Prediction of the shape of extreme inline force and free surface elevation using First Order Reliability Method (FORM)

Amin Ghadirian (amgh@dtu.dk), Henrik Bredmose and Signe Schløer
DTU Wind Energy, Nils Koppels Allé, Building 403, DK-2800 Kgs. Lyngby, Denmark

1. Introduction

The extreme wave loads which are of interest in these cases are estimated by choosing extreme events from linear random sea states and replacing them by either non-linear regular waves (stream function wave theory) or the New Wave theory combined with a stretching method as suggested in the design requirements.

Both of these theories are associated with imitations the most important of which is the symmetry of these waves. FORM, was used in the present work systematically to estimate the extreme wave shapes.

Two parameters of maximum crest height and maximum inline force were used as definers of extreme events.

The results of this process were then compared to the designer wave (wave averaged measurements) of the same criteria (same maximum crest height or maximum inline force).

2. Experiments

The experiments were conducted in the shallow water basin at DHI Denmark at a scale of 1:50.

The full scale diameter of the monopile was 7~m and the water depth was 33~m and 20~m. The monopile was mounted on two force transducers to measure the in-line force and the bending moment.

25 distinct random sea states were tested for a length of between 6 to 70 hours (in lab scale) from which four were selected to investigate in the current paper.

The four sea states were tested both with and without 3D spreading.

3. First Order Reliability Method

Reliability is defined as the probability of failure function, X, being larger than zero where **X** is a vector of stochastic input variables.

First Order Reliability Method (FORM) uses first order Taylor expansion to find the shortest distance between the failure function and center of combined probability distribution of the input variables.

In other words, FORM provides one with the most probable combination of the stochastic inputs that lead to failure and the probability of its occurrence.

This method can be used for structural reliability analysis and for extreme value prediction.

$$\begin{aligned}
\eta^{(1)} &= \textstyle\sum_{j=1}^{N_{freq}} \sum_{i=1}^{N_{dir}} (a_{ij}\cos(\omega_{ij}t) + b_{ij}\sin(\omega_{ij}t)) \\
g &= |\eta_{expected} - \eta^{(1)}| \\
\textit{Represented by} &: \ FORM(\eta_1)
\end{aligned}$$

$$\begin{aligned}
\eta^{(2)} &= \tfrac{1}{4} \textstyle\sum_{i=1}^{N_{freq}} \sum_{j=1}^{N_{freq}} \ldots \\
&\quad \textstyle\sum_{k=1}^{N_{dir}} \sum_{l=1}^{N_{dir}} \ldots \\
&\quad (a_{ik} + ib_{ik})(a_{jl} + ib_{jl})\{\{C^-_{ijkl}\}\cos(\omega_{ik}t - \omega_{jl}t) + \{C^+_{ijkl}\}\cos(\omega_{ik}t + \omega_{jl}t)\} \\
g &= |\eta_{expected} - (\eta^{(1)} + \eta^{(2)})| \\
\textit{Represented by} &: \ FORM(\eta_1 + \eta_2)
\end{aligned}$$

$$\begin{aligned}
F^{(1)} &= \rho A C_M \textstyle\int_{-h}^{0} u_t^{(1)} dz \\
g &= |F_{expected} - F^{(1)}| \\
\textit{Represented by} &: \ FORM(F_1)
\end{aligned}$$

$$\begin{aligned}
F^{(2)} &= \rho A C_M \textstyle\int_{-h}^{0} u_t^{(2)} + u^{(1)}u_x^{(1)} + w^{(} \\
&\quad \rho A C_m \textstyle\int_{-h}^{0} u^{(1)} w_z^{(1)} dz + \\
&\quad 0.5\rho D C_D \textstyle\int_{-h}^{0} u^{(1)}|u^{(1)}| dz + \\
&\quad \rho A C_M \eta u_t^{(1)}(z=0) \\
g &= |F_{expected} - (F^{(1)} + F^{(2)})| \\
\textit{Represented by} &: \ FORM(F_1 + F_2)
\end{aligned}$$

where:

