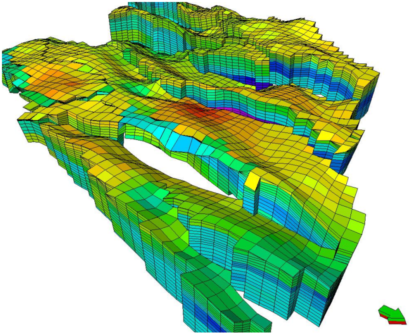
# Generic multiscale framework for reservoir simulation that takes geological models as input

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**T. A&M:** Yalchin Efendiev, Akhil Datta-Gupta

**NTNU:** Vegard Stenerud

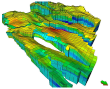
**Stanford:** Lou Durlofsky

**Today:**

Geomodels too large and complex for flow simulation:
Upscaling performed to obtain

- Simulation grid(s).
- Effective parameters and pseudofunctions.

Reservoir simulation workflow



Geomodel

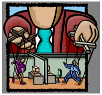
⟶

Upscaling

⟶

Flow simulation

⟶



Management

**Tomorrow:**

Earth Model shared between geologists and reservoir engineers —
Simulators take Earth Model as input, users specify grid-resolution to fit available computer resources and project requirements.

# Objective and implication

**Main objective:**

Build a generic framework for reservoir modeling and simulation capable of taking geomodels as input.

– *generic:* one implementation applicable to all types of models.

**Value:** Improved modeling and simulation workflows.

- Geologists may perform simulations to validate geomodel.
- Reservoir engineers gain understanding of geomodeling.
- Facilitate use of geomodels in reservoir management.

**Equations:**

- Pressure equation
  $c_t \frac{\partial p_o}{dt} + \nabla \cdot v + \sum_j c_j v_j \cdot \nabla p_o = q$
- Mass balance equation for each component

**Primary variables:**

- Darcy velocity $v$
- Liquid pressure $p_o$
- Phase saturations $s_j$, aqueous, liquid, vapor.

**Solution strategy:** Iterative sequential

$$\begin{aligned} v_{\nu+1} &= v(s_{j,\nu}), \\ p_{o,\nu+1} &= p_o(s_{j,\nu}), \end{aligned} \qquad s_{j,\nu+1} = s_j(p_{o,\nu+1}, v_{\nu+1}).$$

(Fully implicit with fixed point rather than Newton iteration).

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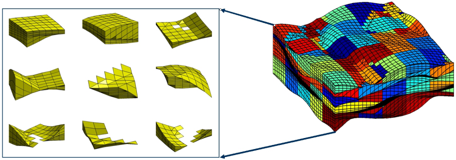
**Advantages with sequential solution strategy:**

- Grid for pressure and mass balance equations may be different.
- Multiscale methods may be used to solve pressure equation.
- Pressure eq. allows larger time-steps than mass balance eqs.

## Pressure equation:

- **Solution grid:** Geomodel — no effective parameters.
- **Discretization:** Multiscale mixed / mimetic method

**Coarse grid:** obtained by up-gridding in index space

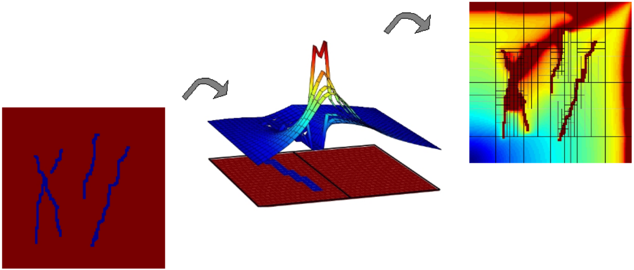


## Mass balance equations:

- **Solution grid:** Non-uniform coarse grid.
- **Discretization:** Two-scale upstream weighted FV method — integrals evaluated on geomodel.
- **Pseudofunctions:** No.

**Multiscale mixed/mimetic method (4M)**
Generic two-scale approach to discretizing the pressure equation:

- Mixed FEM formulation on coarse grid.
- Flow patterns resolved on geomodel with mimetic FDM.



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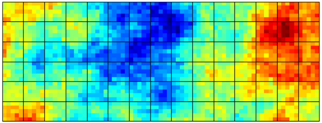

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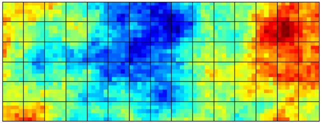

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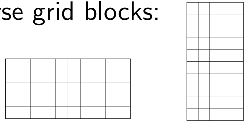
Coarse grid blocks:



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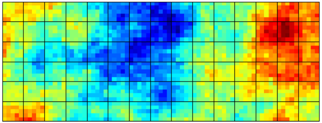

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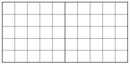

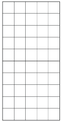

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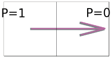


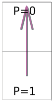

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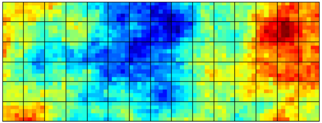

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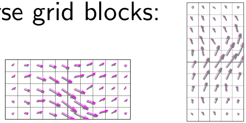

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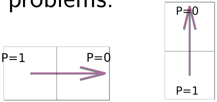
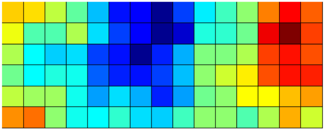


# Multiscale mixed/mimetic method

## Flow based upscaling versus multiscale method

### Standard upscaling:



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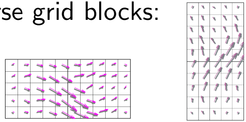

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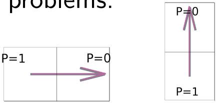

