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Pooling Problems in Natural Gas Transport

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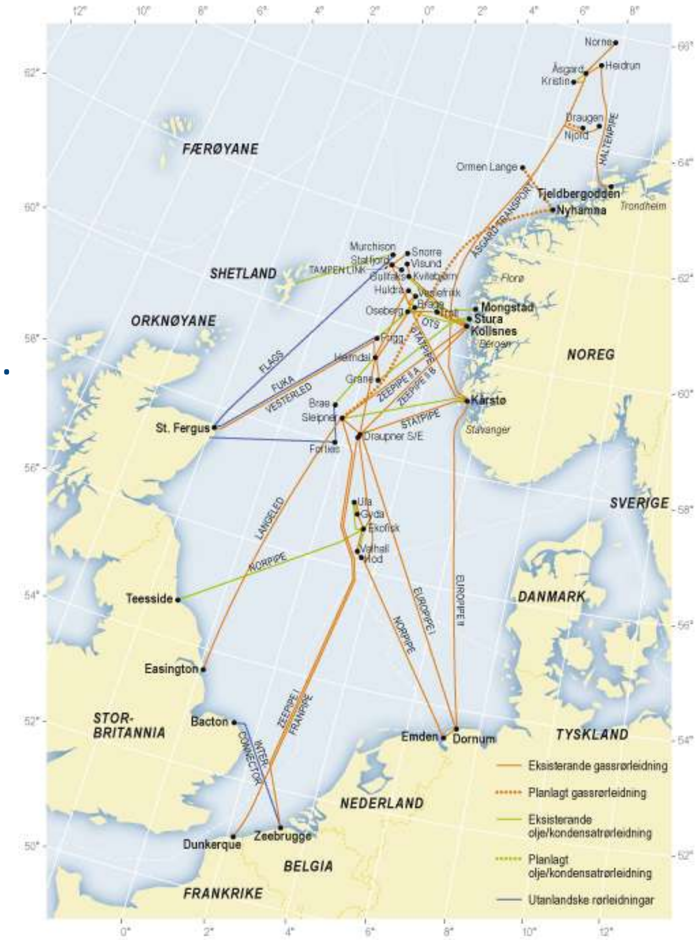
Natural Gas Transport

- Transport natural gas from fields / production nodes via intermediary nodes and processing plants to markets / customers
- One typical problem: find optimal flow
  minimize costs, maximize revenue, maximize flow , …
- Constraints:
  physical / technological, economical / business, …

- May be embedded:
  - Infrastructure investment or network design problems
  - Several time periods



Pooling in Natural Gas Transport

Fields: Quality variations at different sources

- Varying shares of components in gas flow
  CO_2, H_2S, CH_4, C_2H_6, C_3H_8, …

Markets: Quality requirements

Component content, linear blending

- GCV band (composite, approximately linear)
- CO_2 limits
- H_2S limits

Composite quality parameters

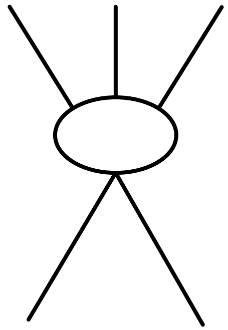
- WI (Wobbe Index)
- SI (Soot Index)

Markets: Price differences (may be quality dependent)

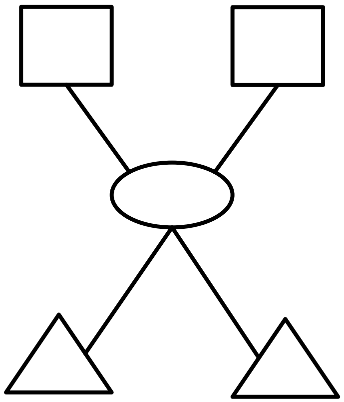
- Maximize profit or revenue, subject to quality constraints

Pooling Formulation

- Pooling:
  - Intermediate nodes
  - Combine all flows into node
  - Divide over pipelines out of node
  - Keep track of composition of the flows



- Original formulation by Haverly (1978) (P):
  2 sources, 1 pool, 2 sinks
  $\rightarrow$ computationally hard, many local optima

