A step towards reduced order modelling of flow characterized by wakes using Proper Orthgonal Decomposition

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Introduction

Problem: High fidelity simulations of flow can be quite demanding, involving up to 10^6–10^9 degrees of freedom and several hours (or days) of computational time, even on powerful and parallel hardware architectures. These techniques can be prohibitive in dealing quickly and efficiently with repetitive solution of PDEs.

Answer: To address the issues, the field of reduced order modelling (ROM) is evolving quickly. We investigate proper orthogonal decomposition (POD) as a potential method for constructing reduced bases for use in ROMs. In the case of flows around cylindrical bodies we found that only a few modes were sufficient to represent the dominant flow structures and their associated energies.

Method

High fidelity simulations were performed of flow around a cylinder, at three different Reynold's numbers (Re $= 265, 2580, 40000$). Simulations were performed with uniform and pulsating inflow boundary conditions,

$$u_{\text{uniform}} = u_\infty = 1\,\mathrm{m/s},$$
$$u_{\text{pulsating}}(t) = u_\infty + \Delta u \sin(2\pi f t)$$

chosen so that $\Delta u = 0.2 \cdot 2\pi f D$, where D is the diameter of the cylinder.

Two-dimensional snapshots were generated from these simulations, representing in each case at least one principal period, sampled at $20\,\mathrm{Hz}$. All snapshots were interpolated on a common, uniform grid and reduced using proper orthogonal decomposition (POD) to an "optimal" ensemble.

Partial Orthgonal Decomposition

Given an ensemble of solutions $\{\varphi_i\}_{i=1}^p$, we seek a set of orthogonal modes $\{\zeta_j\}_{j=1}^p$ such that the reconstructed ensemble truncated at some order N,

$$\varphi_i^{(N)} = \sum_{j=1}^N a_i^j \zeta_j$$

represents the original ensemble "closely", as measured by some norm $\|\cdot\|_a = \sqrt{\langle\cdot,\cdot\rangle_a}$. This gives the covariance matrix $C_{ij} = \langle\varphi_i, \varphi_j\rangle_a$. Its eigenpairs $(\boldsymbol{q}_i, \lambda_i)$ yield the desired modes as

$$\zeta_i = \frac{1}{\sqrt{\lambda_i}} \sum_j q_i^j \varphi_j,$$

The sum of eigenvalues is equal to the trace of $\boldsymbol{C}$, and is interpreted as the average variance in the ensemble. Each eigenvalue λ_i is equal to the average variance captured by its corresponding mode ζ_i throughout the ensemble. Therefore, a condition on N should be

$$\textstyle\sum_{i=N+1}^p \lambda_i / \sum_{i=1}^p \lambda_i \leq \epsilon.$$

We choose to focus on the representation of velocity, so that the covariance function can be written

$$\langle(\overline{\boldsymbol{u}}_i, p_i), (\overline{\boldsymbol{u}}_j, p_j)\rangle_a = \int_\Omega \overline{\boldsymbol{u}}_i \cdot \overline{\boldsymbol{u}}_j.$$

