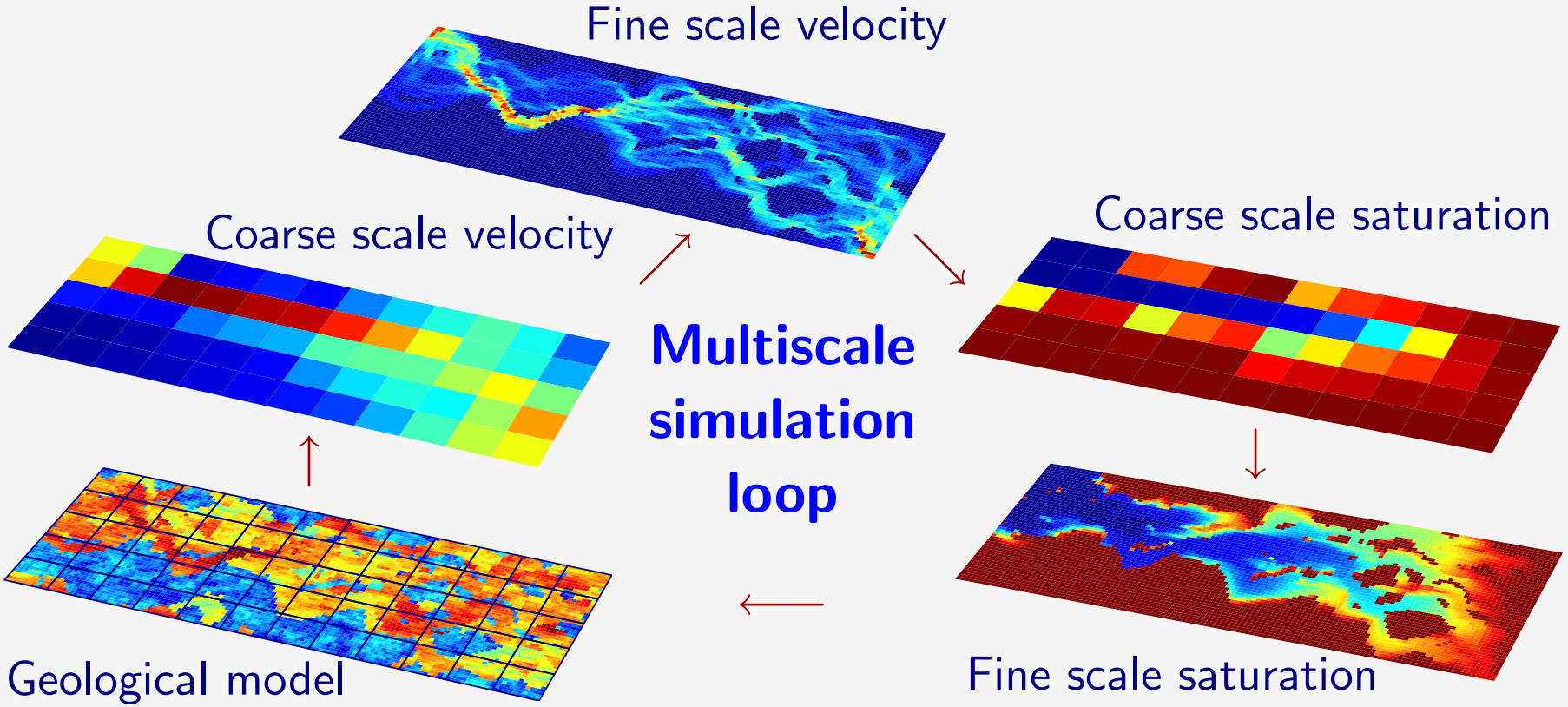
Multiscale simulation of porous media flow

A research project funded by the Research Council of Norway



This work is part of the **GeoScale Research Project**:

> Develop a numerical methodology that facilitates reservoir simulation studies on multi-million cell geological models.

Simulations should run within a few hours on desktop computers.

This work is part of the **GeoScale Research Project**:

> Develop a numerical methodology that facilitates reservoir simulation studies on multi-million cell geological models.

Simulations should run within a few hours on desktop computers.

> A cornerstone in the project is a multiscale mixed finite element method (MsMFEM) that models pressure and filtration velocity.

This work is part of the **GeoScale Research Project**:

> Develop a numerical methodology that facilitates reservoir simulation studies on multi-million cell geological models.

Simulations should run within a few hours on desktop computers.

A cornerstone in the project is a multiscale mixed finite element method (MsMFEM) that models pressure and filtration velocity.

To model the transport we explore two different strategies:
- streamline methods for convection dominated flow.
- an adaptive multiscale finite volume method.

- Real-field geological models: 10^7–10^9 grid cells, conventional simulators handle 10^5–10^6 grid cells.

  Simulations are performed on upscaled models.

- Real-field geological models: 10^7–10^9 grid cells, conventional simulators handle 10^5–10^6 grid cells.

  Simulations are performed on upscaled models.

- Increased demands on reservoir simulation tools: resolution, grid cell geometries, adaptivity,....

  Upscaled models can be inadequate and is a bottle-neck in simulation workflow.

- Real-field geological models: 10^7–10^9 grid cells,
  conventional simulators handle 10^5–10^6 grid cells.

  Simulations are performed on upscaled models.

- Increased demands on reservoir simulation tools:
  resolution, grid cell geometries, adaptivity,....

  Upscaled models can be inadequate and is a
  bottle-neck in simulation workflow.

- Unlikely that the simulation gap will be closed in the foreseeable future: multiple realizations needed to address uncertainty.

Multiscale methods will not eliminate the need for upscaling, but ...

The ability to run simulations on geomodels is needed to validate simulation results, and to enhance our understanding of flow processes in reservoirs with complex structures.

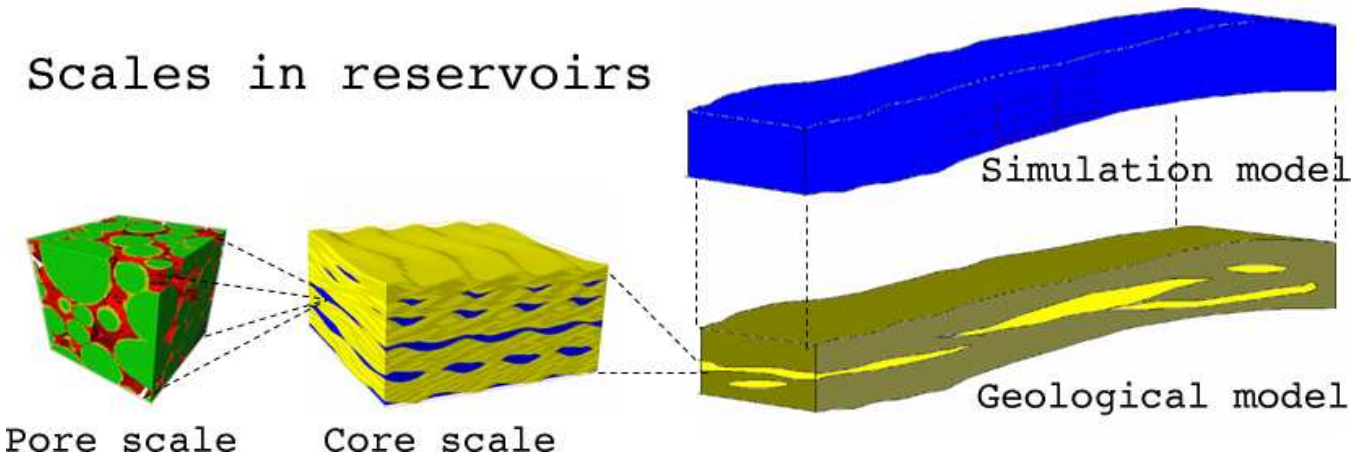
Example: The oil recovery from fractured reservoirs is typically significantly lower than the OR from non-fractured reservoirs.

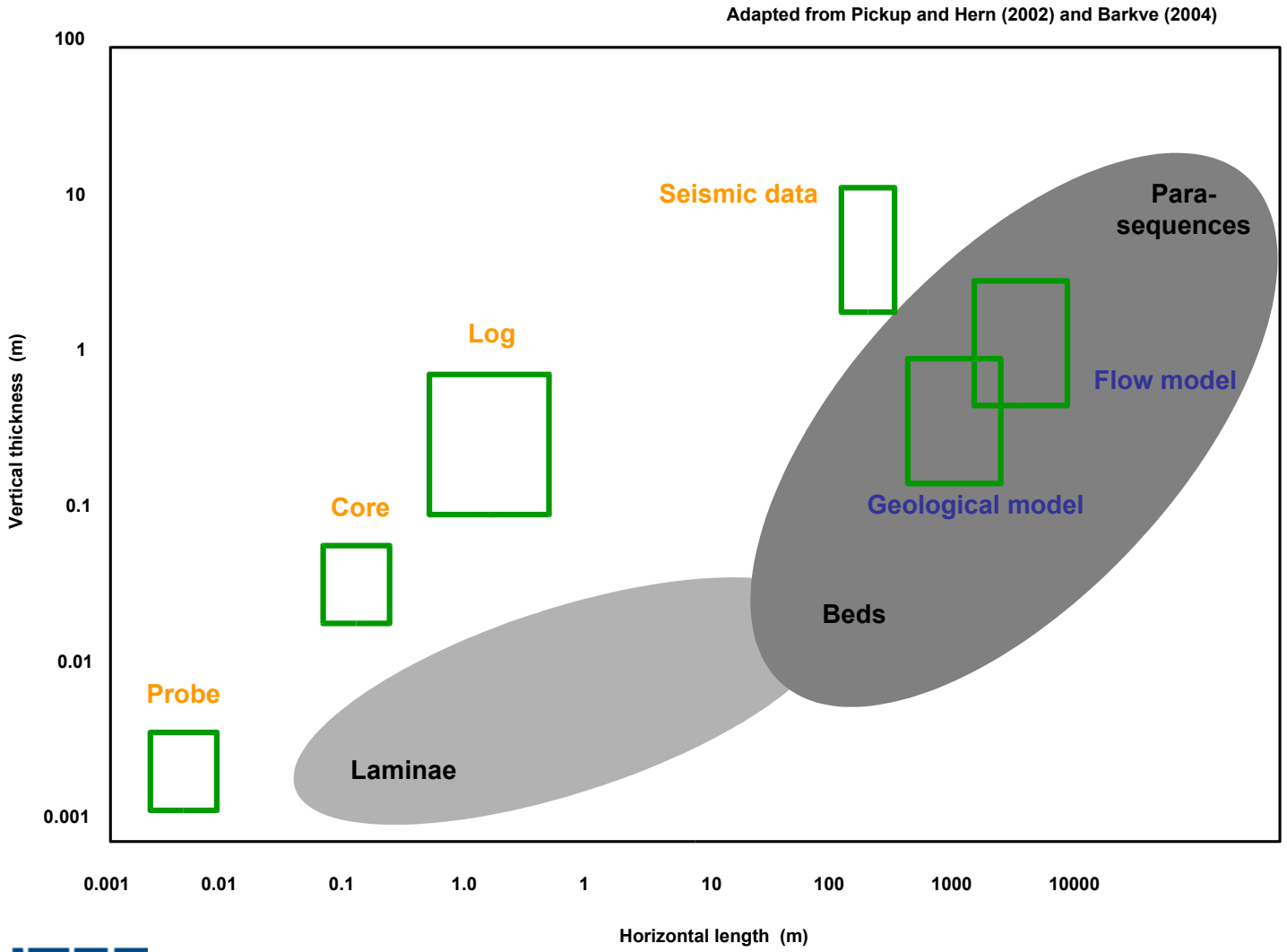
The Geology

Porous media often have repetitive layered structures, but faults and fractures caused by stresses in the rock disrupt flow patterns.

