Modelling of piezoelectric micromachined ultrasound transducers (pMUT) for medical use

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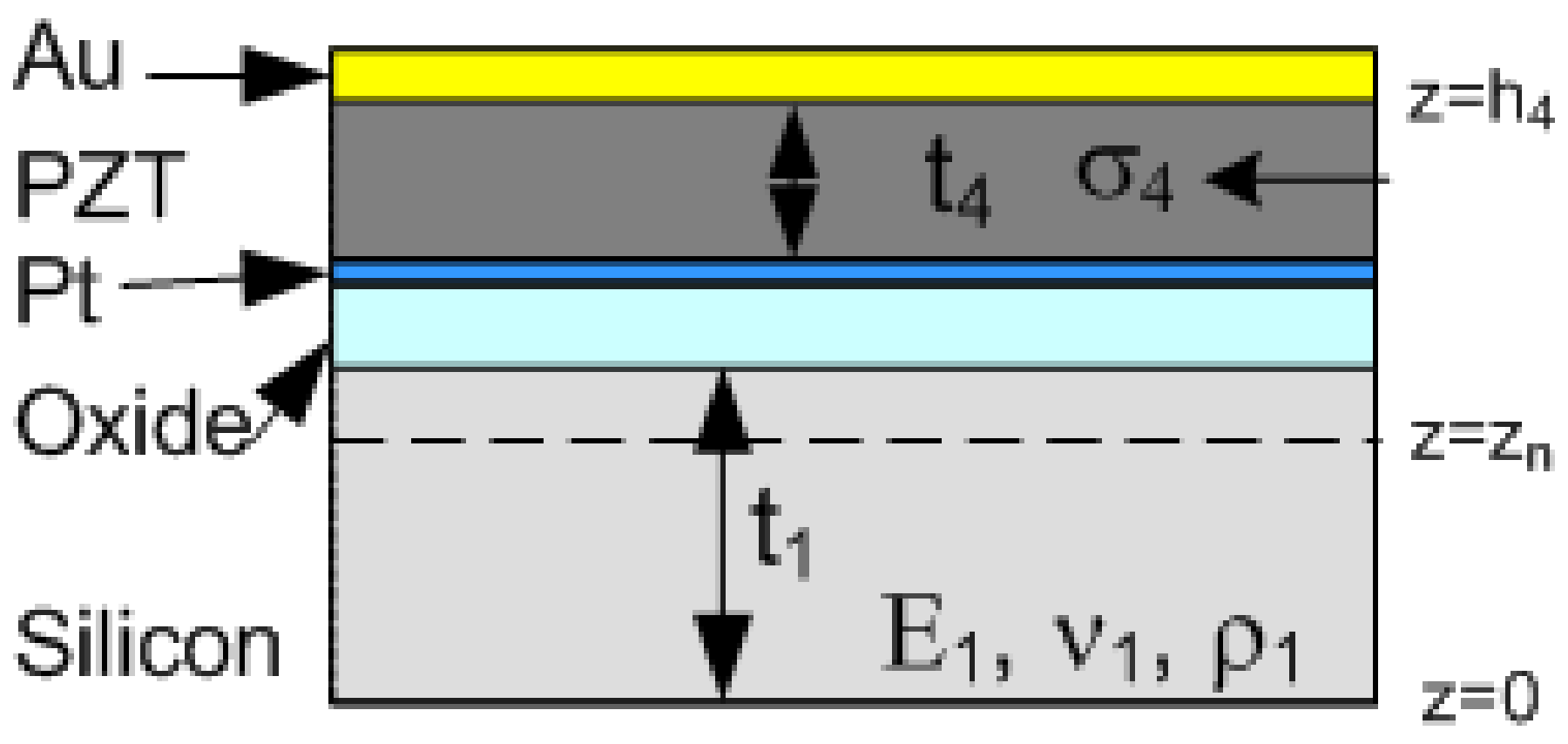
Introduction

PiezoMEMS transducers consist often of multilayered thin-film structures which are difficult to model with finite element modeling (FEM) tools due to high aspect ratio of the geometry.



