Generic multiscale framework for reservoir simulation that takes geological models as input

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Reservoir simulation workflow today:

Geomodel → Upscaling → Flow simulation → Management

Tomorrow:
- Earth Model shared between geologists and reservoir engineers
- Simulators take Earth Model as direct input
- Users allowed to specify grid-resolution at runtime to fit available computer resources and project requirements
Objective and implication

**Main objective:**
Build a generic framework for reservoir modeling and simulation capable of taking geomodels as input.

- *generic:* one implementation applicable to all types of models.

**Value:** Improved modeling and simulation workflows.
- Geologists may perform simulations to validate geomodel.
- Reservoir engineers gain understanding of geomodeling.
- Facilitate use of geomodels in reservoir management.
Discretization

Pressure equation:

- **Solution grid**: Geomodel — no effective parameters.
- **Discretization**: Multiscale mixed / mimetic method

Coarse grid: obtained by up-gridding in index space

Mass balance equations:

- **Solution grid**: Non-uniform coarse grid.
- **Discretization**: Two-scale upstream weighted FV method — integrals evaluated on geomodel.
- **Pseudofunctions**: No.
Multiscale mixed/mimetic method (4M)
Generic two-scale approach to discretizing the pressure equation:

- Mixed FEM formulation on coarse grid.
- Flow patterns resolved on geomodel with mimetic FDM.
Standard upscaling:
Multiscale mixed/mimetic method

Flow based upscaling versus multiscale method

**Standard upscaling:**

![Image of standard upscaling]

↓

Coarse grid blocks:

1. First coarse grid block
2. Second coarse grid block

Multiscale method (4M):

![Image of multiscale method]

⇓

Coarse grid blocks:
Multiscale mixed/mimetic method
Flow based upscaling versus multiscale method

**Standard upscaling:**

Coarse grid blocks:

Flow problems:

P=1  P=0

P=0  P=1
Multiscale mixed/mimetic method
Flow based upscaling versus multiscale method

**Standard upscaling:**

Coarse grid blocks:

Flow problems:
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**Standard upscaling:**

![Standard upscaling diagram]

Coarse grid blocks:

![Coarse grid block diagram]

Flow problems:

![Flow problem diagrams]
**Standard upscaling:**

![Diagram of standard upscaling]

Coarse grid blocks:

![Diagram of coarse grid blocks]

Flow problems:

- $P=1$
- $P=0$
- $P=0$
- $P=1$
Multiscale mixed/mimetic method
Flow based upscaling versus multiscale method

**Standard upscaling:**

![Image of standard upscaling](image1)

**Multiscale method (4M):**

![Image of multiscale method](image2)

- Coarse grid blocks:
  - Standard upscaling:
    - Coarse grid blocks:
      - Flow problems:
        - $P=1$ to $P=0$
        - $P=0$ to $P=1$
Multiscale mixed/mimetic method
Flow based upscaling versus multiscale method

Standard upscaling:

Coarse grid blocks:

Flow problems:

Multiscale method (4M):

Coarse grid blocks:

Flow problems:
Multiscale mixed/mimetic method
Flow based upscaling versus multiscale method

**Standard upscaling:**

- Coarse grid blocks:
- Flow problems:
  - \( P=1 \) \( \rightarrow P=0 \)
  - \( P=0 \) \( \rightarrow P=1 \)

**Multiscale method (4M):**

- Coarse grid blocks:
- Flow problems:
  - \( q=1 \) \( \rightarrow q=-1 \)
  - \( q=-1 \) \( \rightarrow q=1 \)
Multiscale mixed/mimetic method
Flow based upscaling versus multiscale method

**Standard upscaling:**

- Coarse grid blocks:
- Flow problems:
  - \( P=1 \)
  - \( P=0 \)

**Multiscale method (4M):**

- Coarse grid blocks:
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  - \( q=1 \)
  - \( q=-1 \)
  - \( q=1 \)
Multiscale mixed/mimetic method

Hybrid formulation of pressure equation: No-flow boundary conditions

Discrete hybrid formulation: \((u, v)_m = \int_{T_m} u \cdot v \, dx\)

Find \(v \in V, \, p \in U, \, \pi \in \Pi\) such that for all blocks \(T_m\) we have

\[
(\lambda^{-1} v, u)_m - (p, \nabla \cdot u)_m + \int_{\partial T_m} \pi u \cdot n \, ds = (\omega g \nabla D, u)_m
\]

\[
(c_t \frac{\partial p_o}{\partial t}, l)_m + (\nabla \cdot v, l)_m + (\sum_j c_j v_j \cdot \nabla p_o, l)_m = (q, l)_m
\]

\[
\int_{\partial T_m} \mu v \cdot n \, ds = 0.
\]

for all \(u \in V, \, l \in U\) and \(\mu \in \Pi\).

