# Wind Model for Simulation of Thrust Variations on a Wind Turbine

Emil Smilden[a,b], Lene Eliassen[a]
[a]NTNU, [b]AMOS

**NTNU – Trondheim**
Norwegian University of Science and Technology
AMOS

## Abstract

The aerodynamic thrust induced by the air passing through the wind turbine rotor is transferred on to the tower and support structure and must be considered during structural design. This paper provides a computationally simple simulation model for the aerodynamic thrust on a wind turbine. The model is based on an equivalent wind formulation accounting for the effect of wind shear, tower shadow, turbulence and rotational sampling. Wind shear is shown to have a depleting effect on the mean rotor thrust. Both wind shear and tower shadowing cause thrust variations oscillating with the blade passing frequency, the effect of wind shear is however small compared to the effect of tower shadow in this regard. Turbulent wind fluctuations will cause low-frequent thrust variations in addition to thrust oscillating with the blade passing frequency. The equivalent wind model is verified by comparison with results obtained using the software code HAWC2 by DTU Wind Energy.

## Introduction

Wind turbines are dynamically sensitive structures, and especially the first tower vibration mode is prone to excitation by thrust variations induced by the wind passing through the wind turbine rotor [1]. As the blades pass through their arc of motion they will encounter a constantly changing wind field, appearing as imbalances and fluctuations in aerodynamic loading [1]. Turbulence will cause low-frequent load variations [3], and because the rotor frequency is normally higher than the turbulence frequency, turbulence will be sampled by the rotor, appearing as cyclic loads that fluctuates with the blade passing frequency (3P) [1]. In addition, 3P load variations are caused by persistent disturbances of the wind field within the rotor plane due to the presence of the tower, known as tower shadow, and air interacting with the earth surface, known as wind shear. The main contribution of this paper is the development of a wind model for fast simulation of thrust variations on a wind turbine. The model accounts for the effect of wind shear, tower shadow, turbulence and rotational sampling.

## Basic Concept

The wind model is based on the concept of an equivalent wind speed, first presented in [4]. This method is based on the idea of representing the complete wind field encountered by the rotor by a single wind time-series [5]. This time-series can further be used as input to a computationally simple mathematical representation of the rotor aerodynamics for fast calculation of aerodynamic thrust using

$$T_{aero}(t) = \frac{1}{2}\rho A V(t)^2 C_T(\lambda, \beta) \tag{1}$$

where $A$ is the rotor area, $V(t)$ is the wind speed and $C_T(\lambda)$ is the thrust coefficient depending on the tip-speed ratio $\lambda$ and blade pitch angle $\beta$.
The total wind speed is divided into two main components [3]

$$V(t) = V_0 + \tilde{v}_{eq} \tag{2}$$

consisting of the mean wind $V_0$ and the equivalent fluctuating component

$$\tilde{v}_{eq} = \tilde{v}_{ws} + \tilde{v}_{ts} + \tilde{v}_0 + \tilde{v}_3 \tag{3}$$

where $\tilde{v}_{ws}$, $\tilde{v}_{ts}$, $\tilde{v}_0$ and $\tilde{v}_3$ are the equivalent wind components accounting for wind variations caused by wind shear, tower shadow, turbulence and rotational sampling.

## Acknowledgements

This work has been carried out at the Centre for Autonomous Marine Operations and Systems (AMOS). The Norwegian Research Council is acknowledged as the main sponsor of AMOS. This work was supported by the Research Council of Norway through the Centres of Excellence funding scheme, Project number 223254 - AMOS.

