Fast Passivity Enforcement of Rational Macromodels by Perturbation of Residue Matrix Eigenvalues

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Abstract

Residue perturbation (RP) is often used a means for enforcing passivity of rational models. One RP version combines a least squares problem with a constraints part and solves via Quadratic Programming (QP). A major difficulty is that commonly available QP solvers cannot utilize the problem sparsity, leading to lengthy computations. This paper proposes to take the eigenvalues of the residue matrices as free variables. This leads to a more compact problem and thus a fast computation (FRP). The resulting model error is found to be much smaller than when perturbing only the diagonal elements of the residue matrices. It is also shown how to combine the residue matrix eigenvalue perturbation with the recently developed modal perturbation approach (MP), leading to a fast version (FMP). The FMP/MP approaches have the additional advantage of retaining the relative accuracy of the admittance matrix eigenvalues.

Introduction

One major problem with the rational modeling of devices and systems from frequency domain data is that the obtained model is often non-passive. This can lead to unstable simulation results when the model can interact with the adjacent network over its ports.

In [1] was introduced the idea of enforcing passivity by residue perturbation (RP). A linearized relation was calculated between residues and a passivity criterion that is related to the eigenvalues of the real part of the nodal admittance matrix. This relation was used as a constraint in a least squares problem that minimizes the change to the model admittance matrix, **Y**. The resulting problem was solved using Quadratic Programming (QP). Similar ideas were adopted in [2]. Usage of an energy-based cost function has also been proposed, with constraints coming from the Hamiltonian matrix [3]. It has also been proposed [4] to combine the energy-based cost function with the constraint in [1].

In [4], a modal cost function was introduced in the LS problem of RP. This allows to use inverse eigenvalue weighting in the LS problem, thereby preventing that the smallest eigenvalues of **Y** become corrupted by the perturbation. It was shown in [5] that retaining the relative accuracy of the eigenvalues (modes) can be crucial when the model is to be applied in with arbitrary terminal conditions.

In the case of large models (many ports, high order), the RP/MP approaches are demanding in computation time and memory requirements since the commonly available QP solvers cannot utilize sparsity. It has therefore been proposed to reduce the number of free variables by perturbing only a few residue matrices [1] or a few elements in each residue matrix [2]. Unfortunately, this substantially increases the perturbation size. Alternatively, one can use specialized but expensive software that can handle the sparsity (e.g. `CPLEX`) [4],[6], but such software is expensive.

In this paper, it is proposed to reduce the number of free variables by perturbing the eigenvalues of each residue matrix. This leads to fast versions of the RP and MP approaches (FRP, FMP). Usage of the FRP/FMP is demonstrated for a two-port interconnect model and for a six-port transmission line model.

Rational Model

It is assumed that the rational model is given on the pole-residue form (1), approximating the terminal behavior of an admittance matrix **Y**.

$$\mathbf{Y}(s) \cong \mathbf{Y}_{rat}(s) = \sum_{m=1}^{N} \frac{\mathbf{R}_m}{s - a_m} + \mathbf{D} + s\mathbf{E} \tag{1}$$

Matrices $\{\mathbf{R}_m\}$, **D**, and **E** are assumed to be symmetrical, and complex poles and residues come in conjugate pairs. The modeling can easily be achieved using the pole relocating vector fitting algorithm [7], or any of its variants [5],[8]–[10].

Residue Perturbation (RP)

Following the idea in [1], passivity is enforced by perturbing the model parameters, leading to the constrained optimization problem (2).

$$\Delta\mathbf{Y} = \sum_{m=1}^{N} \frac{\Delta\mathbf{R}_m}{s - a_m} + \Delta\mathbf{D} + s\Delta\mathbf{E} \cong \mathbf{0} \tag{2a}$$

$$eig(\mathrm{Re}\{\mathbf{Y} + \sum_{m=1}^{N} \frac{\Delta\mathbf{R}_m}{s - a_m} + \Delta\mathbf{D}\}) > \mathbf{0} \tag{2b}$$

$$eig(\mathbf{D} + \Delta\mathbf{D}) > \mathbf{0} \tag{2c}$$

$$eig(\mathbf{E} + \Delta\mathbf{E}) > \mathbf{0} \tag{2d}$$

The first part (2a) minimizes the change to the admittance matrix elements while the second part (2b) enforces that the perturbed model meets the passivity criterion (3). The third (2c) enforces asymptotic passivity while the last constraint (2d) has been introduced in order to enforce a positive definite **E**, since an **E** with negative eigenvalues can cause an unstable simulation.