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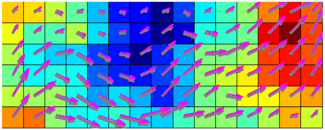

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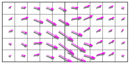

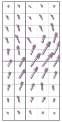

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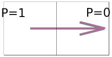


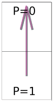

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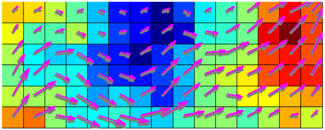

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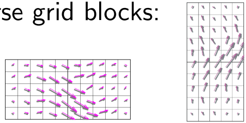

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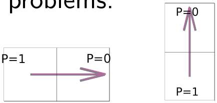
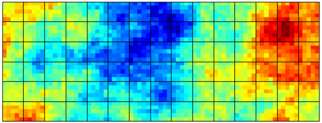



## Multiscale method (4M):



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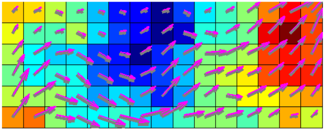

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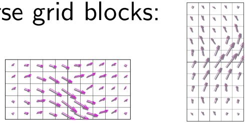

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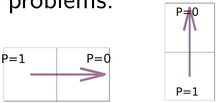


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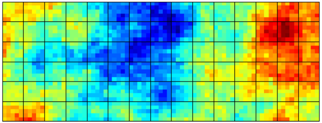

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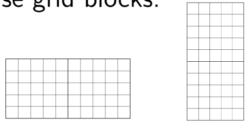

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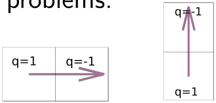

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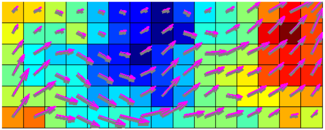

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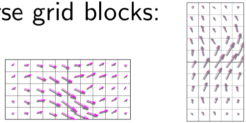

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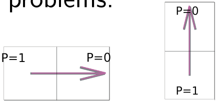


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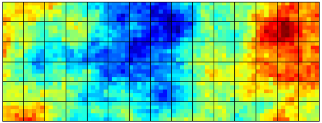

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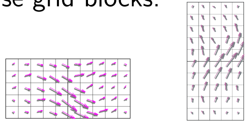

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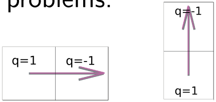

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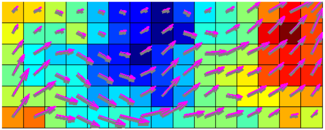

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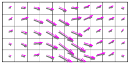

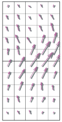

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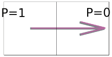


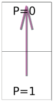


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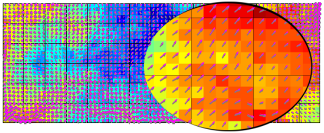

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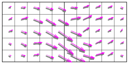

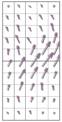

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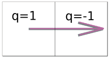
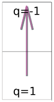


Discrete hybrid formulation: $(u,v)_m = \int_{T_m} u \cdot v \, dx$

Find $v \in V$, $p \in U$, $\pi \in \Pi$ such that for all blocks $T_m$ we have

$$\begin{aligned}
(\lambda^{-1} v, u)_m - (p, \nabla \cdot u)_m + \int_{\partial T_m} \pi u \cdot n \, ds &= (\omega g \nabla D, u)_m \\
(c_t \tfrac{\partial p_o}{dt}, l)_m + (\nabla \cdot v, l)_m + (\textstyle\sum_j c_j v_j \cdot \nabla p_o, l)_m &= (q, l)_m \\
\int_{\partial T_m} \mu v \cdot n \, ds &= 0.
\end{aligned}$$

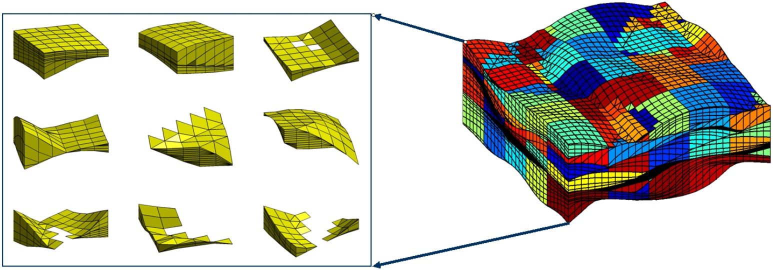
for all $u \in V$, $l \in U$ and $\mu \in \Pi$.

Solution spaces and variables: $\mathcal{T} = \{T_m\}$

| | | |
|---|---|---|
| $V \subset H^{\text{div}}(\mathcal{T})$, | $U = \mathcal{P}_0(\mathcal{T})$, | $\Pi = \mathcal{P}_0(\{\partial T_m \cap \partial T_n\})$. |
| $v$ = velocity, | $p$ = block pressures, | $\pi$ = interface pressures. |

Each coarse grid block is a connected set of cells from geomodel.

**Example:** Coarse grid obtained with uniform coarsening in index space.



**Grid adaptivity at well locations:**

One block assigned to each cell in geomodel with well perforation.

### Definition of approximation space for velocity:

The approximation space $V$ is spanned by basis functions $\psi_m^i$ that are designed to embody the impact of fine-scale structures.

