Bayesian inference in CO_2 storage monitoring: a way to assess uncertainties in geophysical inversions

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Motivation

- Conformance monitoring: convergence between models and monitoring data.
  ➔ Requires quantitative estimates: pressure, saturation, stress changes,…
- Geophysical monitoring can provide quantification of relevant rock physics properties ➔ two-step inversion.
- Inverse problems (two steps) are non linear, highly underdetermined and ill-posed and have non unique solutions.
- Important to quantify/assess the uncertainty related to these measurements:
  ➔ can be achieved with fully Bayesian formulation

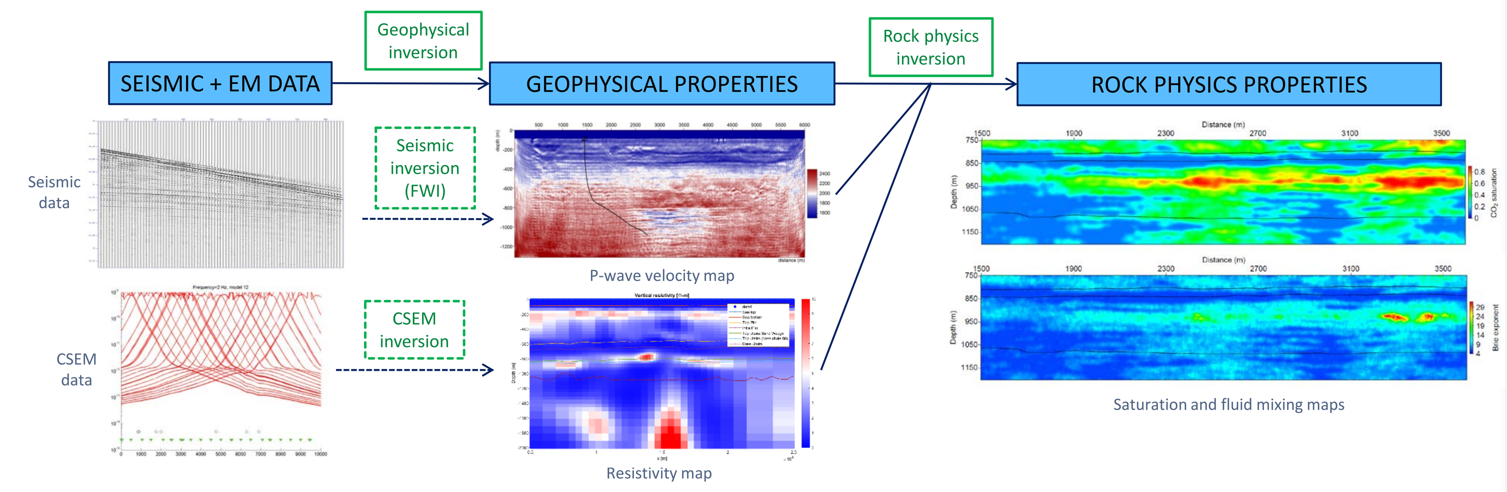


Figure 1: Two-step geophysical quantitative inversion. Figures from Romdhane and Querendez (2014), Bøe et al. (2017), Yan et al. (2018)

Bayesian formulation of inverse problems

- Review book of Tarantola, 2005
- Bayes theorem: $P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$
- Inverse problem formulation: $\boldsymbol{C}_{post}(\boldsymbol{m}) = c\, \boldsymbol{C}_{prior}(\boldsymbol{m}) L(\boldsymbol{m}|\boldsymbol{d_{obs}})$
- Misfit function (L2 norm):

$$L(\boldsymbol{m}|\boldsymbol{d_{obs}}) = [\boldsymbol{d_{obs}} - \boldsymbol{g}(\boldsymbol{m})]^T \boldsymbol{C}_D^{-1} [\boldsymbol{d_{obs}} - \boldsymbol{g}(\boldsymbol{m})]$$

$\boldsymbol{P(B|A)}$ = model posterior probability
$\boldsymbol{P(B)}$= model prior probability
$\boldsymbol{P(A|B)}$= data likelihood knowing the model

$\boldsymbol{m}$ = model vector; c= constant
$\boldsymbol{C}_{post}(\boldsymbol{m})$= posterior probability distribution
$\boldsymbol{C}_{prior}(\boldsymbol{m})$= prior probability distribution
$L(\boldsymbol{m}|\boldsymbol{d_{obs}})$= data likelihood misfit function
$\boldsymbol{d_{obs}}$= observed data; $\boldsymbol{g}(\boldsymbol{m})$= calculated data
$\boldsymbol{C_D}$= data covariance matrix (noise)