$$eig(\mathrm{Re}\{\mathbf{Y}_{rat}(s)\}) = eig(\mathbf{G}_{rat}(s)) > \mathbf{0} \tag{3}$$

The implementation of (2) via first order perturbation leads to the form (4) where $\Delta\mathbf{x}$ holds the perturbed parameters elements. This problem is solved using Quadratic Programming (QP).

$$\min_{\Delta\mathbf{x}} \frac{1}{2}(\Delta\mathbf{x}^T \mathbf{A}_{sys}{}^T \mathbf{A}_{sys}\, \Delta\mathbf{x}) \tag{4a}$$

$$\mathbf{B}_{sys}\,\Delta\mathbf{x} < \mathbf{c} \quad (4b)$$

Matrix $\mathbf{A}_{sys}$ is block diagonal while $\mathbf{B}_{sys}$ is full but with a few rows. The solving of (4) can in Matlab be done using routine `quadprog.m`. This routine treats $\mathbf{A}_{sys}$ as a full matrix.

Modal Perturbation (MP)

In [4] was introduced the Modal Perturbation approach. The motivation is to perturb in such a way that the smallest eigenvalues of $\mathbf{Y}$ are not corrupted. (The same idea is underlying the modal vector fitting (MVF) algorithm [5]).

The admittance matrix $\mathbf{Y}$ is diagonalized

$$\mathbf{Y} = \mathbf{T}\,\mathbf{\Lambda}\,\mathbf{T}^{-1} \quad (5)$$

Equation (5) is post-multiplied with $\mathbf{T}$ and first order derivatives are taken for each eigenpair $(\lambda_i,\mathbf{t}_i)$

$$\Delta\mathbf{Y}\,\mathbf{t}_i + \mathbf{Y}\,\Delta\mathbf{t}_i = \Delta\lambda_i\,\mathbf{t}_i + \lambda_i\,\Delta\mathbf{t}_i \quad (6)$$

Terms involving $\Delta\mathbf{t}_i$ are ignored, and $\Delta\mathbf{Y}$ is replaced with (1), giving

$$\left(\sum_{m=1}^{N}\frac{\Delta\mathbf{R}_m}{s-a_m} + \Delta\mathbf{D} + \Delta\mathbf{E}\right)\mathbf{t}_i(s) = \Delta\lambda_i(s)\,\mathbf{t}_i(s) \cong \mathbf{0}\,,\ i = 1\ldots n \quad (7)$$

Introducing a least squares weighting equal to the inverse of the eigenvalue magnitude gives the final result (8). Details on building the system matrix $\mathbf{A}_{sys}$ and sparsity structures is shown in [4].

$$\left(\sum_{m=1}^{N}\frac{\Delta\mathbf{R}_m}{s-a_m} + \Delta\mathbf{D} + \Delta\mathbf{E}\right)\mathbf{t}_i(s) / |\,\lambda_i(s)\,| = \mathbf{0}\,,\ i = 1\ldots n \quad (8)$$

Perturbation of Residue Matrix Eigenvalues

The number of free variables is substantially reduced by diagonalizing each residue matrix (individually), and perturbing only their eigenvalues

$$\Delta\mathbf{R}_m = \mathbf{S}_m\,\Delta\mathbf{\Gamma}_m\,\mathbf{S}_m^{-1}\,,\ \Delta\mathbf{D} = \mathbf{S_D}\,\Delta\mathbf{\Gamma_D}\,\mathbf{S_D}^{-1}\,,\ \Delta\mathbf{E} = \mathbf{S_E}\,\Delta\mathbf{\Gamma_E}\,\mathbf{S_E}^{-1} \quad (9)$$

This leads to a full but much smaller $\mathbf{A}_{sys}$ (and $\mathbf{B}_{sys}$) in (4). For instance, with n ports and N poles, the number of free variables in (4) is reduced from $M=(n(n+1))N/2$ to $M=nN$. This leads to large savings in computation time since the complexity of the core operations in QP is $O(M^3)$. In the case of complex conjugate residue matrices, the real and imaginary parts are diagonalized separately. Introducing the transformation (9) in RP and MP leads to a fast version (FRP, FMP) of these algorithms. The system matrix ($\mathbf{A}_{sys}$) is full.

