Multiscale methods and streamline simulation for rapid reservoir performance prediction

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Problem

Simulation of immiscible, incompressible two-phase flow, modelled by Darcy's law and a continuity equation for each phase

$$v_i = -k\lambda_i \nabla p_i \tag{1}$$

$$\phi \partial_t S_i + \nabla \cdot v_i = q_i \ . \tag{2}$$

Model reformulation: elliptic pressure equation and hyperbolic saturation equation

$$v = -k\lambda \nabla p, \qquad \nabla \cdot v = q \tag{3}$$

$$\phi \partial_t S + \nabla \cdot (v f(S)) = 0 \tag{4}$$

Motivation

- Gap between geological and simulation models:
  - High-resolution geomodels may have $10^7 - 10^8$ cells
  - Conventional (FV/FD) simulators are capable of about $10^5 - 10^6$ cells
- Upscaling coefficients is not always the answer
  - Loss of details and lack of robustness
- Simulation speed is crucial
  - Large models = high runtime $\Longrightarrow$ parallel implementation?
  - History matching, ranking, process optimization: require fast simulations

Novel approach: multiscale FEM + streamlines

- streamline methods:
  - well-known, fast simulation method that allows simulation of a few million grid-block on single workstations
  - works by convecting phase saturations along streamlines that are given by a mass-conservative velocity field
- multiscale finite element methods:
  - recent approach for solving elliptic equations with strongly heterogeneous coefficients
  - capable of producing conservative velocity fields at multiple scales

Detour: upscaling elliptic problems

Find $a^* = a^*(\bar{x})$ such that if u and u^* solve

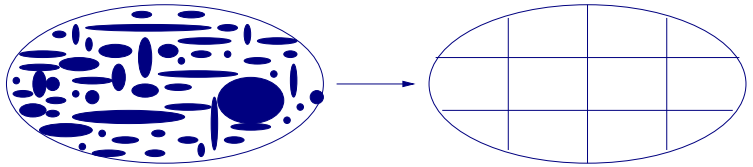
$$\nabla \cdot q = f \qquad \text{and} \qquad \nabla \cdot q^* = \bar{f}\ ,$$

where $q = -a(x)\nabla u$ and $q^* = -a^*(\bar{x})\nabla u^*$, then

$$u^* \sim \bar{u} \qquad \text{and} \qquad q^* \sim \bar{q}\ .$$

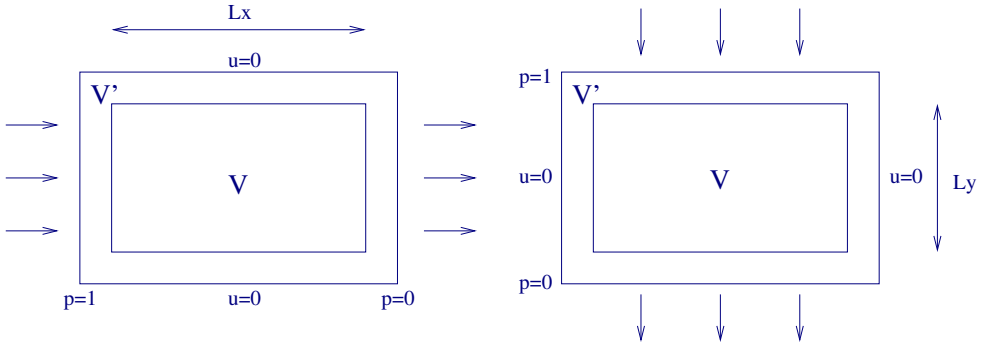
Here the overbar denotes averaged quantities on a coarse grid.

Upscaling permeability:



How do we represent fine-scale heterogeneities on a coarse scale?