Acknowledgements

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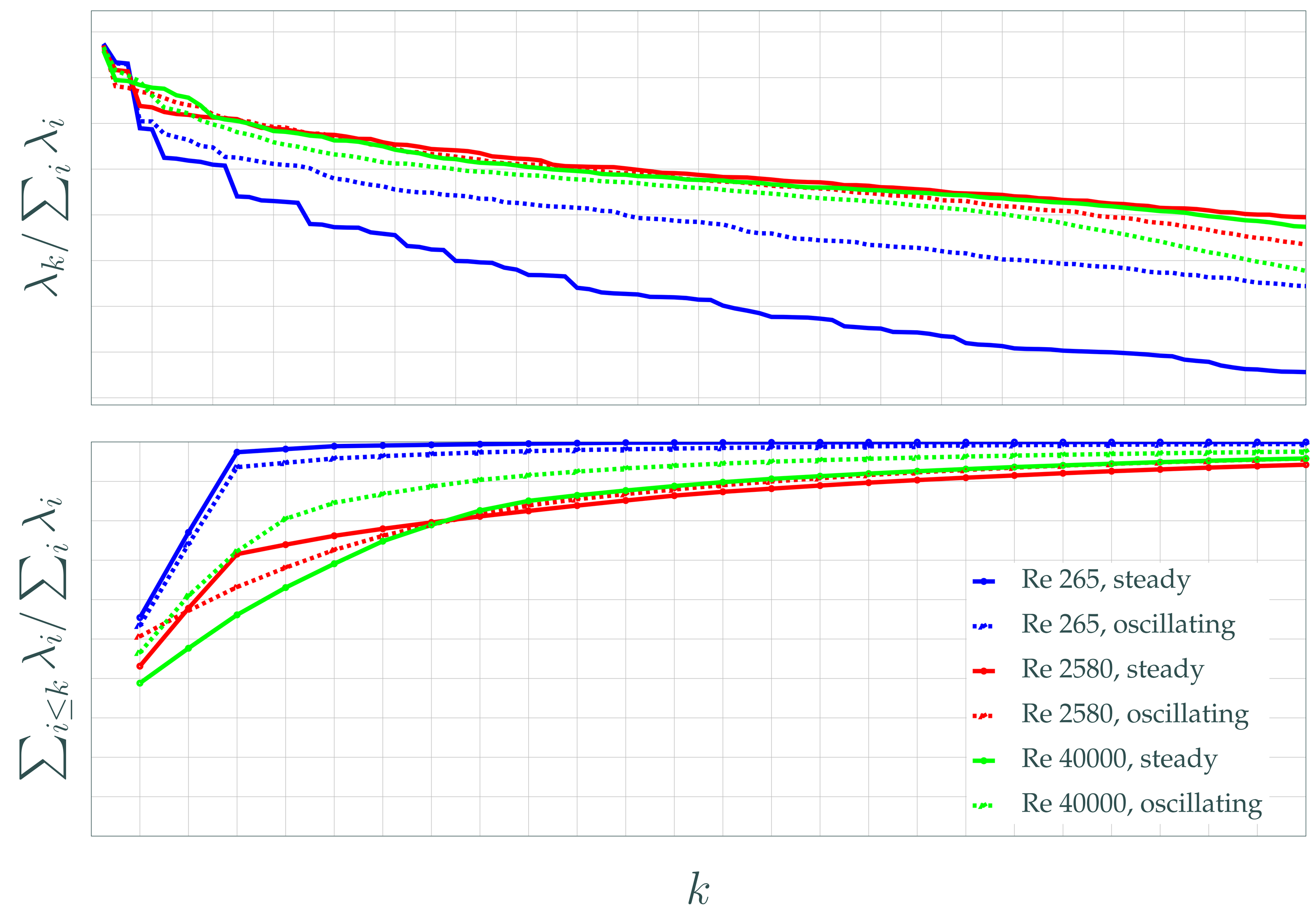