Layer stack consisting of different thin film layers of the pMUT

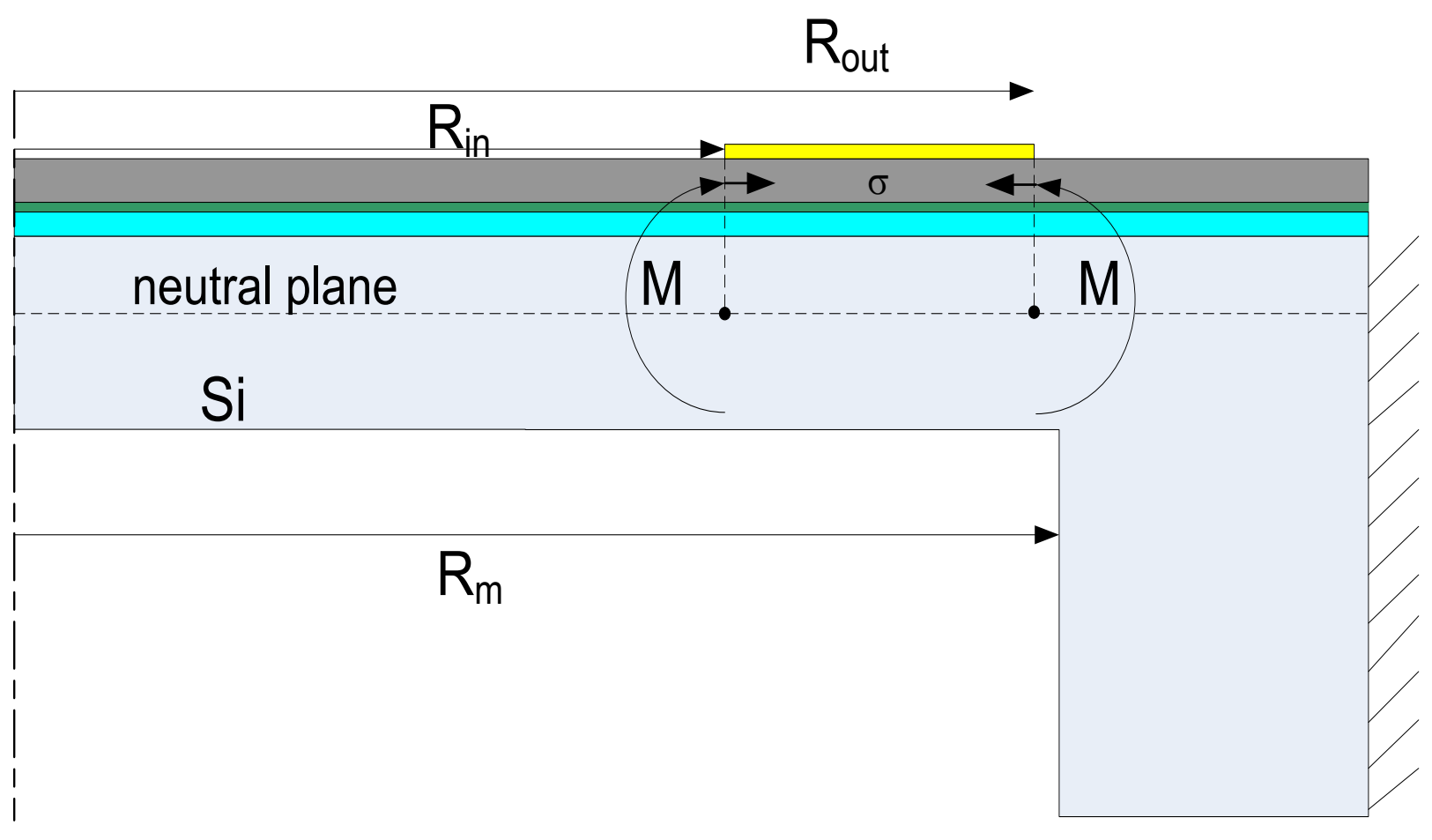
Hence, a set of analytical models for different variables in the transfer function of the pMUT (in liquid medium) has been developed as a checking point for the multiphysics FEM simulations.