### Definition of basis functions:

For each pair of adjacent blocks $T_m$ and $T_n$, define $\psi$ by

$$\begin{aligned} \psi &= -K\nabla u \text{ in } T_m \cup T_n, \\ \psi \cdot n &= 0 \text{ on } \partial(T_m \cup T_n), \end{aligned} \qquad \nabla \cdot \psi = \begin{cases} w_m & \text{in } T_m, \\ -w_n & \text{in } T_n, \end{cases}$$

Split $\psi$: $\quad \psi_m^i = \psi|_{T_m}, \quad \psi_n^j = -\psi|_{T_n}.$

**Basis functions time-independent if $w_m$ is time-independent.**

### Role of weight functions

Let $(w_m, 1)_m = 1$ and let $v_m^i$ be coarse-scale coefficients.

$$v = \sum_{m,i} v_m^i \psi_m^i \quad \Rightarrow \quad (\nabla \cdot v)|_{T_m} = w_m \sum_i v_m^i.$$

$\longrightarrow$ $w_m$ gives distribution of $\nabla \cdot v$ among cells in geomodel.

### Choice of weight functions

$$\nabla \cdot v \sim c_t \frac{\partial p_o}{dt} + \sum_j c_j v_j \cdot \nabla p_o$$

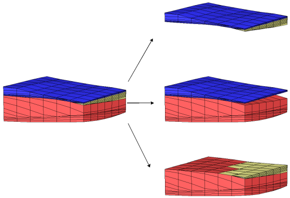
- Use adaptive criteria to decide when to redefine $w_m$.
- Use $w_m = \phi$ ($c_t \sim \phi$ when saturation is smooth).

$\longrightarrow$ **Basis functions computed once, or updated infrequently.**

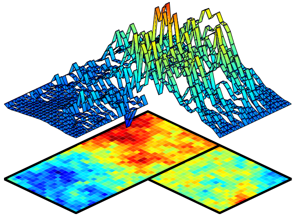
# Multiscale mixed/mimetic method
## Workflow

### At initial time

Detect all adjacent blocks



Compute $\psi$ for each domain



### For each time-step:

- Assemble and solve coarse grid system.
- Recover fine grid velocity.
- Solve mass balance equations.

### Velocity basis functions computed using mimetic FDM

Mixed FEM for which the inner product $(u, \sigma v)$ is replaced with an approximate explicit form ($u, v \in H^{\mathrm{div}}$ and $\sigma$ SPD),
— no integration, no reference elements, no Piola mappings.

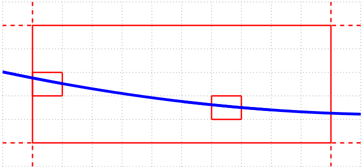
May also be interpreted as a multipoint finite volume method.

### Properties:

- Exact for linear pressure.
- Same implementation applies to all grids.
- Mimetic inner product *needed* to evaluate terms in multiscale formulation, e.g., $(\psi_m^i, \lambda^{-1}\psi_m^j)$ and $(\omega g \nabla D, \psi_{m,j})$.

Grid block for cells with a well

- correct well-block pressure
- no near well upscaling
- free choice of well model.



## Alternative well models

1. Peaceman model:

   $$q_{\text{perforation}} = -W_{\text{block}}(p_{\text{block}} - p_{\text{perforation}}).$$

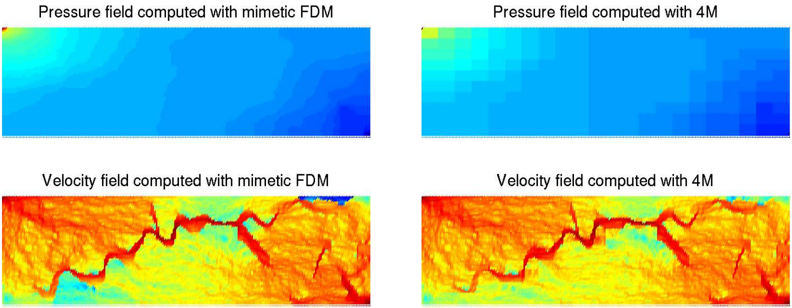
   Calculation of well-index grid dependent.

2. Exploit pressures on grid interfaces:

   $$q_{\text{perforation}} = -\sum_i W_{\text{face}i}(p_{\text{face}i} - p_{\text{perforation}}).$$

   Generic calculation of $W_{\text{face}i}$.

**Example:** Layer 36 from SPE10 (Christie and Blunt, 2001).

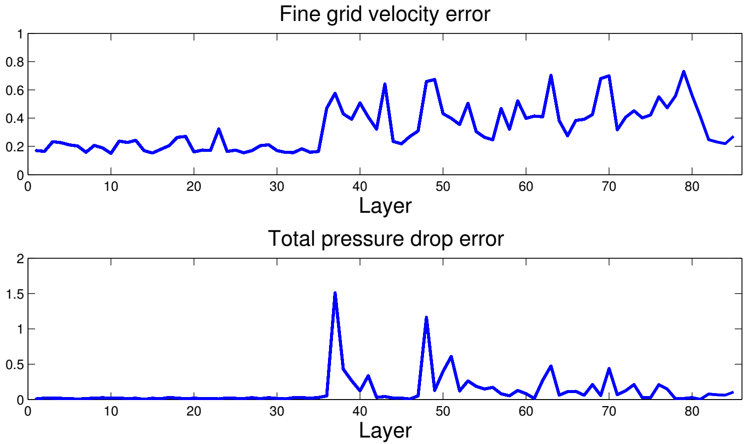



## Primary features

- Coarse pressure solution, subgrid resolution at well locations.
- Coarse velocity solution with subgrid resolution everywhere.

# Multiscale mixed/mimetic method

Example: Pressure and velocity errors for each layer of SPE10. Coarse grid: $5 \times 11$.

