Motivation: Risk assessment of cardiovascular disease

Anatomical aspects of aortic disease have been thoroughly investigated in the last decades by means of CT, MRI and ultrasound. To date, morphological variations can be determined individually and with high sensitivity. In general clinical routine, aortic anatomy and pathology is represented and surveyed statically. There is high potential for techniques, that quantify dynamic morphology and physiology of the aorta during full cardiac cycle. With respect to the coherences between bio-mechanical behavior and aortic disease various open tasks exist such as a more extensive acquisition of risk factors for atherosclerosis, aneurysm formation or aortic dissection. Within the scope of increasing understanding of vascular pathology, development of functional imaging of the aorta becomes more and more important.

Mathematical model and calibration

In this work, we propose a mathematical model of an aortic silicon phantom. As the elasticity of the silicon phantom wall plays a significant role and is reflected in the Windkessel effect in the case of the aortic bow, we model the wall as elastic structure. The physical dynamics for fluid flow and elastic deformation can be modeled by means of partial-differential equations derived by basic laws of continuum mechanics, namely, the conservation of mass and momentum:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0, \quad \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) = - \nabla p + \rho \mathbf{f}, \]

where \( \mathbf{f} \) is the body force. In the case of the aortic phantom, a suitable constitutive law for the fluid stress-strain relation \( \sigma(\varepsilon) \) is given by the incompressible Newtonian fluid model, leading to the incompressible Navier-Stokes equations. The structure stress-strain relation \( \sigma(\varepsilon) \) can be stated by means of the St. Venant Kirchhoff material model.

\[ \sigma(\varepsilon) = \frac{2}{3} \lambda (\varepsilon - I) + 2 \mu \varepsilon, \]

where \( \lambda \) and \( \mu \) are the material constants. Together with according boundary conditions we get the following fluid-structure interaction initial boundary value problem:

\[ \begin{align*}
\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) &= - \nabla p + \mathbf{f}, \\
\rho \frac{\partial \varepsilon}{\partial t} + \nabla \cdot (\rho \varepsilon \otimes \mathbf{v}) &= \sigma(\varepsilon) - \nabla \cdot \mathbf{f},
\end{align*} \]

in \( \Omega \times I \),

\[ \begin{align*}
\mathbf{v} &= 0, & \text{in } \Gamma_I \times I, \\
\mathbf{f} &= 0, & \text{in } \Gamma_0 \times I,
\end{align*} \]

\[ \begin{align*}
\sigma(\varepsilon) &= 0, & \text{on } \Gamma_p \times I, \\
\varepsilon &= 0, & \text{on } \Gamma_0 \times I, \\
\mathbf{f} &= 0, & \text{on } \Gamma_I \times I,
\end{align*} \]

\[ \begin{align*}
\mathbf{v}(x, 0) &= \mathbf{v}_0(x), & \text{in } \Omega, \\
\varepsilon(x, 0) &= \varepsilon_0(x), & \text{in } \Omega.
\end{align*} \]

Calibration

- Structure parameters from silicon tensile test
- MRI measured flowrates for boundary conditions
- IC-type outflow boundary condition

\[ \rho_i - P_{\text{out}} + R \frac{d}{dt} P_{\text{in}} = 0. \]

Numerical simulation, framework embedding and evaluation

- Monolithic ALE solver
- P2/P2/P1 and/or Q2/Q2/P2/1 finite elements
- h-step time discretization

\[ \text{Flowrate}(\text{m}^3/\text{s}) \quad \text{Max WSS}(\text{Pa}) \]

Outlook


References