Modelling of heat transport in two-phase flow and of mass transfer between phases using the level-set method

TGTC-3

Magnus Aashammer Gjennestad and
Svend Tollak Munkejord

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Overview of presentation

Motivation

The mathematical model

Some simulation results

Conclusions

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Basic hypothesis

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Basic hypothesis

A thorough understanding of the processes and phenomena occurring at a small-scale level in the heat exchanger is necessary to obtain an improved understanding of the heat exchanger, its design and operation.

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- Liquefaction of natural gas requires energy. Naturally, we want to make the liquefaction process as energy-efficient as possible.
- A thorough understanding is necessary to design more efficient **heat exchangers**.
- There seems to be a general consensus that **numerical simulation** is one of the **most promising approaches** for studying phenomena such as heat transfer characteristics and condensation/boiling.

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To model temperature-driven flows, we need to introduce a **temperature-dependent buoyancy force**,

$$\boldsymbol{f}_{\mathrm{b}} = \rho \boldsymbol{g} \left(1 - \beta \left(T - T_\infty\right)\right),$$

where β is the thermal expansion coefficient and T_∞ is a reference temperature.

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$$\Gamma = \left\{ \boldsymbol{x} | \phi \left(\boldsymbol{x} \right) = 0 \right\}.$$

The level-set function ϕ is the **signed distance** to the interface,

$$\phi \left(\boldsymbol{x} \right) = \begin{cases} -\min_{\boldsymbol{y} \in \Gamma} \left| \boldsymbol{x} - \boldsymbol{y} \right| & \text{if } \boldsymbol{x} \text{ in Phase 1,} \\ \min_{\boldsymbol{y} \in \Gamma} \left| \boldsymbol{x} - \boldsymbol{y} \right| & \text{if } \boldsymbol{x} \text{ in Phase 2.} \end{cases}$$

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$$\partial_t \phi + \boldsymbol{w} \cdot \nabla \phi = 0.$$

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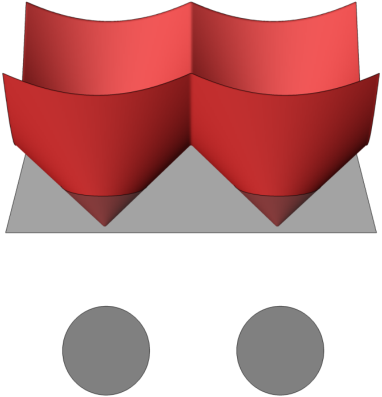
The normal vector field $\boldsymbol{n}$ is defined in terms of ϕ as

$$\boldsymbol{n} = \nabla \phi.$$

An example level-set function



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Any resulting discontinuity in heat flux $\dot{\boldsymbol{q}} = -\kappa \nabla T$ at the interface is used to calculate the mass flux

$$\dot{m} = \frac{[\dot{\boldsymbol{q}} \cdot \boldsymbol{n}]}{\Delta h},$$

where Δh is the specific enthalpy difference of the phase transition.

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The mass flux is used to calculate the jump in velocity $[\boldsymbol{u}]$ at the interface,

$$[\boldsymbol{u}] = \dot{m}\left[\frac{1}{\rho}\right]\boldsymbol{n}.$$

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The interface velocity is found from

$$\boldsymbol{w} = \boldsymbol{u}_1 - \frac{\dot{m}}{\rho_1}\boldsymbol{n}.$$

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Lava lamp

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Photo: Nir Schneider, CC BY 2.0.

Lava lamp

Why simulate the lava lamp?

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Why simulate the lava lamp?

- The lava lamp is a canonical example of a temperature-driven two-phase flow.

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Lava lamp

Why simulate the lava lamp?

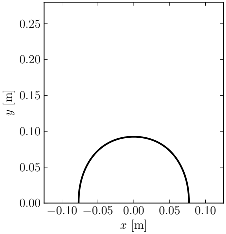
- The lava lamp is a canonical example of a temperature-driven two-phase flow.
- Frequency of *blob exchange oscillations* can be compared to experiment.

Photo: Nir Schneider, CC BY 2.0.

