* for the transformation matrix. Vector fitting means that all elements in a vector share the same set of poles. This has been possible by the introduction of a *new fitting technique—rational fitting by optimal scaling*. The method produces a *state equation realization* (SER) for the entire vector by solving a well-conditioned overdetermined set of linear equations. The fitting routine takes as input the vector as function of frequency plus a set of starting poles. The key idea of the method is to multiply the vector by an "optimal scalar", $\sigma(\omega)$, so that the starting poles give a good fit for the vector. By appending a unity to the original vector, the zeros for $\sigma(\omega)$ are then readily obtained. These zeros are now used as new starting poles, and the original vector is in the end fitted by solving an overdetermined set of linear equations. One strength of the method lies in that the accuracy of the resulting fit is only slightly dependent on the location of the starting poles. Our experience is that *the starting poles can be assumed to be real and logarithmically distributed*. The fitting routine produces a SER with both real and complex poles.

In the paper we implemented in time domain a modal domain model with frequency-dependent transformation matrix, similarly as in the L. Marti model [9]. The new fitting methodology was used to find SERs for the transformation matrix, the modal characteristic admittance matrix and the modal propagation matrix. Application to a 66kV single core cable system showed that *highly accurate results could be achieved with a relatively low number of poles* : 5-6 for each column of the transformation matrix and 7-8 for each element of the modal characteristic and propagation matrix. Of particular interest was the observation that zeros in the right half plane are intrinsically necessary for some elements of the transformation matrix, while traditional Marti-fitting [3] assumes all zeros to lie in the left half plane. Application of the new fitting method, however, automatically produces a SER which has the *correct zeros*. Another observation was that in case of cables where the outer insulation is not much thinner than that of the main insulation (low system voltage), the transformation matrix and in particular the characteristic admittance matrix may have strong peaks. These can, however, be easily fitted using optimal scaling, as *the needed complex poles will automatically be introduced*.

In case of overhead lines the method of modal decomposition with a frequency dependent T seems to be less successful than for cables. Application to a single circuit line showed that T could not be fitted accurately with stable poles only. It was concluded that in case of the characteristic admittance, we may get an unstable modal decomposition. Similar results have also been obtained for other line geometries, including multi-circuit lines. This fact suggests that *a direct phase domain approach may be the right choice for such cases*.

The method of optimal fitting has been implemented as a subroutine where the number of poles is given in the input. This allows the vector and scalar functions to be repeatedly fitted with an increasing number of poles, until an error criterion is met. Consequently, in the case of overhead lines, *columns which turn out to be frequency independent are fitted with a constant vector* (order 0). Thus, in the case of a constant transformation matrix, the proposed simulation model becomes similar to the J. Marti model [3].

8 CONCLUSIONS

A new method has been introduced which allows vectors and scalars to be fitted with rational functions containing complex poles and zeros, in addition to real ones. The fitting process is very robust and there is no problem in specifying a very high order for the fitting if needed.

The new fitting method has been applied with success to cable systems using modal decomposition and frequency dependent transformation matrices. In this model, we need to fit the transformation matrix and the diagonal matrices for propagation and characteristic admittance. The resulting cable model is computationally efficient for the following reasons :

- Accurate fitting can be achieved with a relatively low number of poles. An order of 5-10 is usually sufficient.
- The new fitting method allows each vector of the transformation matrix to be fitted with the same set of poles. This gives approximately a 3-fold increase in computational efficiency in the time step loop for all convolutions involving the transformation matrix.

In the case of overhead lines it is not always possible to obtain an accurate fit with stable poles when relying on modal decomposition with a frequency dependent transformation matrix. A stable model will in such cases be an approximation.

9 ACKNOWLEDGMENTS

Financial assistance by the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. The first author wishes to express his gratitude to the Norwegian Electric Power Research Institute (EFI), Trondheim, Norway, for granting and financing his leave at the University of Toronto.