Moment and deflection due to piezoelectric actuation

The used geometry is a circular multilayered membrane structure with one electrode ring for actuation. We calculate piezoelectric moment per unit length $M_p(V)$ for given voltage and the centre deflection y(V) of the actuated membrane:

$$M_p(V) = e_{31,f}\frac{V}{t_4}\int_{h_3}^{h_4}(z - z_n)\cdot dz$$

Moment due to piezoelectric actuation [1]



Geometry of the pMUT structure

$$y(V) = \frac{M_p(V)\cdot R_{in}^2\cdot\ln\left(\frac{R_m}{R_{in}}\right)}{2D_m}$$

Centre deflection due to piezoelectric actuation [2]

Multi-layer setup

A multilayer setup consisting of a combination of very thin (~100nm) and thicker layers (several µm) N of different materials with big variations in Young's modulus $E_i \Rightarrow \frac{E_i}{1-\nu_i}$ and specific weight (ρ) is necessary to calculate the position of the neutral plane z_n , flexural rigidity D_m and the mass per area µ:

$$z_n = \frac{\sum_{i=1}^{N_l}\left[E_i\left(\int_{h_{i-1}}^{h_i} z\,dz\right)\right]}{\sum_{i=1}^{N_l}(E_i t_i)}$$

Neutral plane

$$D_m := \left[\sum_{i=1}^{N_l}\left[E_i\int_{h_{i-1}}^{h_i}(z - z_n)^2\,dz\right]\right]$$

Flexural rigidty

$$\mu = \sum_{j=1}^{N_l}\rho_j t_j$$

Mass per area

Acoustic modeling

We incorporate the acoustic impedance into a model for an actuated membrane by using the average deflection y_{avg} (1/3 of centre deflection) of the membrane as the coordinate describing the amplitude of the deflection and define the spring constant, k, as the relationship between the average deflection and the total applied force on the membrane:

$$F = P\cdot A = -k\cdot y_{avg} \qquad k = \frac{192\cdot\pi}{R_m^2}D_m$$

Effective mass of the membrane: $m_m = \frac{9}{5}\mu\pi R_m^2$

We let the acoustic impedance relate the force needed to generate the acoustic radiation to the time derivative of the average velocity of the membrane, $F = Z\cdot\dot{y}_{avg}$ Furthermore we approximate the displacement of the membrane by the displacement of a piston defined by the average displacement. Then, we can calculate the acoustic impedance Z(f) [3] :

$$Z(f) = \rho c\pi R_m^2\left(R_1\left(2\frac{2\pi f}{c}R_m\right) + jX_1\left(2\frac{2\pi f}{c}R_m\right)\right)$$

$$R_1(x) = 1 - 2J_1(x)$$

$$X_1(x) = \frac{2H_1(x)}{x} \approx \frac{4}{3\pi}x$$

$\Rightarrow$ $m_w = \frac{8}{3}\rho R_m^3$

Effective mass of water on the membrane

Transfer function and lumped model

Using the previous results and setting $y_{avg} = y_0 e^{j\omega t}$ and $V = V_0 e^{j\omega t}$ we get the **transfer function**:

$$H(\omega) = \frac{y_0}{V_0} = \frac{k\cdot M(V_0)R_{in}^2\ln\left(\frac{R_m}{R_{in}}\right)}{2D_m\left(k + j\omega\cdot Z(\omega) - m_m\omega^2\right)}$$

