Streamlines and a Multiscale Method

Towards Scalable, Robust and Fast Reservoir Simulation

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1 Introduction

A robust reservoir simulator should be able to produce accurate results at a reasonable computational cost for a wide range of reservoir models. Depending on the application, the size and characteristics of the reservoir data will be very different, as will the challenges met by the simulator. E.g., for history-matching purposes the crucial requirement is rapid simulation of relatively small models, while for a detailed study of a water-injection scenario it may be desirable to use a simulation model as close to the geostatistical model as possible. Thus, considering that modern geostatistics can produce models on the order of a billion unknowns, a scalable reservoir simulator must also be capable of producing results for very large models. By combining a streamline method for the convective step with a recent multiscale approach for obtaining a conservative velocity field we hope to take a step towards more scalable, more robust and faster reservoir simulation.

2 Mathematical Reservoir Model

The governing equations for two-phase (oil/water), immiscible, incompressible, isothermal, non-reactive and non-dispersive flow in porous media can be written as [3],

$$-\nabla \cdot \vec{u} = q, \tag{1}$$

$$\phi \frac{\partial S_w}{\partial t} + \vec{u} \cdot \nabla f_w + \nabla \cdot \vec{G}_w = q f_w, \tag{2}$$

where $\vec{u}$ is the total Darcy velocity,

$$\vec{u} = \vec{\vec{K}} \cdot (\lambda_t \nabla P + \lambda_g \nabla D), \tag{3}$$

S_w is the water saturation, q is the volumetric flowrate in the wells, $\vec{\vec{K}}$ is the absolute permeability tensor, P is pressure, D is depth, λ_t and λ_g are the total and total gravity mobilities defined by,

$$\lambda_t = \frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{/\mu_o}, \qquad \lambda_g = g(\frac{k_{rw}\rho_w}{\mu_w} + \frac{k_{ro}\rho_o}{\mu_o}), \tag{4}$$

and f_w and $\vec{G}_w$ are the fractional and gravity fractional flow of water,

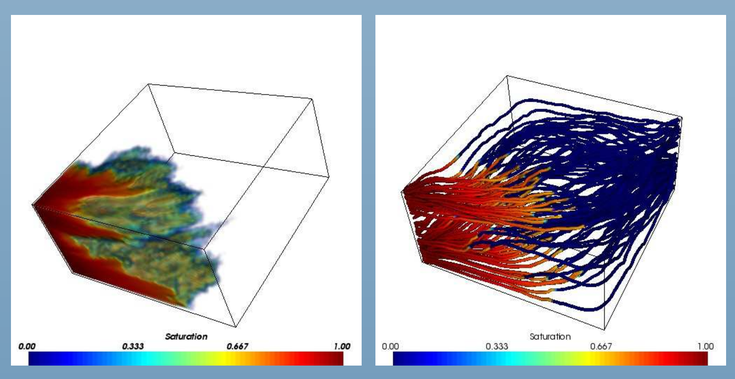
$$f_w = \frac{k_{rw}/\mu_w}{\lambda_t}, \qquad \vec{G}_w = \vec{\vec{K}} \cdot g \nabla D \frac{\frac{k_{rw}}{\mu_w} \cdot \frac{k_{ro}}{\mu_o}}{\lambda_t}(\rho_o - \rho_w). \tag{5}$$

Here k_{rj} is the relative permeability of phase j, ρ_j the density, μ_j the viscosity and g the gravity constant.

3 The Streamline Method

The streamline method is based on an IMPES approach for solving the above equations, where the pressure equation (1) is solved by an implicit scheme and the saturation equation (2) is evolved explicitly in time, assuming constant pressure during the time step. The pressure is then recomputed using the updated phase saturations, and the process is repeated.

However, instead of solving the full, three-dimensional equation (2) directly, it is decoupled into multiple one-dimensional equations along streamlines. This approach intuitively makes sense since the streamlines define the most natural of all possible "flow grids". The idea is to cover the reservoir with a suitable number of streamlines, traced from injectors to producers (in the incompressible case). The phase saturations are then mapped from the numerical grid to the streamlines, and for each streamline a one-dimensional equation is solved to obtain the saturations along the streamline at the end of the IMPES time step. The streamline saturations are then mapped back to the grid by some averaging scheme, thus enabling solution of the pressure equation for the next time step. Figure 1 illustrates the steps of the method.



(a) The initial saturation field is used to evaluate mobilities and compute the global pressures.

(b) The pressure defines the velocity field, and the saturations from (a) are mapped to streamlines.

(c) Saturations are moved forward along the streamlines under the assumption that the streamlines remain fixed during the time step.

(d) Finally the streamline saturations are mapped back onto the grid to yield a new saturation field, and the process can be repeated.

Figure 1: The steps of a streamline method.