The scales that impact fluid flow in oil reservoirs range from

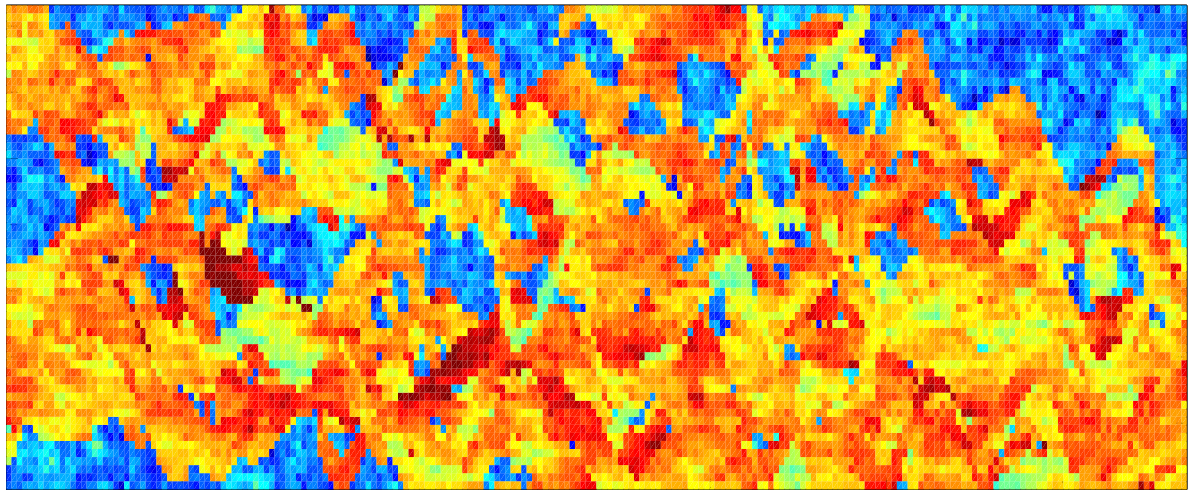
- the micrometer scale of pores and pore channels
- via dm–m scale of well bores and lamina sediments
- to sedimentary structures that stretch across entire reservoirs.




Adapted from Pickup and Hern (2002) and Barkve (2004)
100
10
1
0.1
0.01
0.001
Vertical thickness (m)
Seismic data
Log
Core
Probe
Laminae
Beds
Para-
sequences
Flow model
Geological model
0.001
0.01
0.1
1.0
1
10
100
1000
10000
Horizontal length (m)

Geological models

Geological models give a geometric reservoir description, and a distribution of permeability k, the rocks ability to transmit fluid, and porosity ϕ, the volume fraction open to flow.



Typical porous media structures are often characterized by very large permeability contrasts: $\frac{\max k}{\min k} \sim 10^3 - 10^{10}$.

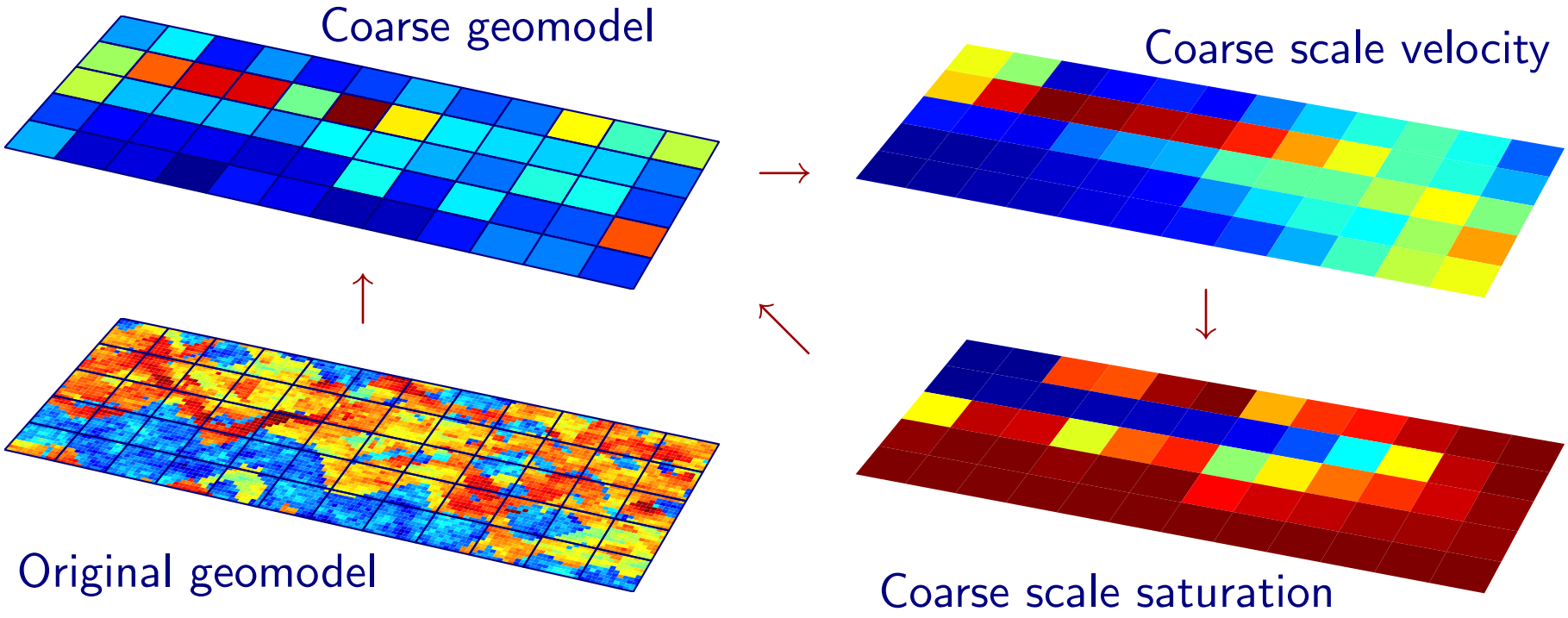
Simulation models

For presentational simplicity we consider a model for incompressible and immiscible two-phase flow without gravity and capillary forces:

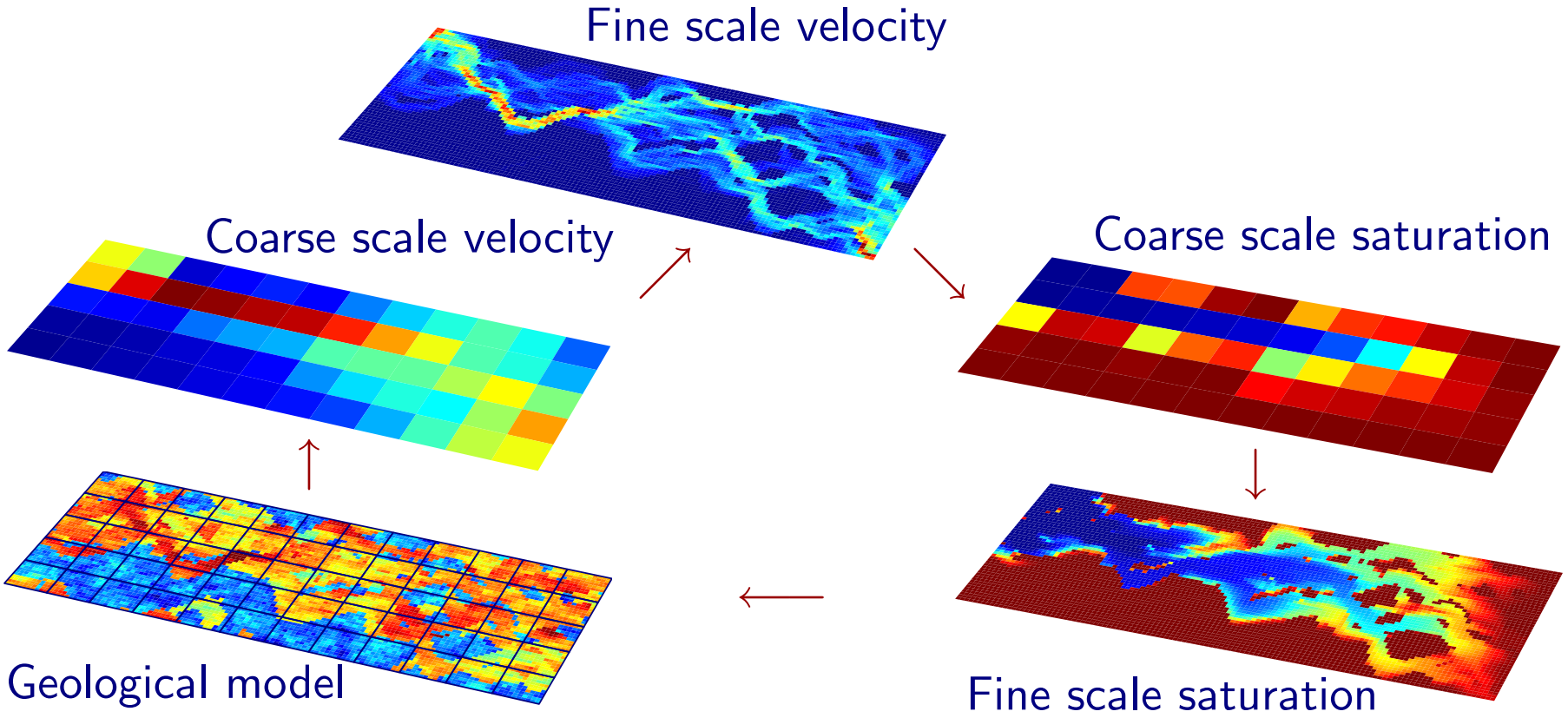
$$\begin{aligned} -\nabla \cdot k[\lambda_w(S) + \lambda_o(S)]\nabla p &= q \\ \phi \partial_t S + \nabla \cdot (f_w v) &= q_w. \end{aligned}$$

Here λ_i is the mobility of phase i, p pressure, S water saturation, $f_w = \lambda_w/(\lambda_w + \lambda_o)$, and $v = v_w + v_o$ the total Darcy velocity.