Full waveform inversion and uncertainty assessment

- Based on Bayesian inversion approach for tomographic methods (Tarantola, 2005; Eliasson and Romdhane, 2017).
- Posterior covariance determined from the Hessian $\boldsymbol{H}$:

$$\boldsymbol{C}_{post} = \boldsymbol{C}_{prior}^{1/2} \left(\boldsymbol{C}_{prior}^{1/2} \boldsymbol{H} \boldsymbol{C}_{prior}^{1/2} + I \right)^{-1} \boldsymbol{C}_{prior}^{1/2}$$

- "Equivalent models" are sampled from posterior Gaussian probability density function. Parameter uncertainty proportional to range of values given by equivalent models.

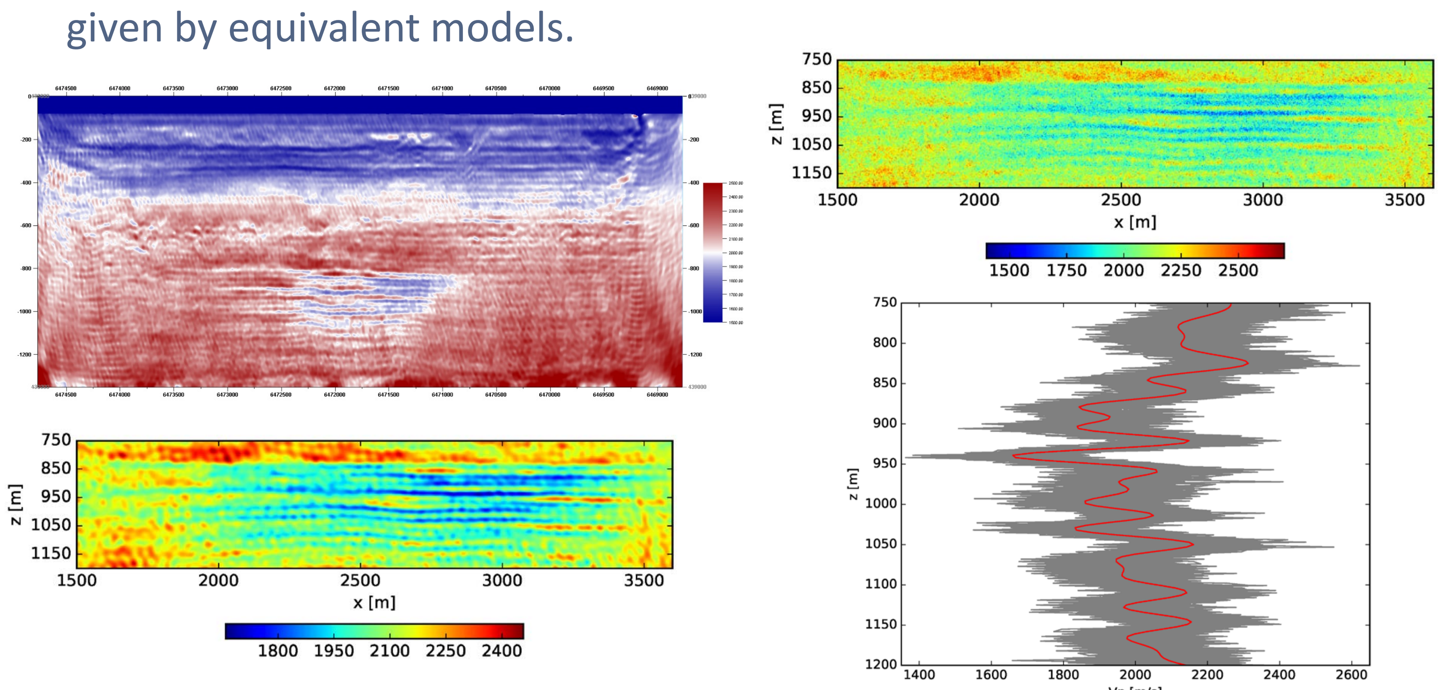
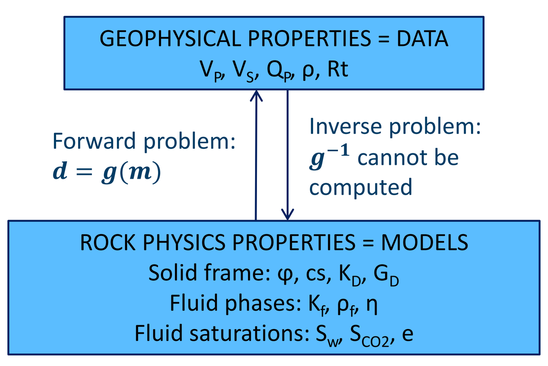


