Multiscale Simulation of Highly Heterogeneous and Fractured Reservoirs

Astrid F. Gulbransen    Vera Louise Hauge
Jostein R. Natvig       Bård Skaflestad

Applied Mathematics, SINTEF ICT
Oslo, Norway

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Reservoir Simulation Group
Direct simulation of geomodels

Research group

- 3 researchers
- 4 postdocs
- 1–2 PhD students
- 3 programmers

Collaboration with national and international partners in industry and academia

Research vision

Direct simulation of complex grid models of highly heterogeneous and fractured porous media — a technology that bypasses the need for upscaling.

http://www.math.sintef.no/GeoScale/
Applications:
- Validation during development of geomodels
- Fast simulations of multiple realizations
- Optimization of production, well placement, etc
- History matching
- Geological storage of CO$_2$

Funding:
- Strategic research grant and PhD/postdoc grants
- Research grants with end-user involvement
- Industry projects
Challenges:

- Industry-standard grids are often nonconforming and contain skewed and degenerate cells.
- There is a trend towards unstructured grids.
- Standard discretization methods produce wrong results on skewed and rough cells.
- The combination of high aspect and anisotropy ratios can give very large condition numbers.
Aim:
To develop a pressure solver with improved accuracy and flexibility.

Solution:
- use a mimetic finite difference method to improve accuracy and to reduce grid sensitivity
- use a multiscale method to balance speed and accuracy.
Model:

\[ \lambda_t^{-1} K^{-1} \mathbf{v} + \nabla p = 0, \quad \text{(Darcy)}, \]
\[ \nabla \cdot \mathbf{v} = q, \]

Seek discrete \( p \) and \( \mathbf{v} \) that maintain

- mass balance
- a discrete form of Darcy’s law

On polyhedral grids, the mimetic method yields exact solutions for linear pressure.

In fact, this is better than many commercial simulators!
Multiscale-streamline simulation of fractured reservoir

The mimetic method (cont’d)

Standard method + skew grids = grid-orientation effects

\( K \): homogeneous and isotropic, symmetric well pattern \rightarrow \) symmetric flow

Streamlines with two-point method

Streamlines with mimetic method
Mixed and mimetic formulation for one grid block:

$$
\begin{bmatrix}
B & C^T \\
C & C^T
\end{bmatrix}
\begin{bmatrix}
v \\
p
\end{bmatrix}
= 
\begin{bmatrix}
-a \\
Q
\end{bmatrix}
$$

By eliminating $v$ we get

$$
CB^{-1} C^T p = Q + CB^{-1} a,
$$

MFEM: $B = \int_K \phi_i \cdot \lambda^{-1} K^{-1} \phi_j \, d\Omega$

Mimetic: $B^{-1} = \lambda_t NKN^T - \text{tr}(K)(1 - UU^T)$
Hybrid formulation:

\[
\begin{bmatrix}
B & C^T & D^T \\
C & \; & \; \\
D & \; & \;
\end{bmatrix}
\begin{bmatrix}
v \\
p \\
a
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
Q \\
0
\end{bmatrix}
\]

Elimination of \( p \) and \( v \) yields a positive definite system for \( a \).

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\[
B = \int_K \phi_i \cdot \lambda^{-1} K^{-1} \phi_j \, d\Omega
\]

Mimetic:

\[
B^{-1} = \lambda_t NKN^T - \text{tr}(K)(1 - UU^T)
\]
Pressure typically varies smoothly while velocity is largely determined by local heterogeneities.
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The multiscale/mixed pressure solver framework
An efficient alternative to upscaling methods

Key Idea
Express fluid flow in reservoir as a linear combination of local flow solutions on pairs of coarse grid blocks.
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Local flows account for small-scale impact on global flow field
  - Each localized flow field is obtained by resolving independent flow problems
  - Any method may be used to discretize these problems

End Result

High-resolution velocity field computable with comparatively few degrees of freedom (local problems resolved once or infrequently)
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Current research problems

- Performance on compressible problems (i.e. with gas)
- Adapting coarse grid to placement of wells
- How to efficiently represent fractures on coarse grids
- How to handle strongly pressure-dependent fluid data
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Modeling of two-phase flow in fractured porous media on unstructured non-uniformly coarsened grids

- We want to determine a coarse grid suitable for saturation simulations that preserves important characteristics of the flow.
- Investigate two coarsening strategies: Non-uniform coarsening and Explicit fracture-matrix separation

**Key ideas:**
- Velocity computed on a fine grid which resolves the fractures
- Saturation computed on the coarse grid

Homogeneous model with 100 fractures

Heterogeneous model with 100 fractures
Non-uniform coarsening algorithm

Two parameters:

- $V_{\text{min}}$: Minimum volume of a coarse block
- $G_{\text{max}}$: Maximum flow through each coarse block

The most important points from the algorithm:

- Group cells of similar flow magnitude into coarse blocks
- Coarse blocks have to be connected
- Avoid too small blocks
- Avoid too large blocks
Non-uniform coarsening algorithm

Coarse grid: Initial step, 152 cells

Coarse grid: Step 2, 47 cells

Coarse grid: Step 3, 95 cells

Coarse grid: Step 4, 69 cells

Note: Random coloring of blocks
Explicit Fracture-Matrix Separation (EFMS)

Step 1: Introduce an initial coarse grid, here $5 \times 5$

Step 2: Separate fracture and matrix part

Step 3: Split non-connected blocks

Initial model: $100 \times 100$ grid cells, 50 fracture lines

Disadvantage: Upscaling factor difficult to tune.
Water saturation equation for a water-oil system:

\[ S_m = S_m \text{ at previous time step} + \left[ \text{Flux in} - \text{Flux out} \right] \]

\( S_m \) = water saturation in coarse grid block \( m \).

- First-order finite volume method discretization
- Fluxes are computed as upstream fluxes with respect to the \emph{fine} grid fluxes on the coarse interfaces
Comparison of coarse grids: NUC, EFMS and Cartesian.

Heterogeneous model with 100 fractures
Saturations solutions at 0.48 PVI.

NUC grid with 206 blocks.

EFMS grid with 236 blocks.

20 × 20 Cartesian grid

Fine grid
Further research

- Capillary diffusion and gravity modeled on non-uniformly coarsened grids
- Compressible flow on non-uniformly coarsened grids
  ⇒ Black-oil model on non-uniformly coarsened grids