Traditional reservoir simulation loop



Reservoir simulation loop using multiscale methods



Multiscale mixed finite element methods

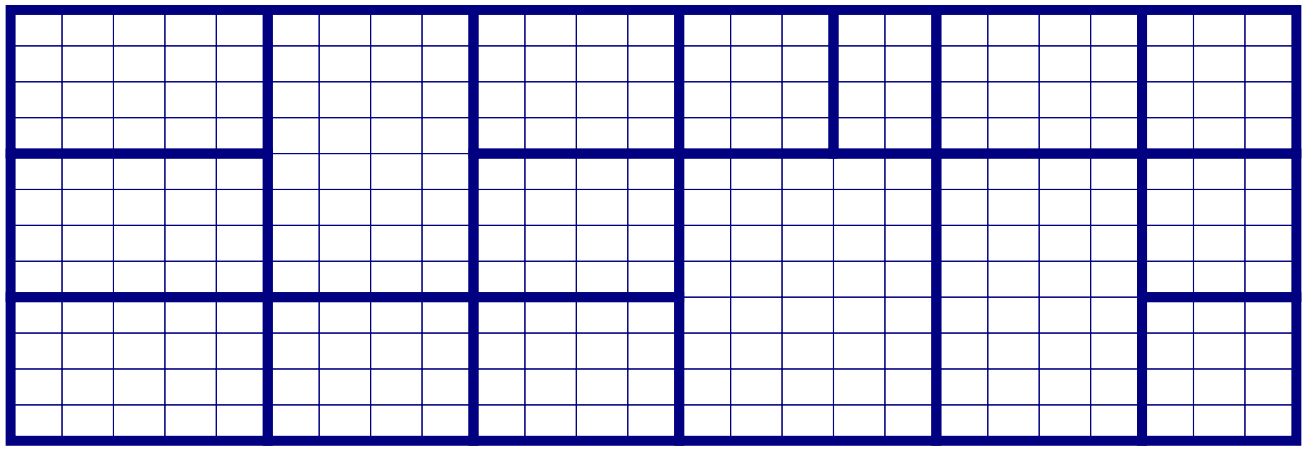
In a mixed FEM formulation one seeks $v \in V$ and $p \in U$ such that

$$\begin{aligned} \int_\Omega k^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= 0 && \forall u \in V, \\ \int_\Omega l \, \nabla \cdot v \, dx &= \textstyle\int_\Omega q l \, dx && \forall l \in U. \end{aligned}$$

Here $V \subset \{v \in (L^2)^d : \nabla \cdot v \in L^2,\ v \cdot n = 0 \text{ on } \partial\Omega\}$ and $U \subset L^2$.

In MsMFEMs the approximation space for velocity $V = \text{span}\{\psi_{ij}\}$ is designed so that it embodies the impact of fine scale structures.

Start with a fine grid $\mathcal{T} = \{T\}$ and introduce a coarsened grid $\mathcal{K} = \{K\}$ with grid blocks of "arbitrary" shape.



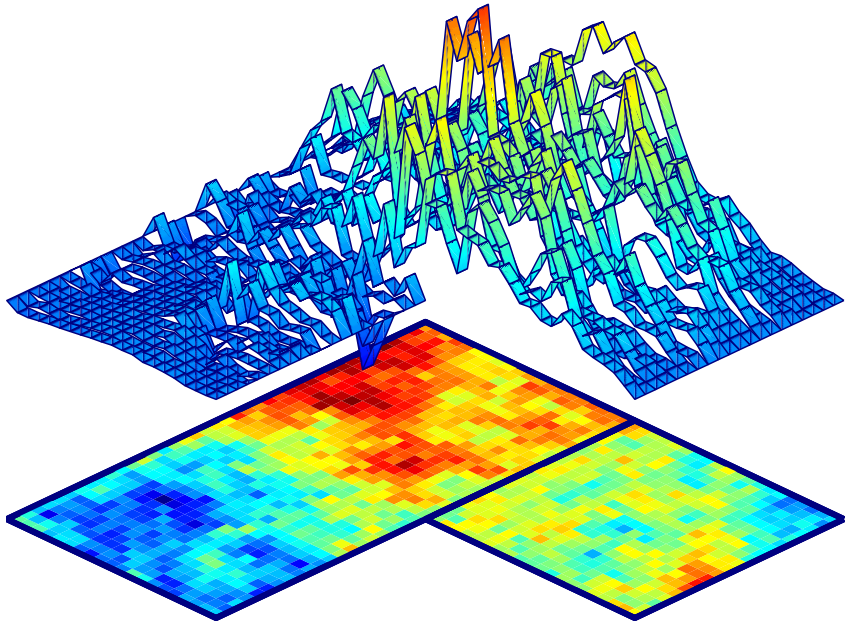
Associate a basis function χ_m for pressure with each grid block:

$$U = \operatorname{span}\{\chi_m : K_m \in \mathcal{K}\} \quad \text{where} \quad \chi_m = \begin{cases} 1 & \text{if } x \in K_m, \\ 0 & \text{else.} \end{cases}$$

Construct a velocity basis function for each interface $\partial K_i \cap \partial K_j$:

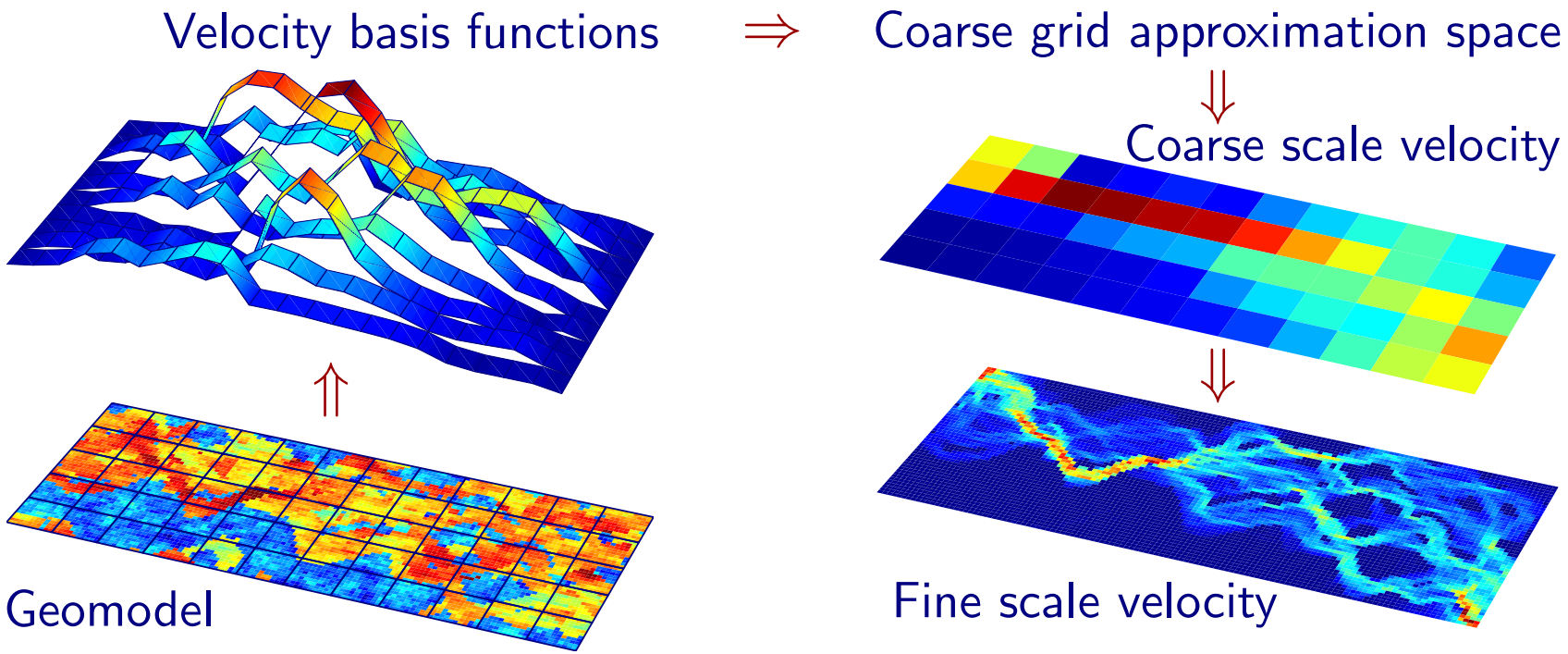
$V = \text{span}\{\psi_{ij}\}$ where $\psi_{ij} = -k\nabla\phi_{ij}$ and ϕ_{ij} is determined by no-flow boundary conditions on $(\partial K_i \cup \partial K_j)\backslash(\partial K_i \cap \partial K_j)$, and

$$\nabla \cdot \psi_{ij} = \begin{cases} q(K_i) & \text{in } K_i, \\ -q(K_j) & \text{in } K_j, \end{cases}$$