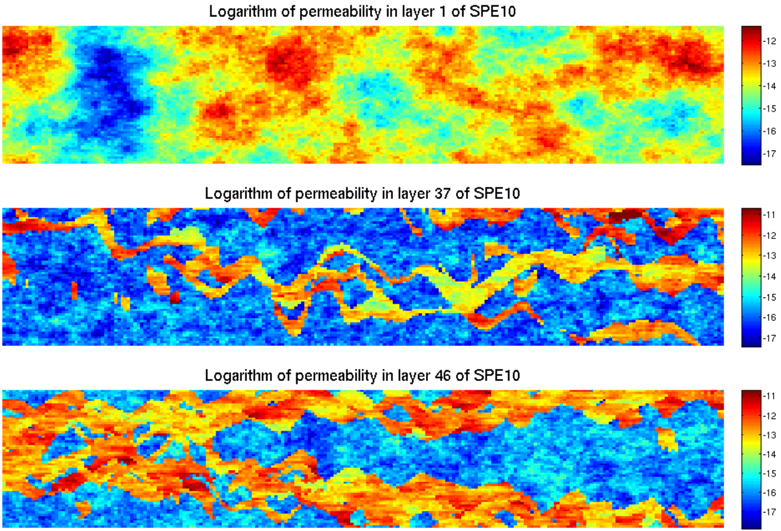



**Observation:**

Consistent velocity accuracy, occasionally large pressure drop error.

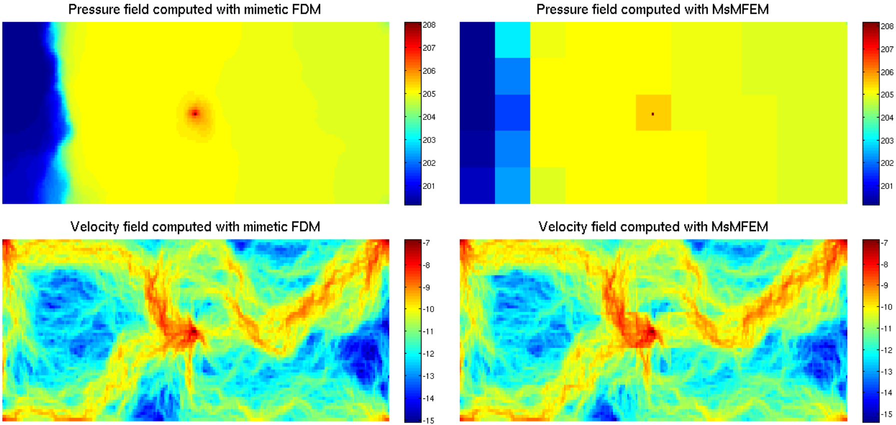
# Multiscale mixed/mimetic method

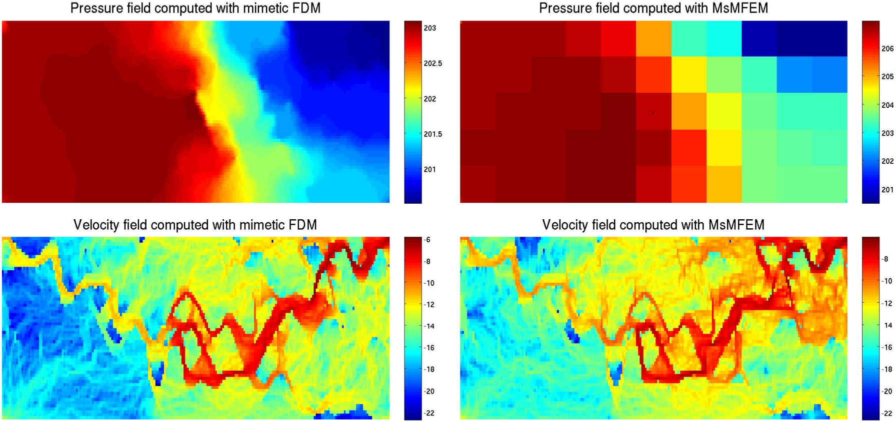
## Layer 1, 37 and 46 of SPE10



# Multiscale mixed/mimetic method

## Pressure and velocity fields for layer 1 of SPE10

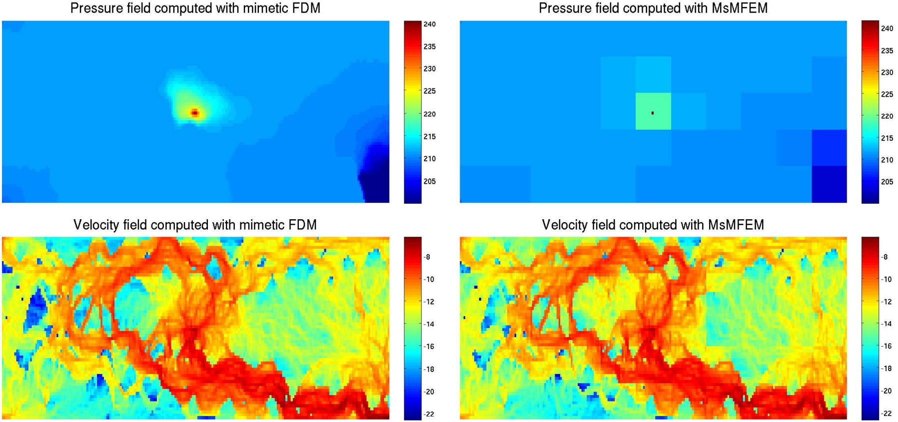




Pressure field computed with mimetic FDM
203
202.5
202
201.5
201
Pressure field computed with MsMFEM
206
205
204
203
202
201
Velocity field computed with mimetic FDM
-6
-8
-10
-12
-14
-16
-18
-20
-22
Velocity field computed with MsMFEM
-8
-10
-12
14
-16
-18
-20
-22

