A Digital Filtering Approach for Time Domain Vector Fitting

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Abstract

Linear macromodels can be extracted from simulated or measured time domain responses using the Time Domain Vector Fitting (TD-VF) algorithm. In this paper we show that data preprocessing by means of a digital filter (FIR) can reduce the model order and avoid false oscillations in the model's response. The procedure is demonstrated for the modeling of a cable from simulated data and for the modeling of power transformers from measurements.

Introduction

High speed electronic devices and complex power systems need an accurate representation of their frequency dependent effects for a reliable electromagnetic transient simulation [1]. Linear devices and subnetworks without excessive delay effects can be efficiently represented by lumped rational macromodels. The model can be identified starting from a set of responses given in the frequency domain or in the time domain that completely describes the terminal behavior of the device. The modeling can be performed by applying the Vector Fitting algorithm (VF) [2] to tabulated data in both the frequency domain (FD-VF) [2-5], time domain (TD-VF) [6] and even the discrete z-domain (ZD-VF) [7].

In this work we are focusing on the extraction of rational models based on truncated time domain responses. When working with truncated responses, TD-VF has been found [8] to be superior to alternative methods such as ARMA and ZD-VF. The obtained model is next subjected to passivity enforcement in order to guarantee a stable simulation, for instance using approaches in [8-12].

The time domain responses are often characterized by fast transitions and high-frequency oscillations that makes it difficult to extract an accurate model without usage of an excessive model order. In order to overcome these difficulties we introduce a numerical low pass filtering approach. The filter, which is implemented as a Finite Impulse Response (FIR) filter, is able to smooth out the high frequency content of the waveform, thereby enabling low-order models that capture the main behavioral information about the device. After a brief review of the TD-VF method we describe in detail the FIR implementation. Finally, we demonstrate the approach to two examples that consider simulated data (high voltage cable) and measured data (power transformer).

Problem Statement

We consider in this work the modeling of a linear time invariant (LTI) system characterized by time domain responses at its ports. For simplicity, we show only results for the scalar case as the generalization to the multi-port case is straightforward. We obtain by simulation or measurement an excitation $u(t_k)$ and output response $y(t_k)$, $0\leq k \leq N_s-1$, being N_s the time domain samples with $t_k=k\Delta t$ and Δt being the time invariant time step length. In the following we introduce the simpler notation $u(k)$ and $y(k)$.

The objective of this work is to determine the rational transfer function $H(s)$ (1) of order N where $s=\sigma+j\omega$ is the Laplace variable. The task is to calculate the poles $\{p_n\}$ and residues $\{r_n\}$ such that the model reproduces as accurately as possible the output time domain response $y(k)$ with the given excitation $u(k)$.

$$H(s)=\sum_{n=1}^{N}\frac{r_n}{s-p_n}+r_0 \qquad (1)$$

Time Domain Vector Fitting (TD-VF)

We use the TD-VF algorithm [6] to identify poles $\{p_n\}$ and residues $\{r_n\}$ of the rational function (1). By transforming the frequency domain formulation of the VF algorithm [2] into the time domain we obtain with successive discretization of the resulting convolution integral [6],[8] the relation (2)

$$y(k)=\sum_{n=1}^{N}m_n\tilde{u}_n(k)-\sum_{n=1}^{N}k_n\tilde{y}_n(k)+m_0u(k) \qquad (2)$$

where $\{k_n\}$ and $\{m_n\}$ are unknowns. The coefficient sequences $\tilde{u}_n(k)$ and $\tilde{y}_n(k)$ are dependent on the numerical integration method employed in the discretization. Assuming trapezoidal integration, these are given recursively in (3) [8] where $\{q_n\}$ is a set of initial poles. (We show only $\tilde{u}_n(k)$ since $\tilde{y}_n(k)$ has the same form). Alternatively, the coefficients can be obtained assuming recursive convolution [6].