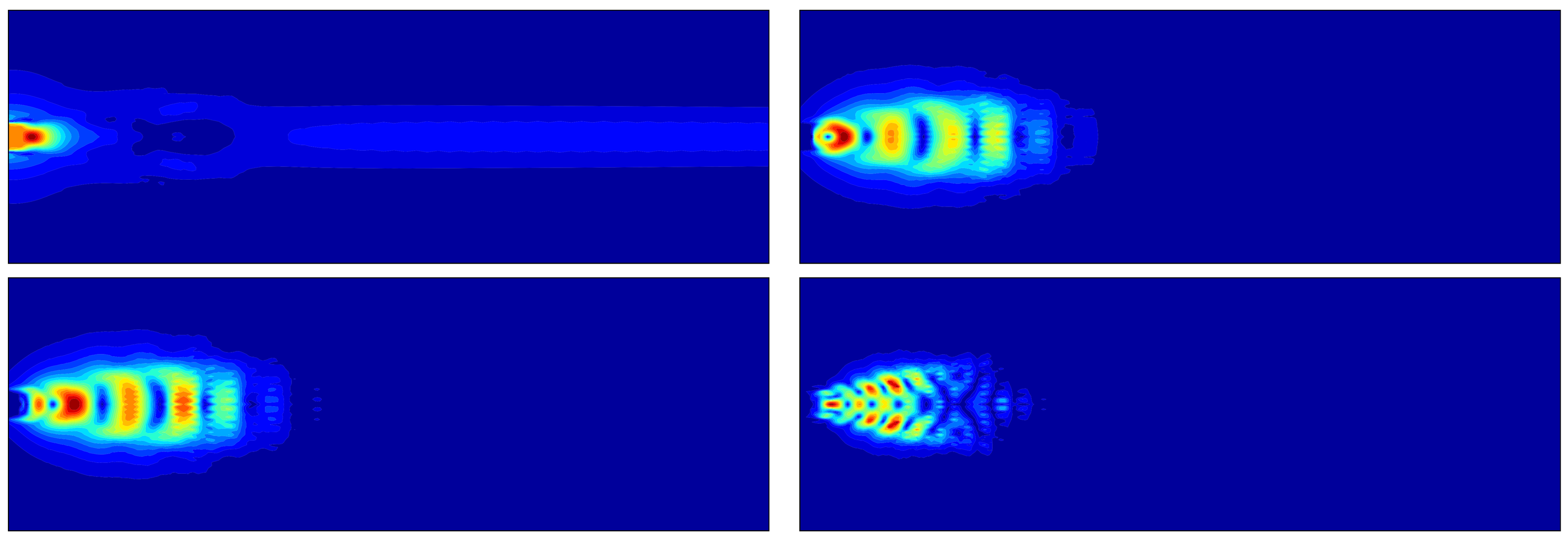
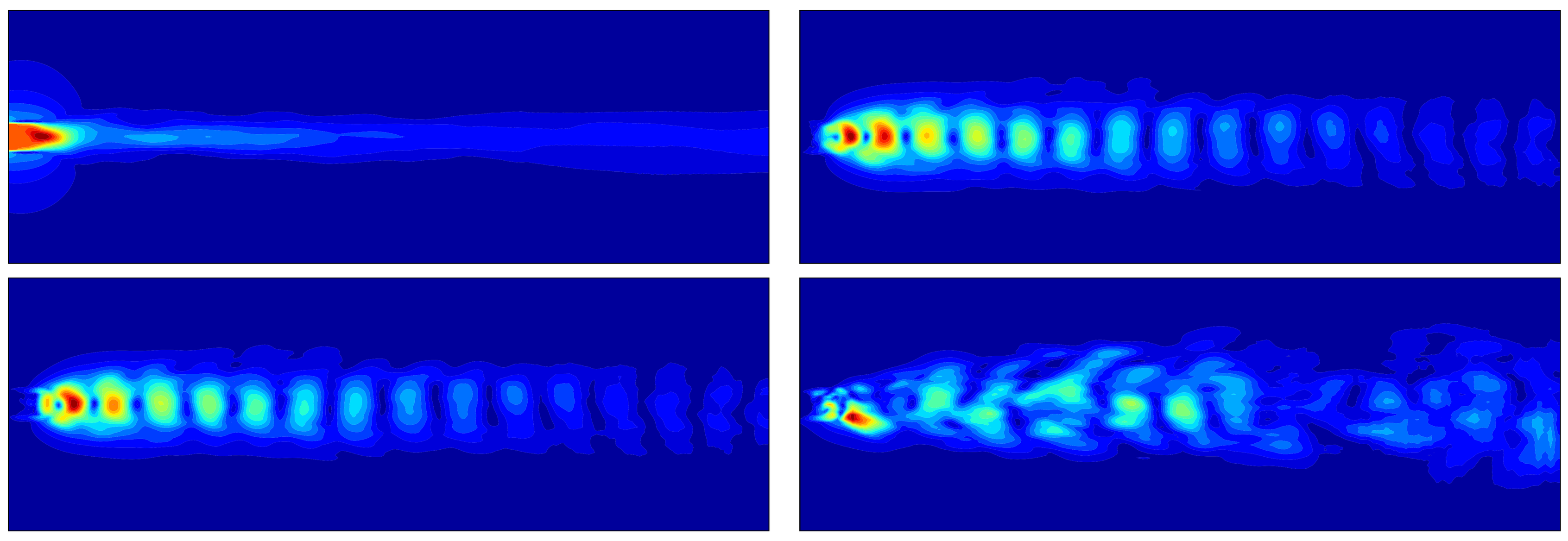


