Operator Splitting of Advection and Diffusion on Non-uniformly Coarsened Grids

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Outline of presentation

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- Discretization of the saturation equation
  - Viscous part and diffusion part
- The two damping strategies
- Numerical examples
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  - Field scale example
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- Concluding remarks
Objectives and strategies

Overall objective:
- Fast flow simulations for high-resolution reservoir models.

Strategy:
- Reduce size of geomodel by using non-uniform grid coarsening.
  ⇒ Flow based grid: Keep important flow characteristics.
- Accompanied by multiscale pressure solvers.
Objective and strategies

Objective of this work:

- Include capillary pressure effects in fast saturation simulations on non-uniform coarse grids.
- Operator splitting to discretize the capillary diffusion separately from the advective term. Assumption: Viscous flow dominant.
- Straightforward projection in the coarse-grid discretization leads to overestimation of diffusion.

Strategy:

- Damping factors for the diffusion operator to correct for the overestimation of diffusion.
Background: Example of coarse grids

SPE10 model 2, layer 46. Original model $60 \times 220$ cells.
Random coloring: Shows shapes and sizes of coarse grid blocks.

Non-uniform coarse grid
319 blocks

Cartesian coarse grid
660 blocks

Non-uniform coarse grid: Flow based, keeps important flow characteristics in the grid.
Simulation results on coarse grids

Reference (13200)  Non-uniform (319)  Coarse Cartesian (660)

Saturation

Log(Velocity)

Note: Details of high-flow channels.
Numerical discretization

Splitting of the saturation equation:

Viscous part: \[ \phi \frac{\partial S}{\partial t} + \nabla \cdot (f_w \nu) = q_w \]

Diffusion part: \[ \phi \frac{\partial S}{\partial t} + \nabla \cdot d(S) \nabla S = 0 \]

Viscous part:
- First-order finite volume method discretization.
- Fluxes are computed as upstream fluxes with respect to the *fine* grid fluxes on the coarse interfaces.
Numerical discretization

Diffusion part:

Time: Semi-implicit backward Euler method:

\[
\phi S^{n+1} = \phi S^{n+1/2} - \Delta t \nabla \cdot d(S^{n+1/2}) \nabla S^{n+1}
\]

Space: Cell-centered finite-difference discretization.

- Fine grid: Two-point flux approximation:

\[
- \int_{\gamma_{ij}} d(S) \nabla S \cdot n_{ij} ds \approx -|\gamma_{ij}| \tilde{d}(S_i, S_j) \frac{S_i - S_j}{|x_i - x_j|}
\]

- Coarse grid: Projection of the fine-grid discretization onto the coarse grid.
Damping of diffusion

Overestimation

- Projection of diffusion operator onto coarse grid
  \[\Rightarrow\] Overestimates diffusion.

- Reason: Saturation gradient computed on fine grid, whereas saturation values represent net saturations in the coarse blocks.
Damping of diffusion: Illustration

Coarse Cartesian grid

Each coarse block consists of $n_x \times n_y$ cells.

Considering a coarse interface in the $x$-direction

Coarse grid diffusion operator:

$$- \sum_{n_y} \Delta y \, d(\gamma_{ij}) \frac{S_i - S_j}{\Delta x} = -\Delta y_c \, d(\Gamma_{ij}) \frac{S_i - S_j}{\Delta x}$$

Desired operator:

$$-\Delta y_c \, d(\Gamma_{ij}) \frac{S_i - S_j}{\Delta x_c}$$

Damping factor of the diffusion term: $\Delta x / \Delta x_c = 1/n_x$
Observation:

Capillary diffusion scales with the ratio in the size of coarse blocks relative to the size of fine cells.

Crude damping factor:

- $(\#\text{coarse blocks} / \#\text{fine cells})^{1/d}$
- Correct factor for square coarse blocks.
- Not sufficient for non-uniform coarse grids with complex geometries.

Fine damping:

- Use directly the geometry information from the fine grid to correct the coarse-grid diffusion operator.
- One factor for each coarse interface $⇒$ More computation.
Numerical examples: Pure capillary diffusion

Transport only driven by capillary diffusion.

Fine grid: 50 × 1 cells  50 × 5 cells
Uniform coarse grid: 10 × 1 blocks  50 × 1 blocks
Crude damping factor: $1/\sqrt{5}$

$\Delta x/\Delta x_c = 0.2$

$\Delta x/\Delta x_c = 1$

Overestimation

Underestimation
Numerical examples: Field scale example

Quarter five-spot, strong capillary diffusion:

$L^2$ error of saturation in different reservoirs

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Numerical examples: Field scale example

Water-cut curves

Homogeneous model with fractures

SPE model without fractures
Numerical examples: Aspect ratio

- Quarter five-spot models with homogeneous permeability field.
- Physical dimensions of 1, 100 and 1000 m in one direction and 1 m in the other (small to large aspect ratios).
Concluding remarks

Projection of the diffusion operator onto coarse grids overestimates the diffusion.

Crude damping sufficient:
- If coarse grid blocks are close to a square, with approximately the same number of fine cells in each direction and aspect ratio of order one.

Fine damping necessary:
- If the coarse grid blocks have large aspect ratios.
- Coarse blocks dissimilar in shape and size.
Thank you for your attention!

Questions?

http://www.sintef.no/GeoScale