- Arithmetic, geometric, harmonic, or power averaging ($(\frac{1}{|V|}\int_V a(x)^p\,dx)^{1/p}$)
- Equivalent permeabilities ($a^*_{xx} = -Q_x L_x/\Delta P_x$)



Multiscale Finite Element Methods (MsFEMs)

Use appropriate "coarse-scale" approximation spaces that are adaptive to the local property of the elliptic differential operator.

MsFEMs are attractive for reservoir simulation:

- The elliptic "parallelization": subgrid resolution at a low cost.
- The flexibility: the natural ability to handle
  - heterogeneous and anisotropic materials,
  - irregular and unstructured grids.
- The ideal foundation for adaptive numerical schemes for the solution of advective transport equations.

The variational formulation (for homogeneous Dirichlet BC)

$$\int \nabla u \cdot a(x) \nabla v \, dx = \int f v \, dx \qquad \forall v \in H_0^1$$

and the mixed formulation (for homogeneous Neumann BC)

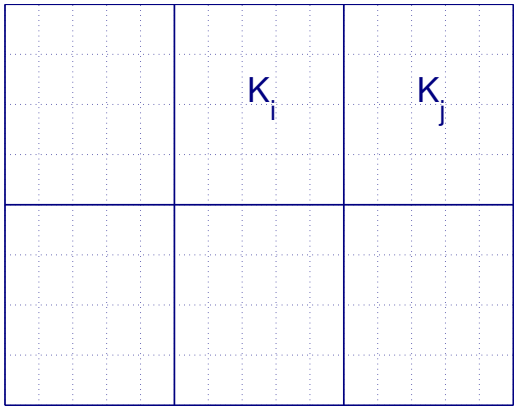
$$\begin{aligned} \int a(x)^{-1} q \cdot p \, dx &= \int u \, \nabla \cdot p \, dx \qquad \forall p \in H_0^{1,div} \\ \int v \, \nabla \cdot q \, dx &= \int f v \, dx \qquad \forall v \in L^2 \end{aligned}$$

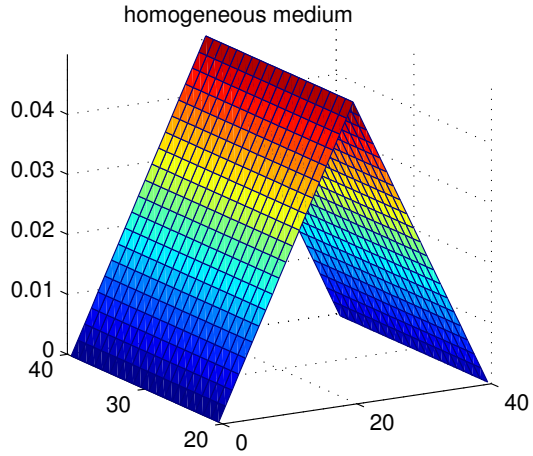
provide a foundation for multiscale finite element methods for elliptic problems.

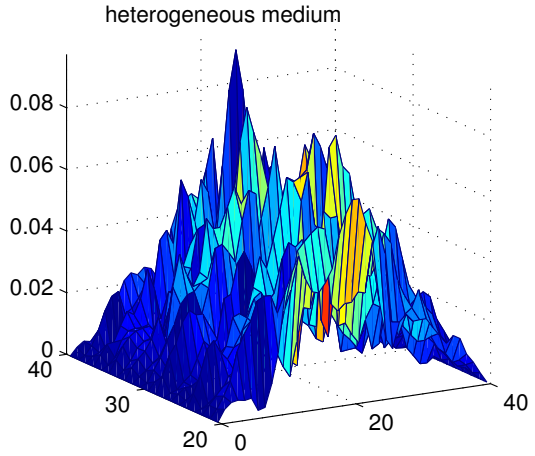
In multiscale methods we seek the solution in low dimensional spaces that are adaptive to the local property of the differential operator.