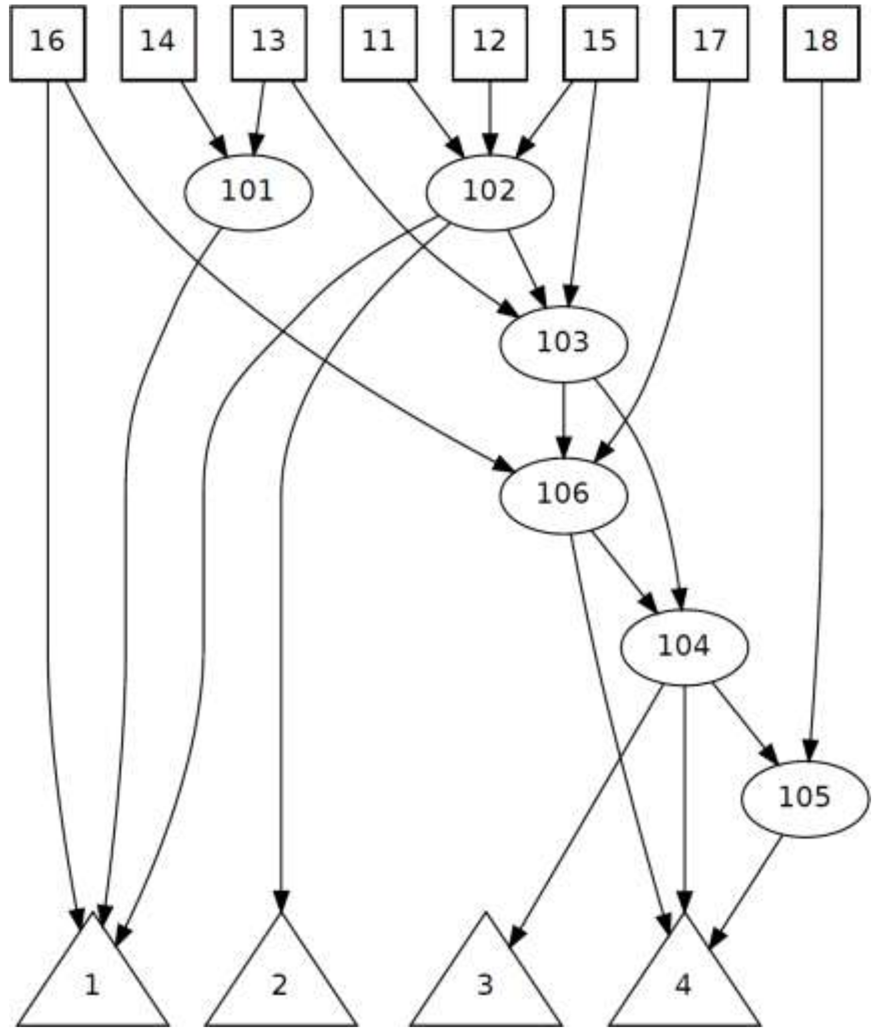


- Several reformulations:
  - Tighter formulations (solve faster): Q, PQ, TP, etc.
  - Generalizations (interconnected pools, network design)

Generalized pooling formulation

We propose a generalized formulation:

- Multiple levels of interconnected pools
- Processing facilities:
  may modify the flow composition
- Composite quality constraints:
  depend on the ratio of several components



Generalized pooling formulation

Part of a larger optimization problem for capacity expansion / natural gas transport (ref. first talk in this session)

