Vehicle Routing in Practice

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Outline

- Background and Context
- The Vehicle Routing Problem
- Industrial Aspects
- Spider - A Generic VRP Solver
- Specific cases
- Ongoing and Further Research
Technology for a better society
An Independent Multi-Disciplinary Contract R&D Organisation
Established in 1950
Among the Largest CRO in Europe

Vision:
Technology for a better Society

Business Concept:
To meet the needs for Research-Based Innovation and Development for the Private and Public Sectors
SINTEF - A Norwegian Contract Research Institute

- Science and engineering – social sciences, health care
- Connected with
  - The Norwegian University of Science and Technology, Trondheim
  - The University of Oslo, Faculty of mathematics and natural sciences
- > 90 % of turnover from industry and public sector contracts
- Annual turnover ~ 200 M€

Activities
- Strategic, long term, basic research
- Contract Research
- Consultancy
- Commercialization and spin-offs
The SINTEF Group

SINTEF's Council
SINTEF's Board

President
Executive
Vice President

Corporate Staff

Research Divisions

SINTEF Health Research
SINTEF Technology and Society
SINTEF ICT
SINTEF Materials and Chemistry
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SINTEF Marine
Marintek
SINTEF Fisheries and Aquaculture
SINTEF Petroleum and Energy
SINTEF Petroleum Research
SINTEF Energy Research
SINTEF Holding
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The Vehicle Routing Problem (VRP)

Given a fleet of vehicles and a set of transportation orders, find a minimum cost routing plan. That is: allocate each order to a vehicle, and for each vehicle, sequence the stops.

- The VRP central to efficient transportation management
- Applications
  - Distribution or pick-up of goods
  - Dial-a-ride
  - Municipal services
  - Repairman problem
  - Newspaper distribution
  - Waste management
  - Gritting, snow clearing
- Very hard, discrete optimization problem
The Capacitated VRP (CVRP)

- Graph $G=(N,A)$
  - $N=\{0,\ldots,n\}$ Nodes
  - 0 Depot, $i\neq 0$ Customers
  - $A=\{(i,j): i,j\in N\}$ Arcs
  - $c_{ij}>0$ Arc cost

- $Demand$ $d_i$ for each $Customer$ $i$
  - either all pickups or all deliveries

- $V$ set of (identical) $Vehicles$, each with $Capacity$ $q$

- Goal
  - Construct a set of minimum cost $Routes$ that start and finish at the $Depot$.
  - Each $Customer$ shall be visited once (no order splitting)
  - Total $Demand$ for all $Customers$ serviced less than $Capacity$ $q$
  - $Cost$: Total $Arc$ $cost$ (also $#$ $Routes$ in richer VRP variants)

- DVRP – constraint on route length
- VRPTW – VRP with time windows
- VRPB – VRP with backhauling (deliveries first, then pickups)
- PDP – pickup and delivery without going through the depot
Mathematical formulation of VRPTW (vehicle flow formulation)

minimize \[ \sum_{k \in V} \sum_{(i,j) \in A} c_{ij} x_{ij}^k \] \quad (1) \quad \text{minimize cost}

subject to:

\[ \sum_{k \in V} \sum_{j \in N} x_{ij}^k = 1, \quad \forall i \in C \] \quad (2) \quad \text{each customer once}

\[ \sum_{i \in C} d_i \sum_{j \in N} x_{ij}^k \leq q, \quad \forall k \in V \] \quad (3) \quad \text{capacity}

\[ \sum_{j \in N} x_{0j}^k = 1, \quad \forall k \in V \] \quad (4) \quad \text{k routes out of depot}

\[ \sum_{i \in N} x_{ih}^k - \sum_{j \in N} x_{kj}^k = 0, \quad \forall h \in C, \quad \forall k \in V \] \quad (5) \quad \text{flow balance for each customer}

\[ \sum_{i \in N} x_{i,n+1}^k = 1, \quad \forall k \in V \] \quad (6) \quad \text{k routes into depot (redundant)}

\[ x_{ij}^k (s_i^k + t_{ij} - s_j^k) \leq 0, \quad \forall (i, j) \in A, \quad \forall k \in V \] \quad (7) \quad \text{start of service and driving time}

\[ a_i \leq s_i^k \leq b_i, \quad \forall i \in N, \quad \forall k \in V \] \quad (8) \quad \text{start of service within TW}

\[ x_{ij}^k \in \{0, 1\}, \quad \forall (i, j) \in A, \quad \forall k \in V \] \quad (9) \quad \text{arc (i,j) driven by k}
The VRP is very hard, computationally

