Toward reservoir simulation on geological grid models

Jørg Espen Aarnes & Knut-Andreas Lie
SINTEF ICT, Dept. of Applied Mathematics

ECMOR IX, 2004 – Cannes, France

Motivation

- Modern reservoir simulators are not able to run routine simulations on geological grid models.
- Upscaling techniques are used to create coarsened grid models for day-to-day simulation.
  - The price to pay is less reliable results.
- Multiscale methods offer the possibility of bridging the gap between the geoscale and the simulation scale.

Model problem (Incompressible two-phase flow)

Incompressible two-phase flow is modeled by the Darcy law:

$$v_i = -k\lambda_i(\nabla p_i - \rho_i G)$$

and the continuity equations for each phase

$$\phi \partial_t S_i + \nabla \cdot v_i = q_i \ .$$

SINTEF

Assuming $S_o + S_w = 1$ we deduce an elliptic equation for pressure

$$v = -k\lambda\nabla p + k\lambda_g G \quad \text{and} \quad \nabla \cdot v = q.$$

and the following advection-diffusion equation for water saturation

$$\nabla \cdot \left[f_w(v + k\lambda_o(\rho_o - \rho_w)G + k\lambda_o \frac{\partial p_{cow}}{\partial S_w}\nabla S_w) \right] = q_w.$$

SINTEF

- The permeability k typically span over many length scales.
  - The velocity field can have a multiple scale structure.
- Details at all scales have a strong impact on the solutions.
  - Conventional numerical methods which are not adaptive to the information at the subgrid scales may give poor accuracy.
  - A prohibitively large number of variables are often needed to resolve all the subgrid scales.
- Hierarchical or multiscale modeling approaches are needed.

The Multiscale Finite Element Methods

The multiscale finite element methods (MsFEMs) is a class of FEMs for (nearly) elliptic problems with multiple scale coefficients.

> *Multiscale methods*: Methods that incorporate fine scale information into a set of coarse scale equations in a way which is consistent with the local property of the differential operator.

The MsFEMs are based upon the construction of appropriate "coarse-scale" approximation spaces that are adaptive to the local property of the elliptic differential operator.

A mixed MsFEM for reservoir simulation

Let the reservoir Ω be partitioned into mutually disjoint elements $\mathcal{K} = \{K\}$ of arbitrary shape and size.

The proposed multiscale method seeks $v \in V$ and $p \in U$ such that

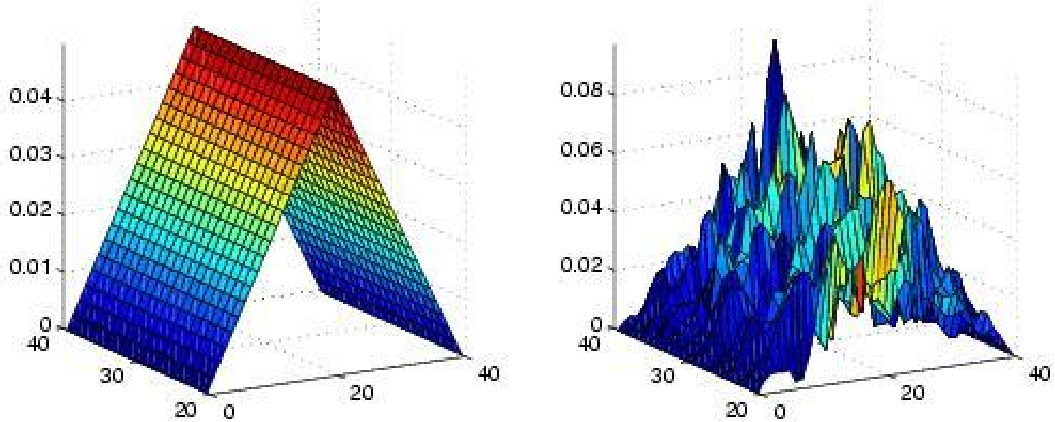
$$\begin{aligned} \int_\Omega (k\lambda)^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= \int_\Omega \frac{\lambda_g}{\lambda} G \cdot u \, dx \\ \int_\Omega l \, \nabla \cdot v \, dx &= \int_\Omega q l \, dx \end{aligned}$$

for all $u \in \{v \in V : v \cdot n = 0 \text{ on } \partial\Omega\}$ and $l \in \mathcal{P}_0(\mathcal{K})$.

