Multiscale Mixed FEM for Near-well Modeling

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Outline

- Motivation
- Multiscale Mixed Finite Element Methods (MsMFEM)
- MsMFEM for near-well modeling
- Numerical experiments
- Conclusions / Further work

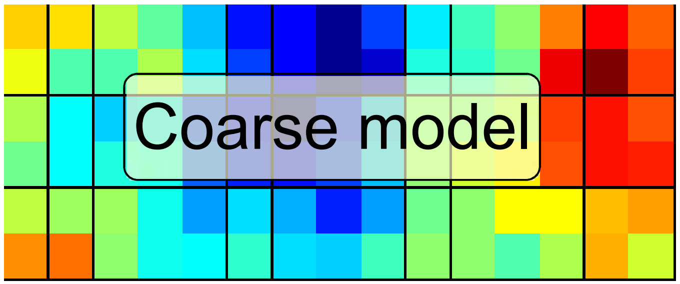
Motivation:

- Near-well region extremely important
- Cannot fully resolve all scales in typical simulation
- Multiscale methods incorporate fine scales in coarse scale equations

Multiscale Methods for Reservoir Simulation

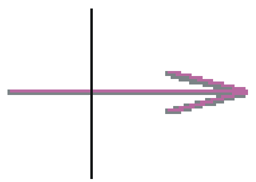
- Multiscale Finite Element Method
  - T. Hou, X. H. Wu, 1997
- Multiscale Mixed FEM
  - Z. Chen, T. Hou, 2003
  - T. Arbogast et al., 2000
  - J. Aarnes et al., 2004 (group at SINTEF)
- Multiscale Finite Volume Method
  - P. Jenny, S. H. Lee, H.A. Tchelepi, 2003

Standard / Multiscale Discretization

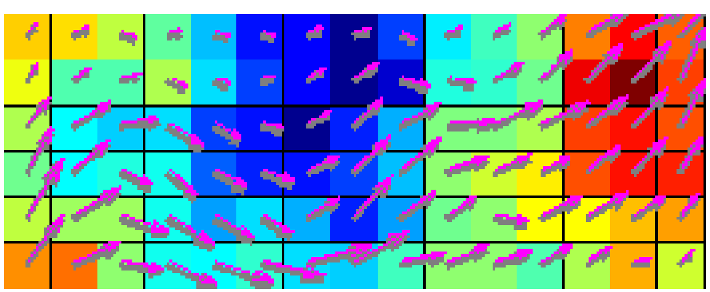


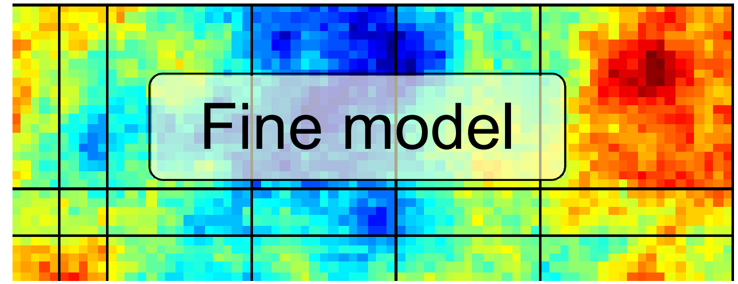
Standard

Local flow:



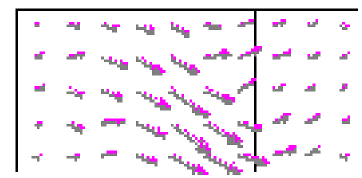
Solve **coarse** equations



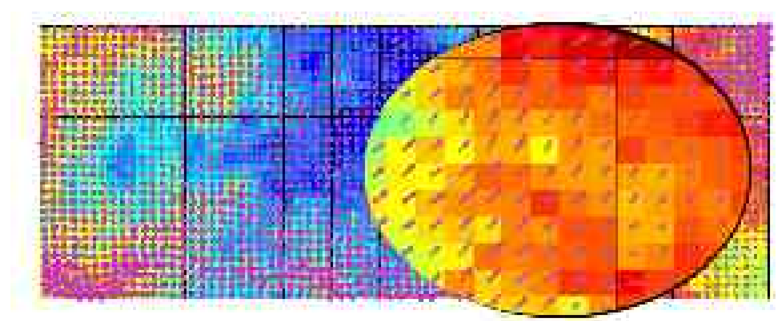


Multiscale

Local flow:



Solve **coarse** equations / form fine scale flow



Mixed Finite Elements (MFEM)

Model equations:

$$v = -\lambda K \nabla p$$

$$\nabla \cdot v = q$$

Choose basis:

$$v = \sum v_i \psi_i$$

$$p = \sum p_j \varphi_j$$

Weak formulation:

$$\int v \cdot (\lambda K)^{-1} \hat{v} - \int p \nabla \cdot \hat{v} = 0$$

$$\int \hat{p} \nabla \cdot v = \int \hat{p} q$$

for all $(\hat{v}, \hat{p}) \in U \times V$.