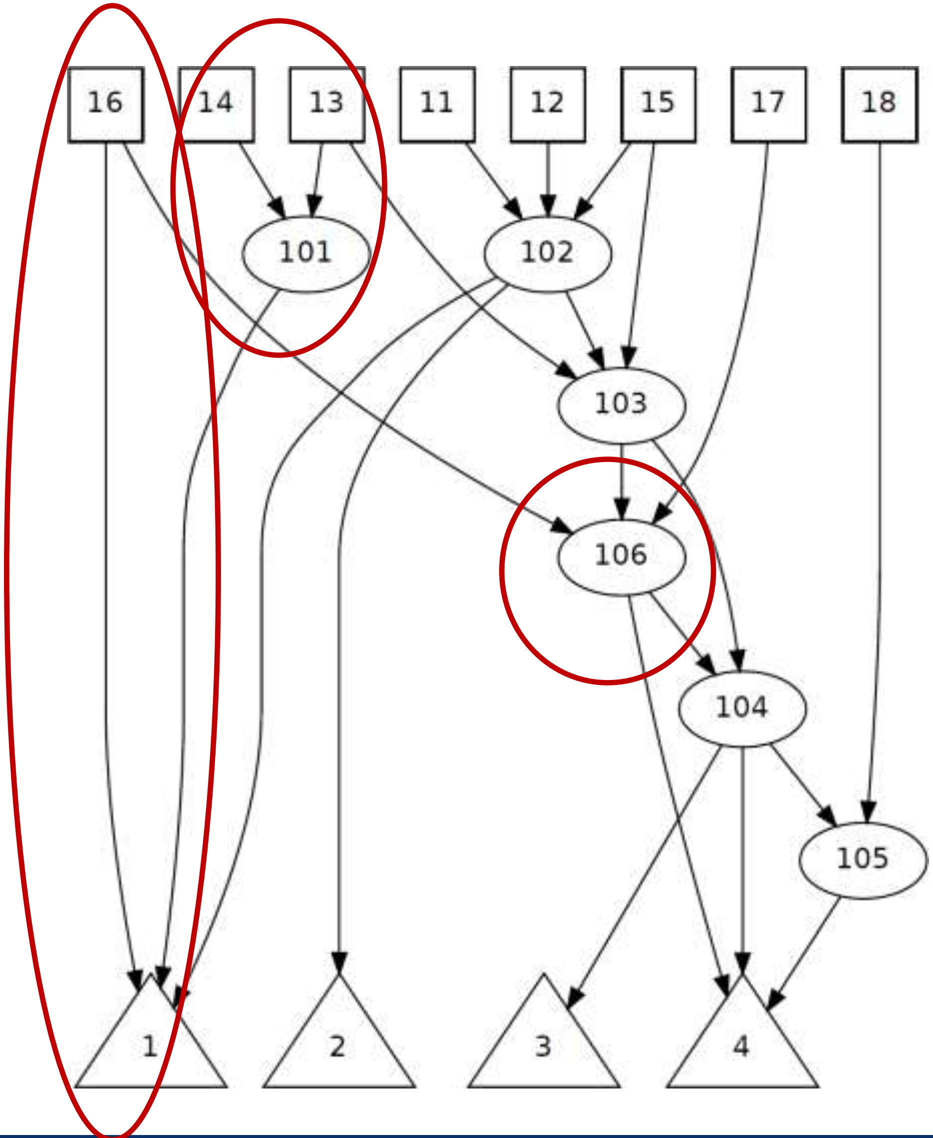
- Single period flow problems
- Multi-period investment problems with embedded pooling problem for each operational period
- Stochastic programs (uncertainty) $\rightarrow$ scenarios

Some properties

- Large-scale problems
- Require fast solution
- May need to solve similar problems many times

Computational Effort

- Maximum flow: "easy" (linear)
- Blending :"easy" (linear)
- Pooling: "hard"
  (non-convex, non-linear)
  - Equal ratio of volume split between pipelines out of a node
    $\frac{f_1^1}{f_2^1} = \frac{f_1^c}{f_2^c}$



Two Approaches

- **Improved formulation**
  - Adding redundant constraints improves solution times
  - Theoretically better precision/accuracy
  - Less mature solvers for large scale problems

- **Discretization**
  - Replace some continuous variables with discrete variables e.g., choose between given split ratios
  - Gives Mixed Integer Linear Program (MILP), also "hard"
  - High quality commercial solvers available

Proposed Solution Approach

- Exploit that not all problems are equally hard, e. g.:
  - No quality constraints
  - Non-binding quality constraints
  - Optimal flow pattern "similar" to single component flow solution

- Save computational effort:
  - Start with solving simpler problem
    - Homogeneous (single-component) flow – no longer a pooling problem
    - Formulation as linear problem $\rightarrow$ can solve efficiently
  - Derive solution of more complex problem
    - Find component flows and split fractions
    - Can employ different discretization schemes

Auxiliary (Linear) Problems

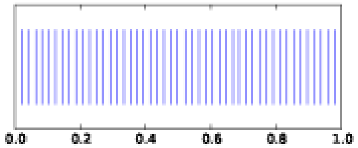
- **A1**: Homogeneous flow problem
  - Maximize total revenue from deliveries to markets
  - Subject to:
    - Production capacity limits
    - Cannot exceed market demand
    - Mass balances throughout the network