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11 APPENDICES

A – Fitting With Simple Fractions

Simple fractions

$$g_i(\omega)=\frac{1}{j\omega-p_i} \tag{A.1}$$

with *real* poles p_i constitute a *basis in the function space* for fitting smooth functions $h(\omega)$. Indeed such functions are smooth, as illustrated in figure A.1 for $p_i=-2.5,-5,-7.5,-10$.

However, the functions $g_i(\omega)$ do not span the whole function space and therefore they are useful only for relatively *smooth* functions. We can see this if we represent $g_i(\omega_k), \omega_k=\omega_1,\omega_2,...\omega_N$, as vectors forming the columns of a matrix G . Figure A.2 shows the singular values obtained by Singular Value Decomposition (SVD) of a 100×100 $G-$ matrix (for 100 points for p_i and ω uniformly distributed from 0 to

10). It is seen that only a fraction of the singular values are not very small. Therefore, *identifying* simple fractions (A.1) for fitting leads to a poorly conditioned problem, especially when the number of simple fractions is high. (Only the orthogonal $U-$matrix of the SVD ($G=USV^H$) provides a well-conditioned basis.) Conversely, a larger set of simple fractions (A.1) that covers the whole frequency range of interest can be *chosen* with great flexibility when fitting any number of smooth functions $h(\omega)$.

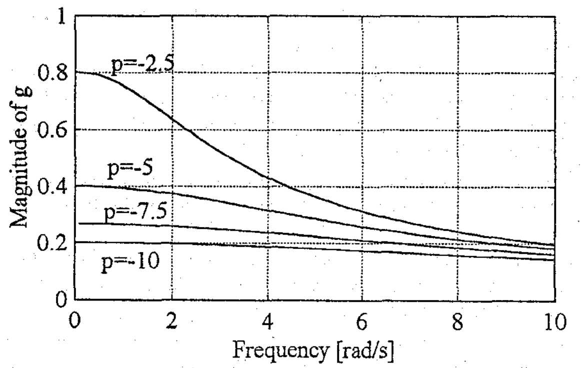

Fig. A.1 *Frequency domain behavior of simple fractions*

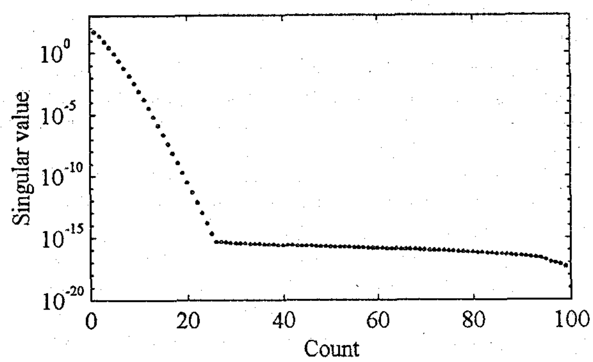

Fig. A.2 *Singular values of G*

B – Computational Efficiency of State Equation Realization Based on Vector Fitting

Consider the scalar-input / vector-output convolution

$$y=h*u \tag{B.1}$$

whose finite State Equation Realization has been found :

$$\dot{x}=Ax+bu \tag{B.2}$$

$$y=Cx+du \tag{B.3}$$

Trapezoidal integration of (A.2) gives :

$$\frac{x_n-x_{n-1}}{\Delta t}=A\frac{x_n+x_{n-1}}{2}+b\frac{u_n+u_{n-1}}{2} \tag{B.4}$$

where Δt is the time step length used in the simulation. Further manipulation gives :

$$x_n=Px_{n-1}+q[u_n+u_{n-1}] \tag{B.5}$$

$$y_n=Cx_n+du_n \tag{B.6}$$

In the following we analyze the time needed in the time step loop for an eigenvector when its elements have been fitted separately, and when they have been forced to share the same set of poles (vector fitting). It is assumed that N poles are used in both cases, and that the system has n conductors.