- Belongs to class of strongly \textbf{NP}-hard optimization problems
- Computing time for \textbf{all VRP optimization algorithms} grows "exponentially" with the number of nodes (probably ... )
- Exact methods are "easy" to design, may take forever
- Currently, exact methods for the C/DVRP can \textbf{consistently} solve instances with \textbf{some 70 customers only} to optimality within a few minutes
- For generic industrial VRP tools, we must give up the quest for optimality and resort to some form of approximation
- What is the optimal plan, anyway?
VRP in Operations Research

- since 1959
- thousands of papers
- mostly generic work on idealized VRP models
- still, one of the successes of OR
- a tool industry has emerged, based on research results
- tremendous increase in ability to "solve" VRP variants
  - general increase of available computing power
  - methodological improvement
- still much to do, VRP research has never been more active
- short road from research results to industrial benefits
VRP in Operations Research

- General, idealized formulations
- Extensions studied in isolation
  - time windows
  - multiple depots
  - inventory constraints
  - ....
- Very fruitful for
  - understanding VRP variants
  - developing highly targeted algorithms
- Not always relevant to real-life problems
- Some application specific work
- Recently: More holistic approach: ”Rich VRPs”
  - General model, many aspects of industrial problems
  - Robust algorithms
Extensions in the VRP-literature

- Location Routing
- Fleet Size and Mix
- VRP With Time Windows
- General Pickup and Delivery
- Dial-A-Ride
- Periodic VRP
- Inventory Routing
- Dynamic VRP
- Capacitated Arc Routing Problem

All kinds of algorithmic approaches
- exact
- approximation methods
  - based on systematic, exact methods
  - metaheuristics

LRP
FSMVRP
VRPTW
GPDP
DARP
PVRP
IRP
DVRP
CARP
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- Spider - A Generic VRP Solver
- Specific cases
- Ongoing and Further Research
Industrial aspects – VRP tool

- Adequate model of the applications
- Wide range of applications
- High quality information
  - addresses
  - distances, times, costs
  - orders
  - fleet and driver information
- Manual planning, user interface
- Ease of use
- Software issues
  - Integration with ERP etc.
  - Extendability
  - Maintainability
  - Documentation
- High quality solution in short time – powerful VRP solver
Industry needs rich VRP models

- Type of service and operation
  - multiple depots, no depot
  - different order types (delivery, pickup, direct, service, ...)
  - order splitting, flexible order volumes
  - node routing, arc routing, mixed
  - multiple tours per day
  - periodic problems
  - connection to inventory and manufacturing

- Constraints
  - capacity, several dimensions, hard and soft
  - time windows, multiple, hard and soft
  - precedences
  - (in)compatibilities

- Realistic distances, times, costs

- Cost components
  - multiple criteria
  - soft constraints and penalties

- Uncertainty and Dynamics
Research on Rich VRPs & related problems at SINTEF

- Industrial Contracts since 1995
- Strategic Research
  - European Commission FP III, IV, V (e.g., GreenTrip 1996-1999)
  - Norwegian Research Council (RCN)
  - Internal Projects, students
- Generic VRP Solver - Spider (1995→)
  - Commercialization from 1999
  - GreenTrip AS → Spider Solutions AS
- TOP Programme 2001-2004 (RCN) [http://www.top.sintef.no/]
  - Basic Research on Rich VRP and related problems
  - VRPTW
  - Shortest Path Problem in Dynamic Road Topologies
- Innovation Projects supported by Research Council of Norway
  - “I Rute” (2001 – 2004) Bulk transportation
  - “DOiT” (2004 – 2007) Stochastic and Dynamic Routing
  - “EDGE” (2005 – 2008) Large Scale Routing
- Ship routing
  - TurboRouter 1995 ->
  - SINTEF Internal Strategic Project (2005-2008)
  - LNGShipping
Invent is a software library that can model and solve a wide class of maritime routing problems. This includes both traditional tramp shipping and industrial shipping problems with inventory management.