## Mathematical model

The equivalent wind speed component accounting for turbulence $\tilde{v}_0(t)$ is calculated by its *Fourier* transform given by

$$\tilde{V}_0(f) = H_0(j2\pi f) \cdot V(f) \tag{4}$$

where the zero harmonics filter $H_0(j2\pi f)$ is found by fitting of a rational transfer function to the admittance function for a general wind turbine rotor, and $V(f)$ is the *Fourier* transform of the fixed-point wind speed calculated by use of the Kaimal spectrum. The equivalent wind speed accounting for turbulence sampling is given by

$$\tilde{v}_3(t) = 2\mathrm{Re}\{\tilde{v}_3(t)\}\cos(3\theta) + 2\mathrm{Im}\{\tilde{v}_3(t)\}\sin(3\theta) \tag{5}$$

where the components of $\tilde{v}_3(t)$ are calculated by their *Fourier* transforms in the same way as for $\tilde{v}_0(t)$. Further, the equivalent wind component accounting for wind shear is given by

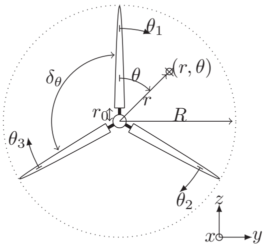

Figure 1: Rotor reference frame

$$\tilde{v}_{ws}(t,\theta) = V_0\left(\frac{\alpha(\alpha-1)}{12}\left(\frac{R-r_0}{H}\right)^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{96}\left(\frac{R-r_0}{H}\right)^3\cos 3\theta\right) \tag{6}$$

where $\alpha$ is the wind shear exponent, $H$ is hub height and the other parameters are defined in Fig. 1. At last, the equivalent wind speed accounting for tower shadow is given by

$$\tilde{v}_{ts}(t,\theta) = \frac{V_0 a^2}{3R}\sum_{n=1}^{3}\left[\frac{-R}{R^2\sin^2\theta_n + b^2}\right] \tag{7}$$

where $a$ is the tower radius and $b$ is the rotor overhang.

## Simulations and discussion

A parameter study was performed to evaluate the importance of including the effect of wind shear and tower shadow in simulations. Further, the equivalent wind model accounting for turbulence and rotational sampling was verified by comparison with results obtained using the software tool HAWC2 by DTU Wind Energy. Simulation parameters are based on the 10MW reference wind turbine of [2]. Fig. (2) shows the effect of both wind shear and tower shadow individually and together. The primary source of thrust variations are tower shadow. Wind shear should still be included due to its depleting effect on mean thrust. Fig. (3) shows the power spectral density for thrust time-series accounting for turbulence. A high energy content is observed at low frequencies, and the peak observed in the spectrum is caused by rotational sampling with peak frequency corresponding to the 3P frequency. The equivalent wind model shows good agreement with the results obtained using HAWC2 except from a small deviation at lower frequencies which is most likely caused by small differences in aerodynamic properties for the two rotors. The second peak in the HAWC2 results is caused by 6P effects which is not modelled by the equivalent wind model.

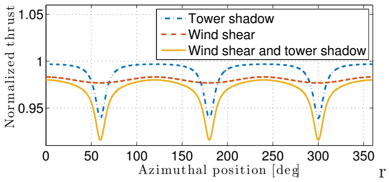

Figure 2: Normalized equivalent thrust accounting for wind shear and tower shadow

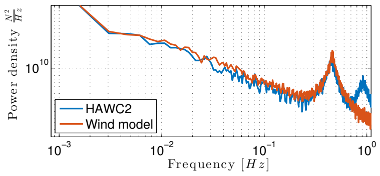

Figure 3: Power spectral density of thrust time-series using HAWC2 and equivalent wind model

## References

[1] László Arany, Subhamoy Bhattacharya, John Macdonald, and S John Hogan. Simplified critical mudline bending moment spectra of offshore wind turbine support structures. *Wind Energy*, 2014.

[2] C Bak, F Zahle, R Bitsche, T Kim, A Yde, LC Henriksen, A Natarajan, and M Hansen. Description of the dtu 10 mw reference wind turbine. *DTU Wind Energy Report-I-0092*, 2013.

[3] Tony Burton, David Sharpe, Nick Jenkins, and Ervin Bossanyi. *Wind energy handbook, 2nd Edition*. John Wiley & Sons, 2011.

[4] W Langreder. Models for variable speed wind turbines. *CREST, Loughborough University of Technology*, 1996.

[5] Poul Sørensen, Anca D Hansen, and Pedro André Carvalho Rosas. Wind models for simulation of power fluctuations from wind farms. *Journal of wind engineering and industrial aerodynamics*, 90(12):1381–1402, 2002.