Mixed discretization:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{v} \\ -\mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{q} \end{pmatrix}$$

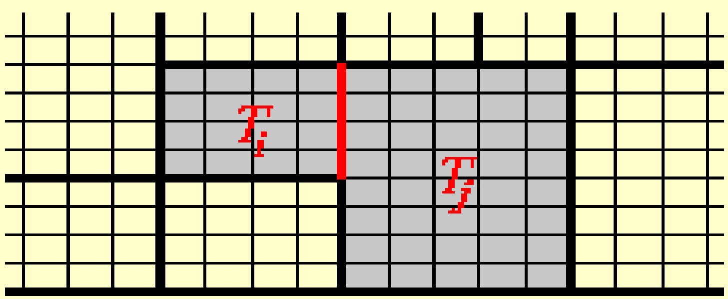
Multiscale MFEM (MsMFEM)

Flux basis function, ψ **:**

$$\psi = -K\nabla p_{\psi}$$

$$\nabla \cdot \psi = \begin{cases} w_i & \text{for } x \in T_i \\ -w_j & \text{for } x \in T_j \end{cases}$$

$$\psi \cdot n = 0 \text{ on } \partial(T_i \cup T_j)$$



Initially compute basis functions

for n=1 **to** N

Solve coarse system based on current saturation

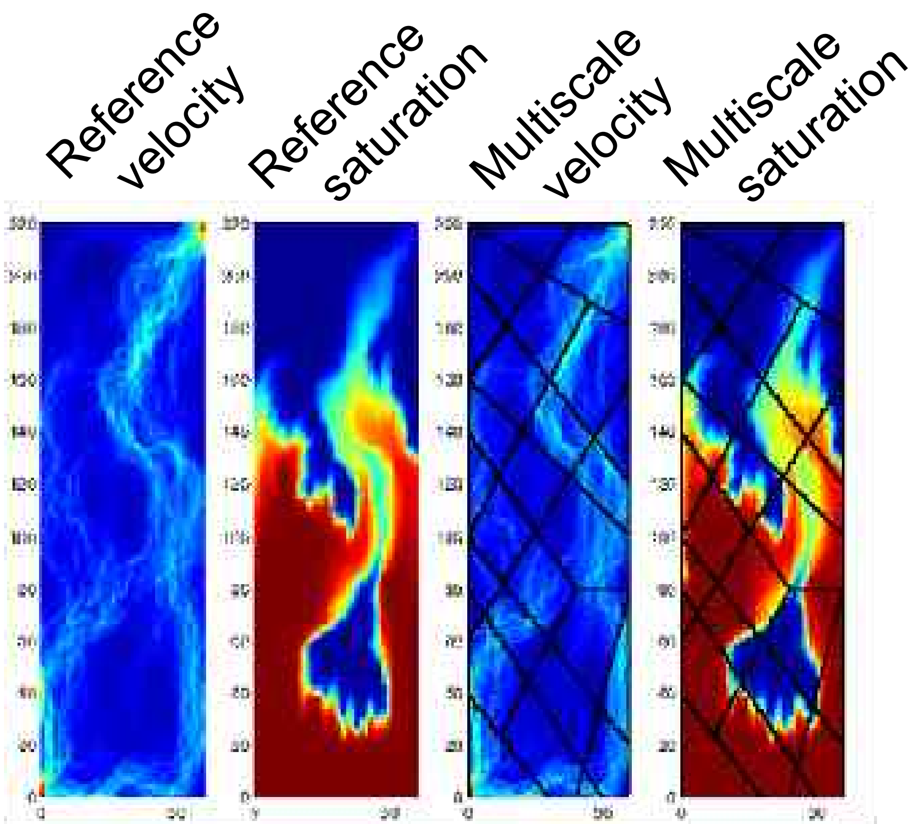
Form fine scale fluxes

Advance fine scale saturation by Δt

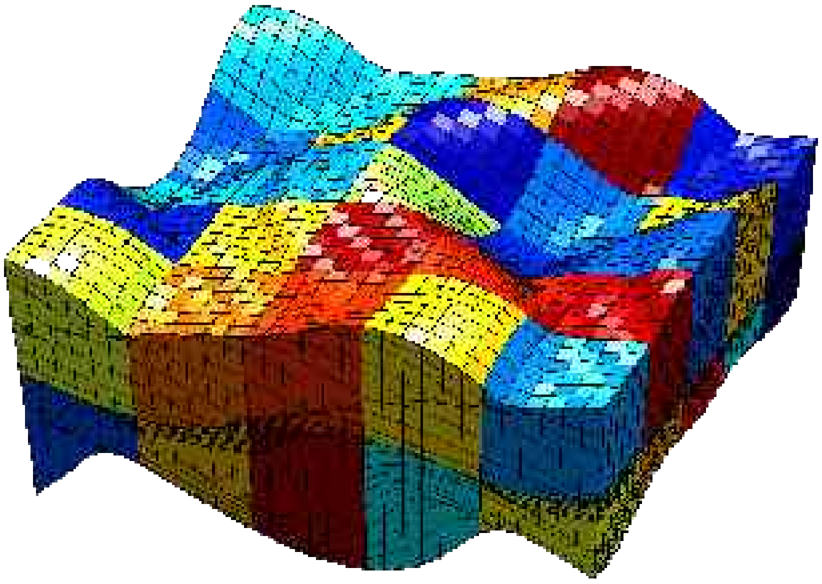
end

Flexible Gridding

Coarse blocks: Any collection of fine grid cells



Fine cells: Both subdivision / MFEM and *mimetic* FDM have been implemented



Options for Improving Accuracy

- Incorporate global information in the basis functions
- Use overlap / border regions for the computation of basis functions
- Occasionally update basis functions to account for mobility effects