Implementation issues

The passivation step in FRP/RP and FMP/MP is combined with passivity checking via the Hamiltonian matrix [2] and the robust iteration scheme in [6]. In the iterations, $\Delta\mathbf{D}$ and $\Delta\mathbf{E}$ are removed from (2) as soon as $\mathbf{D}$ and $\mathbf{E}$ become positive definite. The system matrix $\mathbf{A}_{sys}$ in (2) is built only a single time and is not updated during iterations. The number of constraints in (2b) is kept low by including local minima of violating eigenvalues [6].

Example 1: Single Conductor Interconnect

As a first example we consider the 2×2 terminal admittance matrix $\mathbf{Y}$ of a single conductor interconnect, calculated via the Enhanced Transmission Line Model [11]. The line length is 100 mm, with geometrical data given in [11].

The $\mathbf{Y}$-matrix is fitted by a 50th order pole-residue model, calculated by the (relaxed) vector fitting algorithm. The obtained model has many and quite large passivity violations, as can be seen by the negative eigenvalues in Fig. 1. The rational model is next subjected to passivity enforcement by FMP. As can be seen in Fig. 1, the procedure removes all passivity violations with only a moderate perturbation of the eigenvalues where they are positive. A similar result was obtained with FRP.

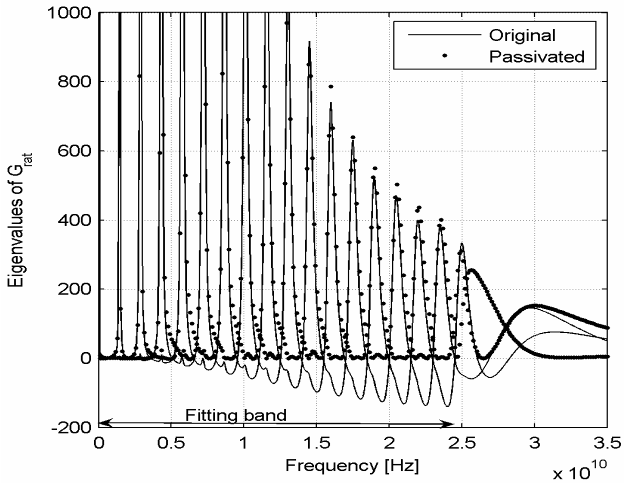

Fig. 1 Passivation by FMP

Example 2: Three Phase Overhead Line

In this example we use the case previously described in [6]. The terminal admittance matrix $\mathbf{Y}$ of the transmission line in Fig. 2 is computed in the frequency domain, from 10 Hz to 10 kHz. A 30th order pole-residue model (1) is calculated for the six-port $\mathbf{Y}$ by fitting all elements simultaneously using vector fitting.

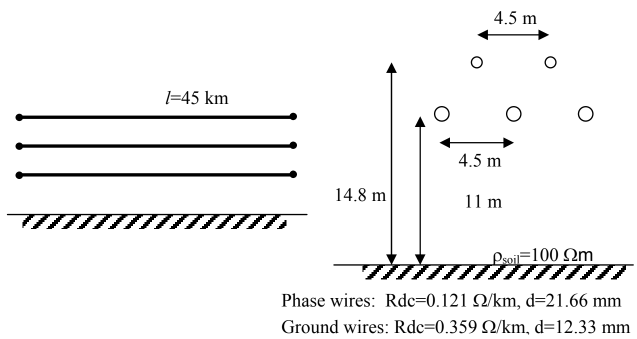

Fig. 2 Three-phase overhead line (132 kV line)

The resulting model is non-passive by criterion (3) as several eigenvalues of $\mathbf{G}_{rat}(s)$ are negative at out-of-band frequencies, see Fig. 3. Thus, the objective is to perturb the model such that all eigenvalues are positive, while at the same time the change to $\mathbf{Y}_{rat}(s)$ is minimal in the fitting range (10 Hz–10 kHz).

The alternative approaches (RP/FRP/MP/FMP) are combined with passivity checking via the Hamiltonian matrix [2] and the robust iteration scheme in [6]. Iterations are run until all passivity violations have been removed.