Let k_a and k_m be the computational time for one addition and one multiplication, respectively. When vector fitting is used, the SER has one input and n outputs, thus

P: $(N\times N)$ (diagonal) q: $(N\times 1)$ (full)
C: $(n\times N)$ (full) d: $(n\times 1)$ (full)

The computational time inferred from (B.5) - (B.6) is then found to be about equal to :

$$K\approx(nN+n+2N)(k_a+k_m)\approx nN(k_a+k_m) \tag{B.7}$$

If each element is fitted separately, each element gives a SER with one input and one output. Since there are n elements we get:

$$K\approx n(N+1+2N)(k_a+k_m)\approx 3nN(k_a+k_m) \tag{B.8}$$

By comparing (B.7) with (B.8) we see that time domain evaluation of a SER based on vector fitting gives about 3 times faster evaluation than that of a SER based on element-by-element fitting.

C – Calculating New Set of Poles by Optimal Scaling

The new set of poles produced by the optimal scaling is equal to the zeros of the particular element h_1 of the scaled vector. These zeros are calculated as follows :
The SER of the scaled vector is given by (20). The SER of h_1 has the same A and b as that of the entire vector, and a row vector c^T and scalar d_1 equal to the corresponding row of C and d for the vector. From (12) and (13) we get the SER of h_1 in the frequency domain :

$$sx=Ax+bu \quad , \quad y=c^Tx+d_1u \tag{C.1}$$

Interchanging input and output gives the SER of the inverse of h_1 :

$$sx=(A-bd_1^{-1}c^T)x+bd_1^{-1}y \quad , \quad u=-d_1^{-1}c^Tx+d_1^{-1}y \tag{C.2}$$

The zeros of h_1 are then calculated as the eigenvalues of the matrix $A'=A-bd_1^{-1}c^T$. Note that d_1^{-1} always exists because h_1 approaches unity at high frequencies.

D – State Equation Realization for T_I^T

Let A, B, C and D represent the SER for T_I. By transposing (15), we find a SER for T_I^T given by $A'=A$, $B'=C^T$, $C'=B^T$, $D'=D^T$.

E – Time Delays For Modal propagation

Each element of the modal propagation matrix H^m (diagonal) can be written as :

$$h^m(\omega)=e^{-(\alpha(\omega)+j\frac{\omega}{v(\omega)})l} \tag{E.1}$$

where α, v, and l denote attenuation, velocity and line length, respectively. It is well known [3] that $h^m(\omega)$ can be fitted by a minimum phase shift function $h_{\min}(\omega)$ plus a time delay, τ :

$$h^m(\omega)\approx h_{\min}(\omega)e^{-j\omega\tau} \tag{E.2}$$

From (E.1) and (E.2) it follows that τ must be chosen so as to satisfy the relation

$$-\frac{\omega}{v(\omega)}l=\angle h_{\min}(\omega)-\omega\tau \tag{E.3}$$

In our implementation, (E.3) is evaluated at a single frequency point, Ω. Thus, the time delay becomes :

$$\tau=\frac{l}{v(\Omega)}+\frac{\angle h_{\min}(\Omega)}{\Omega} \tag{E.4}$$

The phase angle $\angle h_{\min}(\omega)$, which is uniquely defined by the magnitude function $|h_{\min}(\omega)|$, is calculated by equations (24)-(26).

12 BIOGRAPHIES

Bjørn Gustavsen was born in 1965 in Harstad, Norway. He received the M.Sc. degree in Electrical Engineering in 1989 and the Dr. Ing. Degree in 1993, both from The Norwegian Institute of Technology. Since 1994 he has been working at the Norwegian Electric Power Research Institute (EFI), mainly in the field of transient studies. He is currently on leave at the Department of Electrical and Computer Engineering, University of Toronto.