$$\tilde{u}_n(k)=\frac{2+q_n\Delta t}{2-q_n\Delta t}\tilde{u}_n(k-1)+\frac{\Delta t}{2-q_n\Delta t}\left[u(k)+u(k-1)\right] \qquad (3)$$

By writing (2) for each time sample $0\leq k\leq N_s-1$, an overdetermined linear equation is obtained which is solved in the least-squares (LS) sense. The computed unknowns k_n allow us to obtain an improved set of poles $\{q_n\}$ [6]. The final set of poles of $H(s)$ is achieved by iterating the pole relocating procedure until convergence, $\{q_n\}\rightarrow\{p_n\}$. This procedure normally converges in few iterations.

Once we have the poles $\{p_n\}$, the final residues $\{r_n\}$ are computed by solving the (LS) overdetermined linear system (4)

$$y(k)=\sum_{n=1}^{N}r_n\tilde{u}_n(k)+r_0u(k) \qquad (4)$$

Digital Filtering

In situations where the output sequence $y(k)$ is characterized by a non-smooth behavior and fast variations, it can be difficult to calculate a rational model. The reason is that the high frequency content requires to use a very high model order. In order to overcome this problem we reduce the high-frequency content by pre-processing the output response by a digital low pass filter, before applying the TD-VF algorithm.

Ideal Digital Low pass Filter

The ideal low pass filter is represented in the digital frequency domain by the transfer function H_{id}, see Fig. 1. The variable ν ($-0.5 \leq \nu \leq +0.5$) defines the digital frequency axis and the parameter ν_c is the normalized filter cut-off frequency (with respect to the sampling frequency $f_s=(1/\Delta t)$).

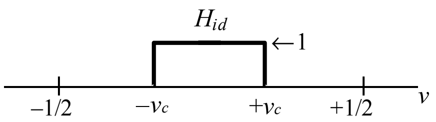

Fig. 1. Low pass filter (ideal and normalized) in the digital frequency domain.

In the discrete time domain, this characterization is equivalent to the impulse response h_{id} sequence (5).

$$h_{id}(m) = \begin{cases} \sin \dfrac{2\pi \nu_c m}{\pi m} & m \neq 0 \\ 2\nu_c & m = 0 \end{cases} \tag{5}$$

Finite Impulse Response (FIR)

In reality, the ideal filter sequence h_{id} (5) is not realizable since it has infinite duration and because it is defined for negative m. For these reasons we adopt in our work the well known Finite Impulse Response (FIR) filtering [13] that makes use of a truncated sequence of the ideal response (5).

Practical FIR Implementation Steps

Given data: response sequence $y(k)$ to be filtered.

1) Starting point: impulse response sequence $h_{id}(m)$ of the ideal digital filter (5).

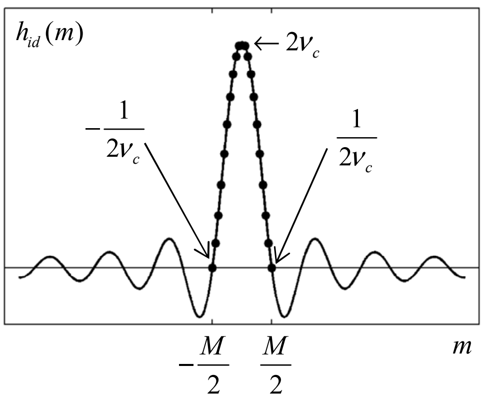

Fig. 2. Truncation of the impulse response $h_{id}(m)$: selection of coefficients.