Figure 2: Final model derived from FWI at f = 39.5 Hz for inline 1874 from Sleipner 2008 vintage (top left). Close-up of the plume-region (bottom left). Random sample ("equivalent model") drawn from the posterior distribution (top right). Extracted depth velocity profiles from 100 samples at x = 2916 m (bottom right). Red line corresponds to the velocity of the final model from FWI.

Rock physics inversion and neighbourhood algorithm



Fast and analytic forward problem/rock physics model (Pride, 2005).

➔ Neighbourhood algorithm (Sambridge, 1999):

- Only 2 control parameters
- Model space guided exploration
- Fit quality and uncertainty

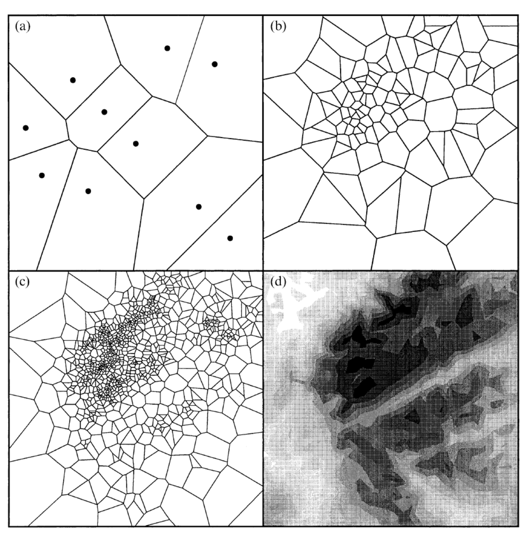


Figure 3 (from Sambridge, 1999): (a) Selection of 10 quasi-uniform random points in the 2D model space. (b) The Voronoi cells about the first 100 samples generated by a Gibbs sampler using the neighbourhood approximation. (c) Similar to (b), but for 1000 samples. (d) Contours of the test objective function.

Sensitivity tests

Inversion of CO_2 saturation, porosity and patchiness exponent from P-wave velocity and resistivity with different levels of noise/uncertainty in the data.