MsMFEM for Near-well Modeling

Goal: Incorporate wellbore flow models into the MsMFEM framework

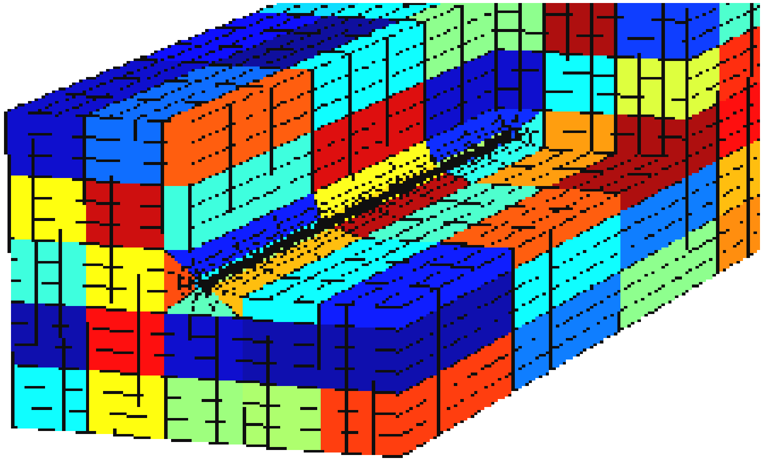
- Obtain detailed flow in the near-well region while solving only a moderate sized system
- Incorporate drift flux model for pressure drop in wellbore

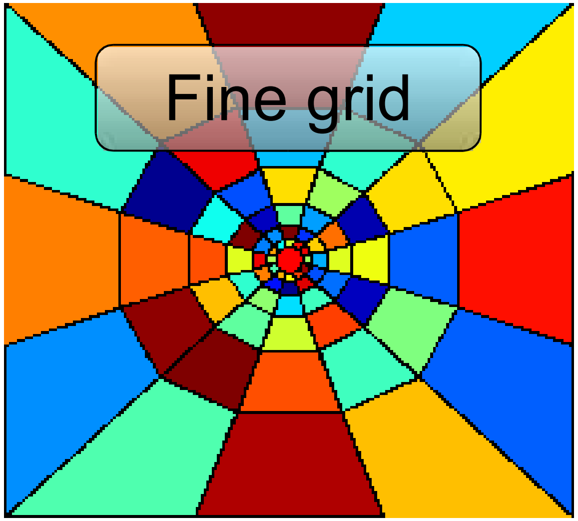
$$\Delta p = \Delta p_f + \Delta p_a + \Delta p_h$$

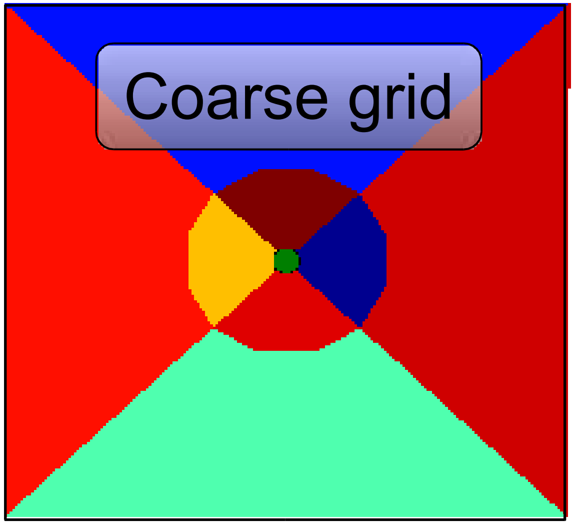
$$\Delta p_f = \frac{2 L f_{tp} \rho_m v_m^2}{D}, \quad \Delta p_a = \frac{2 q_m \rho_m v_m}{A}, \quad \Delta p_h = \rho_m g L \sin\theta$$