- **A2**: Fix total flow volumes, determine component flows
  - Arbitrary objective function, e.g. maximize revenue
  - Subject to:
    - Gas composition at production nodes
    - Component flow out of node determined by split fractions
    - Sum of component flows equals total flow (everywhere in the network)
    - (Quality constraints)

Discretization Schemes for Split Fractions

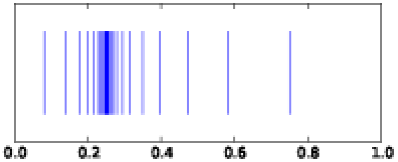
Precision vs. Computational Speed

- D1: Pre-defined split candidates
  e.g., uniformly distributed



- D2: Binary split formulation
  Calculate split as linear combination of ½, ¼, …

- D3: Concentrated split candidates
  Finer discretization close to solution of single-component flow problem

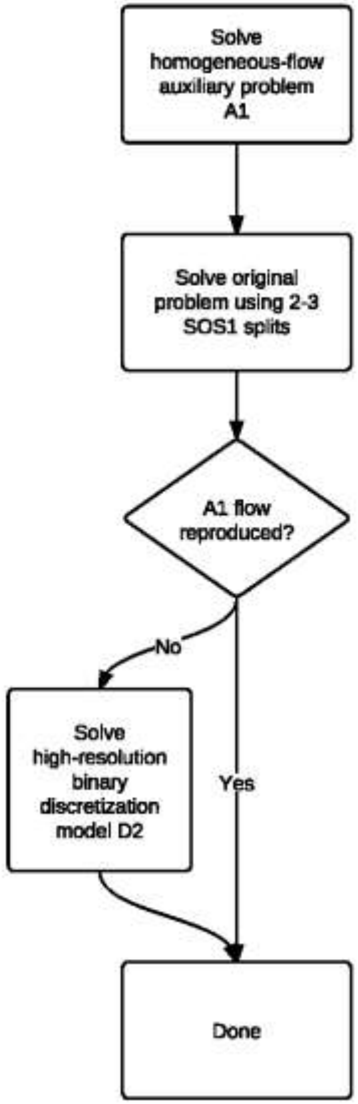


Algorithms to Solve Pooling Problems

- Combine auxiliary problems A1, A2 and discretization schemes D1 – D3 in various ways
- Faster, potentially imprecise formulations vs. slower, more precise formulations
- General idea:
  - Use faster formulations to get an "estimate" of the solution
  - If this estimate is good enough → done
  - If not → improve solution, using available information
- For example:
  - Optimal multi-component flow pattern is similar to homogeneous flow pattern
→ global optimum of pooling problem close to homogeneous solution
  - All quality constraints are satisfied
→ global optimum of pooling problem equal to homogeneous solution

Hybrid Split Algorithm

- Preprocessing – solve homogeneous flow problem A1
- Coarse pooling problem, test with (very) few candidate splits: 0, split ratio from A1, 1
- Only if necessary (total flow ≠ flow from A1):
  - Solve complete pooling problem
  - Use discretization D2 (split ratio = linear combination of ½, ¼, …)



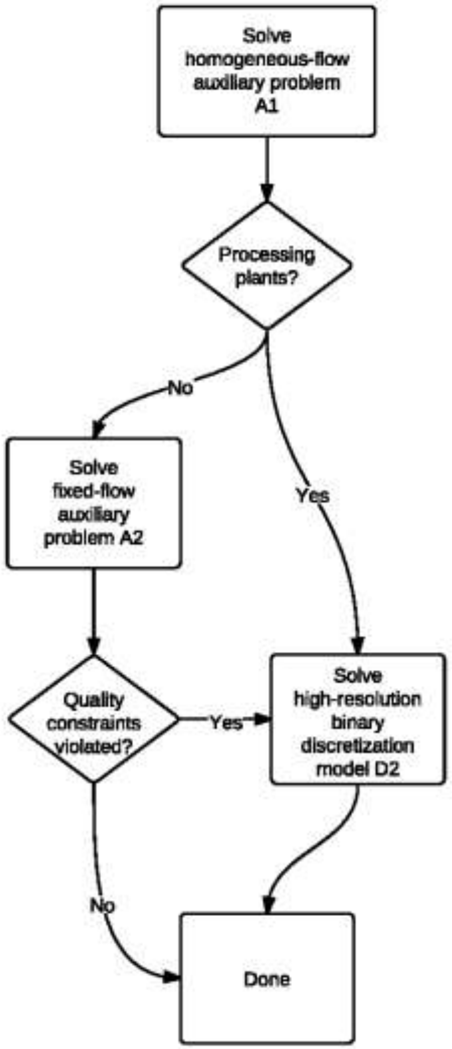
Warm Start

- Solution of homogeneous-flow problem gives upper bound on flows
  - Homogeneous flow = linear combination of component flows
  - Solution of component-flow problems cannot be better than this
- If solution of component-flow problems is "too far" from upper bound
  → may verify solution with finer discretization scheme:
  - Solve pooling problem with concentrated split scheme D3
  - Use this as warm start for solving problem with binary scheme D2

