Upgridding by Amalgamation: Flow-Adapted Grids for Multiscale Simulations

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What is multiscale simulation?

**Generally:**
Methods that incorporate fine-scale information into a set of coarse scale equations in a way which is consistent with the local property of the differential operator.

**Herein:**

- **Multiscale pressure solver (upsampling + downscaling in one step)**
  \[ \nabla \cdot \vec{v} = q, \quad \vec{v} = -\lambda(S)K \nabla p \]

- **Transport solver (on fine, intermediate, or coarse grid)**
  \[ \phi \frac{\partial S}{\partial t} + \nabla \left( \vec{v} f(S) \right) = q \]

**=** Multiscale simulation of models with higher detail
What is multiscale simulation?

Coarse partitioning:

Flow field with subresolution:

Local flow problems:

Flow solutions → basis functions:
Goal:
Given the ability to model velocity on geomodels and transport on coarse grids: Find a suitable coarse grid that best resolves fluid transport and minimizes loss of accuracy.

Formulated as the minimization of two measures:
1. the *projection error* between fine and coarse grid
2. the *evolution error* on the coarse grid
Flow-adapted coarsening

Assumptions:

- a matching polyhedral grid with \( n \) cells \( c_i \),
- a mapping \( \mathcal{N}(c) \) between cell \( c \) and its nearest neighbours,
- a set of flow indicators \( I(c_i) \) in cell \( c_i \)

We seek a coarse grid that:

- adapts to the flow pattern predicted by indicator \( I \),
- is formed by grouping cells into \( N \) blocks \( B_\ell \),
- is described by a partition vector \( p \) with \( n \) elements, in which element \( p_i \) assumes the value \( \ell \) if cell \( c_i \) is member of block \( B_\ell \).
Coarsening principles

▶ Minimize heterogeneity of flow field inside each block

$$\min_{B_j} \left( \sum_{p_i = j} |I_1(c_i) - I_1(B_j)|^p |c_i| \right)^{\frac{1}{p}}, \quad 1 \leq p \leq \infty,$$

▶ Equilibrate indicator values over grid blocks

$$\min \left( \sum_{j=1}^{N} |I_2(B_j) - \bar{I}_2(\Omega)|^p |B_j| \right)^{\frac{1}{p}}, \quad 1 \leq p \leq \infty,$$

▶ Keep block sizes within prescribed lower and upper bounds
Amalgamation algorithm

**Amalgamation of cells:**

- Difficult to formulate a practical and well-posed minimization problem for optimal coarsening $\rightarrow$ *ad hoc algorithms*
- Coarsening process steered by a set of admissible and feasible amalgamation directions

50 × 50 **lognormal permeability:**

- Regular: 25 blocks
- Flow magnitude: 26 blocks
- Isocontours [p]: 26 blocks
Motivation: layered reservoir

Permeability and velocity

Time-of-flight

Partition
Heuristic minimization: algorithmic components

Formulated using a set of:

- **sources** that create a partition vector based upon grid topology, geometry, flow-based indicator functions, error estimates, or expert knowledge supplied by the user, thereby introducing the feasible amalgamation directions

- **filters** that take a set of partition vectors as input and create a new partition as output, by
  - combining/intersecting different partitions
  - performing sanity checks, ensuring connected partitions according to admissible directions, etc
  - modifying partition by merging small blocks or splitting large blocks
Heuristic minimization: algorithmic components

Partition:

▶ prescribed topology or predefined shapes
▶ segmentation of cells $c_i$ into bins $\tilde{B}_\ell$

\[ c_i \subset \tilde{B}_\ell \text{ if } I(c_i) \in [\ell, \ell + 1). \]

assuming indicator function $I$ scaled to the interval $[1, M + 1]$

Intersection:

▶ intersect two or more partitions to produce a new partition
▶ split multiply connected blocks into sets of singly connected cells
Merging:
If block $B$ violates the condition

$$I(B) |B| \geq \frac{N_L}{n} \bar{I}(\Omega) |\Omega|,$$

for a prescribed constant $N_L$, the block is merged with the neighbouring block $B'$ that has the closest indicator value, i.e.,

$$B' = \arg\min_{B'' \subset \mathcal{N}(B)} |I(B) - I(B'')|.$$ 

Refinement:
Refine blocks $B$ that violate the condition

$$I(B) |B| \leq \frac{N_U}{n} \bar{I}(\Omega) |\Omega|,$$

for a prescribed constant $N_U$.
Aarnes, Efendiev & Hauge (2007):

Use flow velocities to make a nonuniform grid in which each coarse block admits approximately the same total flow.
Example: non-uniform coarsening
Amalgamation: admissible directions (neighbourship)

Cell constraints:
- relperm / $p_c$ regions
- facies / rock types
- user supplied