$(a_{i,j}, b_{i,j}) \in \mathcal{N}(0, \sqrt{S_{eta}\partial f \partial\theta})$

4. New Wave and New Force theories

New Wave:

$$\eta_{\text{New Wave}}(\mathbf{X},\tau) = \frac{\alpha_\eta}{\sigma_\eta^2}\sum_n\sum_m \mathrm{Re}\{d_{n,m}\exp(\mathrm{i}(\mathbf{k}_{n,m}\cdot\mathbf{X} - \omega_n\tau))\}$$

where

$$d_{n,m} = S_\eta(\omega_n)\Delta\omega_n\Delta\theta_m$$

And $k_{n,m}$ is the linear wave number vector. Further:

$$\mathbf{X} = \mathbf{x} - \mathbf{x}_1$$

$$\sigma_\eta^2 = \overline{\eta^2} = \int_{\theta=0}^{2\pi}\int_{\omega=0}^{\infty} S_\eta(\omega_n,\theta_m)\mathrm{d}\omega\,\mathrm{d}\theta$$

The force transfer function is defined as

$$\Gamma(\omega,\theta) = \mathrm{i}\rho\pi R^2 C_M\cos(\theta)\omega^2/k$$

So the inline force time series of New Wave is

$$F_{\text{New Wave}}(\mathbf{X},\tau) = \frac{\alpha_\eta}{\sigma_\eta^2}\sum_n\sum_m \mathrm{Re}\{d_{n,m}\Gamma(\omega_n,\theta_m)\exp(\mathrm{i}(\mathbf{k}_{n,m}\cdot\mathbf{X} - \omega_{n,m}\tau))\}$$

New Force:

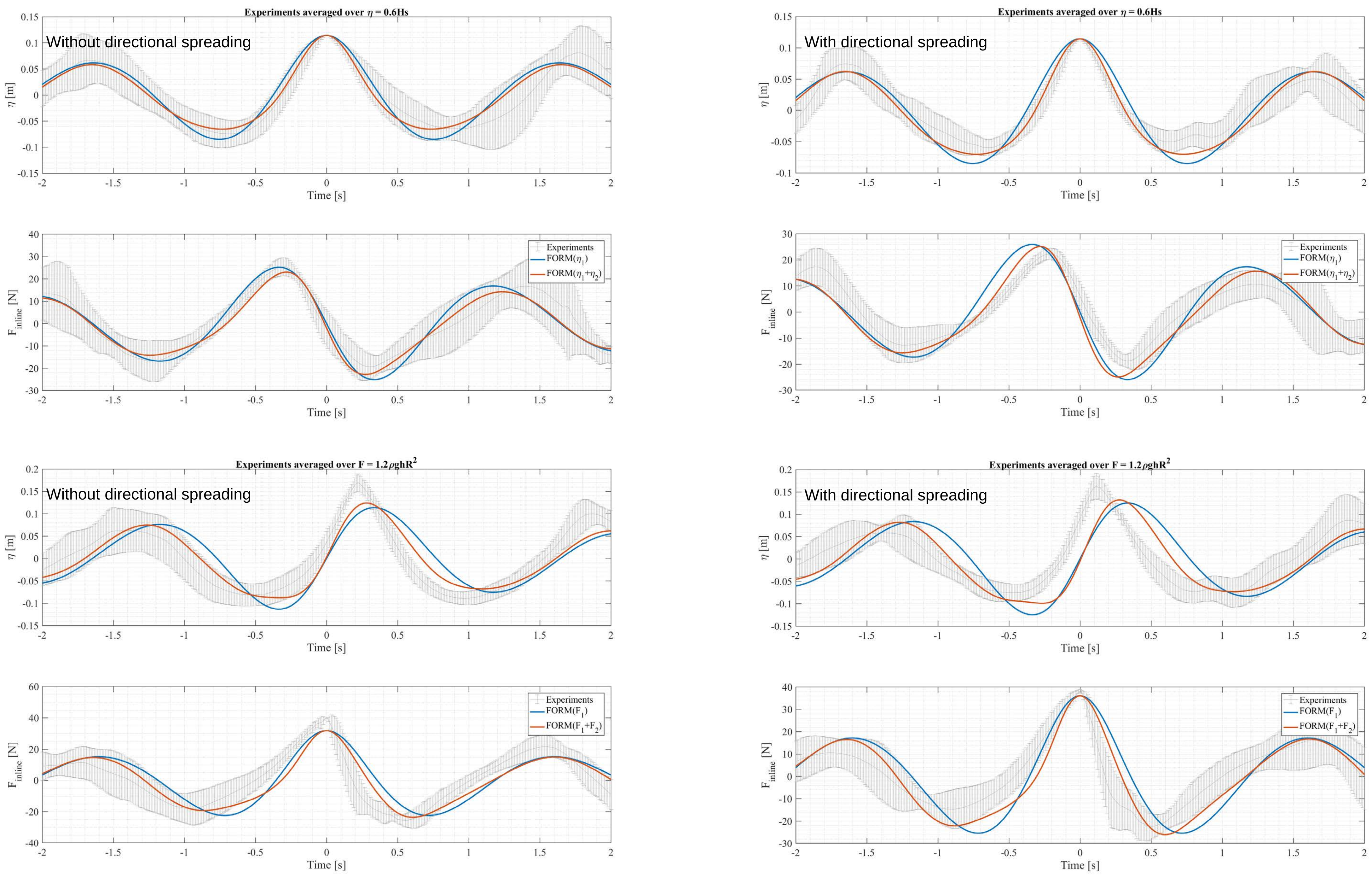
$$S_F(\omega,\theta) = |\Gamma(\omega_n,\theta_m)|^2 S_\eta$$