Adam Semlyen was born in 1923 in Rumania where he obtained a Dipl. Ing. degree and his Ph.D. He started his career there with an electric power utility and held academic positions at the Polytechnic Institute of Timisoara. In 1969 he joined the University of Toronto where he is a professor in the Department of Electrical and Computer Engineering, emeritus since 1988. His research interests include steady state and dynamic analysis as well as computation of electromagnetic transients in power systems.

Discussion

Brandão Faria, Senior Member, IEEE (CETME, Instituto Superior Técnico, Av. Rovisco Pais, 1096 Lisboa Codex, Portugal):

This paper, together with a companion one [1], presents an important and lasting value contribution to the field of transmission line transients computation. The authors are, thus, to be commended by the excellent work they have done. The few comments and questions that next follow have the sole purpose of permitting the clarification of some minor details.

1) On stating that H^m and Y_c^m are diagonal matrices, eqs. (8-9), the authors are tacitly assuming that the product matrix ZY (or YZ) is always diagonalizable. However, unfortunately, that assumption is a false one [2-4]. What kind of strategy should one then adopt in order to keep using the method proposed by the authors if a non-diagonalizable situation is faced at some frequencies?
2) In Section 3.2 it is referred an eigenvector normalization procedure consisting in fixing one element to unity. It is well known that, for bilaterally symmetric transmission line configurations, certain propagation modes may have some elements null in the eigenvector columns. Therefore, some caution must be exercised in the process of fixing elements to unity.
3) The accuracy of the vector fitting technique developed by the authors is shown in a series of illustrations throughout the paper. However, the illustrations refer to the use of different orders of approximation (i.e. different number of poles). It is not clear which criterion should one adhere to in deciding on the minimum number of poles sufficient for achieving a "good" fitting. This question may be decisive for the application of the proposed technique to new computation cases where the "accurate" solution is not known *ab initio*, that is, when a reference is not available for comparison.
4) In Section 6 the authors technique is applied to overhead line configurations with much less success than the one they have got with cable configurations in Section 5. The mathematical reason for this relative failure (explained in the paper) lies on the presence of unstable poles. Can a simple explanation for this be provided based on physical grounds?

The authors reply to these comments will be highly appreciated.

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Manuscript received February 25, 1997.

Taku Noda and Akihiro Ametani (Doshisha University, Kyoto, Japan): We would like to congratulate the authors for developing the new fitting approach - vector fitting - for modal-domain modeling of a transmission line.

The proposed line model is based on the modal decomposition approach. The modal decomposition at a single frequency is always possible and is a perfect theory as shown in [8]. But is it guaranteed that a modal-transformation matrix T_I represents a physically realizable system? If a system is not physically realizable, its numerical implementation would be impossible. For example, a non-causal system cannot be implemented in computer code, because future input and output cannot be obtained.

The authors have concluded that the present line model is suitable for cable modeling but not always for overhead line modeling. But it should be noted that the frequency characteristics of a modal-transformation matrix highly depend on impedance formulas used in the calculations. Impedance formulas used for modeling the single-circuit overhead line of Fig. 13 should be clarified.

The comments of the authors would be highly appreciated.

Manuscript received March 3, 1997.

Bjørn Gustavsen and Adam Semlyen: We wish to thank the discussers for their valuable comments and useful contributions. The following are itemized answers to the problems raised.

Professor Brandão Faria :

Ad 1 Non-diagonalizable matrices ZY (or YZ) are extremely rare and we never encountered one. Even if at certain values of the parameters, notably at some particular frequency, such a situation would occur, a small disturbance of the frequency will lead to a regular matrix. In the integration process implicit in the transition from frequency to time domain, any isolated irregular point would be filtered out. Therefore, we believe that, apart from special, contrived or pathological situations, there should be no concern for non-diagonalizability.