- MsFEM: Variational formulation, $U^{ms} \subset H_0^1$.
- Mixed MsFEM: Mixed formulation, $Q^{ms} \subset H_0^{1,div}$, $U = \mathcal{P}_0$.
- MsDGM: Mixed formulation, $Q^{ms} \subset H_0^{1,div}$, $U^{ms} \subset H_0^1$.

Hence, these methods allow us to recover detailed fine-grid "solutions" from the multiscale coarse-grid solutions.







Base functions are computed numerically and satisfy

$$\psi_{ij} = -k\lambda\nabla\phi_{ij}, \qquad \nabla\cdot\psi_{ij} = \pm\begin{cases}\frac{1}{|K|}, & \text{if } \int_K f\,dx = 0\,,\\ \frac{f}{\int_K f\,dx}, & \text{if } \int_K f\,dx \neq 0\end{cases}$$

in $K_i \cup K_j$, a prescribed boundary condition $\psi_{ij}\cdot n_{ij} = \nu_{ij}$ on $\Gamma_{ij} = \partial K_i \cap \partial K_j$, and no-flow conditions on $\partial(K_i \cup K_j)\backslash\Gamma_{ij}$.

We then seek $v \in Q^{ms} = span\,\{\psi_{ij}\}$ and $p \in U = \hat{p} + \mathcal{P}_0(\mathcal{K})$ such that

$$\begin{aligned} \int_\Omega (k\lambda)^{-1} v \cdot u \, dx - \int_\Omega p \, \nabla \cdot u \, dx &= \int_\Omega \frac{\lambda_g}{\lambda} G \cdot u \, dx \\ \int_\Omega l \, \nabla \cdot v \, dx &= \int_\Omega q l \, dx \end{aligned}$$

for all $u \in Q^{ms}$ and $l \in U$.

Modification of pressure space: the functions $\hat{p} \in L^2(\Omega)$ vanish outside well blocks and satisfy

$$\int_K \hat{p} \, dx = 0, \quad \hat{v} = -k\lambda \nabla \hat{p} + k\lambda_g G, \quad \nabla \cdot \hat{v} = q, \quad \hat{v} \cdot n = \nu_K$$

in the well blocks for some boundary condition $\nu_K \approx v \cdot n$ on ∂K.

Properties of MMsFEM

- Mass conservative
  - Mass conservative velocities on the fine (sub)grid
- Incorporation of small-scale effects into coarse-grid solution
  - A robust alternative to upscaling for computations on the coarse grid
  - A (fast) pressure solver for computations on the fine grid
  - Natural support for adaptive fluid transport computations
- Scalability (and natural parallelisation)
  - Base functions are processed independently

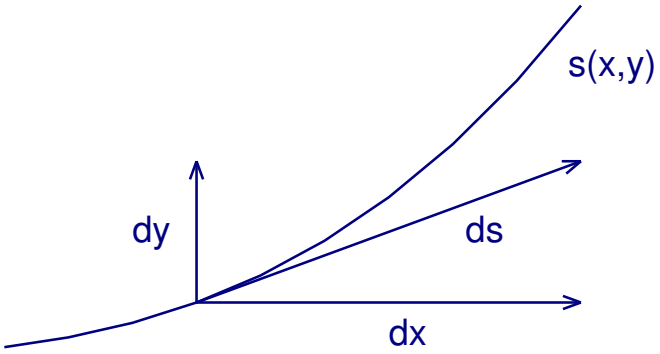
- Flexibility of subgrids
  - No restrictions on subgrids
  - The shape of coarse grid blocks can be (almost) arbitrary
  - Any numerical method can be used for subgrid problems
- Potential speed
  - Computationally efficient if regeneration of all base functions in each time-step is avoided

However: MMsFEM gives an indefinite linear system that may be a bit harder to solve $\longrightarrow$ currently limitations in our prototype solver

Streamline methods

Interpret the saturation equation $\phi\partial_t S + v \cdot \nabla f(S) = 0$ as an equation along streamlines using

$$\frac{v}{|v|} = \left[\frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}\right]^T \qquad \text{or} \qquad v \cdot \nabla = |v|\frac{\partial}{\partial s}$$