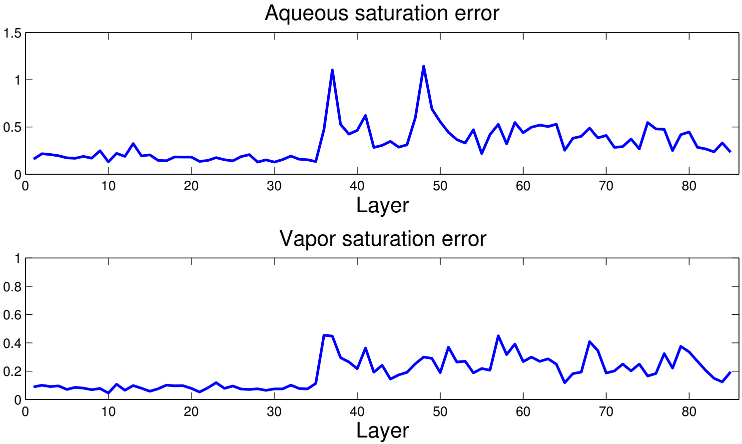
# Multiscale mixed/mimetic method

## Pressure and velocity fields for layer 46 of SPE10



# Multiscale mixed/mimetic method

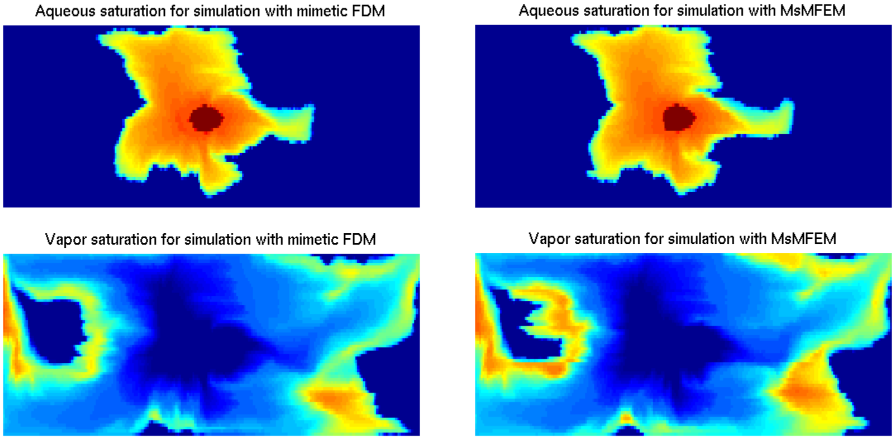
Example: Saturation errors for each layer of SPE10. Coarse grid: $5 \times 11$.




Saturation errors after 0.2 PVI of gas injection and 0.2 PVI of water injection: Aqueous saturation error peaks coorespond to pressure drop error peaks.

# Multiscale mixed/mimetic method

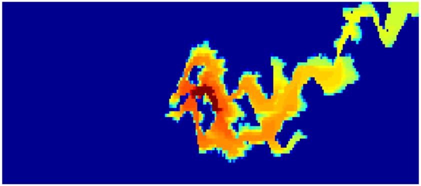
## Saturation fields after 0.2 PVI of gas injection and 0.2 PVI of water injection for layer 1



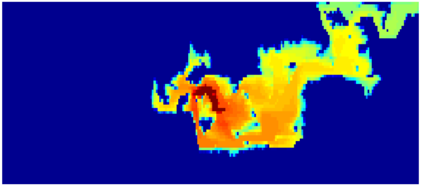
# Multiscale mixed/mimetic method

## Saturation fields after 0.2 PVI of gas injection and 0.2 PVI of water injection for layer 37

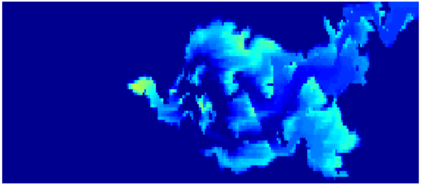
Aqueous saturation for simulation with mimetic FDM



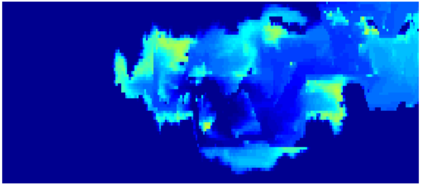
Aqueous saturation for simulation with MsMFEM



Vapor saturation for simulation with mimetic FDM

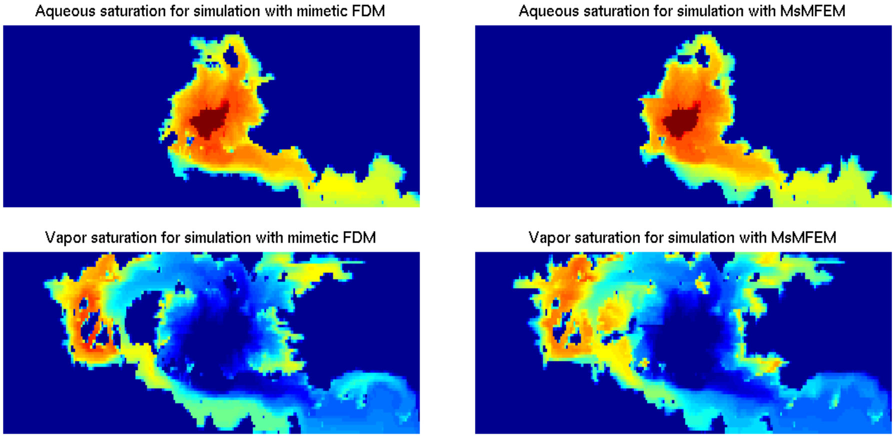


Vapor saturation for simulation with MsMFEM



# Multiscale mixed/mimetic method

## Saturation fields after 0.2 PVI of gas injection and 0.2 PVI of water injection for layer 46



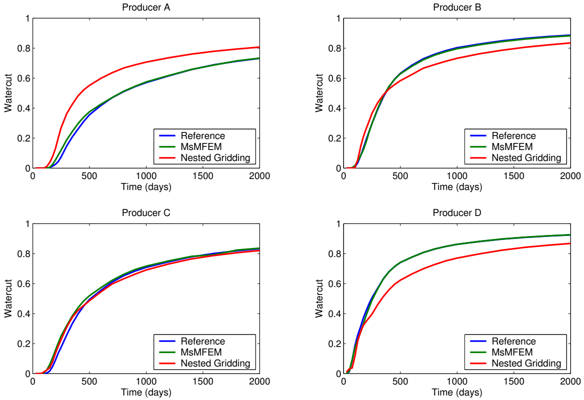
**Model:** SPE10 model 2, 1.1 M cells, 1 injector, 4 producers.