Lava lamp

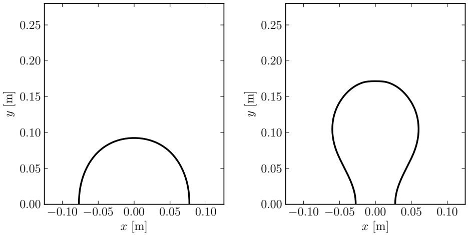
Lava lamp

$t = 500$ s



Lava lamp

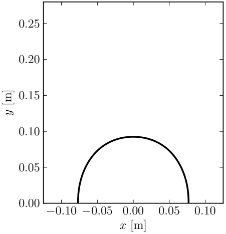
$t = 500$ s

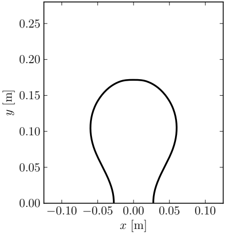


$t = 766$ s

Lava lamp

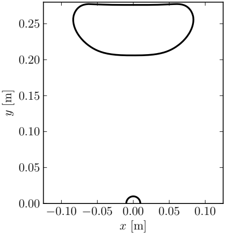
$t = 500$ s





$t = 766$ s

$t = 775$ s



Lava lamp

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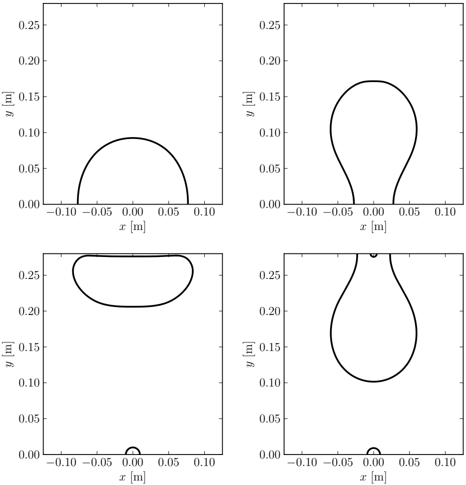


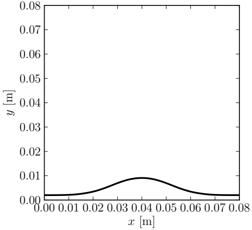
$t = 775$ s

$t = 1214$ s

Boiling film

Boiling film

$t = 0.3$ s



Boiling film

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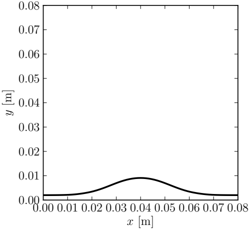
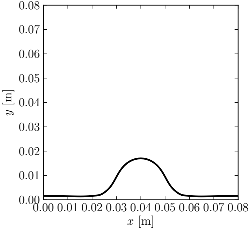
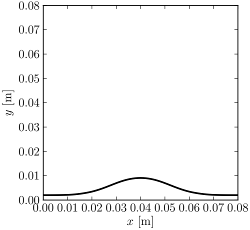


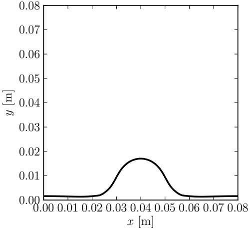


$t = 0.4$ s

Boiling film

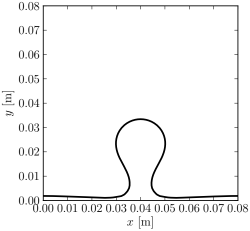
$t = 0.3$ s





$t = 0.4$ s

$t = 0.5$ s



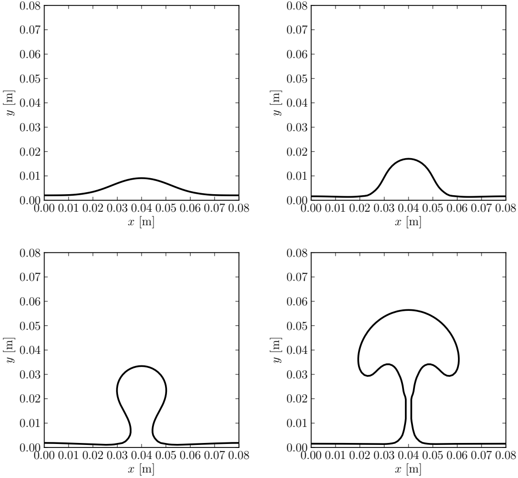
Boiling film

$t = 0.3$ s

$t = 0.4$ s

$t = 0.5$ s

$t = 0.6$ s



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