Vessels can be modeled at tank level with the possibility of detailed tank stowage and cleaning operations.

Inventories can have a time varying production or consumption profile together with an upper and lower limit on the inventory level. The inventories can be combined with traditional orders with laycan and quantity intervals for a pair of pickup and delivery locations.

It is also possible to include contracts with details on required pickup or delivery in given periods, and time varying price curves for income and cost.

Problem instances are described in a general XML format that can be used as input for the automatic planning process.

Invent generates optimized plans for the problems based on advanced heuristic methods that are able to provide high quality solutions in short time.
TurboRouter
Maritime Transport Optimization
AN OCEAN OF OPPORTUNITIES


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Spider - A Generic VRP Solver

- Designed to be widely applicable
- Based on generic, rich model
- Predictive route planning
- Plan repair, reactive planning
- Dynamic planning with stochastic model

- Framework for VRP research
SPIDER - Generalisations of CVRP

- Heterogeneous fleet
  - Capacities
  - Equipment
  - Arbitrary tour start/end locations
  - Time windows
  - Cost structure
- Linked tours with precedences
- Mixture of order types
- Multiple time windows, soft time windows
- Capacity in multiple dimensions, soft capacity
- Alternative locations, tours and orders
- Arc locations, for arc routing, mixed problems and aggregation of node orders
- Alternative time periods
- Non-Euclidean, asymmetric, time-varying travel times
- Compatibility constraints
- A variety of cost components and soft constraints
  - driving time restrictions
  - visual beauty of routing plan, non-overlapping routes
  - levelling
Transportation Order

- Different types:
  - Delivery
  - Pickup
  - Direct (P&D)
  - Service

- Plan structure
  - task
  - task sequence
  - tour
  - plan
Tour

- Time windows and locations given by start/stop tasks
- Selected vehicle and driver (alternative equipages)
- Linked tours
  - (may have different vehicles)
Vehicle and Driver

- Capacity
- Travelling attributes
  - Speed profile
  - Height, weight, length
  - Obey one-way restrictions?
- Driving time regulations
Cost elements

- Travel cost (distance, time, tolls)
- Tour usage cost
  - Cost for starting a tour
  - Cost per order on tour
- Cost for unserviced order
- Waiting time cost
- Cost for alternative locations
- Cost for same/different location
- Cost for breaking work regulations
- Cost for “ugly”, overlapping routes
Constraints

- **Consistency**
  - Complete order
  - Pickup/delivery (Direct orders) same tour and precedence

- **Time**
  - Travel time, Duration
  - Time windows, multiple, hard and soft

- **Vehicle capacities, multiple dimensions, hard and soft**

- **Total capacities over a set of tours**
Constraints

- Compatibility vehicle/order vehicle/location product/compartment
- Orders on same tour
- Corresponding locations (when alternatives)
  - Order: Choose corresponding locations for pickup and delivery task
  - Tour: Choose corresponding locations for start/stop tasks
- Corresponding time periods for sets of orders
  - E.g. Delivery day 1,3,5 or day 2,4,6
Locks

- Prevent optimiser changing part of plan
- Task: Time lock
- Tour: Lock whole or initial part of tour
- Order: Lock (un)assignment
Uniform Algorithmic Approach

- Goals
  - Reach a good local optimum fast
  - Explore interesting parts of search space efficiently

- 3 phases
  - Construction
  - Iterative Improvement: Iterated Variable Neighborhood Descent
  - Tour Depletion