Transformation using time-of-flight τ

$$|v|\frac{\partial}{\partial s} = \phi\frac{\partial}{\partial \tau}$$

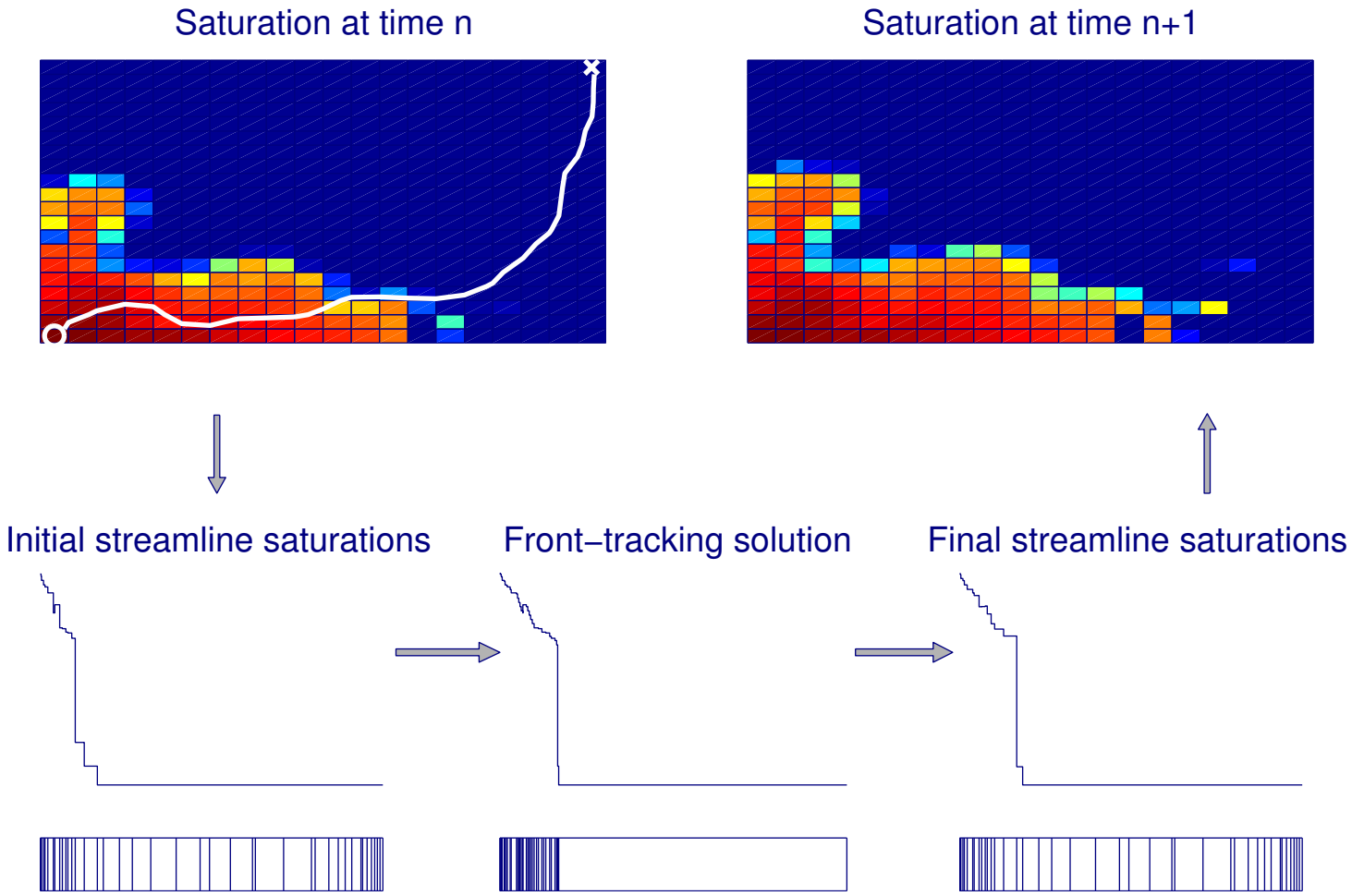
gives a family of 1-D transport equations along streamlines

