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A MULTISCALE MIXED FINITE-ELEMENT METHOD FOR VUGGY AND NATURALLY-FRACTURED RESERVOIRS

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Summary. We present a multiscale method for the Stokes–Brinkman equations used to model vuggy and naturally-fractured (carbonate) reservoirs that contain both porous and free-flow regions.

1 INTRODUCTION

Naturally fractured and carbonate reservoirs are composed of porous material, but will typically also contain relatively large void spaces in the form of fractures, small cavities, and caves (called *vugs* in the geological literature) that can significantly alter the effective permeability and should be accurately accounted for in a geomodel.

The Darcy–Stokes equations have been used to model industrial infiltration processes and coupled surface and subsurface flow, for which porous and free-flow regions are well separated. The Darcy–Stokes model consists of Darcy's law combined with mass conservation in the porous subdomain and the Stokes equations in the free-flow subdomain. To close the model, one needs to specify conditions on the interface between the Darcy and Stokes subdomains. All these conditions require continuity of mass and momentum over the interface, but differ in the way they allow the tangential component to jump across the interface[1].

In carbonate reservoirs, the porous and free-flow regions are not well separated: vugs and rock matrix are intertwined throughout the reservoir, often on multiple scales. This means that the coupled Darcy–Stokes approach is not feasible for several reasons[4]. First of all, one would need precise information about the location and geometry of the interface between vugs and the porous matrix, but also experimentally determined values related to the interface conditions. This may be possible to obtain for an engineered medium or a small rock sample, but is not possible to obtain for a sector or a full reservoir model. Second, explicit representation of the medium on a centimeter scale, as required to resolve vugs and fractures, would make the flow problem computationally intractable. Finally, the free-flow regions may contain loose fill-in material or particle suspensions in the fluids filling the void space.

Recently, Popov et al.[4] proposed to use Stokes–Brinkman equations rather than the Darcy–Stokes equations as a fine-scale model to compute upscaled effective permeabilities. The Stokes–

Brinkman equations can be reduced to the Stokes or Darcy's equations, respectively, by appropriate choice of parameters and give a somewhat coarser model that does not require a precise description of the interface between free-flow and porous regions. This is an advantage for application to real media where the locations of the boundary of the vugs are uncertain. Moreover, the model opens up for a seamless transition between Darcy and Stokes, which may be appropriate e.g., in damaged zones. From a numerical point-of-view it is also attractive to use a single model, where the two regions are represented implicitly through parameters, instead of a two-domain approach involving explicit modelling of the interface between the vuggy and porous regions.

Multiscale simulation[3] is in concept well-suited as an alternative to upscaling, as it allows varying resolution and provides a systematic procedure for coarsening and refining. For Darcy flow, multiscale methods have proved to be more robust than standard upscaling methods and have the advantage that they offer subscale resolution and thus can be used as highly efficient approximate solvers for direct solution of the full fine-scale problem. Multiscale methods have proved to be able to handle industry-standard complexity both with respect to unstructured and corner-point grids and flow physics, but to date multiscale methods have not been applied to simulate flow in naturally-fractured and vuggy media.

The work presented herein is a first step towards making a multiscale framework based on a mixed finite-element formulation[2] for simulating naturally-fractured and vuggy reservoirs. The MsMFE method uses a standard Darcy model to approximate pressure and fluxes on a coarse grid, but captures fine-scale effects through basis functions determined by solving the Stokes–Brinkman equations locally on the underlying fine-scale geocellular grid.

2 A MULTISCALE MIXED FINITE ELEMENT METHOD

The Stokes–Brinkman equations give a unified approach to simulating free-flow and porous regions using a single system of equations for the fluid pressure p and the total velocity $\vec{u}$,

$$\mu \mathbf{K}^{-1}\vec{u} + \nabla p - \tilde{\mu}\Delta\vec{u} = \vec{f}, \qquad \nabla \cdot \vec{u} = q. \tag{1}$$