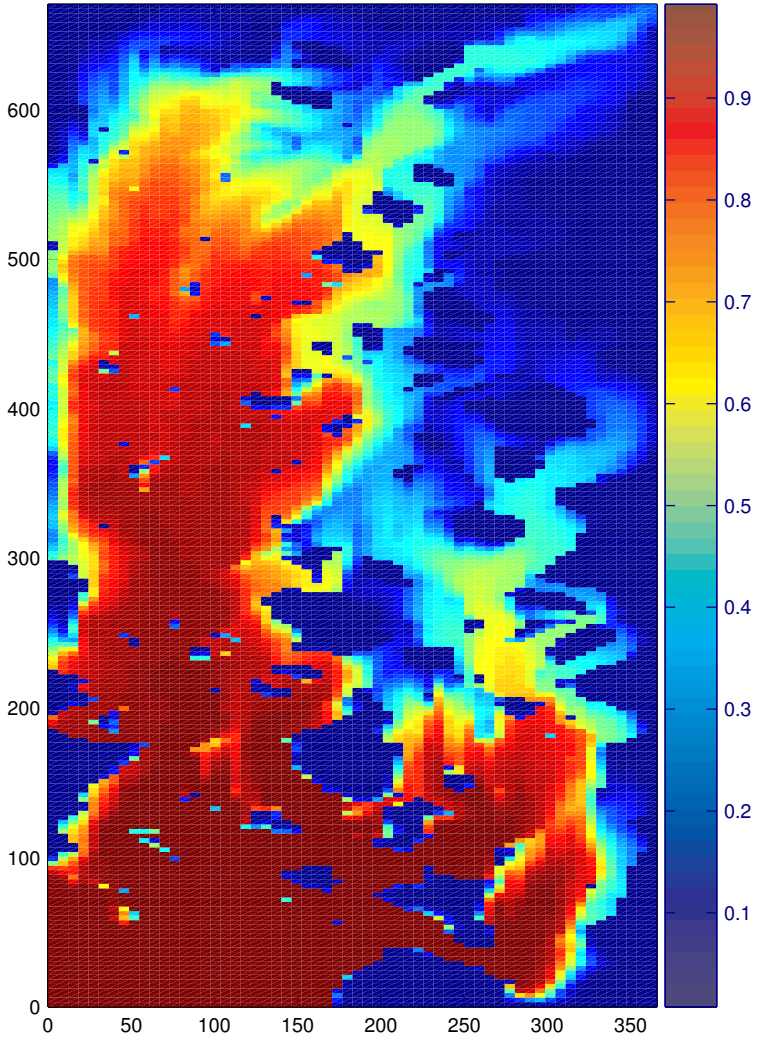


where

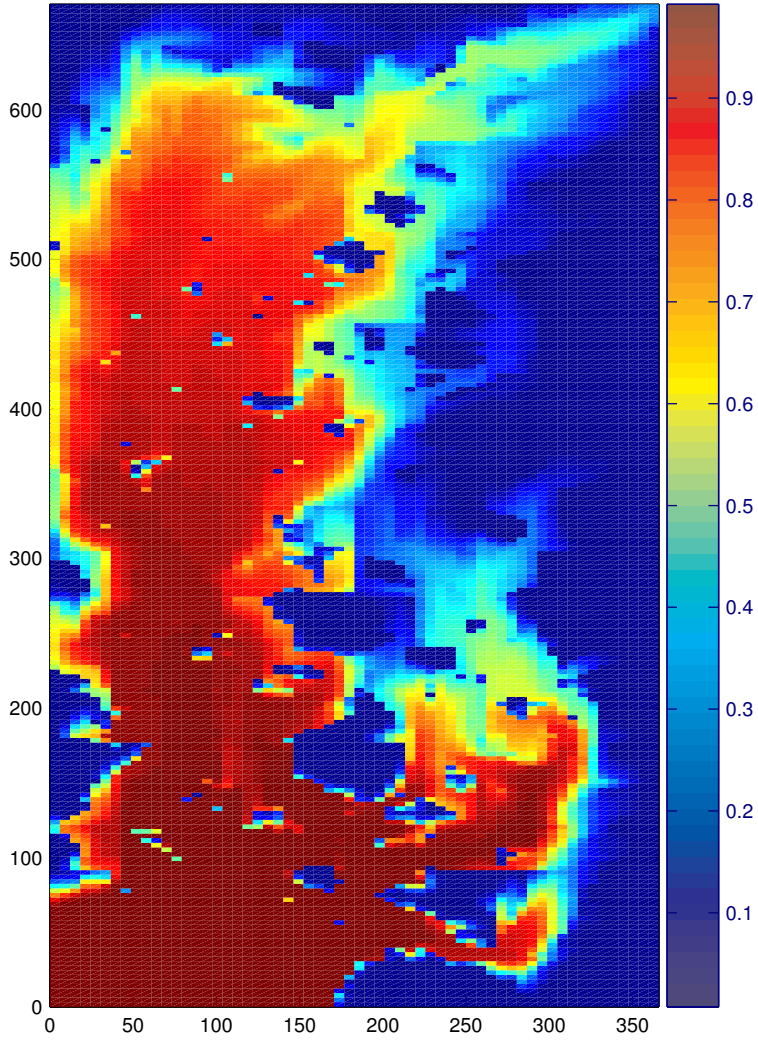
$$q(K) = \begin{cases} \dfrac{|k|}{\int_K |k|} & \text{if } \int_K f\,dx = 0, \\ \dfrac{f}{\int_K f} & \text{if } \int_K f\,dx \neq 0. \end{cases}$$



The fine scale velocity field is expressed as a linear superposition of the basis functions: $v = \sum_{ij} v_{ij}\psi_{ij}$ where the coefficients v_{ij} are obtained from the solution of the coarse scale system.

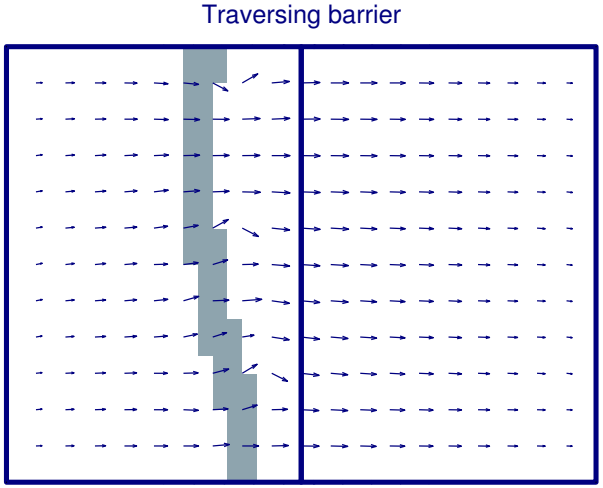


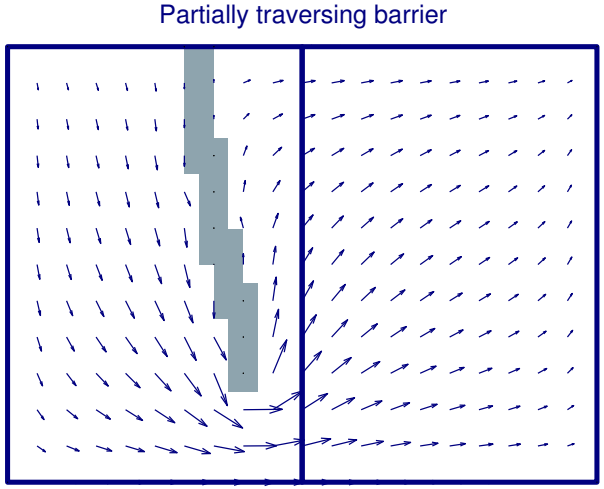
Reference



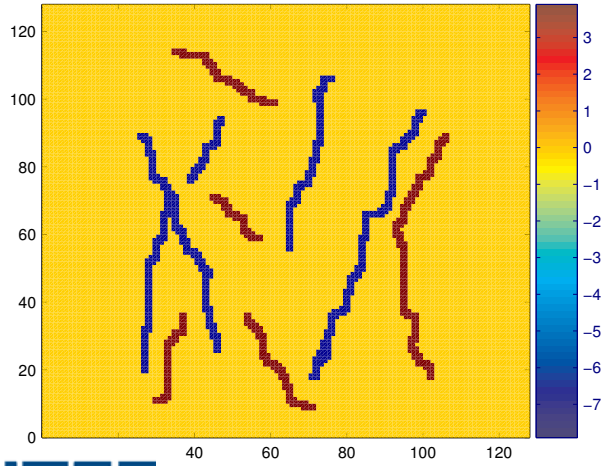
MsMFEM

Large scale barrier structures may not be modeled correctly,





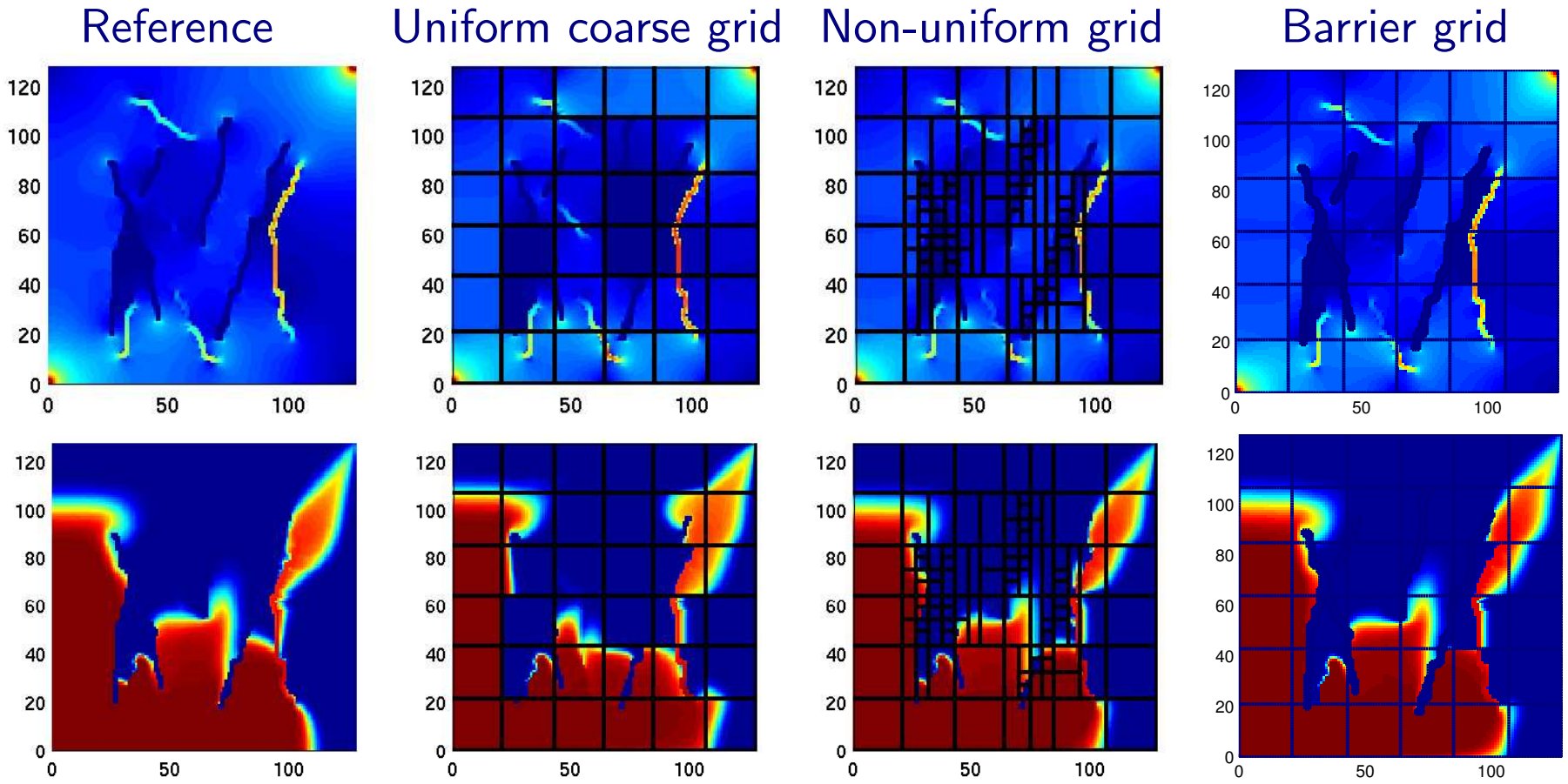
... but the problem is easy to detect and fix automatically.