Well – Reservoir Linkage

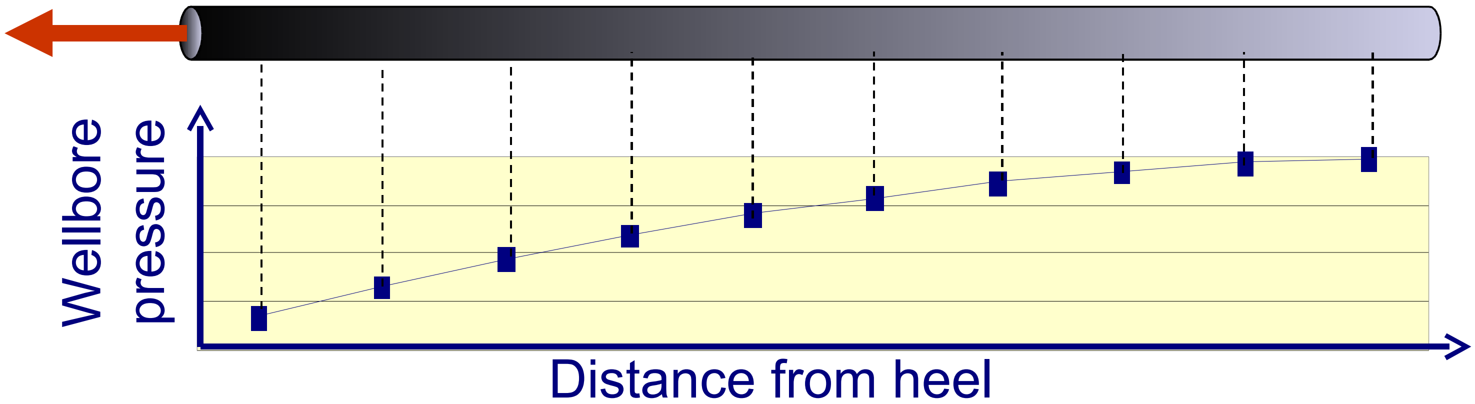
- Fine grid to the annulus/ well segments included in the grid
- Well segments treated as coarse blocks / no well model is used







Wellbore Pressure Loss Treatment



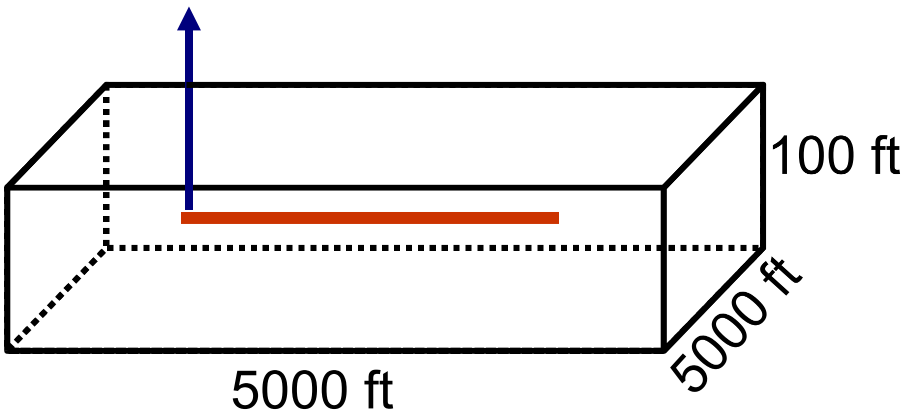
- Frictional pressure loss evaluated using previous and current time steps

$$\frac{\partial p_{n+1}}{\partial L} = -C_{n+1} v_{n+1}^2 \approx -C_n v_n v_{n+1}$$

- Successive substitution to obtain wellbore pressure at first time step

Numerical Experiments: Single-phase

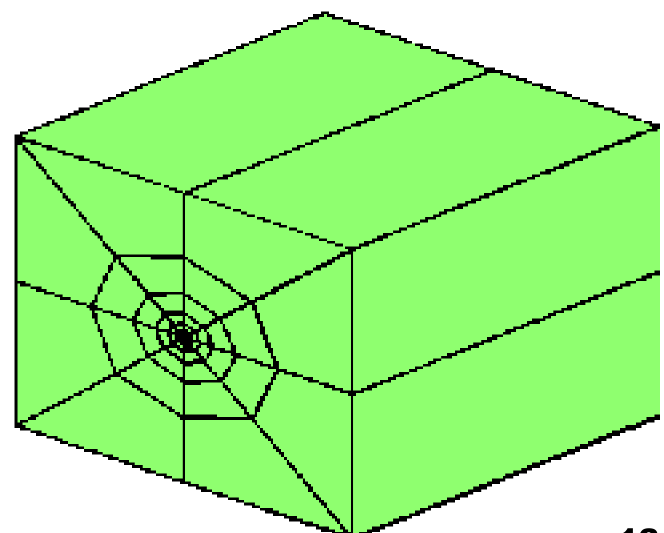
Homogeneous reservoir (validation) : model



Fine cells	18400
Coarse blocks	1900
Well segments	30

Isotropic permeability, *md*	200
Compressibility, psi^{-1}	10^{-4}
Wellbore radius, *inch*	2.0
Well length, *ft*	3000
Pipe roughness, *inch*	0.0012