$$\partial_t S + \partial_\tau f(S) = 0. \tag{5}$$

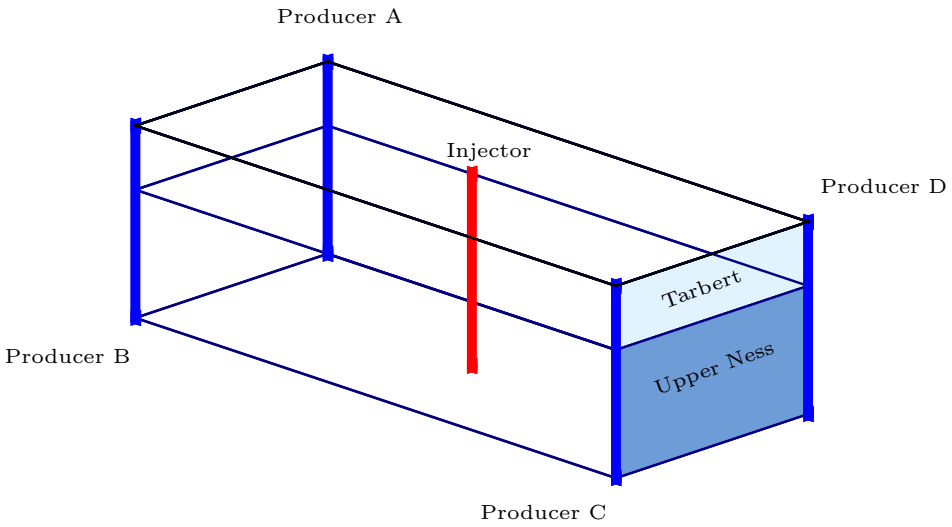
Streamline algorithm

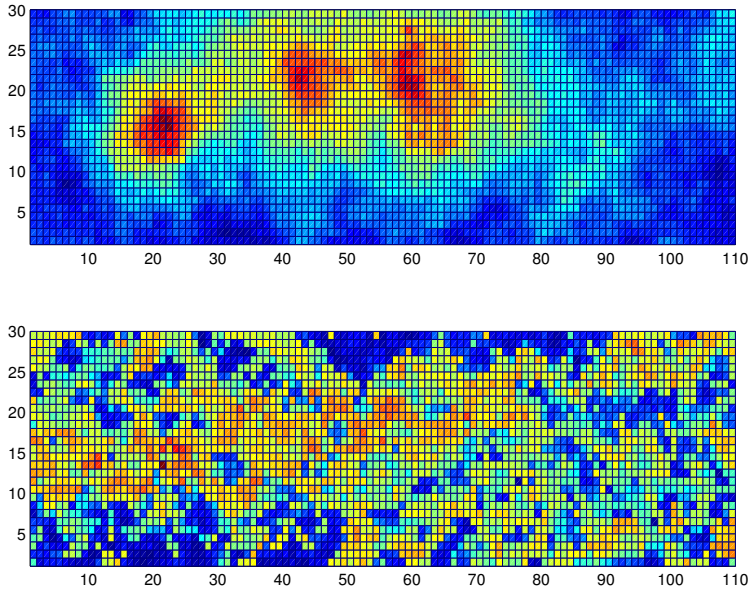
1. Calculate pressure distribution and velocity field on background grid
2. Trace streamlines (using analytical approximation algorithm)
3. Project saturation from physical space to streamlines
4. Solve transport equation (5) along streamlines
5. Project streamline saturations back to grid in physical space