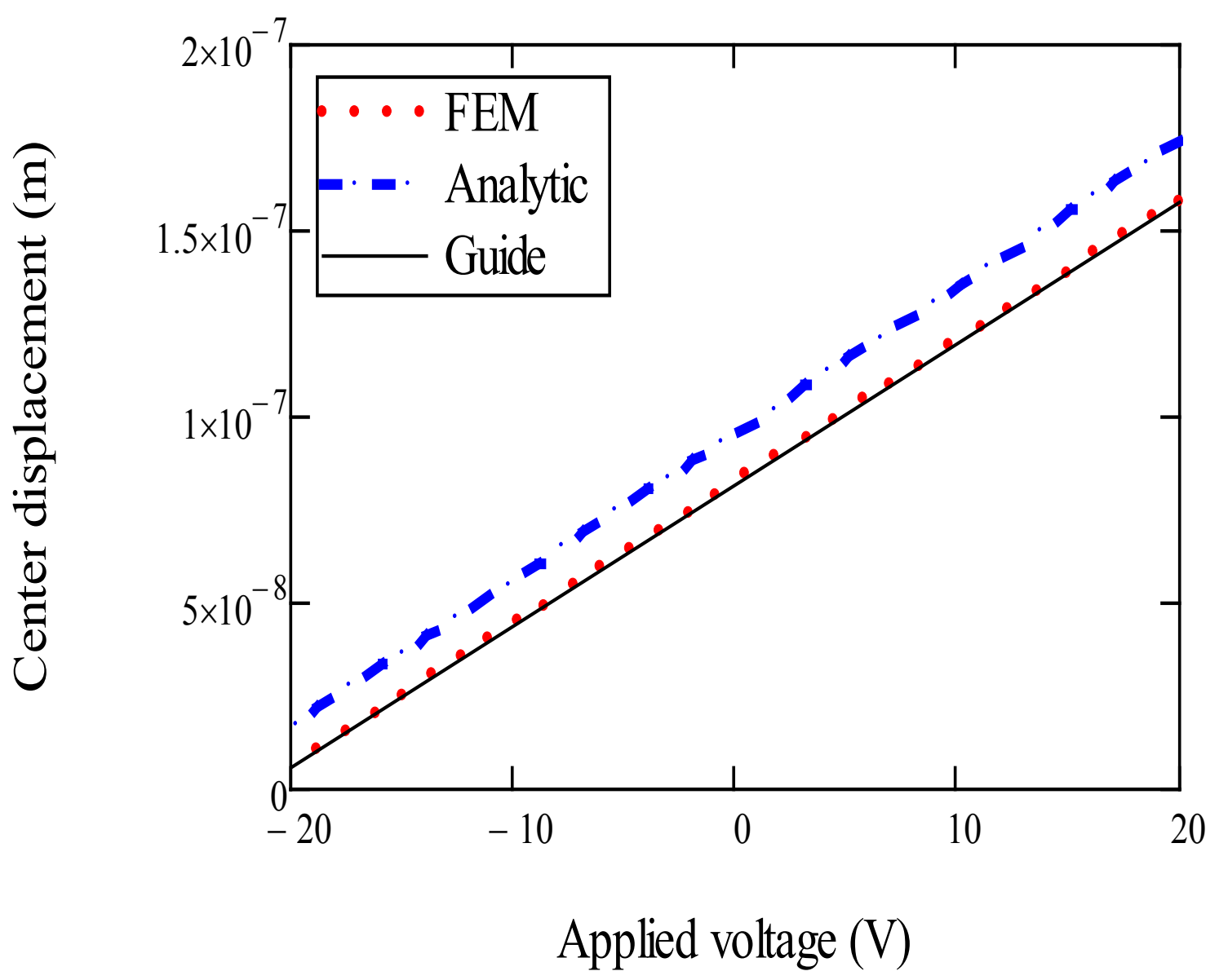
Using the effective mass of the membrane and the surrounding water we get a lumped element model of the **frequency** of the first eigenmode:

$$f_0 \approx \frac{1}{2\pi}\sqrt{\frac{k}{m_m + m_w}}$$

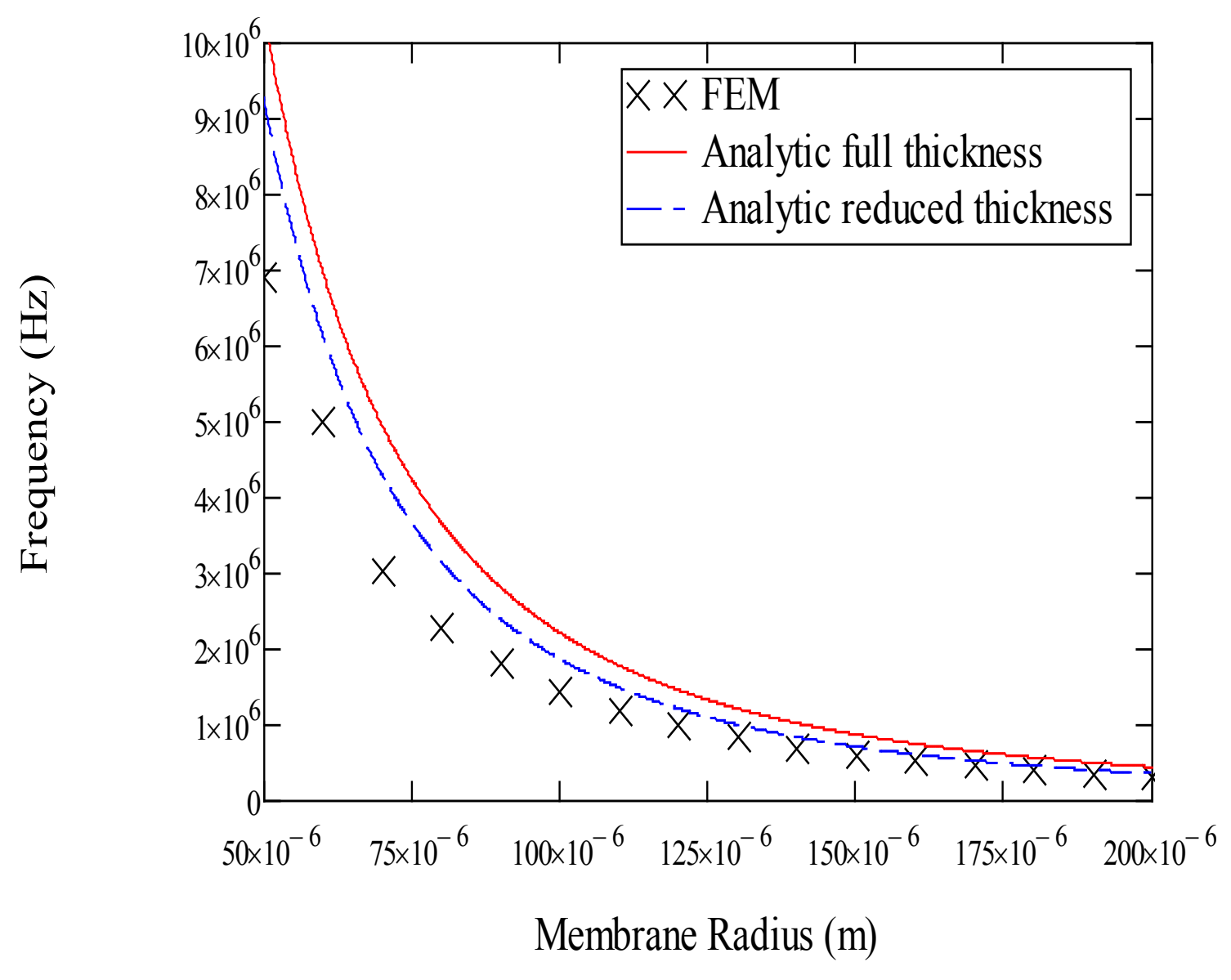
Results and discussion

The results of the analytical modeling for the static centre displacement, a calculation of the eigenfrequency as a function of the membrane radius and a frequency sweep of the centre displacement have been compared to the results of FEM-simulations with COMSOL.