- Based on
  - Iterated Local Search (Martin, Lourenço et al)
  - Variable Neighborhood Descent (Hansen & Mladenovic)
  - Diversification when VND reaches local optimum
Construction of Initial Solution

- Various Sequential Construction Heuristics
- Extended to cover Richer Model
  - Several types of order
  - Non-standard constraints
  - Non-homogeneous fleet
  - Multiple Depots
  - Multiple Tours per Vehicle
- New Constructor
  - Inhomogeneous problems
  - Multiple depots
  - Multiple tours per vehicle
  - More search
Variable Neighborhood Descent - Operators

- Insert
- Remove
- 2-opt
- Or-opt
- 3-opt
- Relocate
- Cross (2-opt*)
- Exchange (Cross-Exchange)
- Tour Depletion
- Change alternative location
- Change alternative time period
- Change vehicle
- ...

- Variety, efficiency, sequence?
Illustration: Exchange

- Inter-tour operator
- Full neighborhood is typically large
- Remedy: Limit on maximum segment length
- Alternative: Focus on promising cut points
Iterated Local Search
(Martin, Lourenço et al)

- Goal: Efficient search in new basins of attraction
- When VND reaches local optimum: diversify
- Random restart
- Alternative initial solution
- Path relinking
- Noising: perturb problem data
- Change objective
- LNS, VLNS
Large Neighborhood Search (Shaw)

- Take away a substantial number of commitments
  - randomly
  - “similar” commitments
- Reconstruct with fast insertion method
  - Cheapest insertion
  - Regret-based insertion
- Accept new solution if
  - better, diverse
  - Threshold Acceptance
  - Simulated Annealing
- Iterate
- Alternative modifications
  - Limited Local Search
  - Full VND Machinery
  - Distance Metrics
Computational Experiments

- Standard test problems in literature for most VRP variants
- "World championship" in Vehicle Routing
- For most instances, the optimal solution is not known
- Testing against the best methods reported in the literature
- "Unfair" comparison: competing with solvers optimized for solving a specific, idealized problem
- Nice to know how well you perform ...
Experimental studies PDPTW
http://www.sintef.no/top

- 354 standard test problems Li & Lim: 100 – 1,000 orders

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- BVH: Bent & Van Hentenryck: Two-stage, Hybrid Local Search
  - 1. Minimize # tours by SA, modified objective
  - 2. Minimize Distance with Large Neighborhood Search
- TS: Tetrasoft, Danish tool vendor, Method unknown
- SR: Stefan Röpke, Univ. Copenhagen
  - Adaptive Large Neighborhood Search
Results on CVRP
- Rounded distances (1)

Augerat et al
- 74 instances, 15-100 customers, limit on # vehicles
  - A (27): 0.15% from optimum, 19 optimal, 0-541 s, average 100 s
  - B (23): 0.11% from optimum, 17 optimal, 1-582 s, average 95 s
  - P (24): 0.23% from optimum, 15 optimal, 0-450 s, average 99 s

Christofides & Eilon (1969)
- 13 instances, 12-100 customers, limit on # vehicles
  - E (13): 0.32% from optimum, 2 optimal, 7 - 487 s, average 178 s

Fisher
- 3 instances, 44 - 134 customers, limit on # vehicles
  - F (3): 0.27% from optimum, 7 optimal, 0-289 s, average 103 s

Gillett & Johnson, 1 instance: G-n262-k25
- best known, -7.09% from previous best known (5685)
G-n262-k25: 5685 vs. 6119
Results on CVRP
- Rounded distances (2)

- Christofides, Mingozi & Toth (1979)
- 5 CVRP instances, 100-199 customers, limit on # vehicles
- 0.99 % from optimum, 21-1255 s, average 404s
- 1 optimal
- 1 new best known -1.6 %
- 1 worse by 2.7 %
- 1 worse by 2.9 %
- M-n200-k16: no feasible solution previously known
- Feasible solution found
  - after 533s: distance 1371
M-n200-k16: First known feasible solution
Computational tests - NEARP