Streamlines are an attractive alternative to FD: fast, parallel, less diffusion, less restrictive time-step conditions


Saturation at time n
Saturation at time n+1
Initial streamline saturations
Front–tracking solution
Final streamline saturations

Numerical examples: SPE10 model 2

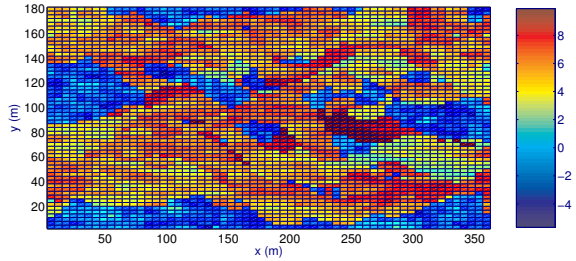




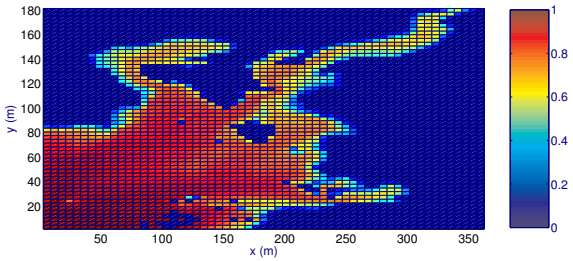
$60 \times 220 \times 85$ grid, $\lambda_w \propto S^2$, $\lambda_o \propto (1-S)^2$, $\mu_o = 3.0$ cP, $\mu_w = 0.3$ cP

2000 days of production at bhp 4000 psi. Injection: 5000 bbl/day.

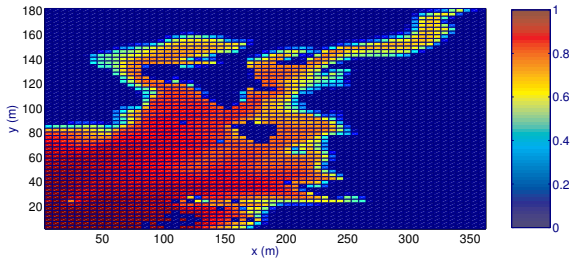
Numerical examples: 2-D quarter five-spots



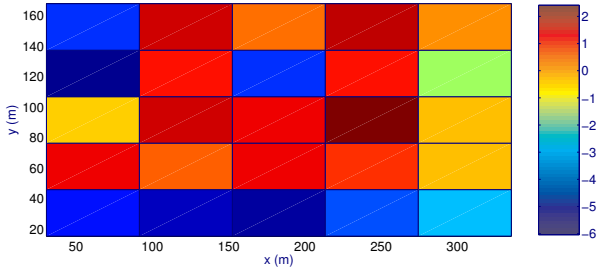
permeability



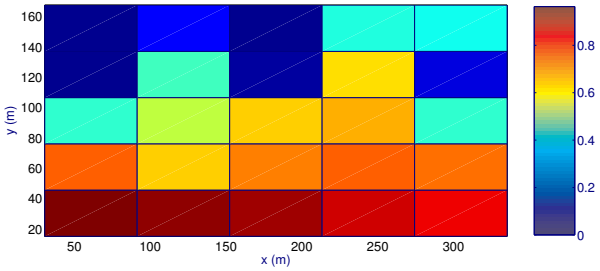
reference solution



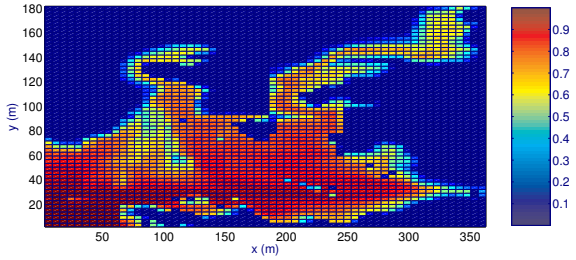
MMsFEM + SL



upscaled perm.