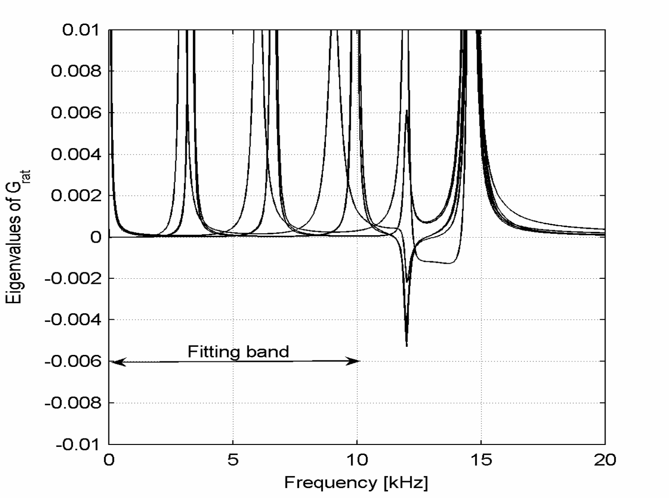

Fig. 3 Eigenvalues of $\mathbf{G}_{rat}(s)$

Fig. 4 shows the change to the eigenvalues of $\mathbf{G}_{rat}(s)$ when perturbed by either RP or FRP. It can be seen that both approaches have resulted in positive eigenvalues and thus a passive model. Despite the rather large correction of the out-of band passivity violation, the perturbation within the fitting band is with both approaches quite small.

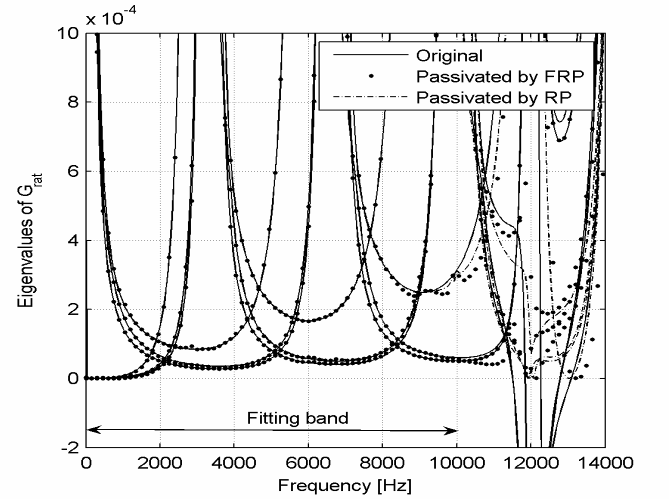

Fig. 4 Eigenvalues of $\mathbf{G}_{rat}(s)$. FRP vs. RP

Fig. 5 shows the deviation from the eigenvalues of the original model $\mathbf{Y}(s)$, in the fitting band. It can be seen that FRP gives only a slightly larger perturbation of the eigenvalues than RP. The increase is remarkably small, considering that the number of free unknowns per residue matrix has been reduced from 21 to 6.

Fig. 6 shows the same result when enforcing passivity using either FMP or MP. As expected, FMP gives a somewhat larger perturbation due to the more constrained solution. When comparing the FMP/MP solution with the FRP/RP solution (Fig. 5), it is noted that the deviation curves are with FMP/MP nearly parallel to the respective eigenvalues whereas those by FRP/RP are nearly "flat". The first result is a direct consequence of the inverse eigenvalue weighting in (8), which is the intended result.