(D2 may be fine grained everywhere between 0 and 1,
D3 is fine grained around split from A1, coarser elsewhere.)

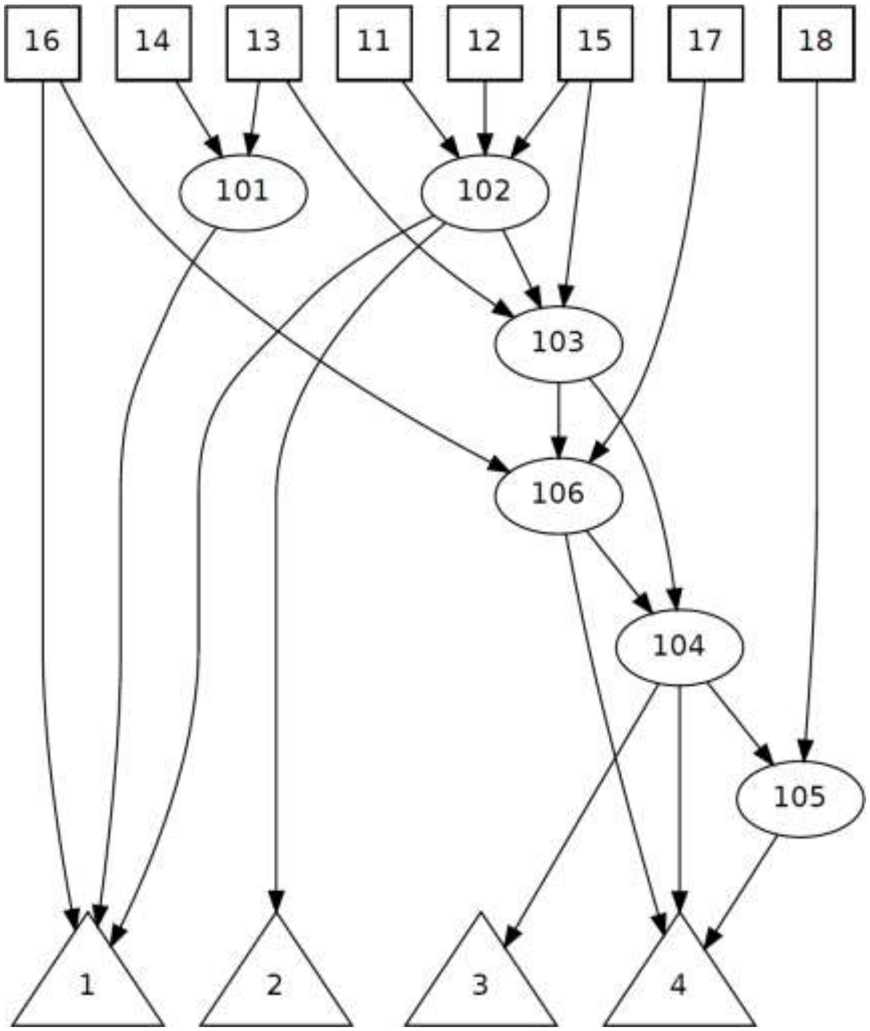
Use Simplest Possible Approach

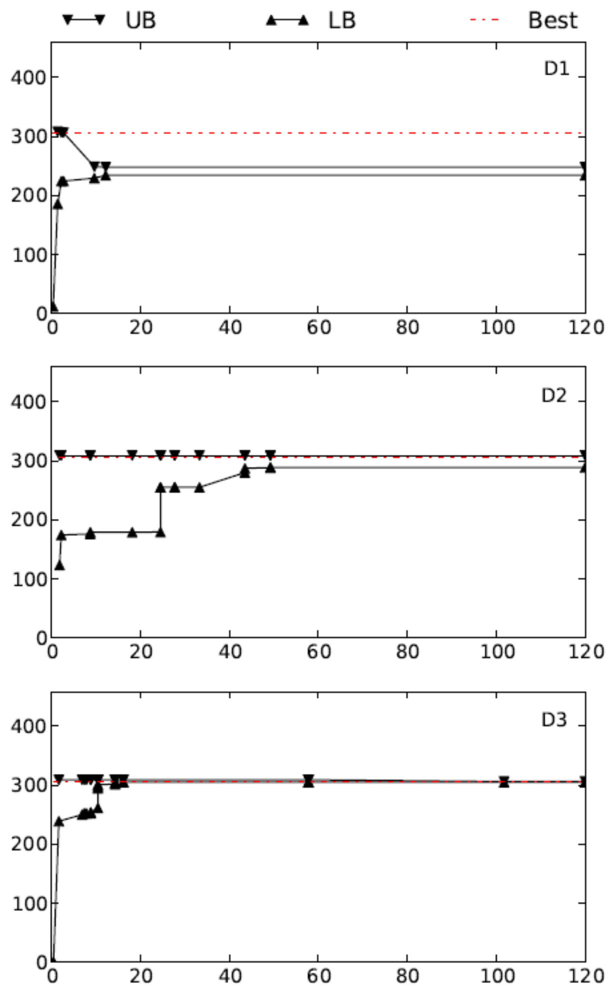
- Get initial solution from homogeneous flow problem
- Processing plants remove some component flows
  → go immediately to high-resolution formulation,
  using binary discretization scheme D2
  (optimal split ratios may be "everywhere")
- Else: determine component flows from homogeneous flows (fixed-flow problem A2)
- Test if quality constraints at markets hold
  - If yes: done
  - Else: solve high-resolution formulation and find new split ratios