Solution spaces and variables: \(\mathcal{T} = \{T_m\}\)

\(V \subset H^{\text{div}}(\mathcal{T}), \quad U = P_0(\mathcal{T}), \quad \Pi = P_0(\partial T_m \cap \partial T_n).\)

\(v = \text{velocity}, \quad p = \text{block pressures}, \quad \pi = \text{interface pressures}.\)
Definition of approximation space for velocity:
The approximation space $V$ is spanned by basis functions $\psi^i_m$ that are designed to embody the impact of fine-scale structures.

Definition of basis functions:
For each pair of adjacent blocks $T_m$ and $T_n$, define $\psi$ by

$$\begin{align*}
\psi &= -K\nabla u \text{ in } T_m \cup T_n, \\
\psi \cdot n &= 0 \text{ on } \partial(T_m \cup T_n), \\
\nabla \cdot \psi &= \begin{cases} 
  w_m & \text{in } T_m, \\
  -w_n & \text{in } T_n,
\end{cases}
\end{align*}$$

Split $\psi$: $\psi^i_m = \psi|_{T_m}$, $\psi^j_n = -\psi|_{T_n}$. 
Multiscale mixed/mimetic method

Workflow

At initial time
Detect all adjacent blocks

Compute $\psi$ for each domain

For each time-step:
- Assemble and solve coarse grid system.
- Recover fine grid velocity.
- Solve mass balance equations.
Example: Layer 36 from SPE10 (Christie and Blunt, 2001).

**Primary features**
- Coarse pressure solution, subgrid resolution at well locations.
- Coarse velocity solution with subgrid resolution everywhere.
Model: SPE10 model 2, 1.1 M cells, 1 injector, 4 producers.

Coarse grid: 5 × 11 × 17
- Reference
- 4M
- Upscaling + downscaling

4M + streamlines: ~ 2 minutes on desktop PC.

Water-cut curves at producers A–D

![Producer A](image1)
![Producer B](image2)
![Producer C](image3)
![Producer D](image4)
Modeling transport on the fine grid (e.g. geomodel) may be a bottle neck or infeasible.

**Question:** Can we derive a coarse grid formulation that exploits the key information in the high resolution velocity field?
Coarse grid formulation of mass balance equations
Utilizing high resolution velocity fields and avoiding pseudofunctions

Modeling transport on the fine grid (e.g. geomodel) may be a bottle neck or infeasible.

**Question:** Can we derive a coarse grid formulation that exploits the key information in the high resolution velocity field?

**Yes,** by using a coarse grid that resolves flow patterns.

![Logarithm of permeability: Layer 37 in SPE10](image1)
![Logarithm of velocity on geomodel](image2)
![Logarithm of velocity on non-uniform coarse grid: 208 cells](image3)
![Logarithm of velocity on Cartesian coarse grid: 220 cells](image4)
Coarse grid formulation of mass balance equations
Utilizing high resolution velocity fields and avoiding pseudofunctions

Modeling transport on the fine grid (e.g. geomodel) may be a bottle neck or infeasible.

**Question:** Can we derive a coarse grid formulation that exploits the key information in the high resolution velocity field?

**How:** Separate, clean, refine, cleanup.
Grid generation procedure
Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

**Separate:** Define $g = \ln |v|$ and $D = (\max(g) - \min(g))/10$.

Region $i = \{ c : \min(g) + (i - 1)D < g(c) < \min(g) + iD \}$.

Initial grid: connected subregions — 733 blocks
Grid generation procedure
Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

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Initial grid: connected subregions — 733 blocks

Merge: If $|B| < c$, merge $B$ with a neighboring block $B'$ with

$$\frac{1}{|B|} \int_B \ln |v| \, dx \approx \frac{1}{|B'|} \int_{B'} \ln |v| \, dx$$

Step 2: 203 blocks
Grid generation procedure
Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

**Refine:** If criteria — \( \int_B \ln |v| \, dx < C \) — is violated, do

- Start at \( \partial B \) and build new blocks \( B' \) that meet criteria.
- Define \( B = B \setminus B' \) and progress inwards until \( B \) meets criteria.

Coarse grid: Step 3

Step3: 914 blocks
Grid generation procedure
Example: Layer 1 SPE10 (Christie and Blunt), 5 spot well pattern

**Refine:** If criteria — $\int_B \ln|v|dx < C$ — is violated, do
- Start at $\partial B$ and build new blocks $B'$ that meet criteria.
- Define $B = B \setminus B'$ and progress inwards until $B$ meets criteria.