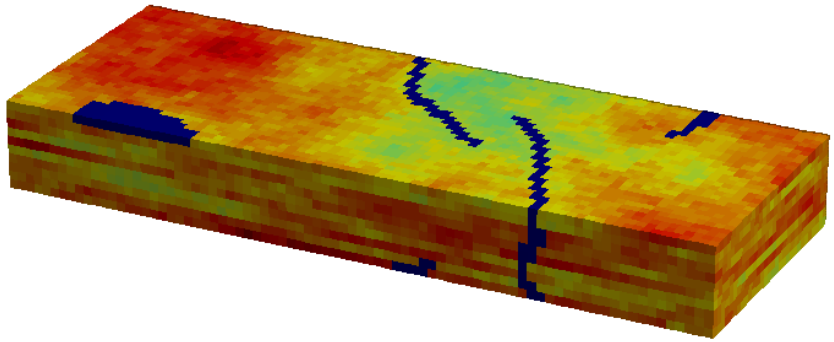


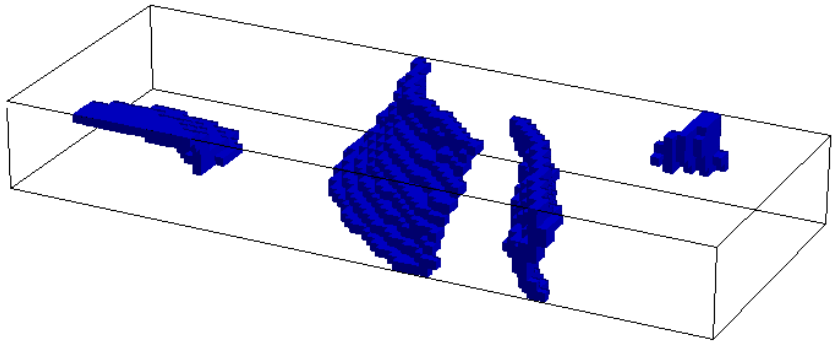
$$k_{\text{red}} = 10^{4}$$
$$k_{\text{yellow}} = 1$$
$$k_{\text{blue}} = 10^{-8}$$
$$\text{Fine grid} = 128 \times 128.$$

SINTEF

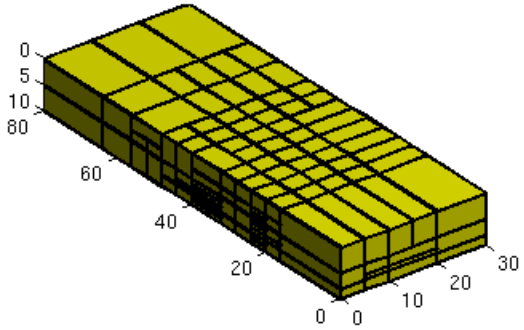
Grid refinement and grid adaption is straight forward



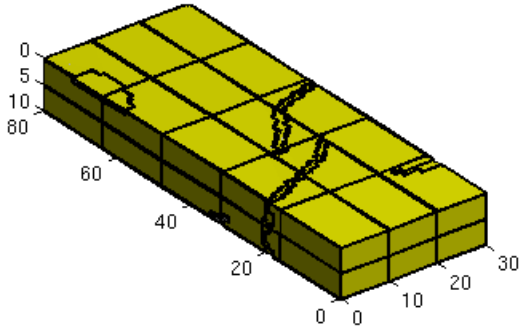




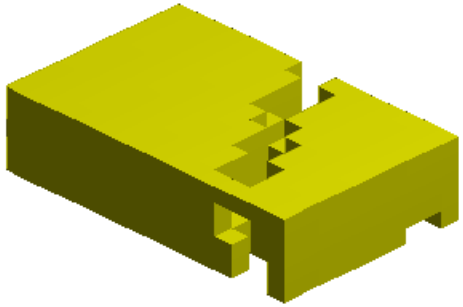
Non-uniform grid, hexahedral cells

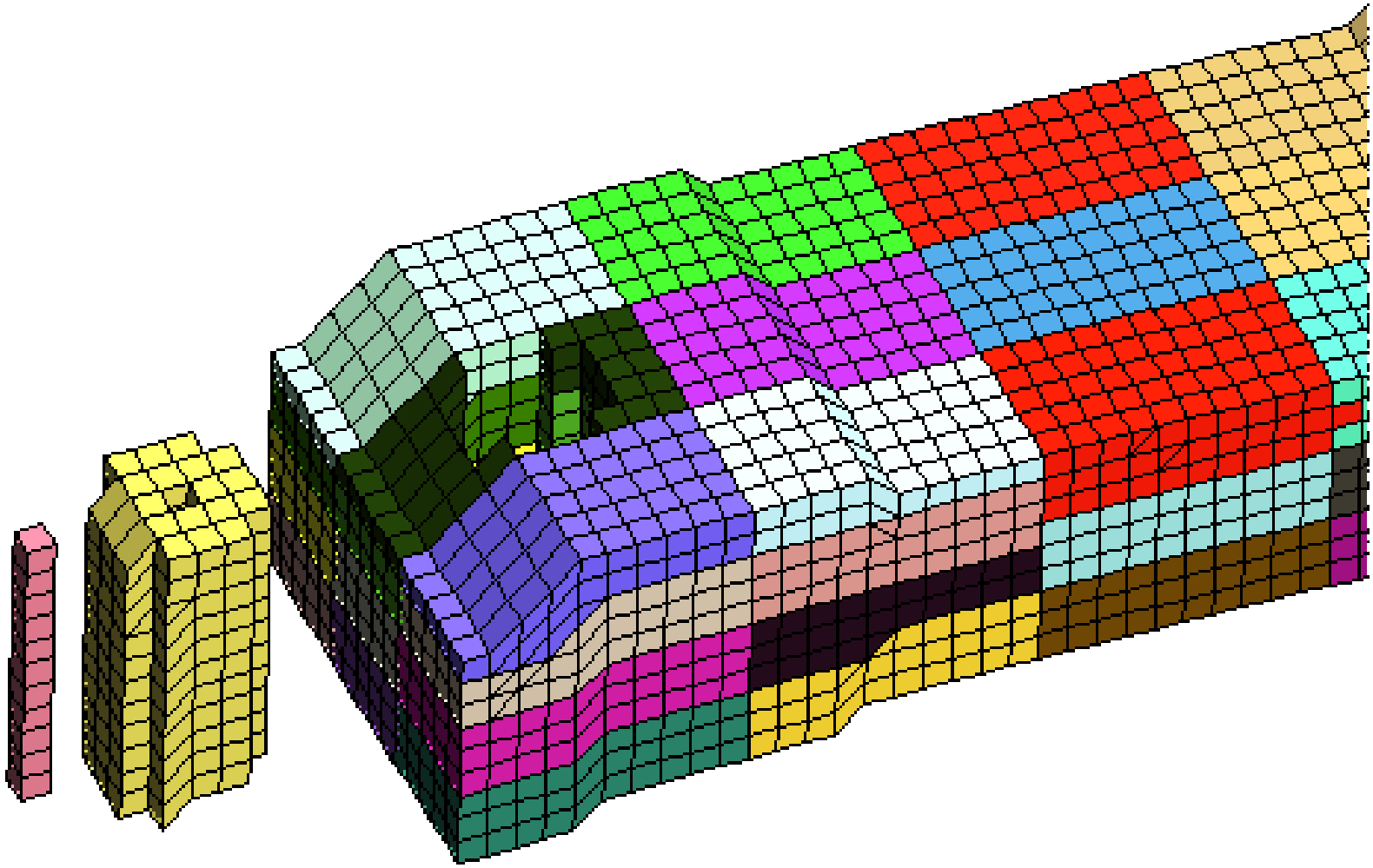


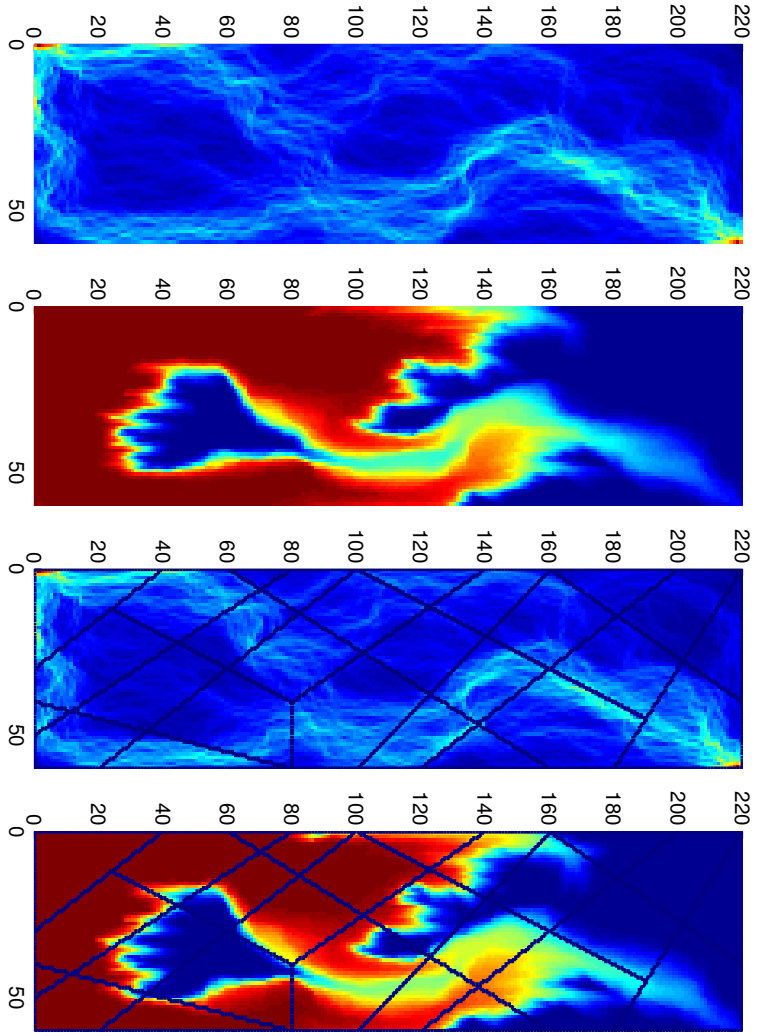
Non-uniform grid, general cells



General grid-cell







MsMFEMs enjoy the following prop.:

They are accurate: flow scenarios match closely fine grid simulations.