2) Truncation of the h_{id} impulse response: we select only the (M+1) coefficients that defines the first lobe of the ideal '*sinc*' sequence, see Fig. 2. This is done in order to avoid oscillations in the filtered response. The filter order M, which is chosen as an even number, is obtained by (6) where the function 'ceiling' returns the upper closest integer.

$$M = 2 \text{ ceiling}\left(\frac{1}{2\nu_c}\right) \tag{6}$$

3) Computation of the coefficients of the (translated) FIR discrete response h_{FIR}: these correspond (7) to the coefficients of the translated (with delay $M/2$) and truncated h_{id}. Note that the causality is hence guaranteed.

$$h_{FIR}(n)\Big|_{n=0}^{n=M} = h_{id}(m)\Big|_{m=-M/2}^{m=+M/2} \tag{7}$$

4) Application of the filter FIR: we obtain the output filtered response sequence $y^f(k)$ as the discrete convolution (8) between the data sequence $y(k)$ and h_{FIR}.

$$y^f(k)\Big|_{k=0}^{k=N_s-1} = \sum_{j=0}^{M} h_{FIR}(j)\, y(k-j) \tag{8}$$

5) Removal of the delay ($M/2$) of the filtered response (9). Note that the first few samples of the filtered response (corresponding to negative time) are discarded.

$$y^f(k) = y^f(j)\Big|_{j=M/2+1}^{N_s} \tag{9}$$

Choice of Cut-off Frequency

The normalized cut-off frequency ν_c is determined according to the specific application. Whenever is possible to reduce ν_c, the reduced high-frequency content of the signal often allows a successful application of TD-VF with fewer poles.

The choice of ν_c has an impact on the selection of the initial poles $\{q_n\}$ of the TD-VF. In particular, these are assumed linearly spaced in the interval (10), where T_w is the observation window length of the original time domain samples.

$$\left(\frac{1}{T_w}, \frac{1}{\Delta t}\nu_c\right) \tag{10}$$

Application Example 1: Simulated Data

To demonstrate the efficiency of the filtering approach, we consider in this example an open-ended high voltage underground cable, as shown in Fig. 3. The cable is of coaxial design with a copper core conductor and copper screen. The purpose is to identify a rational model of the cable with respect to the connected cable end when the far end is open.

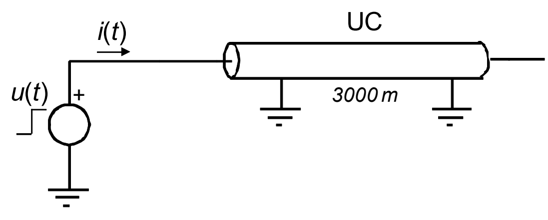

Fig. 3. High voltage cable

Using the circuit solver PSCAD, which is based on numerical integration with a fixed time step length [1], we simulate the step response current $i(t)$ that flows into the cable for a time duration of 120 µs with time step Δt=0.12 µs. The resulting current is characterized by a sequence of very steep wave fronts (high frequency content).

We use two alternative input datasets for the TD-VF. The first one is based on the original response $i(t)$ whereas the second one is based on the a filtered current $i(t)$ obtained by the FIR filter with a normalized cut-off frequency ν_c=0.045. We use 20 pole relocation iterations in TD-VF.

Fig. 4 shows the model response obtained by TD-VF when filtering has not been applied. The results is shown for two alternative model orders (N=40, N=100). Comparison with the original simulated response shows that the model performance is quite unsatisfactory due to spurious Gibbs-like oscillations.

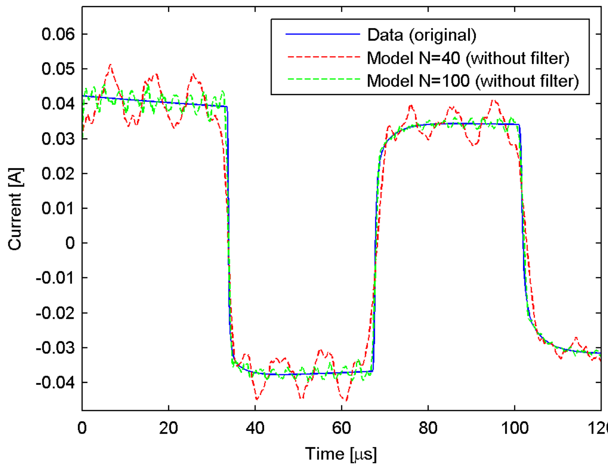

Fig. 4. Fitting model to data (without filter) using two alternatives orders. N=40, N=100.