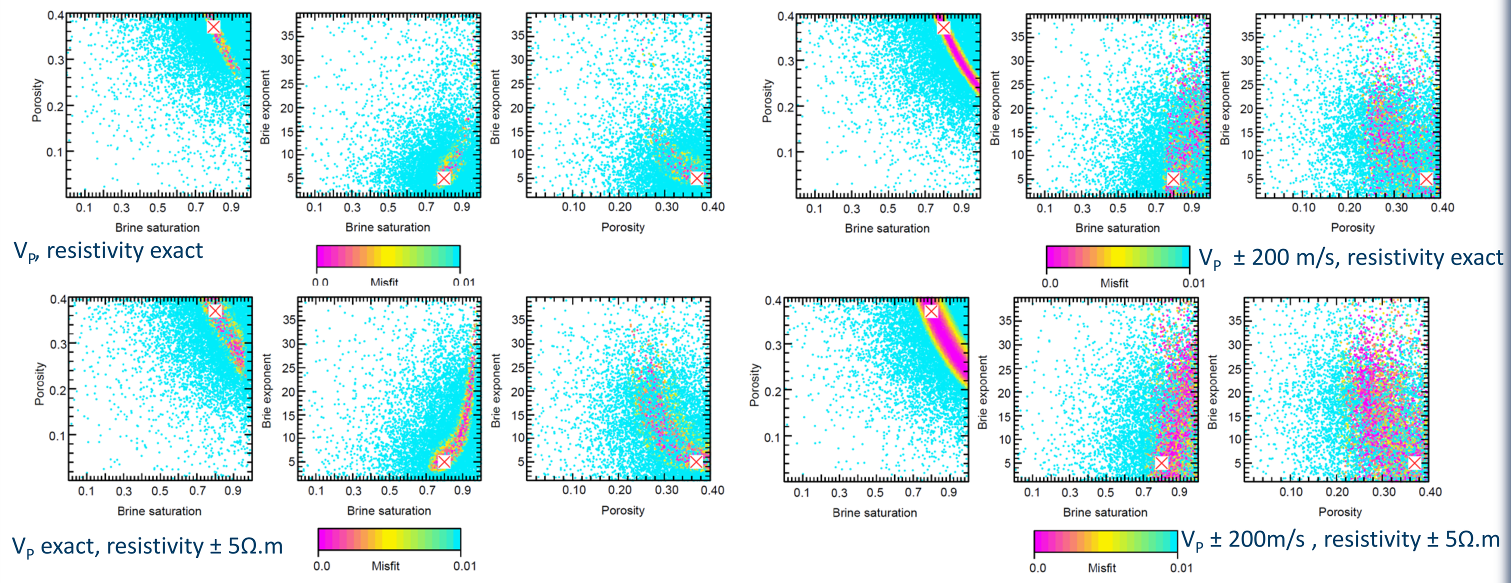


Figure 4: 2D slices of 3D model space where the inverted parameters are CO_2 saturation, porosity and Brie exponent. Each dot corresponds to a model with a misfit given by the color scale (absolute values). The red crosses stand for the true model.

2D Sleipner case studies

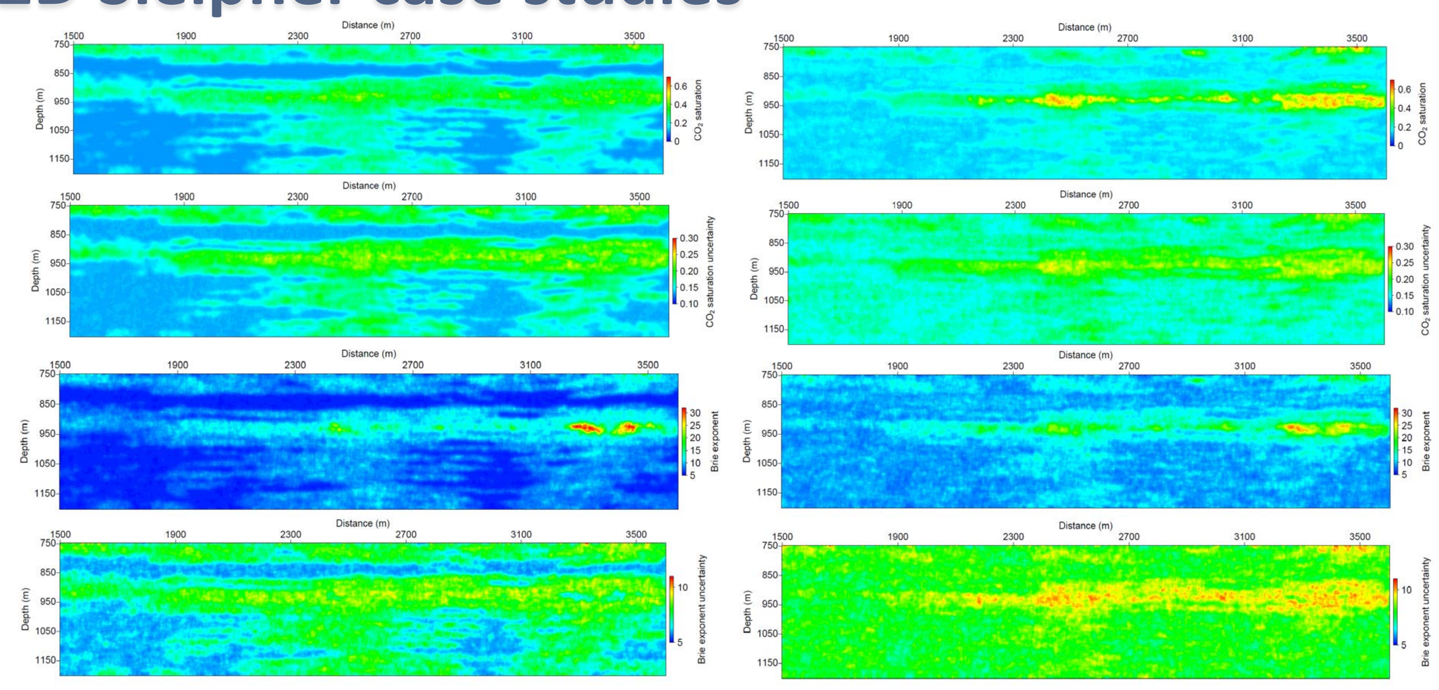


Figure 5: Inversion of CO_2 saturation and patchiness exponent for the inline 1838 with no uncertainty on data (left panels) and 100 m/s uncertainty on P-wave velocities (right panels). P-wave velocity is used as input.