upscaled solution



nested gridding

Nested gridding: upscale ($k\lambda$), solve for pressure and then subgrid problem for velocities

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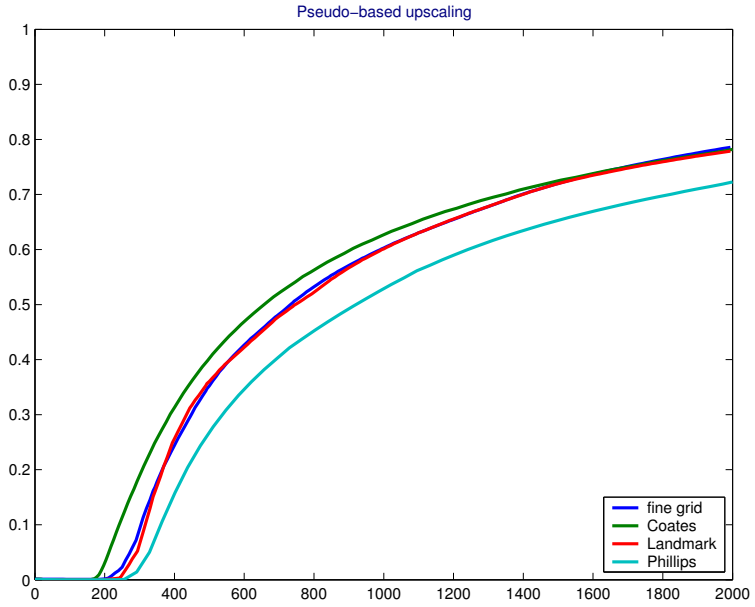
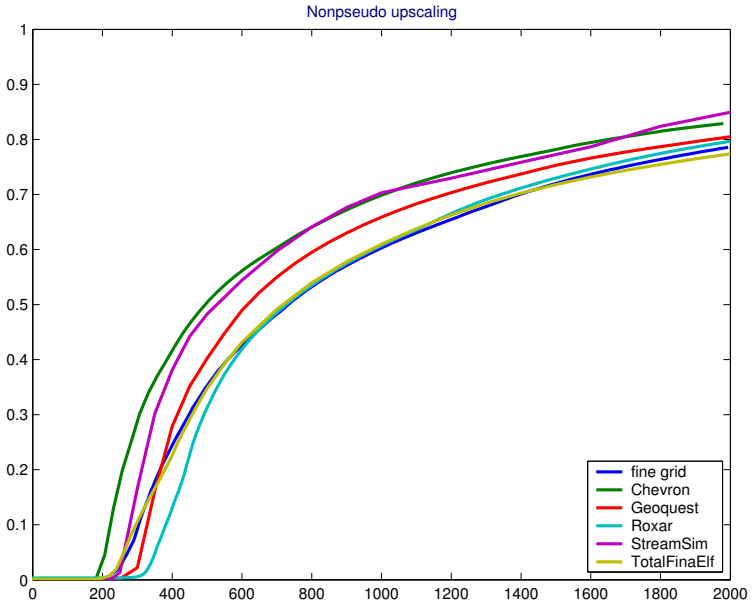
Relative L^1 error in saturation for Tarbert and Upper Ness

Upsc. factor	Global BC	Local BC	Nested gridding	Upscaled
2×2	1.013e-02	1.094e-02	3.094e-02	6.801e-02
3×3	1.219e-02	1.732e-02	3.814e-02	1.005e-01
5×5	1.421e-02	2.889e-02	5.033e-02	1.497e-01
10×10	2.523e-02	5.377e-02	7.643e-02	2.415e-01
2×2	3.644e-02	4.132e-02	1.672e-01	2.383e-01
3×3	4.360e-02	6.705e-02	1.868e-01	2.864e-01
5×5	4.881e-02	9.659e-02	2.373e-01	3.812e-01
10×10	8.477e-02	1.981e-01	4.848e-01	7.217e-01