Fig. 5 shows the same result when the simulated current response has been processed by the FIR before applying TD-VF. Clearly, a much better result is now obtained as the response is essentially free of oscillations. The only disadvantage is the inevitable loss of steepness for the wave fronts. We also note that the response is *not* shifted in time, thanks to the removal of the delay in Step 5). This result is quite different from what would be achieved by a physical low-pass filter which results in distorted and shifted responses.

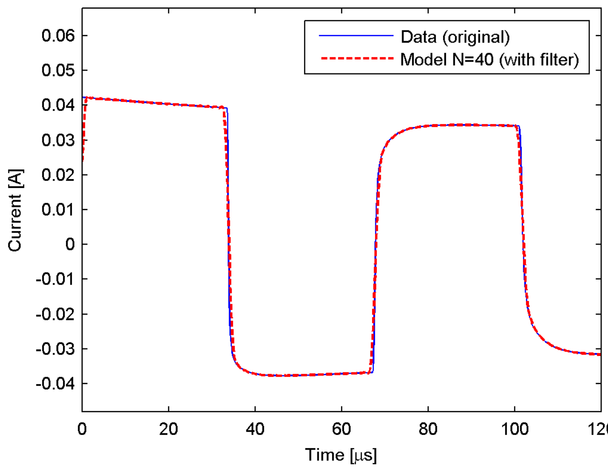

Fig. 5. Fitting model to data (with filter). N=40.

A Remark About Passivity

The passivity of the model, i.e. the inability to produce energy, has not been considered in this example in order to obtain a direct and fair comparison of the results. However, we have noticed that the model constructed without the filter approach has several passivity violations while the model obtained via the filter approach happens to be passive. We have found a similar result in many cases; the usage of the filter often helps to build passive or nearly passive models.

Application Example 2: Measured Data

We consider now a measured dataset obtained from a 3-phase power transformer used in high-voltage power distribution systems, see Fig. 6. A near step voltage is applied to port 1 with ports 2 and 3 grounded. The task is to obtain a model for the voltage transfer to the open port 4, i.e. the voltage ratio $R_{41}(s) = V_4(s)/V_1(s)$.

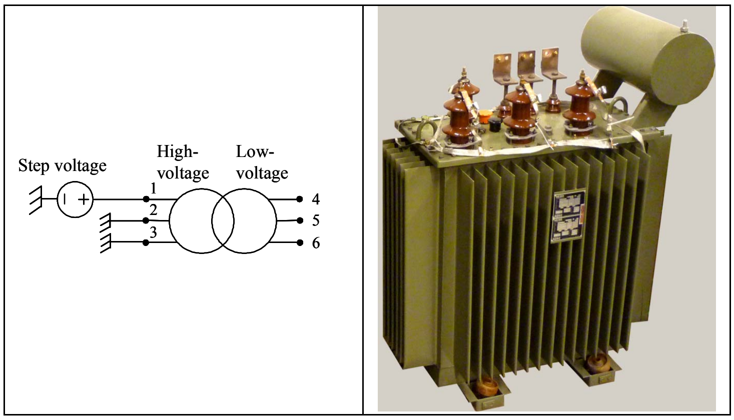

Fig. 6. High-voltage power transformer (six-port device)

Fig. 7 shows the measured excitation V_1 and the response V_4. The time record has a duration of 10 µs with a sampling step length Δt=1 ns. The responses are seen to be quite noisy.