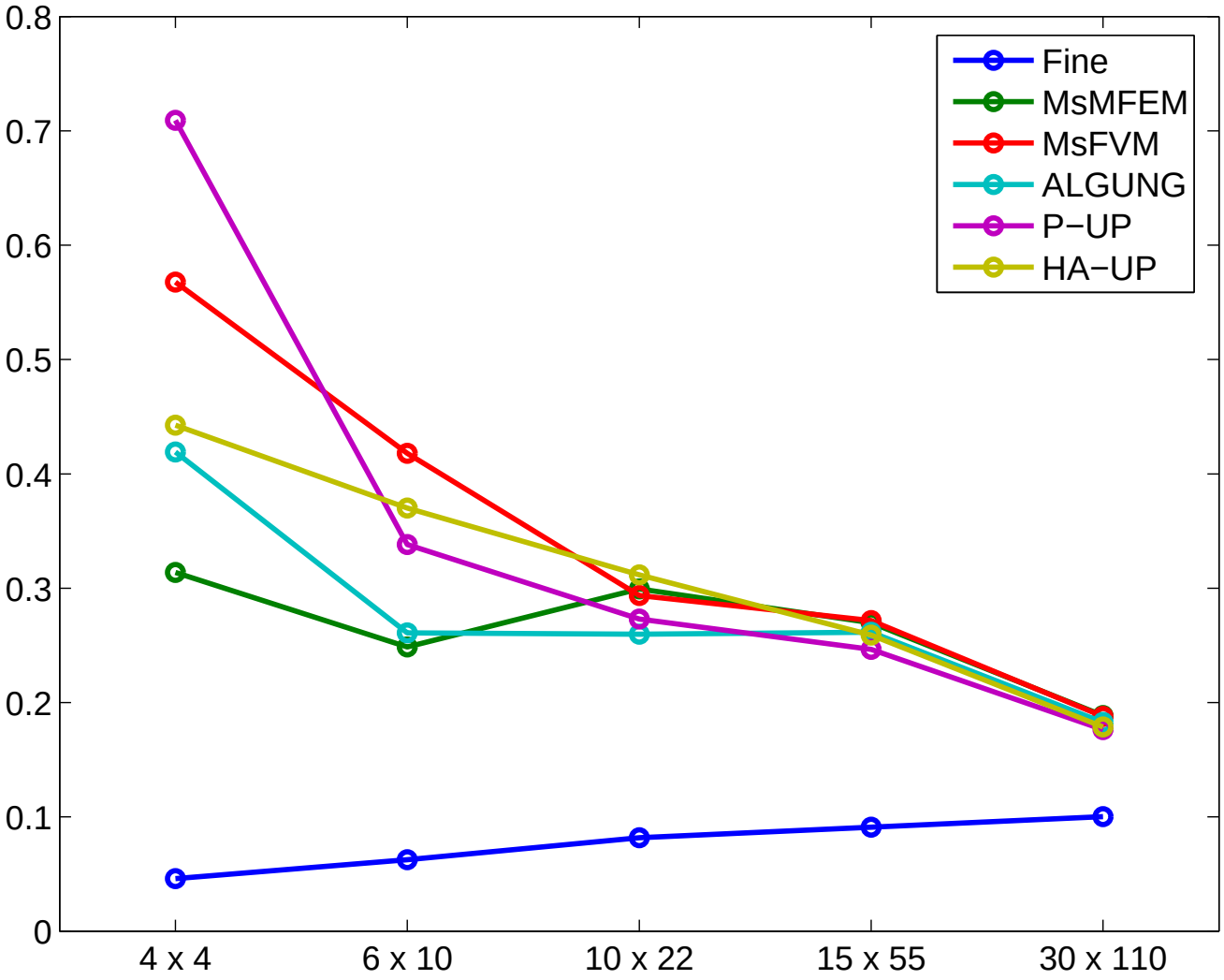
They are efficient: basis functions need to be computed only once.

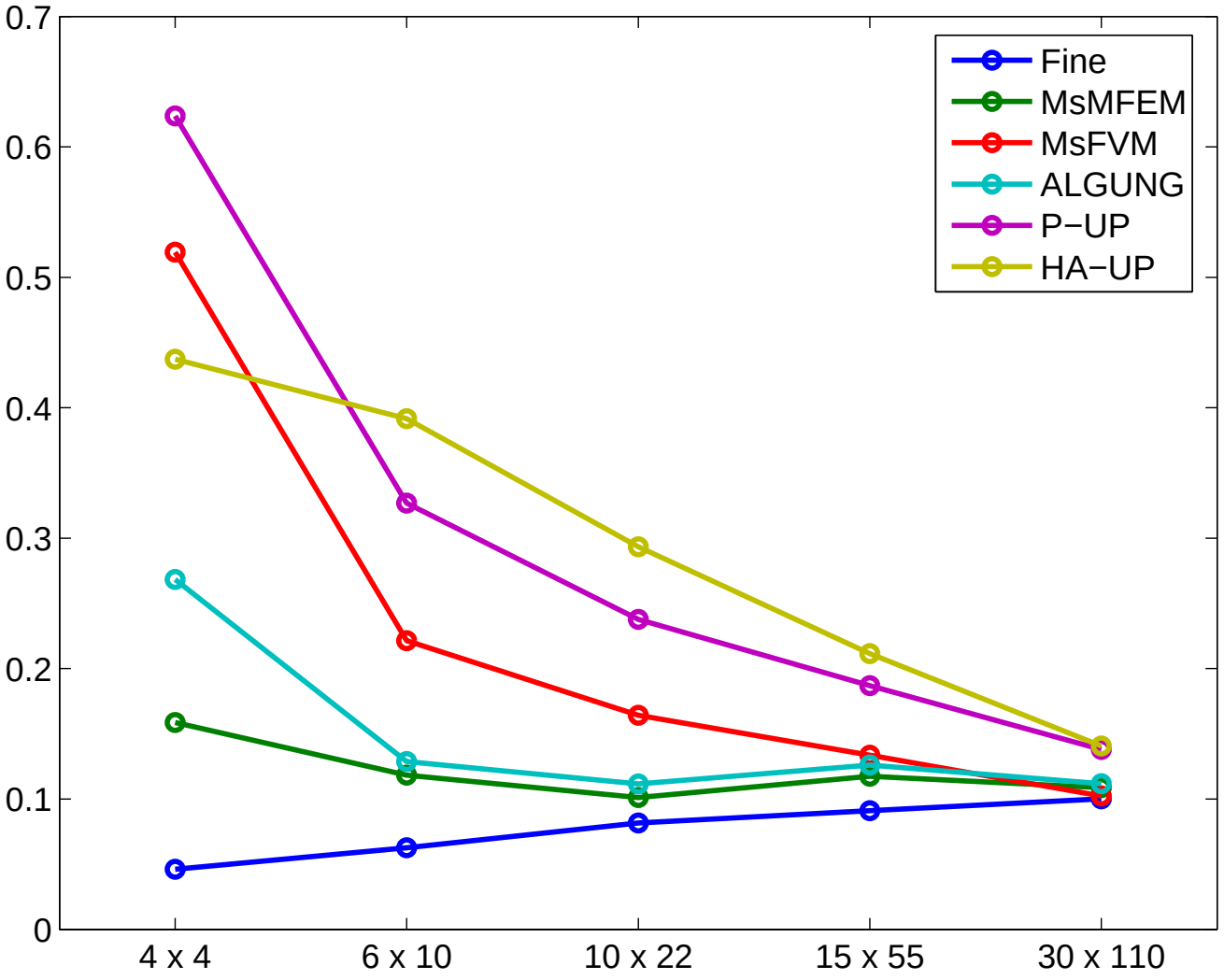
They are flexible: unstructured and irregular grids are handled easily.

They are robust: suitable for modeling flow in porous media with very strong heterogeneous structures.

The MsMFEM provide velocities on coarse and fine grids.

Can the MsMFEM be used as an upscaling method?
yes, but to capitalize on the enhanced resolution provided by the MsMFEM we need to solve the saturation equation on the fine grid.


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
4 x 4
6 x 10
10 x 22
15 x 55
30 x 110
Fine
MsMFEM
MsFVM
ALGUNG
P−UP
HA−UP


0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
4 x 4
6 x 10
10 x 22
15 x 55
30 x 110
Fine
MsMFEM
MsFVM
ALGUNG
P−UP
HA−UP

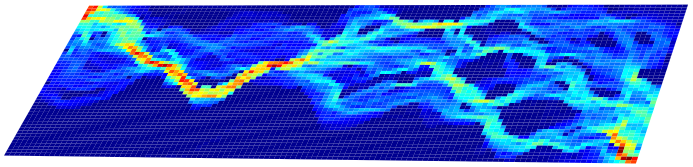
Conclusion I: Multiscale methods for elliptic equations provide a robust and efficient tool to get accurate velocity fields on fine grids,

... but solving the saturation equation on multi-million cell geomodels becomes a bottle-neck in large flow simulations.

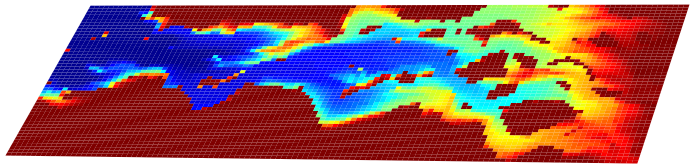
Is it possible to develop a similar multiscale methodology for solving the saturation equation more efficiently?

A multiscale method for the saturation equation

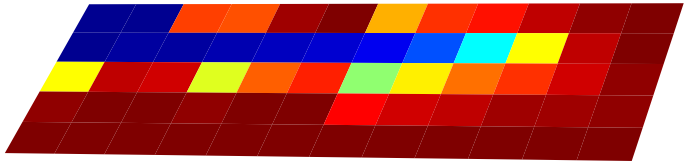
Fine scale velocity at t_{n+1}



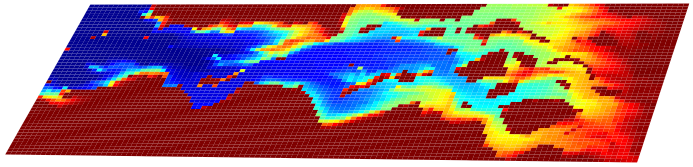
Fine scale saturation at t_n



Coarse scale saturation at t_{n+1}



Fine scale saturation at t_{n+1}



Assume that S^n is a saturation field on the fine grid $\{T\}$ at $t = t_n$, and denote non-degenerate fine grid interfaces by $\gamma_{ij} = \partial T_i \cap \partial T_j$.

1: For each K in the coarse grid, do

$$\bar{S}^{n+1}|_K = \bar{S}^n|_K + \frac{\triangle t}{\int_K \phi \, dx} \left[\int_K q_w \, dx - \sum_{\gamma_{ij} \subset \partial K} F_{ij}(S^n) \right],$$

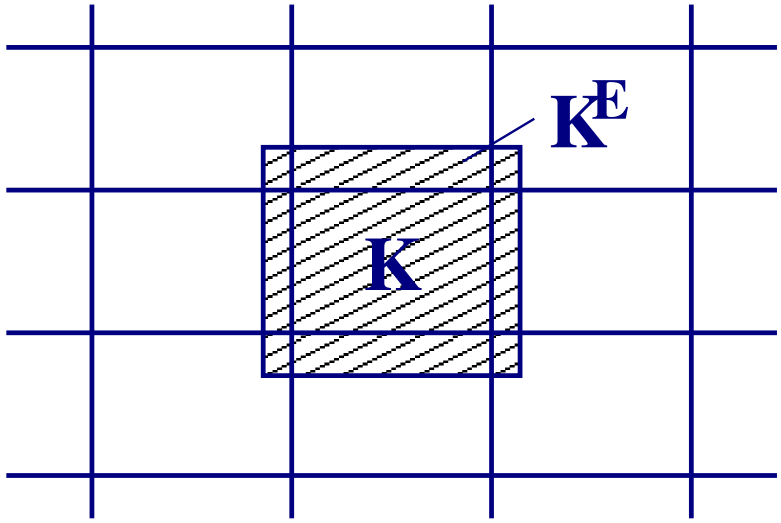
where $F_{ij}(S) = \max\{f_w(S_i)v_{ij}, -f_w(S_j)v_{ij}\}$.

2: Map $\bar{S}^{n+1}|_K$ onto the fine grid: $S^{n+1}|_K = I_K(\bar{S}^{n+1})$.

Here $I_K(\bar{S}) = \chi_K(x, t(\bar{S}))$, where χ_K is a solution of

$$\phi \frac{\partial \chi_K}{\partial t} + \nabla \cdot [f_w v^0] = q_w \quad \text{in } K^E = K \cup \{T : \partial K \cap \partial T \neq \emptyset\},$$

subject to the following constraints:



Fixed velocity: $v^0 = v(x, t_0)$.