Coarse grid:
$5 \times 11 \times 17$

- Reference
- 4M
- Upscaling + downscaling

**4M+streamlines:**
$\sim$ 2 minutes on desktop PC.

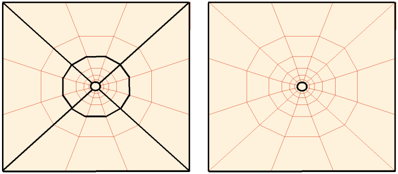
Water-cut curves at producers A–D

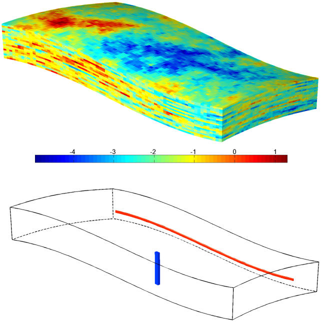


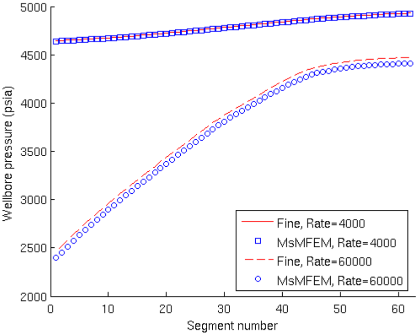
Krogstad and Durlofsky, 2007:

Fine grid to annulus,
block for each well segment

- No well model needed.
- Drift-flux wellbore flow.

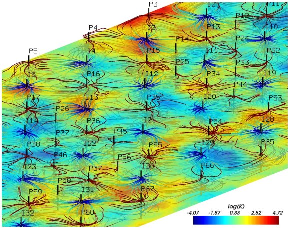


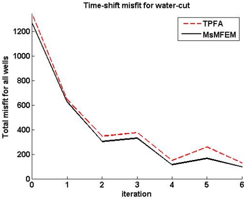


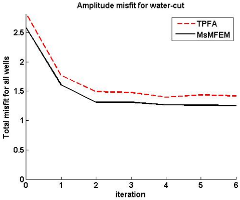
Stenerud, Kippe, Datta-Gupta, and Lie, RSS 2007:

- 1 million cells, 32 injectors, and 69 producers
- Matching travel-time and water-cut amplitude at producers
- Permeability updated in blocks with high average sensitivity
  $\longrightarrow$ Only few multiscale basis functions updated.




Time-residual




Amplitude-residual

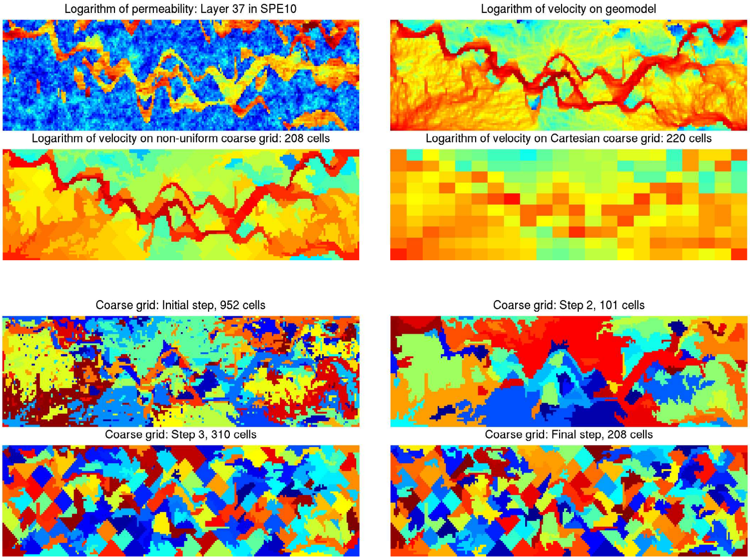
**Computation time:** $\sim$ 17 min. on desktop PC. (6 iterations).

**Task:** Given ability to model velocity on geomodels, and transport on coarse grids:

Find a suitable coarse grid that resolves flow patterns and minimize accuracy loss.

# Coarse grid for solving mass balance equations

Example: Layer 37 SPE10 (Christie and Blunt), 5 spot well pattern.