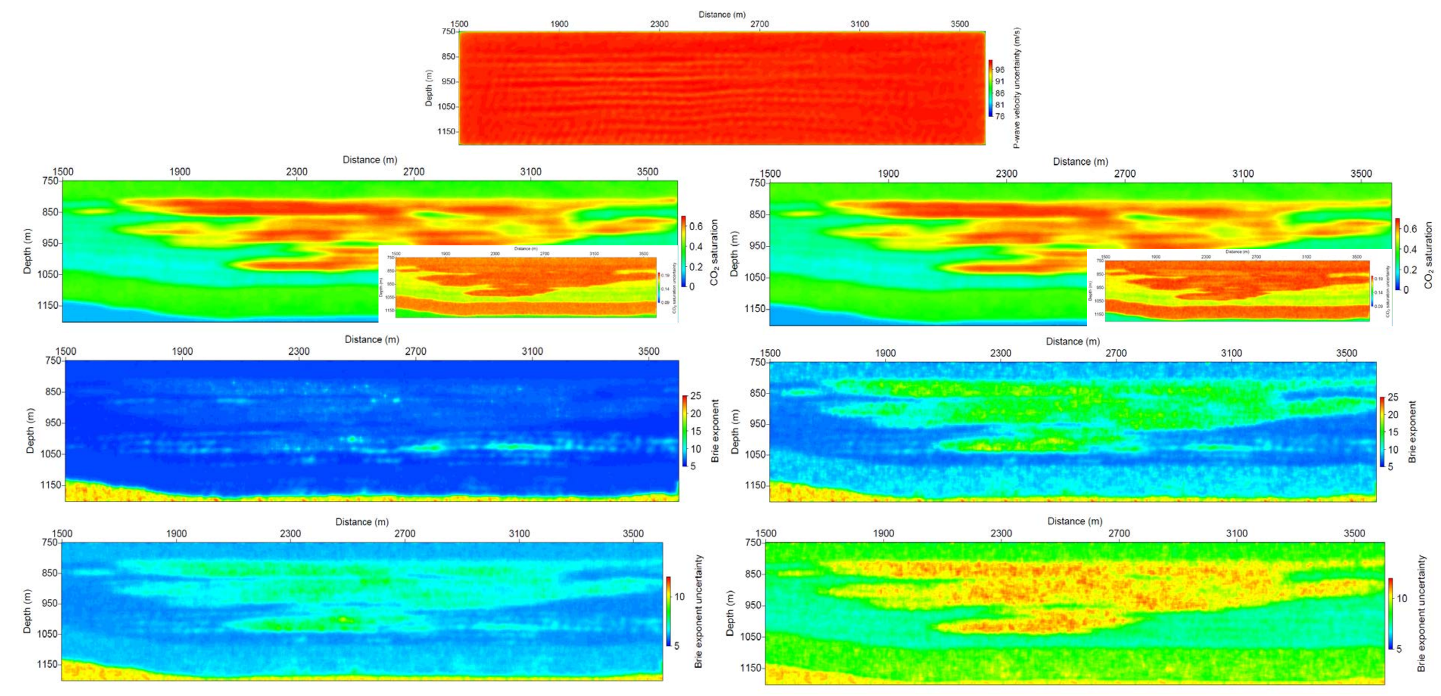


Figure 6: Inversion of CO_2 saturation and patchiness exponent for the inline 1874 with no uncertainty on data (left panels) and uncertainty on P-wave velocity (from uncertainty analysis after FWI, top figure) and 10 Ω.m uncertainty on R_t (right panels). P-wave velocity and resistivity are used as input.

Conclusions and way forward

- Successful propagation of uncertainty between the two inversion steps.
- Bayesian formulation allows to account for noise/uncertainty in the data and prior model distributions.
- Effect of uncertainty in geophysical properties is observed in the final results with an increase of CO_2 saturation and patchiness exponent uncertainties.
- Prior model distribution and spatial correlation need to be implemented in the rock physics inversion step.

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