Here, $\mathbf{K}$ is a permeability tensor that is equal to the Darcy permeability in the porous subdomain, μ is the viscosity of the fluid, $\tilde{\mu}$ is an effective viscosity, $\vec{f}$ denotes body forces, and q denotes fluid sources. Equation (1) reduces to Darcy or Stokes flow by appropriate choice of parameters ($\tilde{\mu} = 0$ for Darcy and $\mathbf{K} = \infty$ and $\tilde{\mu} = \mu$ for Stokes). To model vuggy media, however, we will set $\tilde{\mu} = \mu$. In the absence of body forces, we may then rewrite the equation as

$$\nabla p = -\mu \mathbf{K}^{-1}\vec{u} + \tilde{\mu}\Delta\vec{u}. \tag{2}$$

A comparison of the magnitude of the two velocity terms on the right-hand side shows that the first term dominates the second by several orders of magnitude for typical reservoirs. In other words, (2) can be seen as Darcy's equation with a small viscosity perturbation. Using a nonzero $\tilde{\mu}$ in the porous regions also means that we need not represent interfaces and impose interface conditions explicitly.

Our mixed finite-element method is based on a two-grid formulation with a fine grid (here a Cartesian 2D grid for simplicity) and a coarse grid, where each block Ω_i consists of a connected

collection of cells from the fine grid. To formulate the method, we start from the variational formulation

$$\begin{aligned} b(\vec{u},\,\vec{\psi}) - c(p,\,\vec{\psi}) &= 0,\\ c(\vec{u},\,\pi) &= (q,\,\pi). \end{aligned} \tag{3}$$

Here, $(\vec{\psi},\pi)$ are generalizations of the lowest-order Raviart–Thomas basis functions (RT0): π is constant on each coarse block, whereas $\vec{\psi}$ accounts for local variations of flow velocity. These variations are due to subgrid heterogeneities in the porous regions, increased flow velocities resulting from free-flow regions on the subgrid scale, and geometrical effects in the case of non-square blocks. The multiscale basis functions are computed by solving a local flow problem

$$\mu\mathbf{K}^{-1}\vec{\psi}_{ij} + \nabla\varphi_{ij} - \tilde{\mu}\Delta\vec{\psi}_{ij} = 0, \qquad \nabla\cdot\vec{\psi}_{ij} = \begin{cases} w_i(\vec{x}), & \text{if } \vec{x}\in\Omega_i,\\ -w_j(\vec{x}), & \text{if } \vec{x}\in\Omega_j,\\ 0, & \text{otherwise,}\end{cases} \tag{4}$$

over a neighbourhood Ω_{ij} containing two coarse blocks Ω_i and Ω_j. A flow is driven across the interface $\partial\Omega_i\cap\partial\Omega_j$ by the source function w. To produce a unit average flow, $w_i(\vec{x})$ is normalized over Ω_i. Moreover, w_i scales with trace($\mathbf{K}$) to avoid unnaturally large velocities in low-permeable fine cells within the coarse blocks. The flow problem (4) is localized by specifying zero Neumann boundary conditions and solved using the Taylor–Hood mixed elements.

Direct assembly of (3) gives a saddle-point problem. We therefore introduce an additional set of unknowns, the pressure λ at the cell faces, and solve the following coarse-scale hybrid system

$$\begin{bmatrix} \boldsymbol{\Psi}^{\mathsf{T}}\boldsymbol{B}_D^f\boldsymbol{\Psi} & \boldsymbol{C} & \boldsymbol{D}\\ \boldsymbol{C}^{\mathsf{T}} & \mathbf{0} & \mathbf{0}\\ \boldsymbol{D}^{\mathsf{T}} & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}^c\\ -\boldsymbol{p}^c\\ \boldsymbol{\lambda}^c\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \boldsymbol{q}^c\\ \mathbf{0}\end{bmatrix}. \tag{5}$$

Here $\boldsymbol{\Psi}$ contains the multiscale basis functions and the vectors $\boldsymbol{u}^c$, $\boldsymbol{p}^c$, and $\boldsymbol{\lambda}^c$ contain the coarse-scale degrees-of-freedom. The matrix $\boldsymbol{B}_D^f$ is a fine-scale mass matrix for the case of pure Darcy flow defined with Taylor–Hood elements rather than RT0, and is a block diagonal matrix of 9×9 blocks for 2D Cartesian grids. If we use no overlap (i.e., $\Omega_{ij}=\Omega_i\cup\Omega_j$), the B-part of (5) is block diagonal with respect to the coarse blocks, and can be reduced to a symmetric positive-definite system for $\boldsymbol{\lambda}^c$. When overlap is used, one is in general better off using the original mixed formulation of the system. In either case, the fine-scale velocities are computed as $\boldsymbol{v}^f=\boldsymbol{\Psi}\boldsymbol{v}^c$.