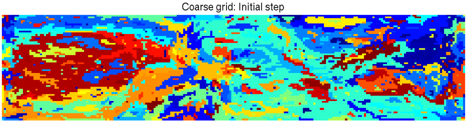


# Grid generation procedure

## Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

**Separate:** Define $g = \ln|v|$ and $D = (\max(g) - \min(g))/10$.

$$\text{Region } i = \{c : \min(g) + (i-1)D < g(c) < \min(g) + iD\}.$$




**Initial grid: connected subregions — 733 blocks**

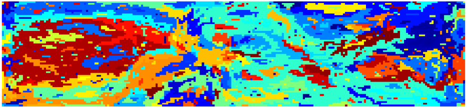
# Grid generation procedure

Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

**Separate:** Define $g = \ln|v|$ and $D = (\max(g) - \min(g))/10$.

$$\text{Region } i = \{c : \min(g) + (i-1)D < g(c) < \min(g) + iD\}.$$

Coarse grid: Initial step

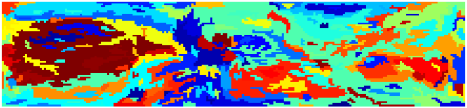


**Initial grid: connected subregions — 733 blocks**

**Merge:** If $|B| < c$, merge $B$ with a neighboring block $B'$ with

$$\frac{1}{|B|}\int_B \ln|v| dx \approx \frac{1}{|B'|}\int_{B'} \ln|v|\, dx$$

Coarse grid: Step 2

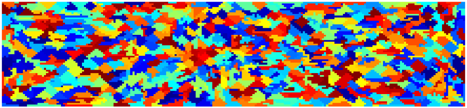


**Step 2: 203 blocks**

**Refine:** If criteria — $\int_B \ln |v| dx < C$ — is violated, do

- Start at $\partial B$ and build new blocks $B'$ that meet criteria.
- Define $B = B \backslash B'$ and progress inwards until $B$ meets criteria.

Coarse grid: Step 3



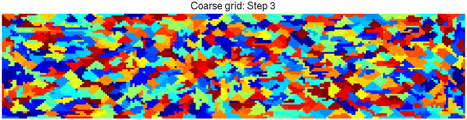
**Step3: 914 blocks**

# Grid generation procedure

## Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

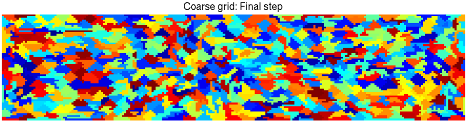
**Refine:** If criteria — $\int_B \ln|v|dx < C$ — is violated, do

- Start at $\partial B$ and build new blocks $B'$ that meet criteria.
- Define $B = B\backslash B'$ and progress inwards until $B$ meets criteria.



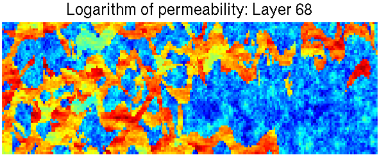

**Step3: 914 blocks**

**Cleanup:** Merge small blocks with adjacent block.

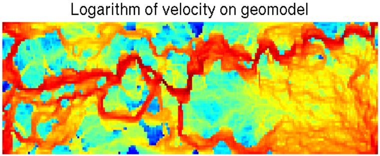


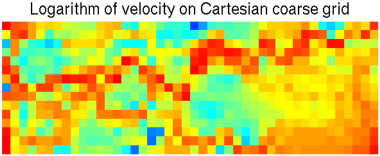

**Final grid: 690 blocks**

# Layer 68 SPE10, 5 spot well pattern

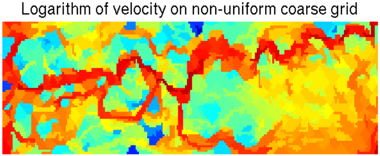



Geomodel: 13200 cells

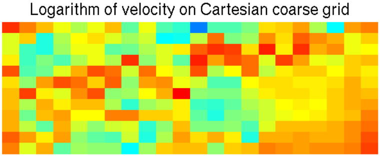


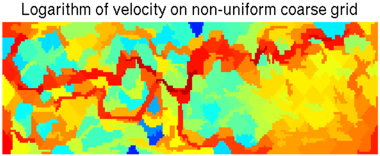

Coarse grid: 660 cells




Coarse grid: 649 cells




Coarse grid: 264 cells




Coarse grid: 257 cells

## Experimental setup:

**Model:** Incompressible two-phase flow (oil and water).

**Initial state:** Completely oil-saturated.

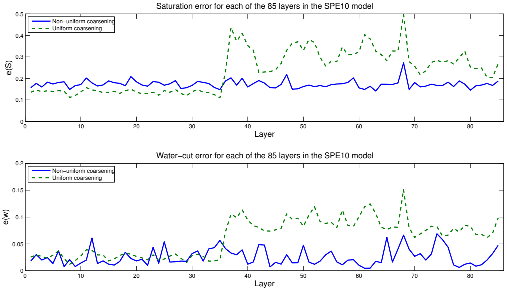
**Relative permeability:** $k_{rj} = s_j^2, \quad 0 \leq s_j \leq 1.$

**Viscosity ratio:** $\mu_o/\mu_w = 10.$

## Error measures: (Time measured in PVI)

**Saturation error:** $e(S) = \int_0^1 \frac{\|S(\cdot,t) - S_{\text{ref}}(\cdot,t)\|_{L^1(\Omega)}}{\|S_{\text{ref}}(\cdot,t)\|_{L^1(\Omega)}}\, dt.$

**Water-cut error:** $e(w) = \|w - w_{\text{ref}}\|_{L^2([0,1])}/\|w_{\text{ref}}\|_{L^2([0,1])}.$




Geomodel: $60 \times 220 \times 1$

Uniform grid: $15 \times 44 \times 1$

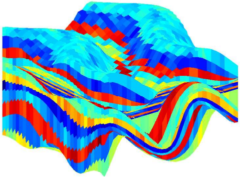
Non-uni. grid: 619–734 blocks

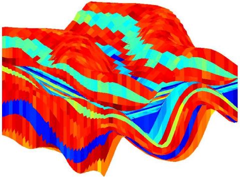
**Observations:**

- First 35 layers smooth $\Rightarrow$ Uniform grid adequate.
- Last 50 layers fluvial $\Rightarrow$ Uniform grid inadequate.
- Non-uniform grid gives consistent results for all layers.