$V = \text{span}\{\psi_{ij} : \text{meas}(\partial K_i \cap \partial K_j) > 0\}$ where ψ_{ij} is defined by

$$\psi_{ij} = -k\lambda\phi_{ij}, \quad \nabla \cdot \psi_{ij} = \pm \begin{cases} \frac{1}{|K|} & \text{if} \quad \int_K f\,dx = 0, \\ \frac{f}{\int_K f\,dx} & \text{if} \quad \int_K f\,dx \neq 0, \end{cases}$$

and no-flow boundary conditions on $(\partial K_i \cup \partial K_j) \backslash (\partial K_i \cap \partial K_j)$.



SINTEF

$U = \hat{p} + \mathcal{P}_0(\mathcal{K})$ where $\hat{p}$ is defined by

$$\hat{p} = \begin{cases} 0 & \text{if} \quad \int_K f \, dx = 0, \\ \pi(K) & \text{if} \quad \int_K f \, dx \neq 0, \end{cases}$$

and $\pi(K)$ is related to $v = \sum_{ij} v_{ij}\psi_{ij}$ by

$$\pi(K_i) = \sum_j v_{ij} \left(\phi_{ij} - \int_{K_i} \frac{\phi_{ij}}{|K_i|} \, dx \right).$$

Some of the features that makes this MsFEM an attractive tool for reservoir simulation are:

- The elliptic "parallelization": subgrid resolution at a low cost.
- The flexibility: the natural ability to handle
  - heterogeneous and anisotropic materials,
  - irregular and unstructured grids.
- The ideal foundation for adaptive numerical schemes for the solution of advective transport equations.

The saturation equation

- The pressure equation stands for the majority of the CPU time in reservoir simulation.
  - The proposed MsFEM provides a basis geoscale simulation.
- Is it possible to exploit all information inherent in the MsFEM solution when solving the saturation equation?
- If so, is it worthwhile – will the simulated saturation profiles be closer to the true flow scenario in the reservoir?

Alternative 1: Streamline simulation

- Streamline methods offer the possibility of rapid reservoir performance predictions, and are accepted as a complementary technology to traditional FD based simulation.

- Streamline methods are disputed because
  - They require a sequential IMPES formulation.
  - They do not account for capillary pressure.
  - The speed advantage may diminish for compressible flow.

Alternative 2: Coarse scale FD simulation

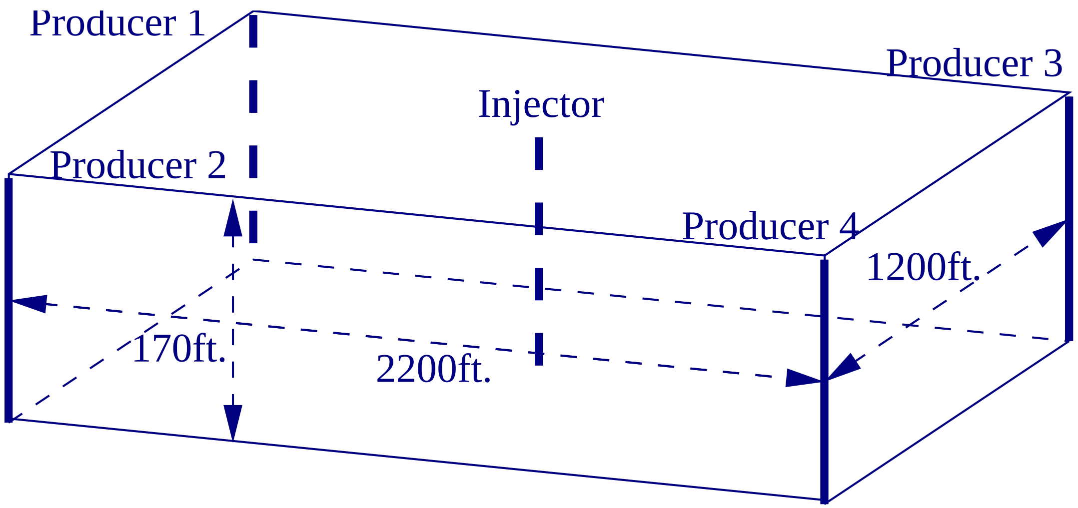
- FD methods allow us to incorporate complex physics, but available computing resources prevents geoscale simulation.
  - Coarse scale FD simulation is the only valid option.
- Some of the primary drawbacks with this approach are:
  - The need to compute pseudofunctions.
  - Important subgrid information is neglected.
  - Strong numerical diffusion.