3.1 Mathematical Formulation

Mathematically the transformation from three to one dimension is performed by introducing the *time-of-flight*, denoted τ, along a streamline,

$$\tau(s) = \int_0^s \frac{\phi(\zeta)}{|\vec{u}|}\,\mathrm{d}\zeta, \tag{6}$$

which is just the time it takes a particle to reach a distance s along the streamline. Inserting the differential form of (6) into (2) then yields the following one-dimensional equation along the streamline:

$$\frac{\partial S_j}{\partial t} + \frac{\partial f_j}{\partial \tau} + \frac{1}{\phi}\nabla \cdot \vec{G}_j = \frac{q f_j}{\phi} \tag{7}$$

This equation is considerably easier to solve than the original three-dimensional version (2), and can be solved by any suitable numerical scheme.

3.2 Properties

Two of the most important properties of the streamline method are:

- **Speed.** The method is very fast, and allows routine simulations of million grid block cases even on single workstations.
- **Scalability.** Low memory requirements and completely independent processing of streamlines makes the streamline method scalable both on serial and parallel computer architectures.

It is also worth noting that the application area of the streamline method is not restricted to the situation described above. In fact, streamlines have been successfully applied to much more general cases, e.g., multiphase, dispersive, compositional displacement [5, 2] and unstructured grids [4].

4 A Mixed Multiscale FEM Method

The basic idea of a multiscale method is to incorporate fine scale information into a solution on a coarser scale. Figure 2 shows a schematic view of the overall strategy. The starting point (A) is a coarse grid superimposed on a fine grid, with the relevant data given on the fine grid. The first step (1) is to process the subgrids within each coarse block to obtain some quantity representative of the local behaviour. Having done this (B), step (2) combines these quantities to yield a global solution on the coarse scale (C). From here it is possible to continue with the computation on the coarse scale, in which case the multiscale method may be seen as a generalization of upscaling. However, often it is both desirable and feasible to also perform step (3), which is to recover a solution on the fine grid.

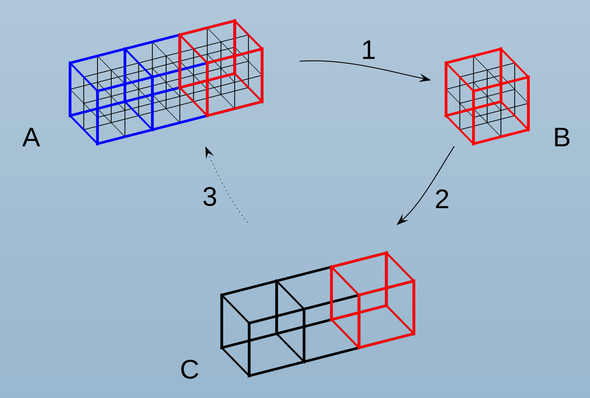


Figure 2: Schematic view of a multiscale method.

4.1 MMsFEM

The MMsFEM methods are mixed FEMs that use an approximation space $\mathcal{V}_h$ for the Darcy velocity $\vec{u}$ that captures the local behaviour of the differential operator $L = -\nabla \cdot \vec{\vec{K}}\lambda_t \nabla$. The approximation space is spanned by a set of basis functions $\vec{\psi} = -\vec{\vec{K}}\lambda_t \nabla \phi$, where ϕ satisfies $L\phi = f_K$ inside each coarse block K for some appropriate source term f_K and boundary condition ν_K. By a special choice of such basis functions the MMsFEM yields a conservative velocity field suitable for streamline tracing also on the fine grid.

Letting Ω be the reservoir domain and Π_h the space of piecewise constants, the mixed FEM formulation of the pressure equation (1) reads:

Find $(\vec{u}_h, P_h) \in \mathcal{V}_h \times \Pi_h$ *such that,*

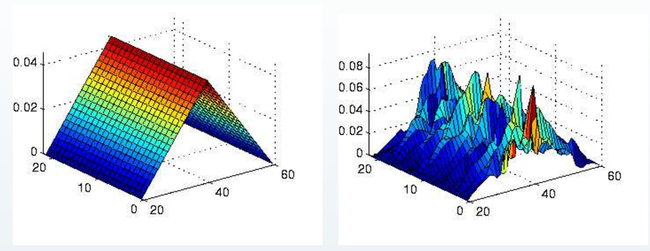
$$\begin{aligned} \int_\Omega (\vec{\vec{K}}\lambda_t)^{-1}\,\vec{u}_h \cdot \vec{v}_h\,\mathrm{d}\vec{x} &= \int_\Omega P_h \nabla \cdot \vec{v}_h\,\mathrm{d}\vec{x} - \int_\Omega \frac{\lambda_g}{\lambda_t} D \nabla \cdot \vec{v}_h\,\mathrm{d}\vec{x} \quad && \forall \vec{v}_h \in \mathcal{V}_h, \\ \int_\Omega Q_h \nabla \cdot \vec{u}_h\,\mathrm{d}\vec{x} &= \int_\Omega q Q_h\,\mathrm{d}\vec{x}, && \forall Q_h \in \Pi_h. \end{aligned} \tag{8}$$