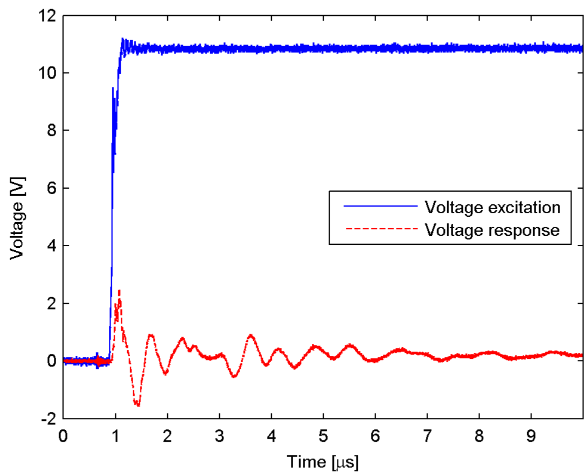

Fig. 7. Measured voltage excitation on port 1 and voltage response on port 4.

A model is constructed using an order N=40 with 30 pole relocating iterations. Fig. 8 and Fig. 9 (expanded view) compares the model's response with that of the measurement. It can be seen that in this case, a very good result is achieved whether the filter is applied or not. Here, the limited frequency content of the response allows the standard version of the TD-VF by itself to perform very well. The result also confirms excellent noise immunity of the TD-VF method.

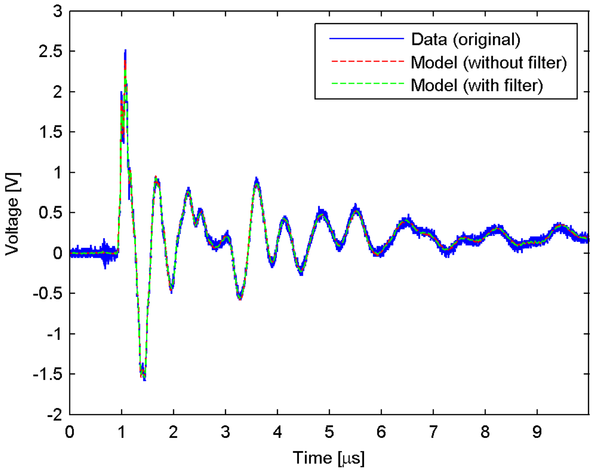

Fig. 8. Fitting model to data (with and without filter). N=40.

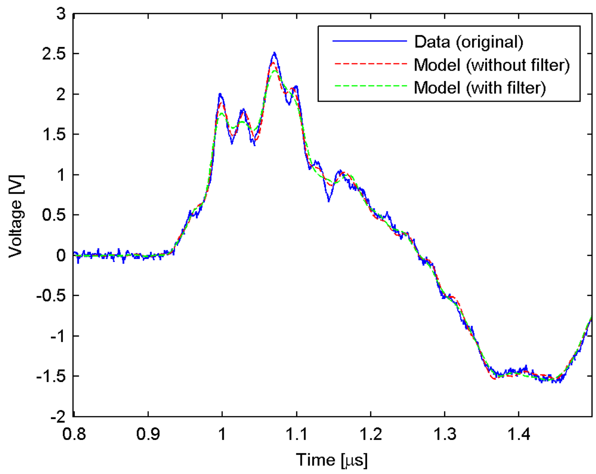

Fig. 9. Expanded view of Fig. 8.

Conclusions

In this work a FIR-type digital filter has been introduced as a data pre-processor for TD-VF. The filtering offers several advantages by reducing the required model order and by avoiding false oscillations in the models' response. In addition it often reduces the severity of passivity violations in the obtained model. The filter is carefully designed so as to avoid false delays in the response.

We have considered datasets coming from both simulations and measurements. In the first case, the response was characterized by steep edges due to wave reflections in a cable. Here, the filtering approach greatly reduced the presence of false oscillation in the model's response. In the second case, the response was contaminated by measurement noise. Here, the standard version of the TD-VF performed well even without usage of the filter.

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