- Prins & Bouchenoua CBMix (23 instances)
- No lower bounds, no proven optima, only one competitor
- UB error 0.94%
- 8 best known solutions (6 new)
Products - architecture

Geomatikk AS
- Web solution
  - Web-server (Servlet, C++/Java)
    - Guider (topology)

SPIDER Solutions AS
- Designer
  - Planner (VRP solver, COM)
    - Guider (topology)

Distribution Innovation AS
- DI web solution
  - Spider Server
    - Planner (VRP solver, COM)
      - Guider (topology)
Spider Designer - Applications

- Distribution of bread (Bakers)
- Mail collection and distribution (Posten Norge)
- Local pickup and delivery (Schenker)
- Newspaper distribution, 1st tier (Aftenposten, Dagbladet)
- Newspaper distribution, last mile (Aftenposten, Stavanger Aftenblad)
- Collection of milk from farms (TINE)
- Distribution of fodder to farms (Landbruksdistribusjon)
- Distribution of fuel oil (Hydro Texaco)
- Location analyses, depot (obnoxious facility location, Norsk Gjenvinning AS)
- Distribution of blood (Ullevål sykehus)
- Distribution of groceries (REMA 1000)
- Distribution of magazines (Bladcentralen)
- Distribution of ice cream (Diplom Is; Hennig Olsen)
- Dial-a-ride, elderly, hospital patients (Nor-Link)

- Savings 2-35%, depending on application
Challenges for Routing Technology

- Industrial awareness
- Range of applications
- Information availability and quality
- User interfaces
- Model adequacy and flexibility
- Software engineering
- Robustness of solution method
- Solution quality for large-size and complex problems
Challenges for Spider

- Electronic road network - GIS
- Instance robustness
  - all kinds of instances
- Scalability
  - very large size problems
- Extendability
  - new operators, new VND sequence

- Exploiting modern commodity computers
  - multi-core, heterogeneous, GPUs
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Scalability – Solving ”Huge” VRPs

- Faster VRP solver
  - heuristic investigation of neighborhoods
  - clever sequencing of constraint and objective component evaluation
  - more powerful metaheuristics
  - hybrid methods, even combining exact methods and heuristics

- Problem Decomposition
  - geographically
  - temporally
  - based on product
  - clustering methods will be useful
  - subproblem

- Abstraction and Aggregation
  - clustering methods will be useful

- Exploit modern parallel and heterogeneous computer architectures
- Multi-level search and collaborative solvers
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Products - architecture

Geomatikk AS

Web solution

Web-server
(Servlet, C++/Java)

Guider
(topology)

SPIDER Solutions AS

Designer

Planner
(VRP solver, COM)

Guider
(topology)

Distribution Innovation AS

DI web solution

Spider Server

Planner
(VRP solver, COM)

Guider
(topology)
- Newspaper distribution
- City of Oslo
- 500k inhabitants
- 200k households
- 35k modules
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### Rutevalg

- **Distribusjon**: M1-5

### Velg geografi
- **Rutesøk**: 
- **Region**: 
- **Område**: 
- **Forfall**: 03 Oslo Syd - RNO#1218: Gr.(3) -2C

### Velg måltall / tidsmodus
- **Måltall**: LE, LT, OM, RL, TB, D%, LEV, Δ OM, Δ RL
- **Tidsmodus**: Snitt, Man, Tirs, Ons, Tors, Fre, Lor, Søn

### Forfall

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### Distribution Innovation

#### Routes

- **Search Route**
- **Module**
- **Pending**
- **Routemeasures**
- **Optimization**

#### Routeutvalg

- **Distribusjon**: M1-6

#### Velg geografi

- **Rutesøk**: 
- **Region**: Velg-
- **Omradé**: Velg-
- **Forfall**: Ø3 Oslo Syd - RNO#1218: Gr.(3) -2C

#### Velg måltall / tidsmodus

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- **Tidsmodus**: Snitt, Man, Tirs, Ots, Tors, Fre, Lør, Søn

### Tabell

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**SINTEF**

**ICT**
Problem description

- Last mile part of two-echelon distribution
- (Open) DVRP with extensions

Objectives
- total duration
- route balancing
- clustering, route separation, ”visual beauty”