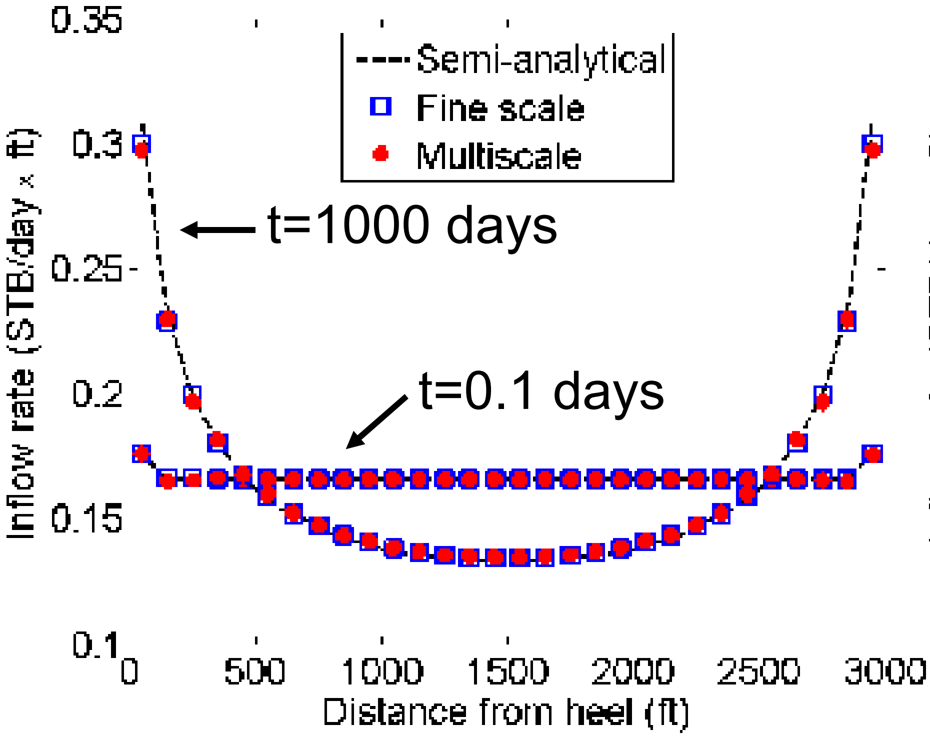
Coarse block



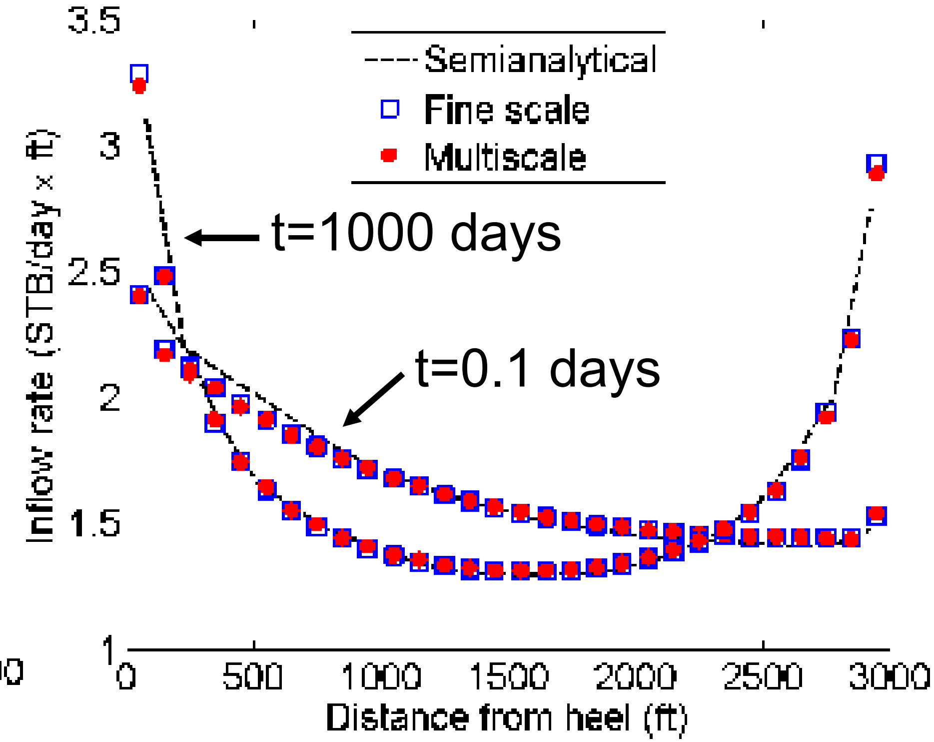
Numerical Experiments: Single-phase

Homogeneous reservoir (validation) : results

Production rate:
500 STB/day

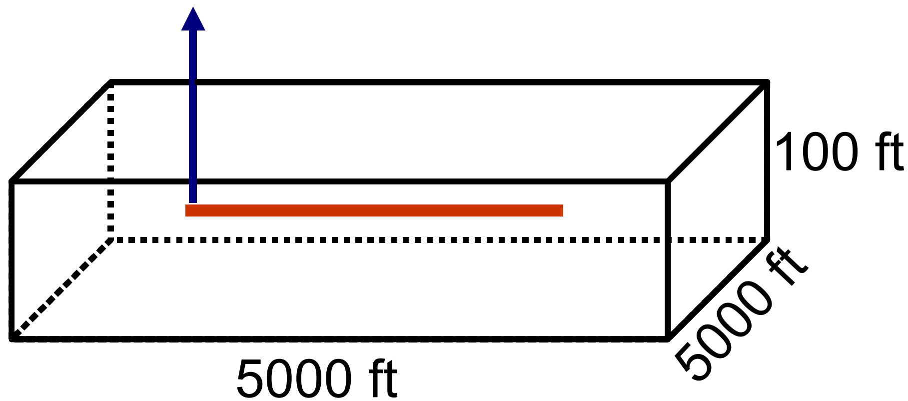


Production rate:
5000 STB/day



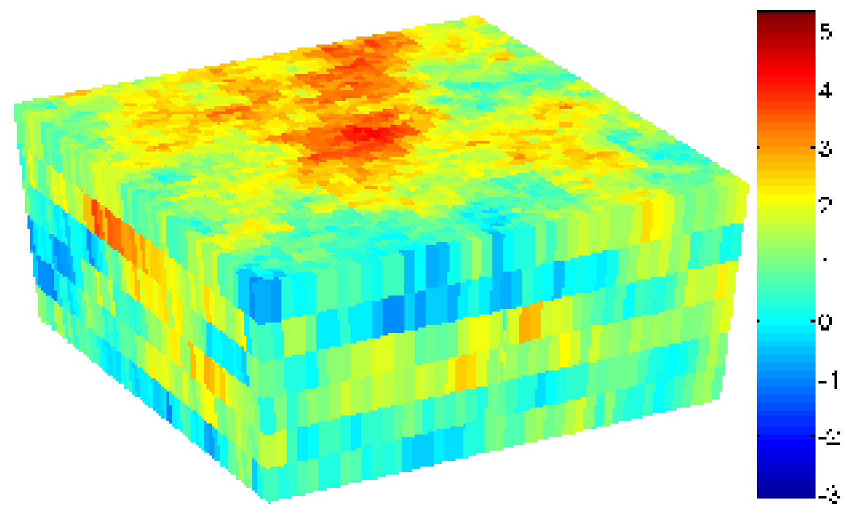