Initial conditions: $\chi_K^0 = S^0$.

Boundary conditions: $f_w = 1$ on $\{\gamma_{ij} \subset \partial K^E : T_i \subset K^E,\ v_{ij} < 0\}$.

To ensure that the multiscale method is mass conservative we must choose $t(\bar{S})$ so that $\int_K I_K(\bar{S})\phi \, dx = \bar{S} \int_K \phi \, dx$.

- Analysis (performed by Y. Efendiev) shows that the proposed multiscale method should be accurate away from sharp fronts.
- Sharp fronts occur mainly in transient flow regions.
  These regions are characterized by $\alpha \leq \bar{S} \leq \beta$.
- To enhance the accuracy of the multiscale method one can solve the saturation equation on a fine grid in transient flow regions.
- Domain decomposition type localization procedures provide a natural environment for the development of adaptive schemes.

Domain decomposition method for the saturation equation:

1: For $T_i \in K^E$, compute:

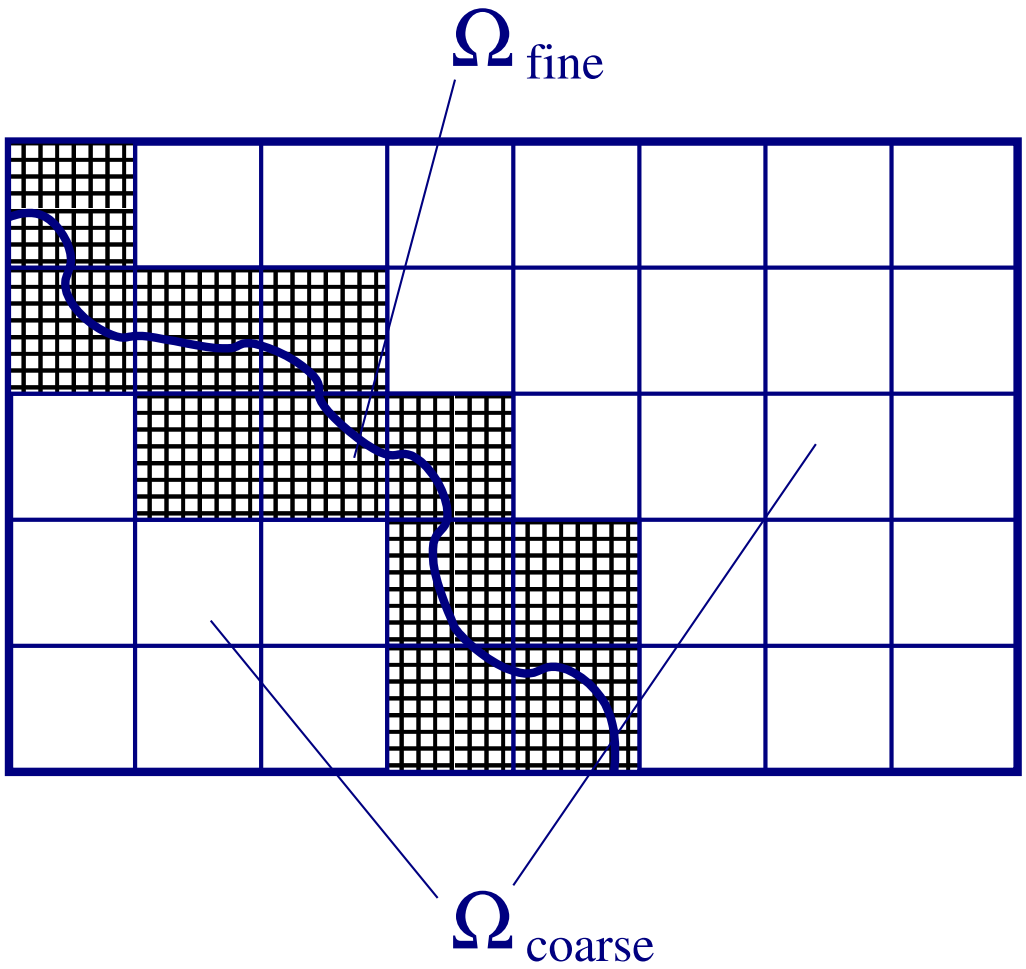
$$S_i^{n+1/2} = S_i^n + \frac{\triangle t}{\phi_i |T_i|} \left(\int_{T_i} q_w(S^{n+1/2})\, dx - \sum_{j \neq i} F_{ij}^* \right),$$

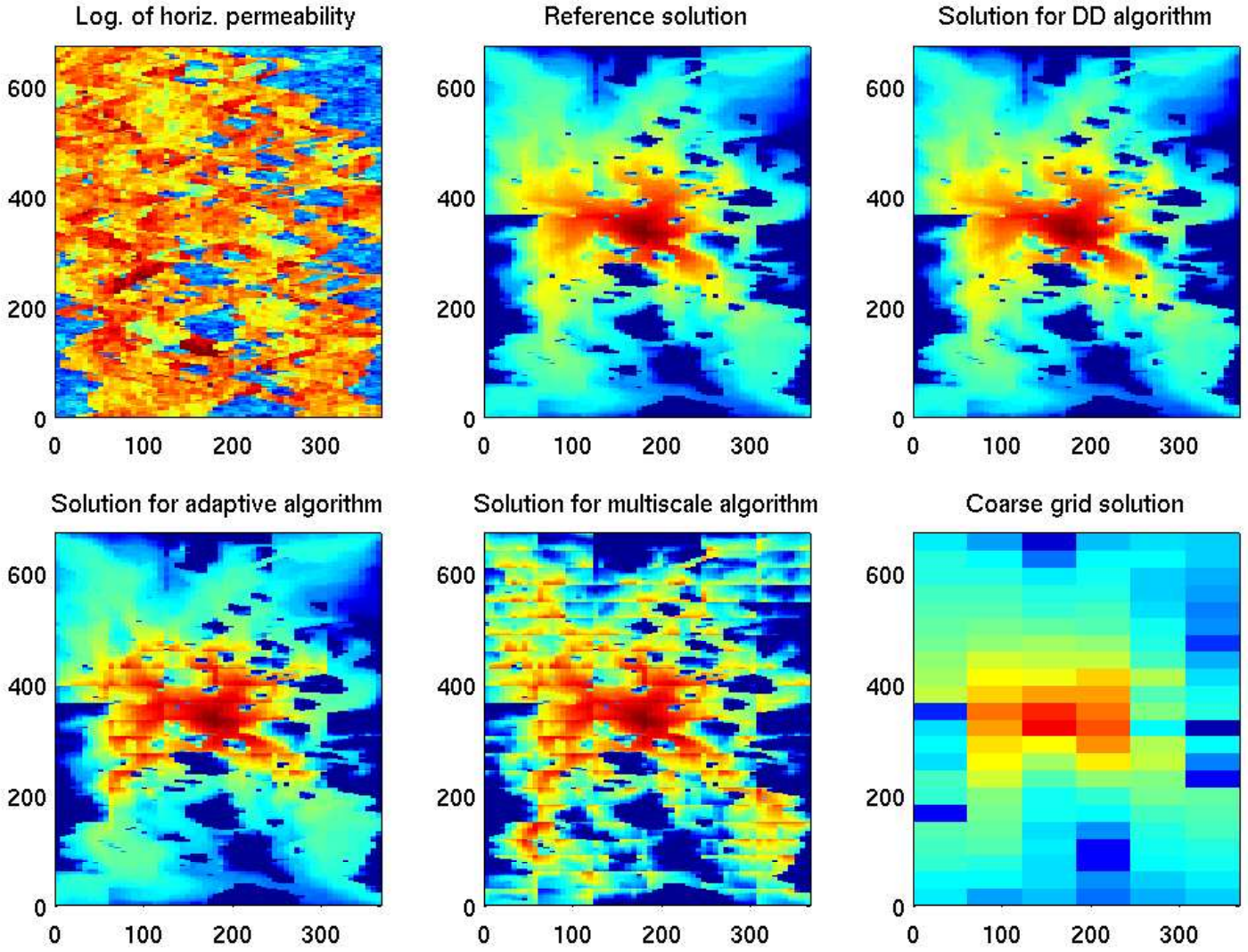
where $F_{ij}^* = \begin{cases} F_{ij}(S^n) & \text{if } \gamma_{ij} \subset \partial K \text{ and } v_{ij} < 0. \\ F_{ij}(S^{n+1/2}) & \text{otherwise.} \end{cases}$

2: For $T_i \in K$, set $S_i^{n+1} = S_i^{n+1/2}$.

Adaptive algorithm:

- Use DD method in regions with rapid transients (Ω_{fine}).
- Use the multiscale method in regions with slow transients (Ω_{coarse}).




Log. of horiz. permeability
Reference solution
Solution for DD algorithm
Solution for adaptive algorithm
Solution for multiscale algorithm
Coarse grid solution
0
200
400
600
0
100
200
300

Benchmark: 10th SPE comparative solution project

- **Fine grid:** $1.122 \cdot 10^6$ cells, **Coarse grid:** 2244 blocks.
- **MsMFEM** for pressure eq., **MsFVM** for saturation eq.

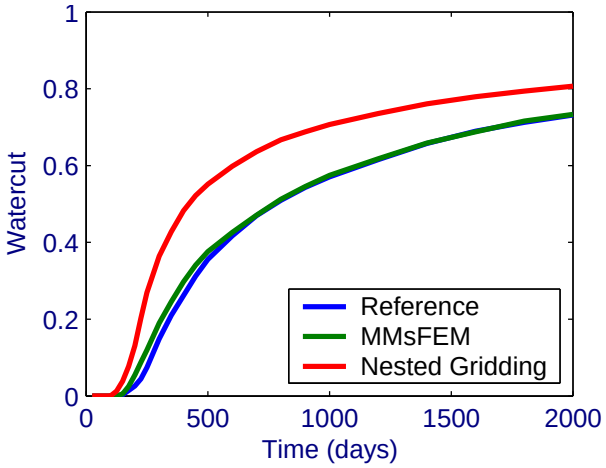
To assess solution accuracy we employ the following norms:

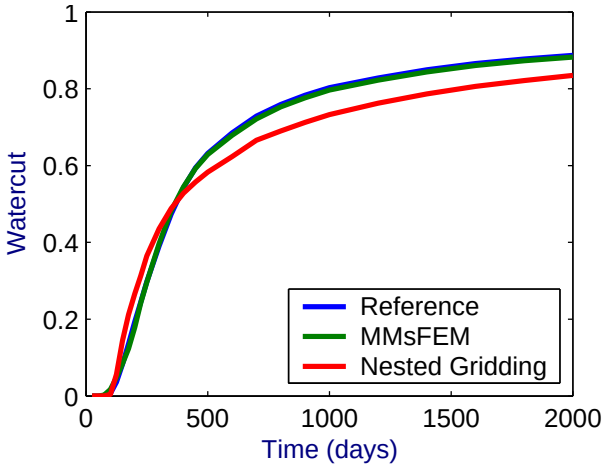
$$e_F(S) = \frac{\|S_{\text{ref}} - S\|_{L^2_\phi}}{\|S_{\text{ref}} - S^0_{\text{ref}}\|_{L^2_\phi}}, \qquad e_C(S) = \frac{\|\bar{S}_{\text{ref}} - \bar{S}\|_{L^2_\phi}}{\|\bar{S}_{\text{ref}} - \bar{S}^0_{\text{ref}}\|_{L^2_\phi}}.$$