Alternative 3: Multiscale FD simulation

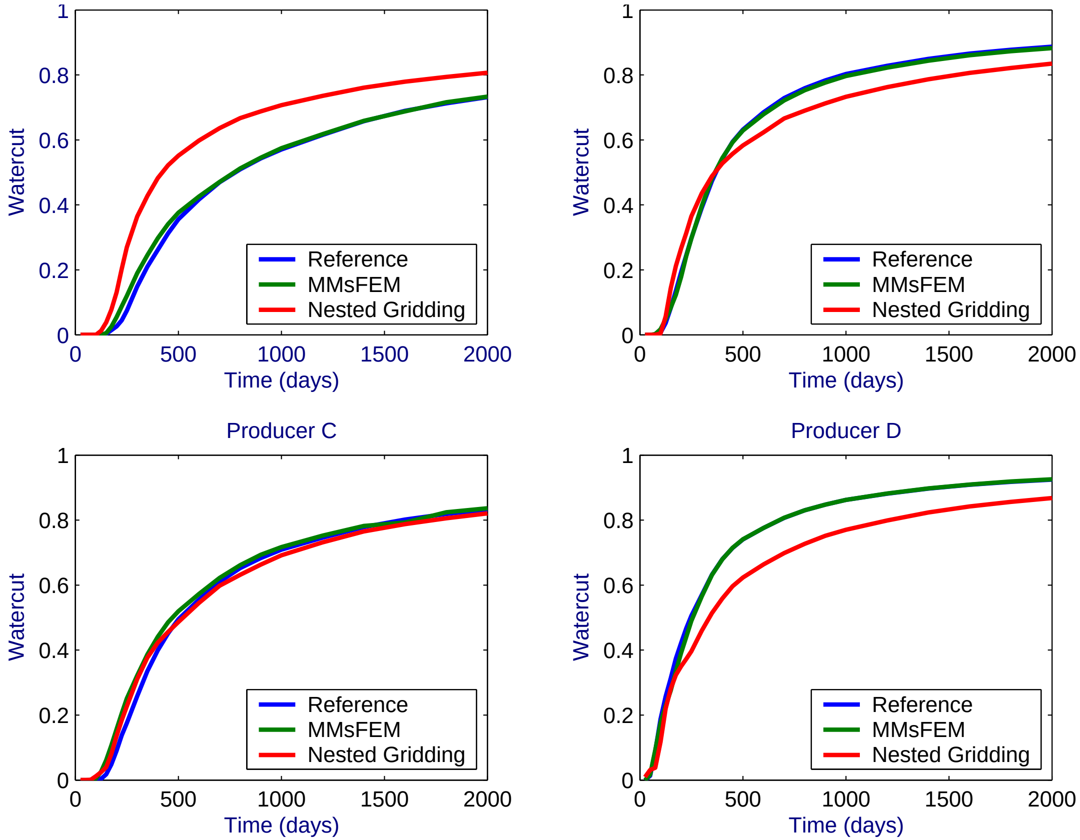
- Multiscale method for the advection–diffusion equations may take many different forms. Some examples are:
  - Methods based on the separation of scales.
  - Methods that resolve the subgrid scales locally and the large scales on a coarse grid.
  - Methods that incorporate fine scale information into an equation modeling the transport at the coarse scale level.
- The pros and cons of these methods are yet to be established.

Numerical Test Case (two-phase flow)

We show results for the multiscale–streamline methodology on the second test case used in the 10th SPE comparative solution project.