$$F_{\text{New Force}}(\mathbf{X},\tau) = \frac{\alpha_F}{\sigma_F^2}\sum_m\sum_n \mathrm{Re}\{S_F\Delta\omega\Delta\theta\exp(\mathrm{i}(\mathbf{k}_{n,m}\cdot\mathbf{X} - \omega_n\tau))\}$$

Free surface elevation time series of the New Force is

$$\eta_{\text{New Force}}(\mathbf{X},\tau) = \frac{\alpha_F}{\sigma_F^2}\sum_m\sum_n \mathrm{Re}\left\{\Gamma^*(\omega_n,\theta_m)S_\eta\Delta\omega\Delta\theta\exp(\mathrm{i}(\mathbf{k}_{n,m}\cdot\mathbf{X} - \omega_n\tau))\right\}$$

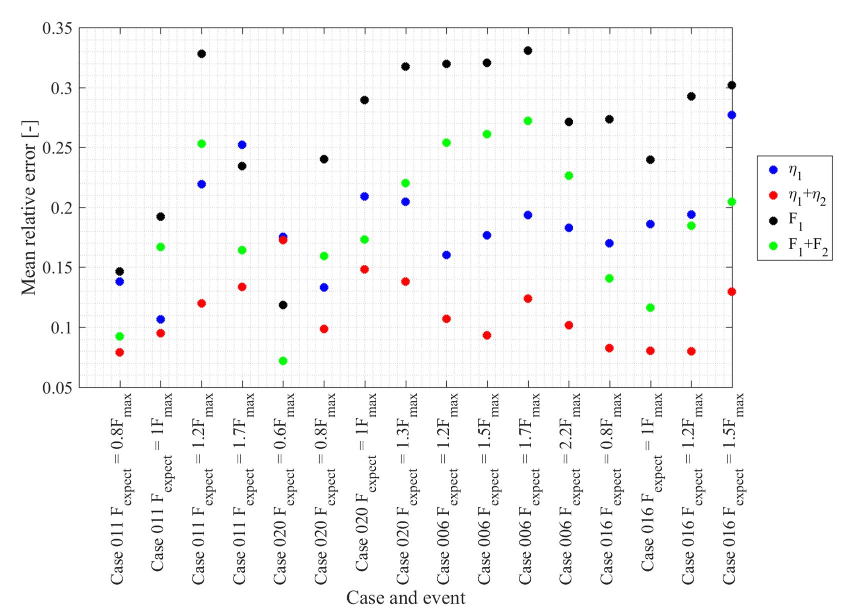
5. Results



6. Conclusions

In summary, a relatively good agreement between the First Order Reliability Method results of free surface elevation including the second order effects, and the wave averaged measurements was observed. It can be concluded that with a more nonlinear model a better agreement between the numerical results and the measurements is expected.

The inline force time series reproduced using the numerical method were not as consistent with the measurements as the free surface elevation time series. This was explained with the negligence of the drag terms above still water level. Hence a more nonlinear model, can reduce this discrepancy too.



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Acknowledgments

The present research was partly funded by the DeRisk project of Innovation Fund Denmark, grant number 4106-00038B. Further funding was provided by Statoil, DHI and DTU. All funding is gratefully acknowledged.

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FORSKNING, TEKNOLOGI & VÆKST I DANMARK
DTU Wind Energy
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