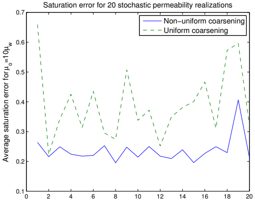
# Example 2: Geomodel = unstructured corner-point grid

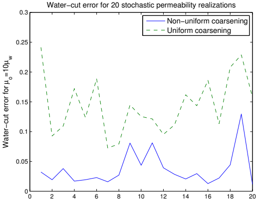
20 realizations from lognormal distribution, Q-of-5-spot well pattern, upsc. factor $\sim$ 25





$\Leftarrow$ 2 realizations.
Geomodel:
15206 cells




Uniform grid:
838 blocks

Non-uni. grid:
647–704 blocks

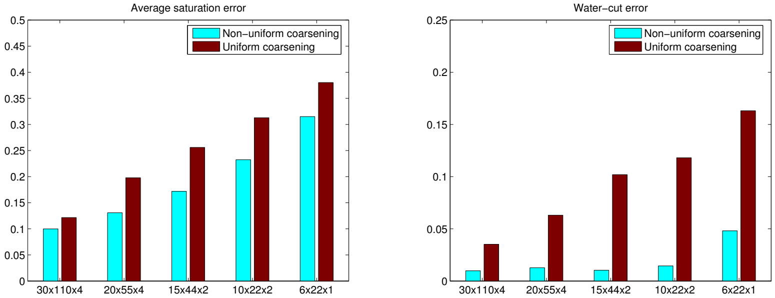
**Observations:**

- Coarsening algorithm applicable to unstructured grids
  — accuracy consistent with observations for SPE10 models.
- Results obtained with uniform grid (in index space) inaccurate.

# Example 3: Geomodel = four bottom layers from SPE10

Robustness with respect to degree of coarsening, 5-spot well pattern

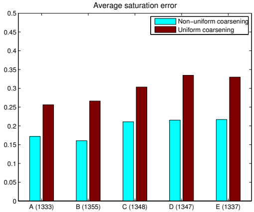
| | Number of cells in grid (upscaling factor 4–400) | | | | |
|---|---|---|---|---|---|
| Uniform grid | 30x110x4<br>13200 | 20x55x4<br>4400 | 15x44x2<br>1320 | 10x22x2<br>440 | 6x22x1<br>132 |
| Non-U. grid | 7516 | 3251 | 1333 | 419 | 150 |

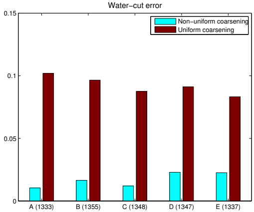



**Observations:**

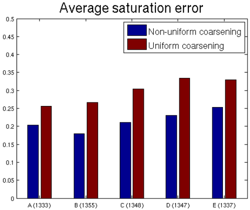
- Non-uniform grid gives better accuracy than uniform grid.
- Water-cut error almost grid-independent for non-uniform grid.

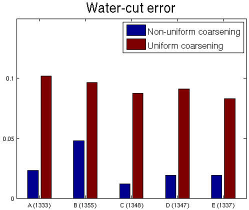
Grid generated with respective well patterns.




Grid generated with pattern C



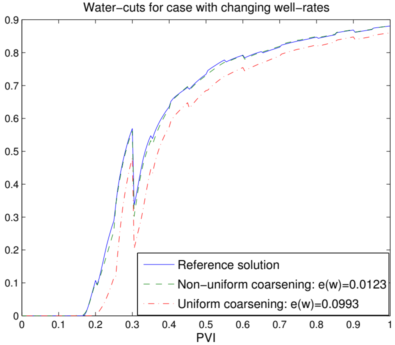



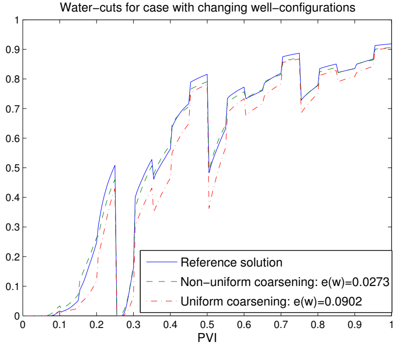

**Observation:**

Grid resolves high-permeable regions with good connectivity
— Grid need *not* be regenerated if well pattern changes.

# Example 5: Geomodel = four bottom layers from SPE10

Robustness with respect changing well positions and well rates, upscaling factor $\sim 40$




5-spot, random prod. rates
grid generated with equal rates

well patterns: 4 cycles A–E
grid generated with pattern C

**Observations:**

- NU water-cut tracks reference curve closely: 1%–3% error.
- Uniform grid gives $\sim$ 10% water-cut error.

# Conclusions

**Multiscale mixed/mimetic method:**

- Reservoir simulation tool that can take geomodels as input.
- Solutions in close correspondence with solutions obtained by solving the pressure equation directly.
- Computational cost comparable to flow based upscaling.

**Applications:**

- Reservoir simulation on geomodels
- Near-well modeling / Improved well models
- History matching on geomodels

**Potential value for industry:**

Improved modeling and simulation workflows.

# Conclusions

**Coarse grid for mass balance equations:**

- A generic semi-automated algorithm for generating coarse grids that resolve flow patterns has been presented.
- Solutions are significantly more accurate than solutions obtained on uniform coarse grids with similar number of cells.
- Water-cut error: 1%–3% — pseudofunctions superfluous.
- Grid need **not** be regenerated when flow conditions change!

**Potential application:**

User-specified grid-resolution to fit available computer resources.

**Relation to other methods:**

Belongs to family of flow-based grids[a]: designed for flow scenarios where heterogeneity, rather than gravity, dominates flow patterns.

[a] Garcia, Journel, Aziz (1990,1992), Durlofsky, Jones, Milliken (1994,1997)

# I have a dream ...