Numerical Experiments: Single-phase

Heterogeneous reservoir : model



Fine cells	18400
Coarse blocks	1900
Well segments	30

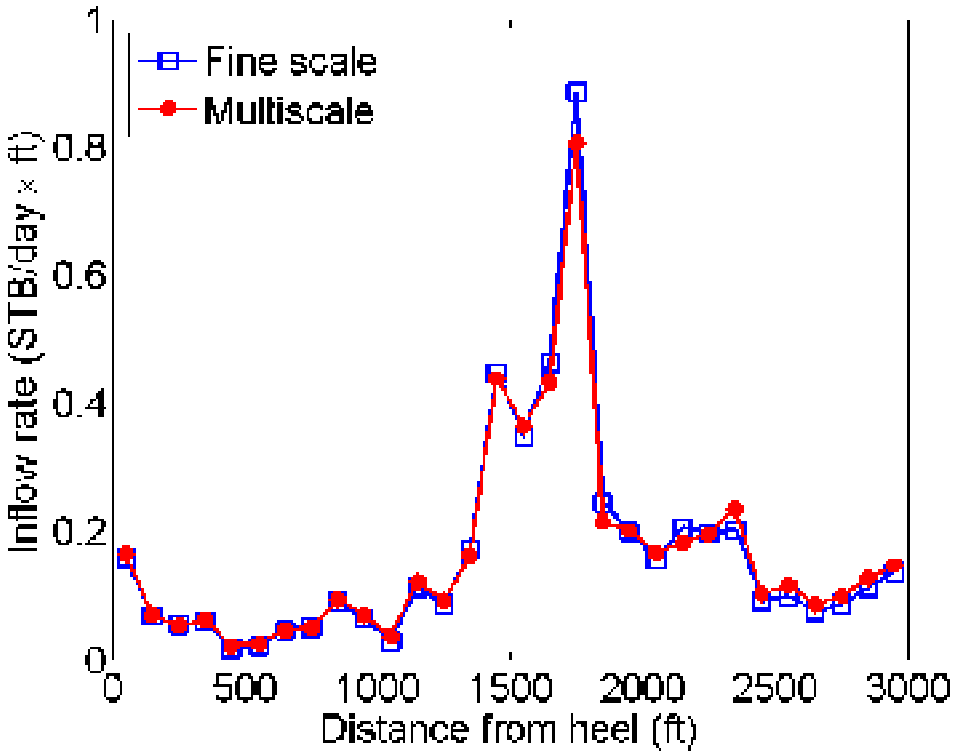
Isotropic permeability, *md*	10^{-3}-10^{5}
Compressibility, *psi* $^{-1}$	10^{-4}
Wellbore radius, *inch*	2.0
Well length, *ft*	3000
Pipe roughness, *inch*	0.0012