- $60 \times 220 \times 85$ cells.
- Sim. time: 2000 days.
- Prod. at 4000 psi bhp.
- No-flow BC.

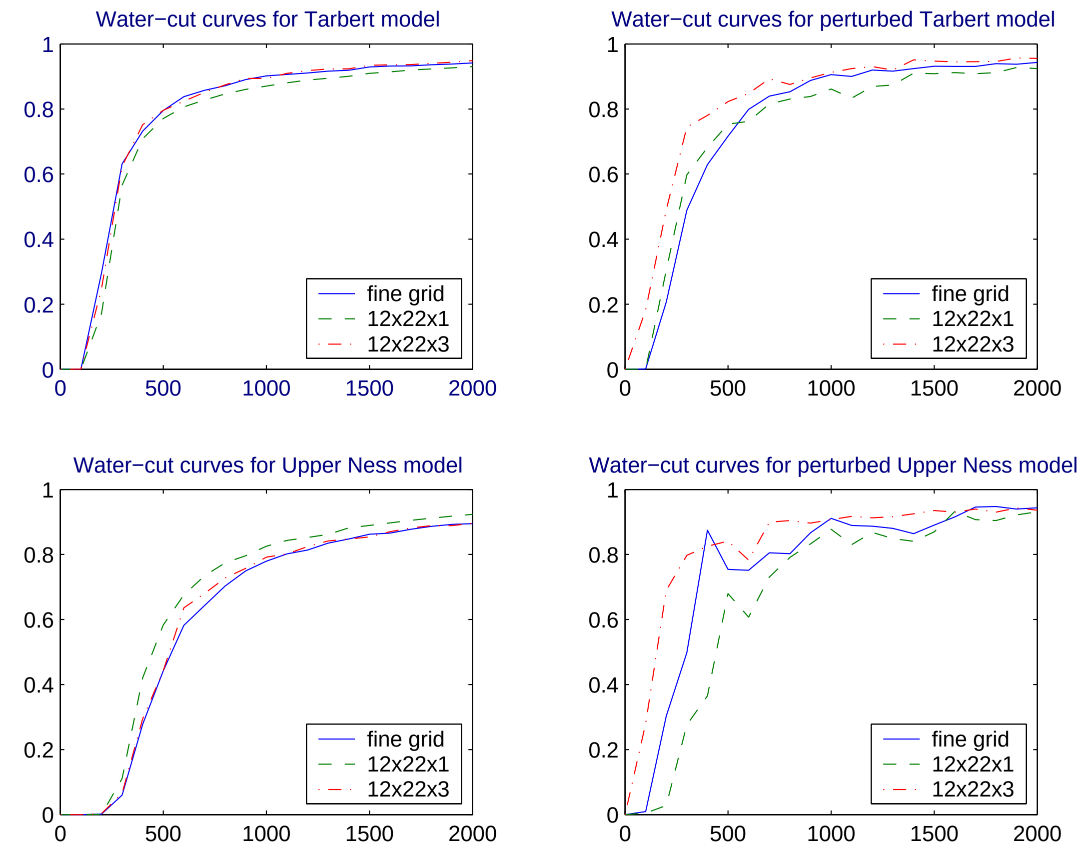


Water-cut curves for

- Reference solution
- Mixed MsFEM
- NG-method

using Alternative 1: Streamline simulation.

SINTEF



Accumulated producer water-cut curves for

- Reference solution
- MMsFEM (12x22x1)
- MMsFEM (12x22x3)

for the 3 top layers and the 3 bottom layers.

SINTEF

Conclusions

- Improved reservoir descriptions demand better and faster simulation tools that can handle grid models with $> 10^6$ cells.
- The mixed MsFEM has shown promising results in terms of computing accurate velocity fields at a low cost.
  - The well-model for the mixed MsFEM should be improved.
- Streamline methods is a natural option for solving the saturation equation, but coarse scale FD methods and multiscale FD methods may also be good alternatives.

Announcement

A Workshop on multiscale modeling with applications to fluid flow and material science will take place in Oslo, Norway, October 18-20.

The final day will put emphasis on the use of multiscale methods in reservoir management.

Please visit

www.cma.uio.no/conferences/2004/multiscale_workshop.html

for further information.