A Review of the Morison Equation for Calculating Hydrodynamic Loads on Vertically-Oriented Cylinders

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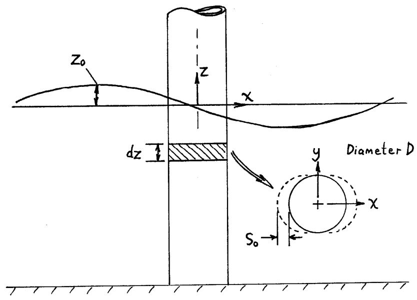
A Definition of the Problem



Find the net force of the fluid on the structure, *F(z,t)*.

Key Points

The loads on the structure are a function of **several flow processes** (waves, current, structural motion) which act simultaneously and interact nonlinearly.

Calculation of loads is heavily empirical. There is a lot of laboratory data at flow parameters (like Reynolds number) that are not representative of full-scale structures. There have been field measurements on full-scale structures, but here the flow parameters are somewhat uncertain. Connecting the two is not easy; design values should not be considered "final" or broadly applicable.

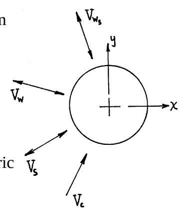
The value used for **fluid damping should be calibrated independently of the primary drag coefficient** C_D, and should be guided by full-scale data.

There are several important, outstanding issues that are not considered in this presentation:
- free surface effects, run-up, draw-down, impact (slamming), and ringing ("burst motions")
- negative damping, "lock-in", the interaction of vortex shedding and structural vibration
- forces on members at an angle to the oncoming flow, or parallel to the free surface

Flow Processes

It is convenient to think about the hydrodynamic loading in terms of flow processes. Multiple processes – wind-generated waves, remote swell, current, and structural motion – are active simultaneously, and their (nonlinear) interaction results in the fluid force on the structure.

(For the present discussion, we shall assume that each process can be described by a single dominant trigonometric term; in reality, multiple harmonics are involved.)



The net flow velocity vector may exhibit large fluctuations in both direction and magnitude. If the flow separates, forming a wake of shed vortices, then there is a ``memory effect"; the pressure about the structure is a function not only of the instantaneous flow velocity vector, but also its time history.

$$V = V_{w0}\sin(2\pi f_w t + \psi_w)(\cos\theta_w \mathbf{i} + \sin\theta_w \mathbf{j})$$
$$+V_{w0,s}\sin(2\pi f_{ws} t + \psi_{ws})(\cos\theta_{ws} \mathbf{i} + \sin\theta_{ws} \mathbf{j})$$
$$+\dot{s}_0\sin(2\pi f_s t + \psi_s)(\cos\theta_s \mathbf{i} + \sin\theta_s \mathbf{j})$$
$$+|V_c|(\cos\theta_c \mathbf{i} + \sin\theta_c \mathbf{j})$$

Morison Equation

How do we predict loads on the structure? For large-volume structures (Kc = $V_{w0}/f_w\, D < 1$ or 2), potential theory is used to calculate the wave forces, with an empirical drag force (the second term in the equation below) superposed to account for a steady current.

Typical ocean wavelengths are over 40 m, therefore wind turbine towers will typically be considered small-volume structures. In this case, the Morison equation is used. This equation is a little bit of theory combined with a lot of empiricism:

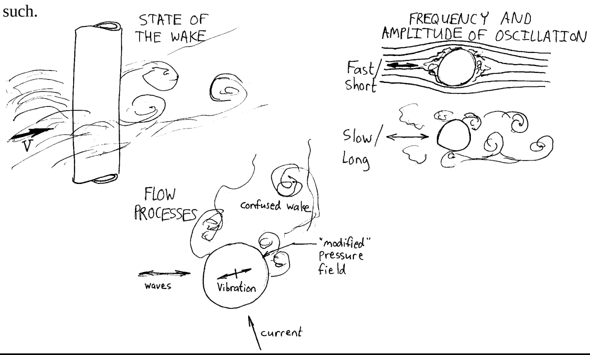
$$dF = \rho\frac{\pi D^2}{4}\; dz\; C_M\; \dot{V} + \frac{1}{2}\rho D\; dz\; C_D\; |V|\; V$$

The Morison equation states that the fluid force is a superposition of a term in phase with the acceleration of the flow (inertia), and a term whose dominant component is in phase with the velocity of the flow (drag). It accounts for some flow nonlinearity, by way of the drag term.

The Morison equation is deterministic. In itself it does not account for the history of the flow (the state of the wake), the frequency with which the flow oscillates back and forth, nor the fact that the instantaneous velocity vector *V* arises as a superposition of several flow processes.

Morison Equation: Empirical Coefficients

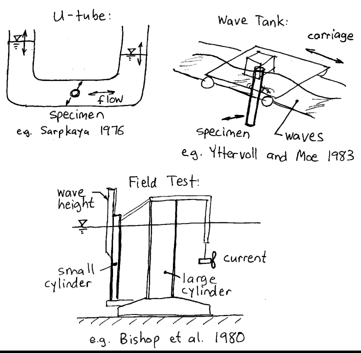
The effect of the history of the flow on the fluid force *dF* must be accounted for entirely by the coefficients C_M and C_D. In other words, the coefficients are a function of the state of the wake, the flow processes which are active, the frequency of flow oscillation, and such.