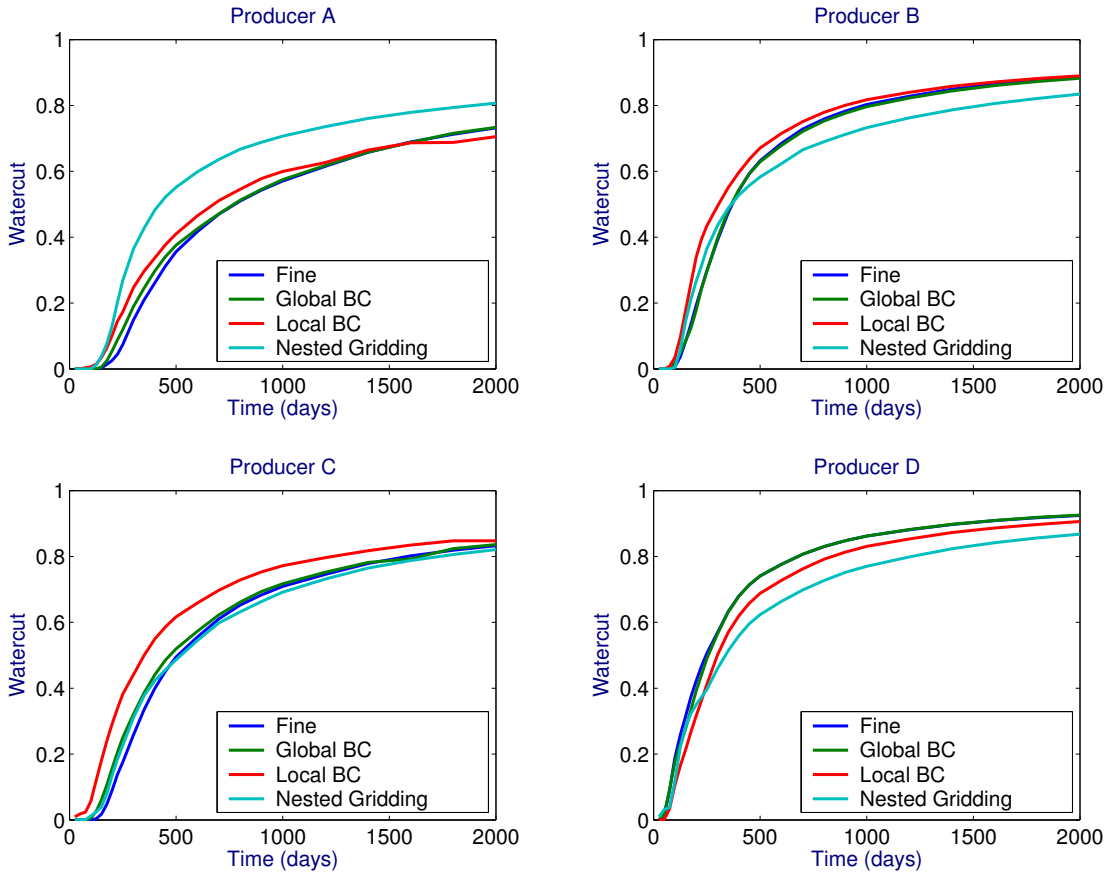
Global BC: ν_{ij} given by initial pressure solution
Local BC: ν_{ij} scaled according to flux and K

Numerical results: SPE10 benchmark





Producer A; data taken from http://www.spe.org/csp



Coarse grid: $5 \times 10 \times 17$

Final remarks

- Novel method:
  - robust alternative to upscaling
  - has potential for large geomodels
- Extensions and further work
  - non-orthogonal (corner-point) and unstructured grids
  - threephase and multicomponent flows
  - efficiency and parallellisation
  - numerical linear algebra for indefinite systems
  - fractures