**Step3:** 914 blocks

**Cleanup:** Merge small blocks with adjacent block.

**Final grid:** 690 blocks
Layer 68 SPE10, 5 spot well pattern

Logarithm of permeability: Layer 68

Geomodel: 13200 cells

Logarithm of velocity on Cartesian coarse grid

Coarse grid: 660 cells

Logarithm of velocity on non-uniform coarse grid

Coarse grid: 649 cells

Logarithm of velocity on Cartesian coarse grid

Coarse grid: 264 cells

Logarithm of velocity on non-uniform coarse grid

Coarse grid: 257 cells
**Performance study**

**Model:** Incompressible and immiscible two-phase flow (oil and water) without effects from gravity and capillary forces.

**Initial state:** Completely oil-saturated.

**Parameters:** \( k_{r,j} = s_j^2 \), \( 0 \leq s_j \leq 1 \), and \( \mu_o/\mu_w = 10 \).

**Coarse grid formulation**

Two-scale first order upstream-weighted finite volume method:

\[
\Delta S_{w,i} = \frac{\triangle t}{\int_{V_i} \phi} \left( \int_{V_i} q_w \, dx - \int_{\partial V_i} f_w(S_w) v_w \cdot n \, ds \right)
\]

**Error measures:** \( t = \text{PVI}, \ w = \text{water-cut}, \ r = \text{reference solution}. \)

\[
e(S) = \int \left( \| S(\cdot, t) - S_r(\cdot, t) \|_{L^1} / \| S_r(\cdot, t) \|_{L^1} \right) \, dt.
\]

\[
e(w) = \| w - w_r \|_{L^2} / \| w_r \|_{L^2}.
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Performance study

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Two-scale first order upstream-weighted finite volume method:

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\]

**Error measures:** \( t = \text{PVI}, \ w = \text{water-cut}, \ r = \text{reference solution}. \)

\[
e(S) = \int \left( \frac{\|S(\cdot, t) - S_r(\cdot, t)\|_{L^1}}{\|S_r(\cdot, t)\|_{L^1}} \right) dt.
\]

\[
e(w) = \|w - w_r\|_{L^2} / \|w_r\|_{L^2}.
\]
Example 1: Geomodel = individual layers from SPE10
5-spot well pattern, upscaling factor ∼ 20

Observations:

- First 35 layers smooth ⇒ Uniform grid adequate.
- Last 50 layers fluvial ⇒ Uniform grid inadequate.
- Non-uniform grid gives consistent results for all layers.
Example 2: Geomodel = unstructured corner-point grid
20 realizations from lognormal distribution, Q-of-5-spot well pattern, upsc. factor \( \sim 25 \)

\[ \Leftarrow 2 \text{ realizations.} \]

Geomodel: 15206 cells

Observations:

- Coarsening algorithm applicable to unstructured grids — accuracy consistent with observations for SPE10 models.
- Results obtained with uniform grid (in index space) inaccurate.
Example 3: Geomodel = four bottom layers from SPE10
Robustness with respect to degree of coarsening, 5-spot well pattern

<table>
<thead>
<tr>
<th>Number of cells in grid (upscaling factor 4–400)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform grid</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Non-U. grid</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Observations:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-uniform grid gives better accuracy than uniform grid.</td>
</tr>
<tr>
<td>Water-cut error almost grid-independent for non-uniform grid.</td>
</tr>
</tbody>
</table>
Example 4: Geomodel = four bottom layers from SPE10
Dependency on initial flow conditions, upscaling factor $\sim 40$

Grid generated with respective well patterns.

Grid generated with pattern C

**Observation:**
Grid resolves high-permeable regions with good connectivity — Grid need *not* be regenerated if well pattern changes.
Conclusions

**Multiscale mixed/mimetic method:**
- Reservoir simulation tool that can take geomodels as input.
- Solutions in close correspondence with solutions obtained by solving the pressure equation directly.
- Computational cost comparable to flow based upscaling.

**Applications:**
- Reservoir simulation on geomodels
- Near-well modeling / Improved well models
- History matching on geomodels

**Potential value for industry:**
Improved modeling and simulation workflows.
Conclusions

Coarse grid for mass balance equations:
- A generic semi-automated algorithm for generating coarse grids that resolve flow patterns has been presented.
- Solutions are significantly more accurate than solutions obtained on uniform coarse grids with similar number of cells.
- Water-cut error: 1%–3% — pseudofunctions superfluous.
- Grid need not be regenerated when flow conditions change!

Potential application:
User-specified grid-resolution to fit available computer resources.

Relation to other methods:
Belongs to family of flow-based grids\(^a\): designed for flow scenarios where heterogeneity, rather than gravity, dominates flow patterns.

I have a dream ...

... that one day

geologists and reservoir engineers decide to communicate and see their contributions as part of a larger picture, and that multiscale methods are used for what they are worth.