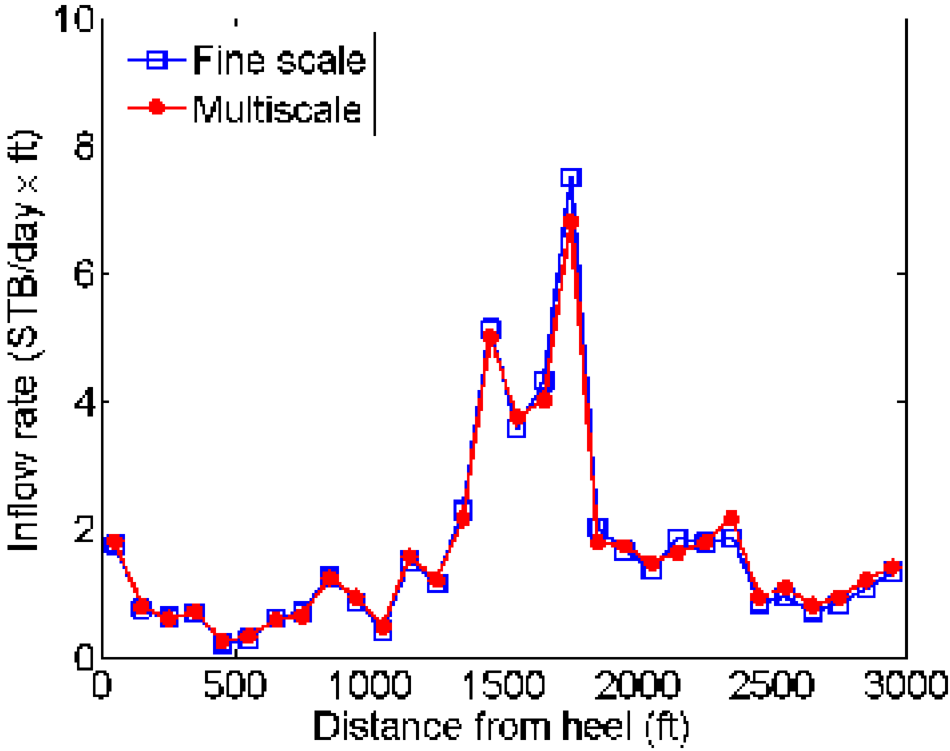
Numerical Experiments: Single-phase

Heterogeneous reservoir : Time = 1000 days

Production rate:
500 STB/day



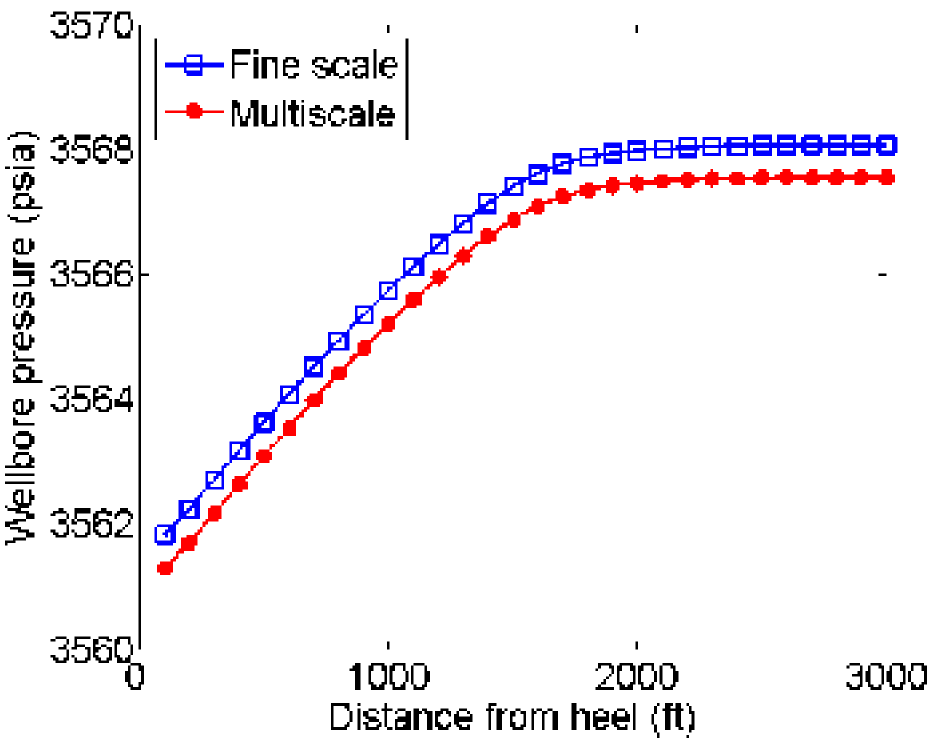
Production rate:
5000 STB/day



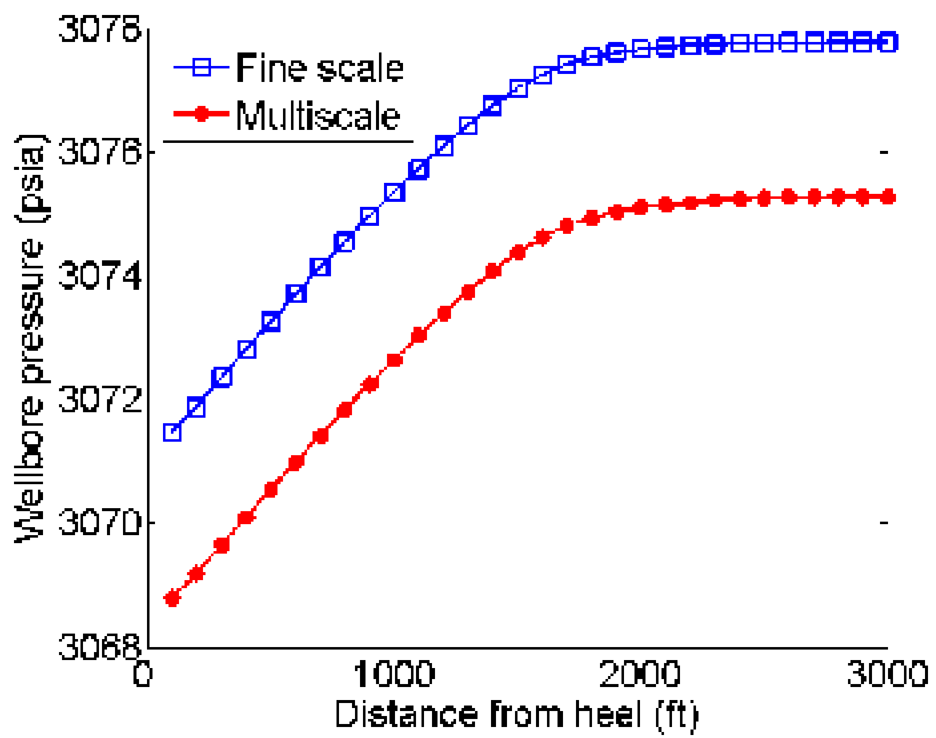
Numerical Experiments: Single-phase

Heterogeneous reservoir : 5000 STB/day

Time = 0.1 days



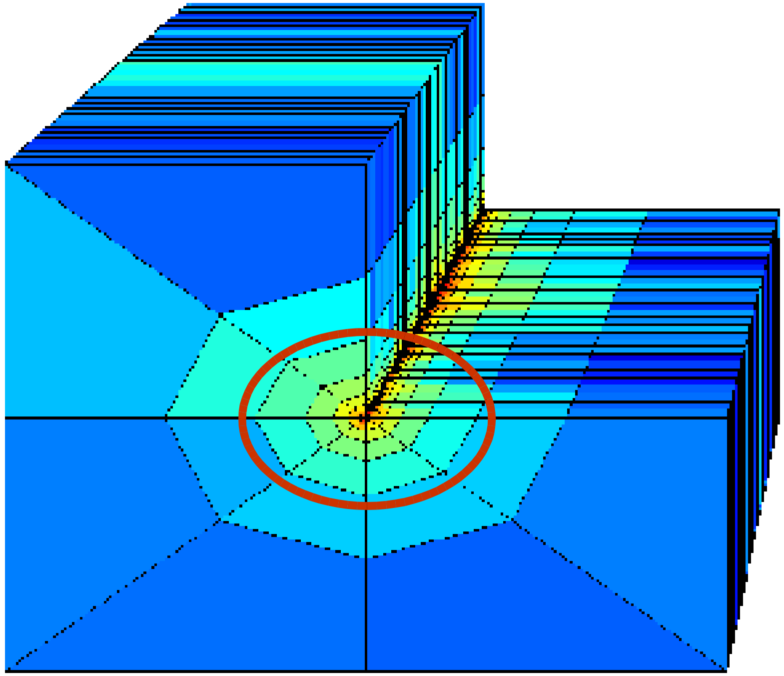