Here $\bar{S}$ denotes the coarse grid saturations corresponding to S, and

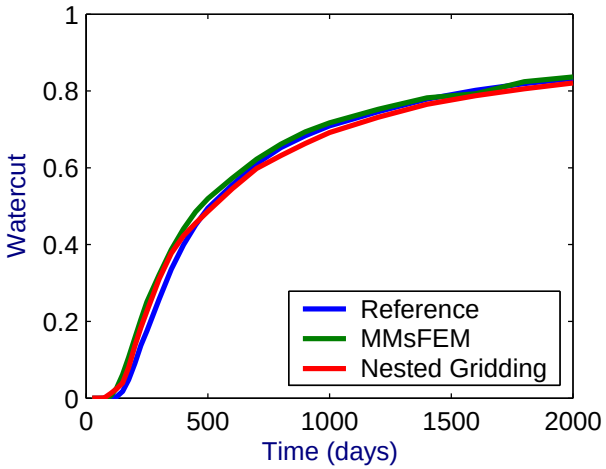
$$\|S\|^2_{L^2_\phi} = \int_\Omega (S\phi)^2 \, dx.$$

Water-cut curves for MsMFEM + streamline simulation:

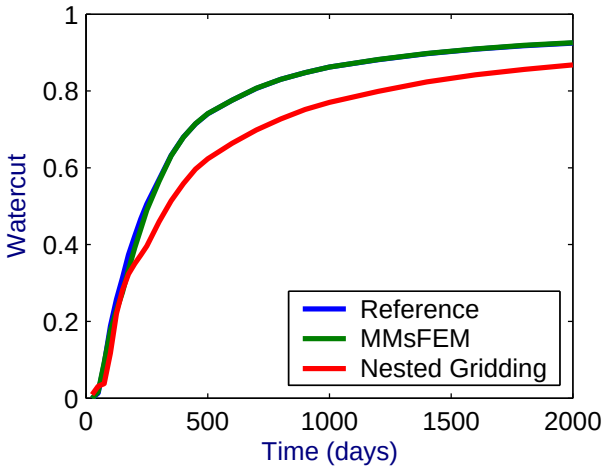




Producer C

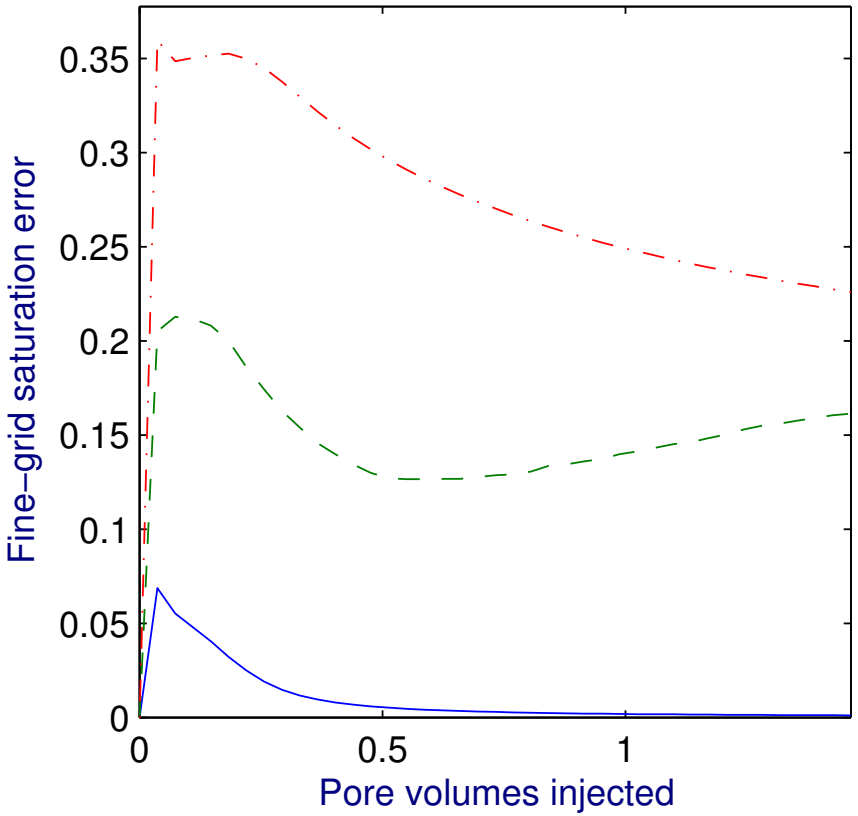


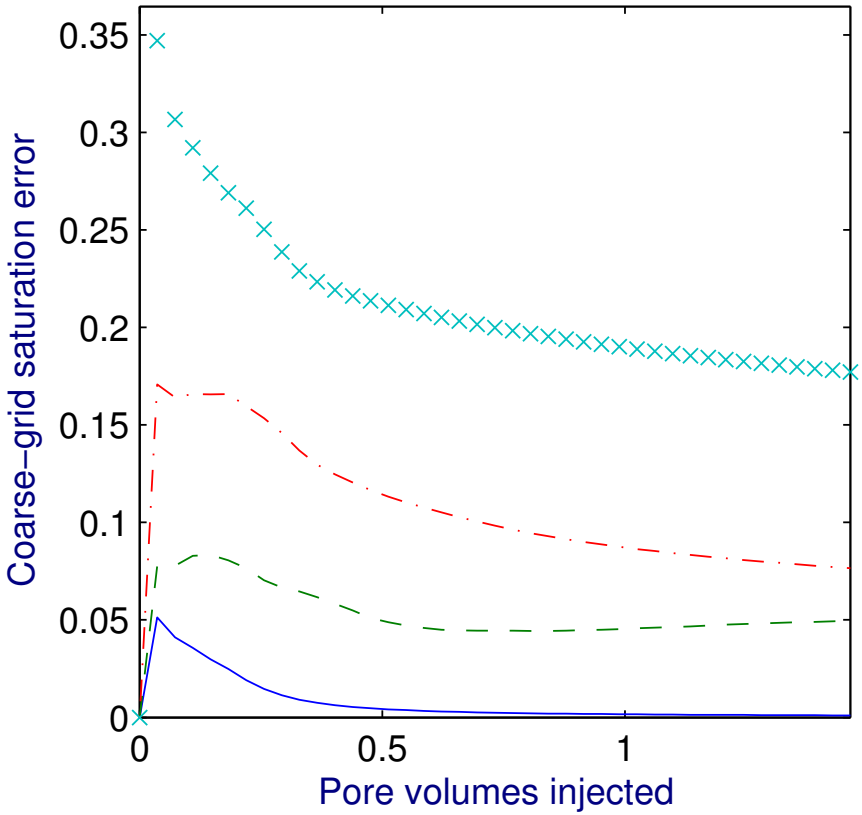
Producer D



SINTEF

Accuracy of saturation profiles obtained using AMsFVM:





Water-cut curves

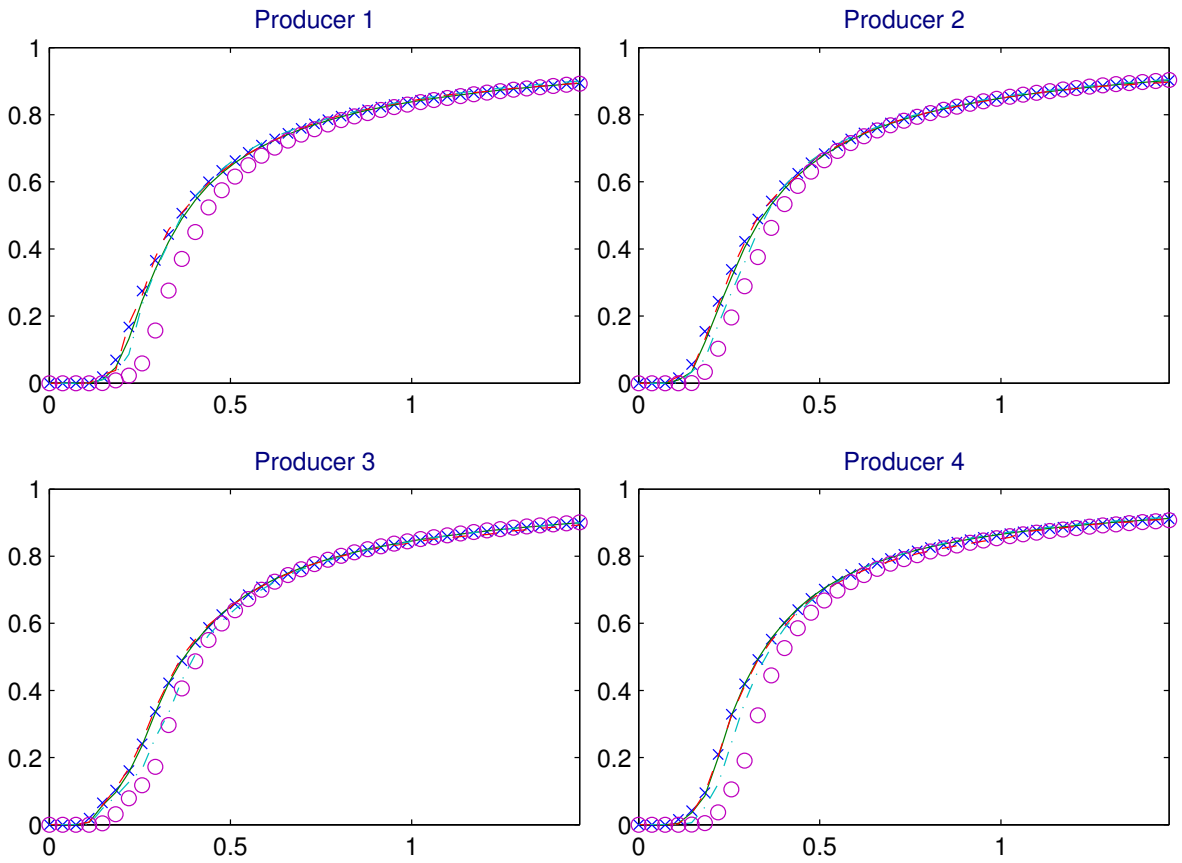
x: Reference solution.

–: DD method.

--: Ad. Ms. method

-·: Multiscale method

o: Coarse FV method



Conclusions and Acknowledgments

To run simulations directly on geological models require faster and more flexible simulators than what we have available today. Multiscale methods, as the ones presented, have a natural flexibility, and provide a tool for running high-resolution reservoir simulations, possibly on geological models.

Collaborators:

SINTEF

Stein Krogstad
Vegard Kippe
Knut-Andreas Lie

Texas A&M

Yalchin Efendiev