3 NUMERICAL EXPERIMENTS

Our first example considers the inclusion of several circular vugs in a homogeneous porous medium represented on a 90×90 grid. Figure 1 shows the MsMFE solution computed on a coarse 3×3 grid, compared with a reference solution obtained by solving the Stokes–Brinkman equations on the fine grid. We observe that the MsMFE solution resolves the global flow pattern correctly and that details around the vugs are well captured.

In the second example, we consider a combination of vugs and fractures that connect some of the vugs and provide long-range correlation in the reservoir. Figure 2 shows the MsMFE

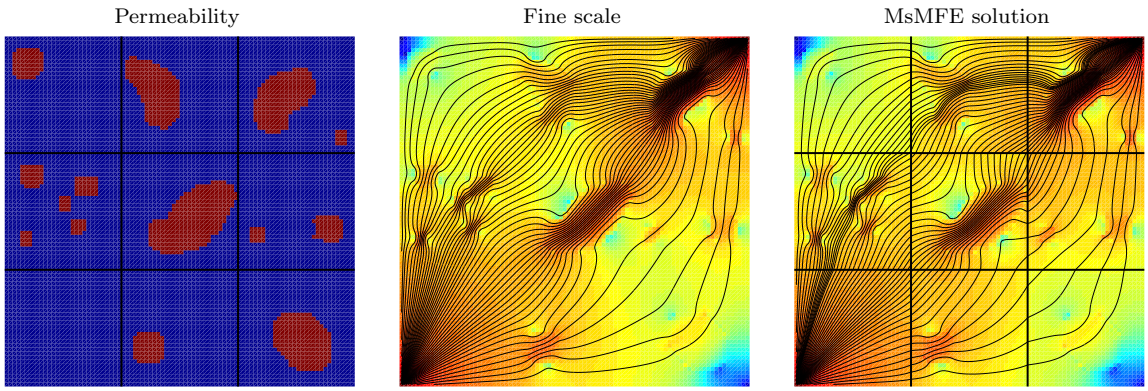

Figure 1: Case 1 with ratio of permeability in vugs and background equal $10^8 : 1$.

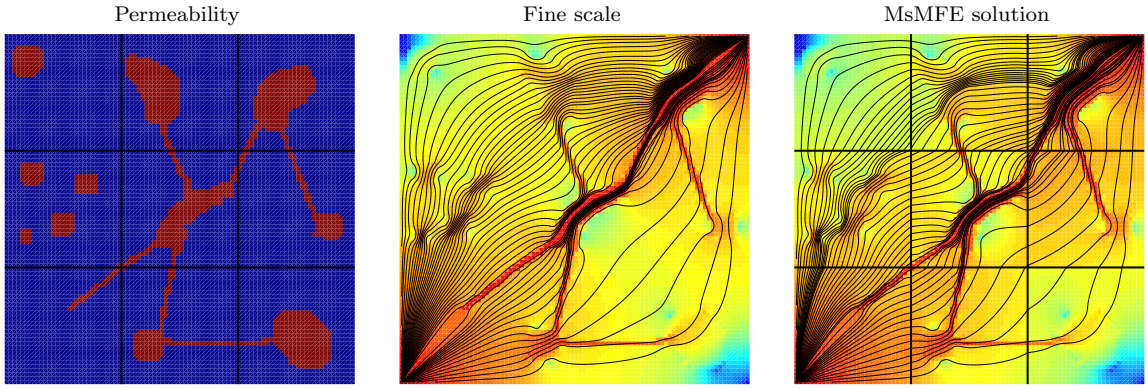

Figure 2: Case 2 with ratio of permeability in vugs/fractures and background equal $10^8 : 1$.

solution compared with the fine-scale reference solution. Also in this case, the MsMFE method is able to accurately reproduce the fine-scale solution.

These preliminary results indicate that the multiscale method is able to resolve the interaction of porous and free-flow regions, capture fine-scale details, and resolve long-range correlation. The MsMFE method may therefore be a promising approach for simulating flow in naturally-fractured and vuggy media.

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