Ad 2 We agree with the cautionary note of the discusser regarding the selection of an element of the vector to be normalized to unity. In our implementation, the elements of T were scanned as function of frequency. The element with the largest minimum value was normalized to unity. We wish to add that only in the case of transformation matrices is an element equal to unity obtainable by normalization. In most other applications of *vector fitting* an element equal to unity is appended to the given vector. This permits a transitional scaling and smoothing of the vector, an important step in the state equation approximation process.

Ad 3 Vector fitting is used to find a rational function approximation of a given function. In deciding the minimum number of poles needed for a given accuracy, a trial and error approach will have to be used.

Ad 4 We cannot explain the unstable modal decomposition encountered for overhead lines on physical grounds. We can only point to the fact that several eigenvalues for overhead lines lie very close over a wide range of frequencies, while this is usually not the case for underground cables. It is interesting to note that in figure 12 the increasing "peak" in the characteristic admittance, observed with decreasing thickness for the inner insulation, is due to the two eigenvalues getting close. Eventually we arrived at a situation where the transformation matrix could no longer be fitted with stable poles only.

The robustness, accuracy, and efficiency of vector fitting makes it attractive for many other applications in addition to the one presented in the paper. In a more general setting, the transfer functions are in general not strictly proper but could be just proper or even improper (order or numerator equal or greater than that of the denominator). It turned out to be a simple matter to generalize the vector fitting program to be applicable to all these cases. These would often occur in the identification of transformers or of external system equivalents over a wide band of frequencies.

Dr. Taku Noda and Professor Akihiro Ametani :

The discussers raise the interesting question whether it is guaranteed that a modal transformation matrix T will represent a physically realizable system. All we can say is that the successful state equation realizations obtained for T indicate physical realizability. In addition, there is a plausibility argument to support this: since both the phase and modal domain input-output relations (transfer functions) are causal, the link T between them should have the same property. The only pertinent difficulty we have experienced is that the realization may result unstable in the case of overhead lines. This instability cannot be removed by scaling.

For modeling the single circuit overhead line we used the complex depth method [A] for the ground return impedance and approximate formulae [11,B] for the internal conductor impedance.

We would finally take advantage of this opportunity to present a more efficient implementation of the time domain integration than the one in the paper. The approach, suggested by Dr. Thor Henriksen (EFI), can be summarized as follows :

We start with equations (B.5) and (B.6) :

$$x_i = Px_{i-1} + q(u_i + u_{i-1}) \tag{a}$$

$$y_i = Cx_i + du_i \tag{b}$$

Replacing in (a) and (b) the state vector x_i with the modified vector

$$x_i' = x_i - qu_i \tag{c}$$

gives

$$x_i' = Px_{i-1}' + (Pq+q)u_{i-1} \tag{d}$$

$$y_i = Cx_{i-1}' + (Cq+d)u_i \tag{e}$$

Equation (d) can be evaluated more efficiently by using a similarity transformation via the matrix

$$S = \mathrm{diag}(Pq+q) \tag{f}$$

Since P is diagonal, the final result takes the form

$$x_i'' = Px_{i-1}'' + eu_{i-1} \tag{g}$$

$$y_i = Rx_i'' + Gu_i \tag{h}$$

where e is a vector of ones and

$$R = CS \tag{i}$$

$$G = Cq + d \tag{j}$$

We next look at the computational effort for updating the past history source Rx_i'' in the time step loop. Let k_a and k_m denote the computational effort for one addition and multiplication, respectively. Assume that we have fitted a column of n elements using N poles.

If (g) and (h) are evaluated separately for each element, we find the total computational cost to be:

$$K = 2nN(k_a + k_m) \tag{k}$$

If (g) and (h) are evaluated taking advantage of all elements having the same state vector we get:

$$K = (n+1)N(k_a + k_m) \approx nN(k_a + k_m) \tag{l}$$

Comparing (k) with (l) shows that the columnwise realization gives a two-fold increase in computational efficiency as compared to element-by-element realization.

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