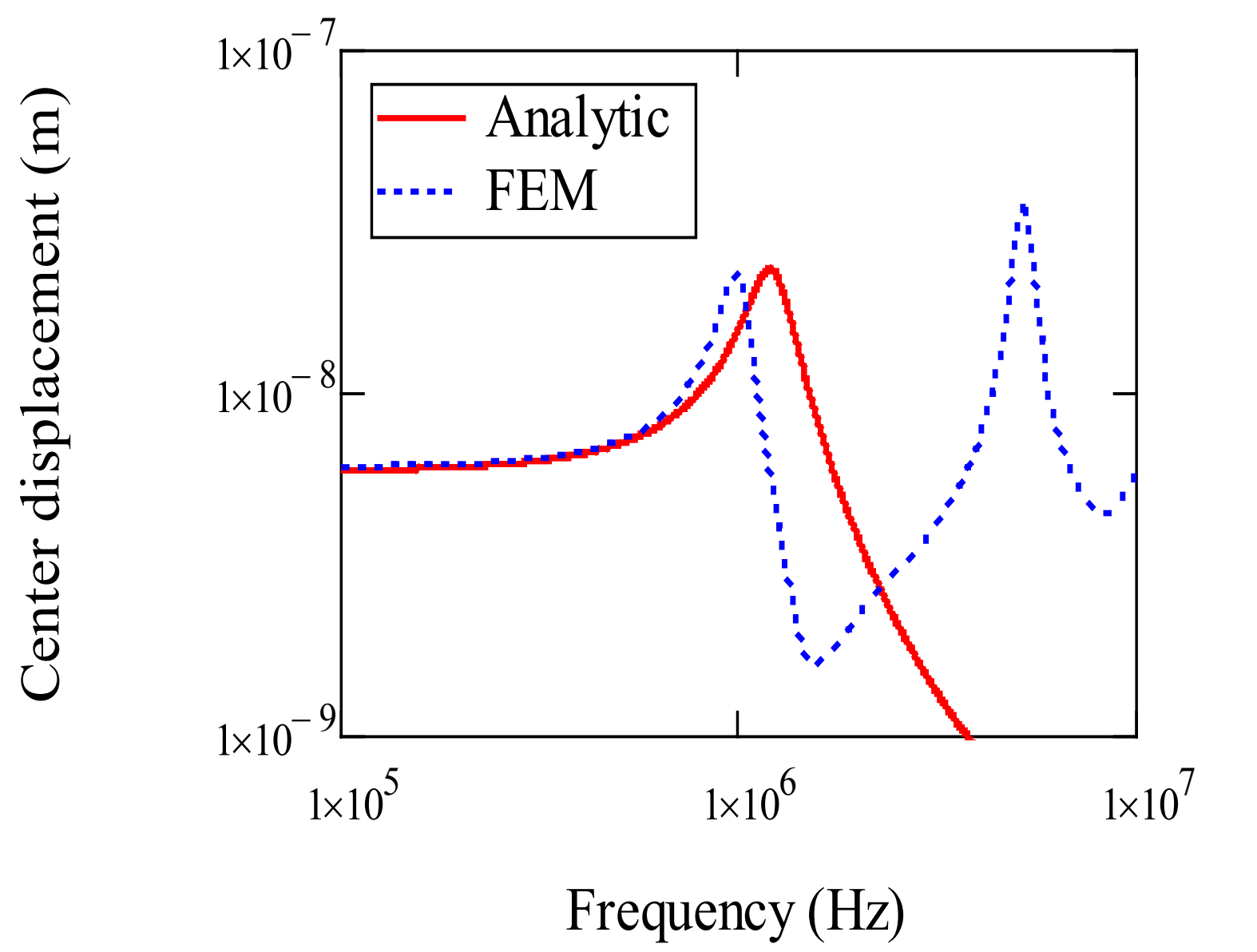
Static centre displacement



Eigenfrequency as function of membrane radius



Frequency sweep of centre displacement



The comparison of the results shows only minor deviations between the analytic and finite element models. The FE models, however, show additional effects (e.g. higher order eigenfrequencies) compared to the applied analytical models. By using both types of models a cross check of the validity of results is possible, which is highly important for such real multiphysics models.

References & Acknowledgements

References:
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[2]Bakke, Th, et al. , A novel ultra-planar, long-stroke, and low voltage piezoelectric micromirror , J of Micromechanics and Microengineering, 20(6), 2010
[3] Kinsler, L. E:, Fundamentals of acoustics, 4 th. Ed. Wiley, 2000, 548p.

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