Face constraints:
- faults
- user supplied

Horizons

Geometry:
- distance

Topology:
- face neighbours
- edge neighbours
- point neighbours
- ...
Amalgamation: extended neighbourship (topology)

5-neighborhood

9-neighborhood
Amalgamation: restricted neighbourship (topology)

Upper row: $\mathcal{N}(c_{ij}) = \{c_{i,j}\pm1\}$
Lower row: $\mathcal{N}(c_{ij}) = \{c_{i,j}\pm1, c_{i\pm1,j}, c_{i\pm1,j\pm1}\}$
Amalgamation: restricted neighbourship (facies)

Facies distribution Cartesian PEBI

Constraining to facies / saturation regions:

▶ useful to preserve heterogeneity
▶ useful to avoid upscaling $k_r$ and $p_c$ curves
Amalgamation: restricted neighbourship (satnum)

Realization from SAIGUP study, coarsening within six different saturation regions
Amalgamation: restricted neighbourship (faults)

5 × 5 partition
46 blocks

6 × 5 partition
52 blocks

unconstrained

constrained

52 blocks
58 blocks

Water-cut curves

Fine grid
5x5-based coarse grid
6x5-based coarse grid

Fine grid
5x5-based coarse grid with barrier
6x5-based coarse grid with barrier
Amalgamation: feasible directions (indicators)

Feasible directions

Cell

error estimates

a priori

sensitivity

a posteriori

ad hoc flow-based

transmissibilities

velocity

flux

multipliers

...
Example: flow-based indicators

General observations:

▶ Time-of-flight is typically a better indicator than velocity
▶ Velocity is a better indicator than vorticity
▶ Vorticity is a better indicator than permeability
▶ ... 

However, for smooth heterogeneities, the indicators tend to overestimate the importance of flow.
Example: hybrid methods

**Velocity + Cartesian partition:**

- $n \times m$
- intersect $\rightarrow$ merge $\rightarrow$ refine
- $\log |\vec{v}|$

**Time-of-flight + Cartesian partition:**

- $n \times m$
- intersect $\rightarrow$ merge $\rightarrow$ refine
- $-\log(\tau \tau_r)$
**Example: hybrid methods**

**Satnum + velocity + Cartesian:**

- $n \times m$
- \( \log |\vec{v}| \)
- \( \text{intersect} \rightarrow \text{merge} \rightarrow \text{refine} \)
Flow-adapted coarsening: summary

- Developed a general and flexible framework
  - Heuristic algorithms: good rather than optimal grid
  - Algorithmic components: partition, intersection, merging, refinement
  - Key concepts: flow indicator, admissible and feasible directions
- Systematic way of generating fit-for-purpose grids
  - Several existing methods appear as special cases
- Inclusion of geological information and expert knowledge important
  - Facies, saturation regions, surfaces, faults, etc.
  - Predefined shapes and topologies
**Coarse-grid discretisation**

**Bi-directional fluxes (upwind on fine scale):**

\[
S_{\ell}^{n+1} = S_{\ell}^{n} - \frac{\Delta t}{\phi_{\ell}|B_{\ell}|} \left[ f(S_{\ell}^{n+1}) \sum_{\partial B_{\ell}} \max(v_{ij}, 0) 
- \sum_{k \neq \ell} \left( f(S_{k}^{n+1}) \sum_{\Gamma_{k\ell}} \min(v_{ij}, 0) \right) \right].
\]

This gives a **centred scheme** on the coarse scale.

**Net fluxes:**

\[
S_{\ell}^{n+1} = S_{\ell}^{n} - \frac{\Delta t}{\phi_{\ell}|B_{\ell}|} \sum_{k \neq \ell} \max \left( f(S_{\ell}^{n+1}) \sum_{\Gamma_{k\ell}} v_{ij}, 
- f(S_{k}^{n+1}) \sum_{\Gamma_{k\ell}} v_{ij} \right).
\]

This gives an **upwind scheme** on the coarse scale.
Coarse-grid discretisation: numerical diffusion

Layer 37 from SPE10
Coarse-grid discretisation: matrix structure

Flow-adapted grid

Cartesian grid

Coarse-grid discretisation: numerical errors

Average errors over all layers of the two formations in SPE10
Average errors over all layers of the two formations in SPE10
Dynamical adaption

- Inaccurate representation of strong displacement fronts can lead to significant errors.
- **Idea:** Refine dynamically around strong fronts.
- For a Buckley–Leverett displacement:
  - Unswept region ahead of the displacement: coarse grid.
  - Swept region behind the front: coarse grid.
  - At the front: fine or intermediate grid.
Example: layers from SPE10

After injection of 0.1 PVI

adaptive  coarse  fine grid

Layer 22 from SPE10
Example: layers from SPE10

After injection of 0.5 PVI

adaptive  coarse  fine grid

Layer 22 from SPE10
Example: layers from SPE10

Layer 37 from SPE10