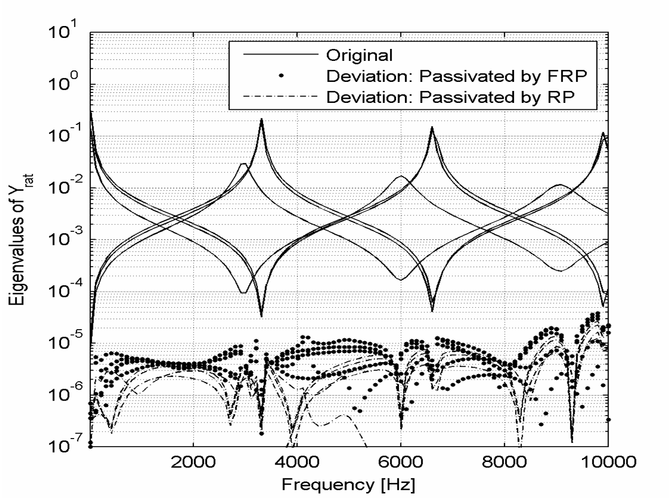

Fig. 5 Eigenvalues of $\mathbf{Y}_{rat}(s)$ in fitting range. FRP vs. RP

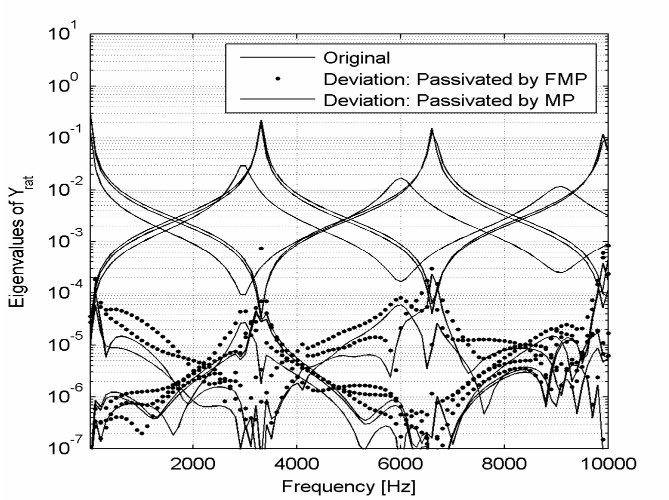

Fig. 6 Eigenvalues of $\mathbf{Y}_{rat}(s)$ in fitting range. FMP vs. MP

Discussion

Table 1 shows some key numbers related to the first passivity iteration in Example 2, solved by `quadprog.m` in Matlab. In all cases, $\mathbf{B}_{sys}$ in (4b) has 24 constraints (rows).

- Usage of FRP/FMP over RP/MP reduces the number of free variables from 630 to 180. This reduces the computation time by more than 90% for solving the QP problem by Matlab's `quadprog.m`.
- The change to $\mathbf{Y}(s)$ in the fitting range ($\|\Delta\mathbf{Y}\|_2$) is higher by only a factor of about two when using FRP/RP over FMP/MP. Usage of FMP over FRP leads to a larger perturbation, since FMP sacrifices accuracy of large eigenvalues at the expense of small eigenvalues. However, it was shown in [5] that retaining the *relative* accuracy of the eigenvalues can be crucial in situations where the model is to be applied with arbitrary terminal conditions. This is particularly relevant for models with a large eigenvalue spread, for instance a transmission line at low frequencies.

Table 1 Comparison of RP, FRP, MP, FMP.

	size($\Delta\mathbf{x}$)	Time [sec]	$\|\|\Delta\mathbf{Y}\|\|_2$	cond($\mathbf{A}_{sys}^T\mathbf{A}_{sys}$)
RP	630	19.4	1.02*E*–4	1.01*E*5
FRP	180	1.68	1.68*E*–4	1.99*E*5
MP	630	23.1	6.12*E*–4	6.89E13
FMP	180	1.96	1.34*E*–3	4.14*E*8

It has previously been proposed to reduce the problem size by using only a few of the residues as free variables. In Fig. 7 is compared the change to the eigenvalues of $\mathbf{G}_{rat}(s)$ when perturbing by either FRP or RP, when in RP using residues from the diagonal elements of the residue matrices. This, however, leads to a large perturbation of the model. The larger perturbation is also evident in Fig. 9, which compares deviation curves for the eigenvalues of $\mathbf{Y}(s)$.

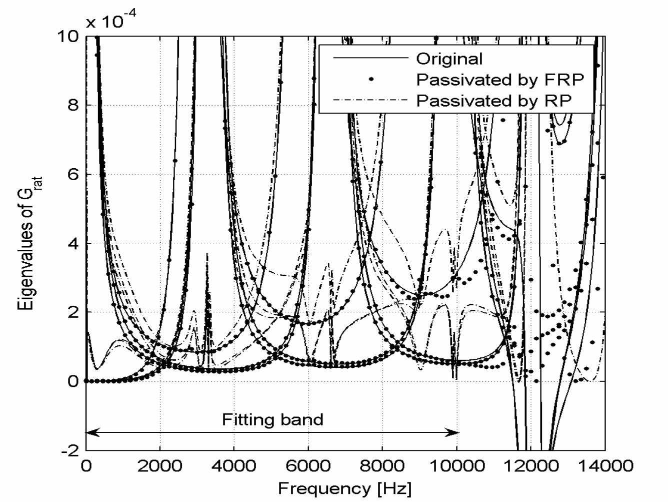

Fig. 7 Eigenvalues of $\mathbf{G}_{rat}(s)$ when in RP taking diagonal elements of residue matrices as free variables