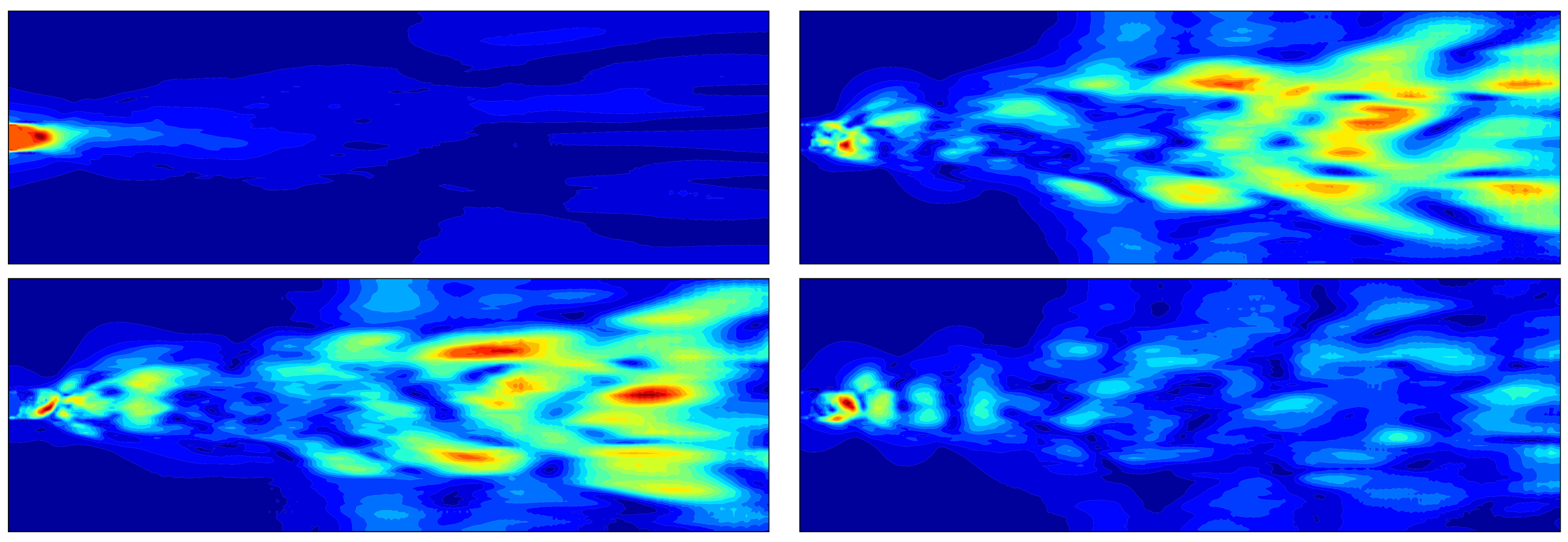
Energy spectrum and cumulative energy spectrum for the six different cases.



First modes for Re $= 265$.



First modes for Re $= 2580$.



First modes for Re $= 40000$.

Discussion

In all cases, about 30 modes suffice to cover 90% of the energy content. For low Reynold's number cases, the number of considerably smaller. For the other cases, the energy decay is consistent, suggesting this decay rate may be representative for a wider range of parameters. The first mode is always "laminar" and the following two modes appear to be phase-shifted principal oscillations. Higher modes provide turbulent content.

For the kinds of flows considered here, POD appears an attractive method for constructing the reduced bases required by ROMs.