Constraints
- route duration
- # routes
Additional niceties

- Determination of vehicle type (pedestrian, car)
- Combined routes
- Links that may be used by one route only
- ...
Extended problem

- Integrated problem
- Distribution from print shop to subscriber
- Location routing: Determination of pickup points
Demand aggregation based on road topology, proximity

- Distance/time, capacity threshold
- Issues on traversal possibilities, constraints
- Typical reduction factor of 5-20
- Needs extension to arc model (Node Edge Arc Routing Problem, NEARP)
Case: Oslo data from Aftenposten

33.200 orders reduced to 5600 aggregates
Decomposition approaches

- Geographical
- Cluster-first route second
  - Capacitated Clustering Problem – with balancing
  - Routing
Outline

- Background and Context
- The Vehicle Routing Problem
- Industrial Aspects
- Spider - A Generic VRP Solver
- Specific cases
- Ongoing and Further Research
Important trends in VRP research

- Richer models, larger instances
- Exact methods
- Self-adaptation
- Hybrid and collaborative methods
- Parallel and heterogeneous computing
Parallel computing

- Some tasks in EA and LS are embarrassingly parallel
- "Simple" parallelization through multi-threading
  - fine-grained to medium level granularity
  - very interesting for routing technology
  - not so interesting for VRP research, no literature
- More interesting parallelization
  - Coarse-grained, asynchronous
  - Multi-search
  - Collaborative search
  - Parallel multi-level
- Recent review by Crainic, 80 references
- Additional 30 papers
- Heterogeneous computing not really investigated
Parallel and heterogeneous computing

- CPU Clock frequency has stagnated
  (The Beach Law does not hold anymore ...)
- Moore’s Law still valid
- Multi-core and heterogeneous commodity computers
- Fine-grained and coarse-grained parallelism
Heterogeneous Computer

- A *heterogeneous computer* is a tightly coupled system of processing units with distinct characteristics.

- A modern desktop or laptop computer is an example of such a system, as most systems include both a task-parallel, multi-core CPU and one or more data-parallel processors in the form of programmable graphics processing units (GPUs).

![CPU (Intel Nehalem) and GPU (NVIDIA GeForce 280) diagrams]
Graphics Processing Unit (GPU)

- Background: Game industry
- Massive parallel architecture (>500 cores per chip)
- Typically 10-50 times speedup compared to CPU applications
NVIDIA Fermi GPU

- 3 billion transistors
- 512 cores
- Up to 1 Terabyte of GPU memory
- 1.5 Teraflops performance
- 40 nm manufacturing process

NVIDIA GTX 480
The Collab project

- "Iteration level" parallelization of Spider
- Experimental VRP Solver
- [http://www.sintef.no/Projectweb/Collab/](http://www.sintef.no/Projectweb/Collab/)

- Special session "[Metaheuristics on graphics hardware](http://www.sintef.no/Projectweb/Collab/)" under the META’10 conference in Djerba Island, October 28-30 2010, deadline May 15 [http://www2.lifl.fr/META10/](http://www2.lifl.fr/META10/)
Summary

- The Vehicle Routing Problem is a key to efficient transportation
- The VRP is a very hard optimization problem
- Thousands of VRP papers, mostly generic, idealized
- OR success story, tool industry
- Industry needs rich models, powerful algorithms
- Still challenges, VRP research has never been more active
- Generic VRP Solver SPIDER
  - rich, generic model
  - resolution algorithm based on metaheuristics
  - results

- Still a lot of work to be done ...
Chapter (pp 397-435) in 
Hasle, Geir; Lie, Knut-Andreas; Quak, Ewald (Eds.) 
Geometric Modelling, Numerical Simulation, and Optimization: 
Applied Mathematics at SINTEF 
http://www.springer.com/
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- Advances in Vehicle Routing
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  - Arne Løkketangen
  - Geir Hasle
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- See May issue of Transportation Science for call
Vehicle Routing in Practice

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