Morison Equation: Empirical Coefficients

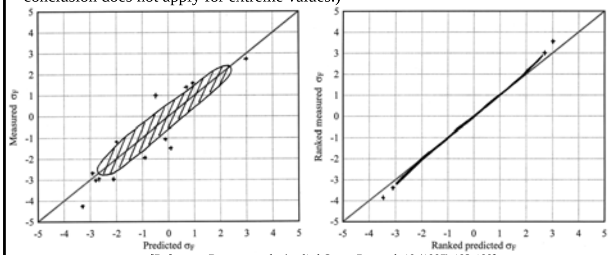
Coefficients are determined by either a laboratory experiment or measurements on a field test rig mounted in the ocean.

Flow conditions in the laboratory are controlled, while in the field there is always some uncertainty as to the local flow conditions. However, the results of laboratory experiments are seldom directly applicable to the design of full-scale structures; typically, the Reynolds number is much too low.



Morison Equation: Empirical Coefficients

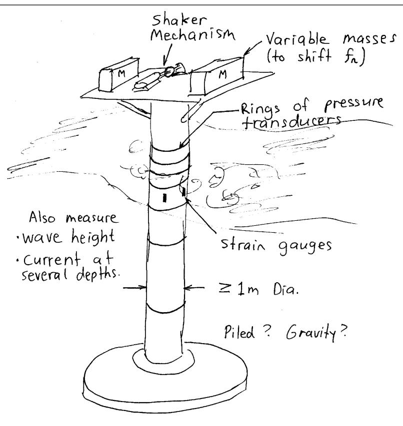
When an experiment is performed, and the coefficients in the Morison equation are calibrated to the experiment, then a good correlation is obtained, particularly if experimental and calculated load cycles are ranked lowest to highest. (This conclusion does not apply for extreme values.)



[Reference: Burrows et al., Applied Ocean Research 19 (1997) 183-199]

There is a need for further *field measurements* regarding the interaction between fluid flow and structural motion, particularly the appropriate value of C_D with which to calculate fluid damping of structural motion, under various flow conditions.

An Experiment I Would Like to See



Morison Equation: Multiple Flow Processes

Write the Morison equation such that the multiple flow processes are explicit:(1)

$$dF = \rho \frac{\pi D^2}{4}\ dz\ a_w + \rho \frac{\pi D^2}{4}\ dz\ (C_M - 1)\ (a_w - \ddot{s})$$
$$+\frac{1}{2}\rho D\ dz\ C_D\ |u_w + u_c - \dot{s}|\ (u_w + u_c - \dot{s})$$

But, each process is acting with its own amplitude, frequency, and phase. Why should we be able to describe the effects of the simultaneous wave, current, and structural motion processes through just one drag coefficient and one added mass coefficient? Propose:

$$dF = \rho \frac{\pi D^2}{4}\ dz\ C_{M1}\ a_w - \rho \frac{\pi D^2}{4}\ dz\ (C_{M2} - 1)\ \ddot{s}$$
$$+\frac{1}{2}\rho D\ dz\ |c_{d0}u_c + c_{d1}u_w - c_{d2}\dot{s}|\ (c_{d0}u_c + c_{d1}u_w - c_{d2}\dot{s})$$

This equation says that the processes interact, but they do so with different strengths.

(1): Swell and wind-generated waves have been combined into one "wave" term.

Morison Equation: Multiple Flow Processes

Attempting to derive firm values for all those empirical coefficients would be clumsy and difficult. Is the separate-coefficient form of the Morison equation useful for anything?

Yes. Consider a case in which the amplitude of the structural velocity is small in comparison with the combined amplitude of the wave and current velocities, say, $s_0 < 0.2\ (u_{w0} + u_{c0})$. Then, neglecting terms of $O(s^2)$, the drag term of the separate-coefficient Morison equation can be written as:

$$dF_D = \frac{1}{2}\rho D\ dz$$
$$\times\left[|c_{d0}u_c + c_{d1}u_w|\ (c_{d0}u_c + c_{d1}u_w) - 2|c_{d0}u_c + c_{d1}u_w|\ (c_{d2}\dot{s})\right]$$

If we assume (following current practice) that we can derive a single drag coefficient C_D that is representative of the combined effects of c_{d0} and c_{d1}, then we can write the drag term:

$$dF_D = \frac{1}{2}\rho D\ dz\ \left[\ C_D\ |u_c + u_w|\ (u_c + u_w) - 2C_{D2}\ |u_c + u_w|\ \dot{s}\right]$$

Damping

$$dF_D = \frac{1}{2}\rho D\ dz\ \left[\ C_D\ |u_c + u_w|\ (u_c + u_w) - 2C_{D2}\ |u_c + u_w|\ \dot{s}\right]$$

This equation is useful, because it gives us the means to – and, in fact, says that we should – calibrate our structural damping independently from the calibration of the primary loading. This has been corroborated by experiment, for example Yttervoll and Moe (1983).

Because the loading associated with the $|u_c + u_w|\ (u_c + u_w)$ term may be several times the magnitude of the loading associated with the $|u_c + u_w|\ (ds/dt)$ term, it is advisable to determine, or at least validate, the value of C_{D2} based upon damping measurements, rather than a least-squares fit to force data.