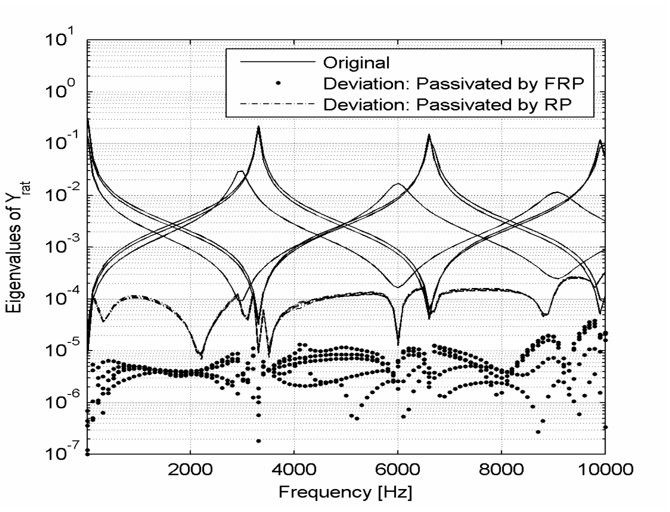

Fig. 8 Eigenvalues of $\mathbf{Y}_{rat}(s)$ in fitting range. FRP vs. RP with diagonal elements of residue matrices as free variables

Conclusions

Enforcing passivity by perturbing residue matrix eigenvalues instead of residue matrices, offers several advantages,

- The number of free variables is greatly reduced, thereby reducing computation time and memory requirements
- Large scale problems can be solved without the need for specialized sparse QP solvers.
- When combined with residue perturbation (RP), or modal perturbation (MP), the resulting fast approach (FRP/FMP) gives only a slightly larger model perturbation.

Usage of FMP/MP over FRP/RP has the additional advantage of retaining the relative accuracy of the eigenvalues of the admittance matrix, thereby making the passivity enforcement less likely to result in inaccurate model behavior when applied is situations with arbitrary terminal conditions.

Acknowledgement

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References

[1] B. Gustavsen, and A. Semlyen, ”Enforcing passivity for admittance matrices approximated by rational functions”, *IEEE Trans. Power Systems*, vol. 16, no. 1, pp. 97-104, Feb. 2001.

[2] D. Saraswat, R. Achar, and M.S. Nakhla, “Enforcing passivity for rational function based macromodels of tabulated data”, *Proc. Electrical Performance of Electronic Packaging* (EPEP), pp. 295-298, 2003.

[3] S. Grivet-Talocia, “Passivity enforcement via perturbation of Hamiltonian matrices”, *IEEE Trans. Circuits and Systems I*, vol. 51, no. 9, pp. 1755-1769, Sept. 2004.

[4] B. Gustavsen, “Passivity enforcement of rational models via modal perturbation”, *IEEE Trans. Power Delivery*, submitted.

[5] B. Gustavsen, and C. Heitz, “Rational modeling of multiport systems by modal vector fitting” *Proc. IEEE Workshop on Signal Propagation on Interconnects*, 2007. Submitted.

[6] B. Gustavsen, “Computer code for passivity enforcement of rational macromodels by residue perturbation”, *Proc. IEEE Workshop on Signal Prop. on Interconnects*, pp. 115-118, 2005.

[7] B. Gustavsen, and A. Semlyen, “Rational approximation of frequency domain responses by vector fitting”, *IEEE Trans. Power Delivery*, vol. 14, no. 3, pp. 1052-1061, July 1999.

[8] S. Grivet-Talocia, “Package macromodeling via time-domain vector fitting”, *IEEE Microwave and Wireless Components Letters*, vol. 13, no. 11, pp. 472 474, Nov. 2003

[9] B. Gustavsen, “Improving the pole relocating properties of vector fitting”, *IEEE Trans. Power Delivery*, vol. 21, no. 3, pp. 1587-1592, July 2006.

[10]D. Deschrijver, B. Haegeman, and T. Dhaene, “Orthonormal vector fitting: A robust macromodeling tool for rational approximation of frequency domain responses”, *IEEE Trans. Advanced Packaging*, accepted.

[11]A. Maffucci, G. Miano, and F. Villone, “An enhanced transmission line model for full-wave analysis of interconnects in non-homogenous dielectrics”, *Proc. IEEE Workshop on Signal Propagation on Interconnects*, 2004, pp. 21-24.