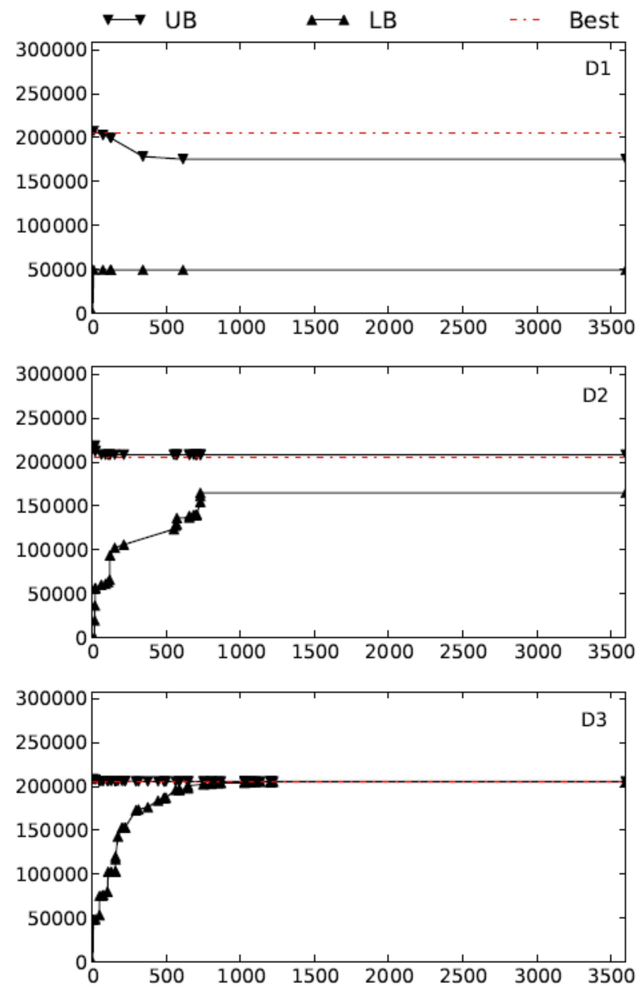
Computational results

- Evaluate solution quality for five test cases
  - Based on real industry cases from NCS
  - 3 single-period cases – after 120 seconds
  - 2 large multi-period cases – after 1 hour
- Also: time to reach a given gap between pre-computed estimate of optimum and best solution found
  - True global optimum (upper bound): from homogeneous-flow problem
  - Estimate for component-flow problem from very fine-grained D2, several days





(a) Case 1, over 120 seconds.



(b) Case 4, over 3600 seconds.

Thank you for your attention!

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