Denote the coarse grid faces by $\Gamma_{ij} = \partial K_i \cap \partial K_j$. Then $\mathcal{V}_h = \mathrm{span}\{\vec{\psi}_{ij}\}$ where each $\vec{\psi}_{ij}$ is associated with the face Γ_{ij}, and defined by,

$$(\nabla \cdot \vec{\psi}_{ij})_{|K} = -\nabla \cdot \vec{\vec{K}}\lambda_t \nabla \phi_{ij} = \begin{cases} \frac{1}{|K|}, & \text{in non-well blocks,} \\ \frac{q}{\int_K q\,\mathrm{d}\vec{x}} & \text{in well blocks.} \end{cases} \tag{9}$$

Each basis function $\vec{\psi}_{ij}$ is non-zero in the interior of blocks K_i and K_j, as well as on the interface Γ_{ij} itself where the boundary condition ν_{ij} must be satisfied. Away from the wells it is reasonable to expect that the velocity is determined by the pressure drop across the face, thus ν_{ij} is assumed proportional to the face transmissibility. In well blocks the radial flow pattern is also taken into account, thus ν_{ij} is taken to be proportional to the product of the face transmissibility and the radial distance to the well. The boundary conditions are also scaled to ensure compatibility, i.e., to ensure that $\int_{\Gamma_{ij}} \nu_{ij}\mathrm{d}\,s = 1$.

The resulting MMsFEM method can be viewed as an extension of the Raviart-Thomas mixed FEM to the case where the coefficients can vary within each element. Figure 3 illustrates this for a 2D example.



(a) Homogeneous

(b) Heterogeneous

Figure 3: x-component of a 2D basis function for homogeneous and heterogeneous coefficients. In the homogeneous case the basis reduces to the lowest order Raviart-Thomas basis, but in the heterogeneous case it is changed by the small-scale variations.

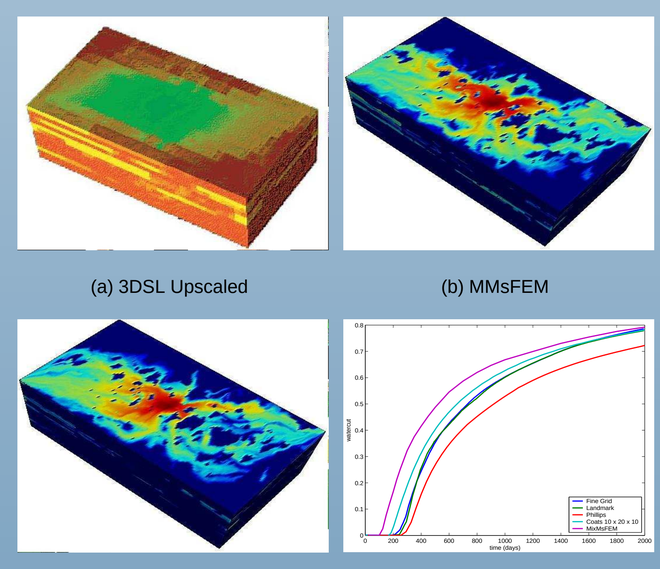
4.2 Properties

Some of the most important properties of the MMsFEM are:

- **Mass conservative on the fine grid.** The particular choice of basis functions described above yields a mass-conservative fine grid velocity field which can be used for streamline tracing.
- **Automatic incorporation of small-scale effects into a coarse grid solution.** Thus the method may be viewed as a robust alternative to upscaling if subsequent computations are performed on the coarse scale.
- **Subgrid flexibility.** The method does not limit the subgrids in any way, and they can be tuned to model physical features such as fractures or faults. Also, any numerical method (not only FEMs) can be used for the subgrid problems.
- **Scalability.** The basis functions are processed individually, and the method is thus very well suited for parallel implementation. Also, on a single processsor system the memory requirement is reduced since the global problem only has to be solved on the coarse scale.
- **Potential speed.** Can be computationally very efficient if recomputation of the basis functions at every time step is avoided, e.g. by using some adaptive scheme based on a measure of the changing mobility field.

5 Example

The data is Model 2 from the 10th SPE Comparative Solution Project [1]. Dimensions are $60 \times 220 \times 85$, and there is a single injector in the middle and producers in the four corners. Figure 4 compares an upscaled solution to the results obtained with the MMsFEM/Streamline method. The MMsFEM coarse grid only has a resolution of $5 \times 11 \times 17$, but still the solution has a very high level of detail and is close to the reference. The watercut curves show that the error is comparable to that of the pseudo-based upscaling techniques.



(a) 3DSL Upscaled

(b) MMsFEM

(c) Fine Grid Reference

(d) Watercuts

Figure 4: Comparison of MMsFEM and upscaling.

6 Concluding Remarks

By combining a multiscale method for the pressure equation (1) with a streamline method for the saturation step (2) we obtain a simulation strategy that is very scalable and may help bridge the gap between geostatistical and reservoir simulation models. The approach is also robust in the sense that many different situations can be handled efficiently by tuning the subgrids to model various physical features. Experiments with adaptive recomputation of the basis functions have shown that less than 10% of the functions needs to be recomputed at every time step, thus MMsFEM can accelerate the already very fast streamline method for large reservoir models.

References

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