Time = 1000 days

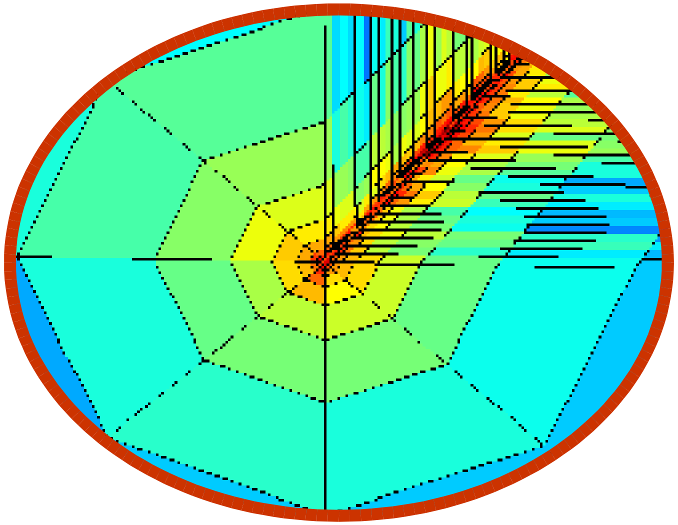


Numerical Experiments: Single-phase

Heterogeneous reservoir : results

Velocity profile around well:





Conclusions / Further Work

- Extended MsMFEM to include wellbore flow models
- Presented numerical examples for single-phase flow
- Further develop and test two-phase capability
- Apply to complex wells