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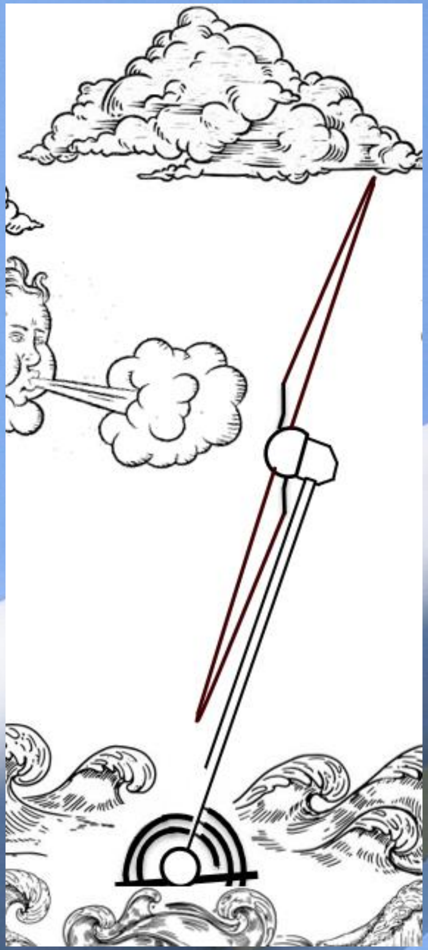


Fast response methods for floating wind turbines

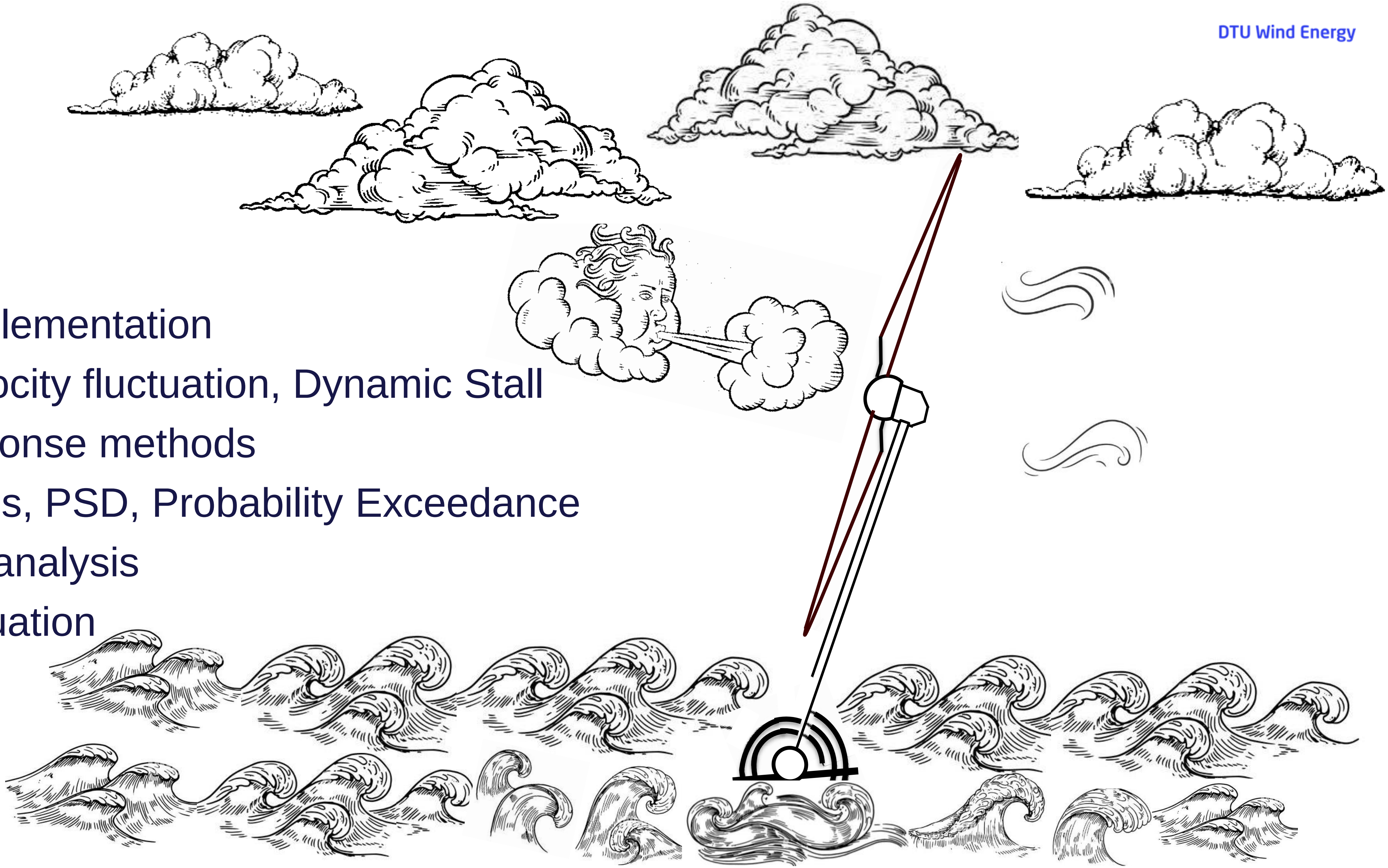
Bogdan Pamfil[a], Henrik Bredmose[a], Taeseong Kim[a], Wei Yu[b]

[a]Wind Turbine Division, Wind Turbine Design Division, Department of Wind and Energy Systems, Technical University of Denmark, Kongens Lyngby, Denmark

[b]Wind energy section, Faculty of Aerospace Engineering, Delft University of Technology, Delft, Netherlands

Content :

- Motivation
- Model implementation
- Inflow Velocity fluctuation, Dynamic Stall
- Fast Response methods
- Time series, PSD, Probability Exceedance
- Accuracy analysis
- CPU evaluation



Motivation

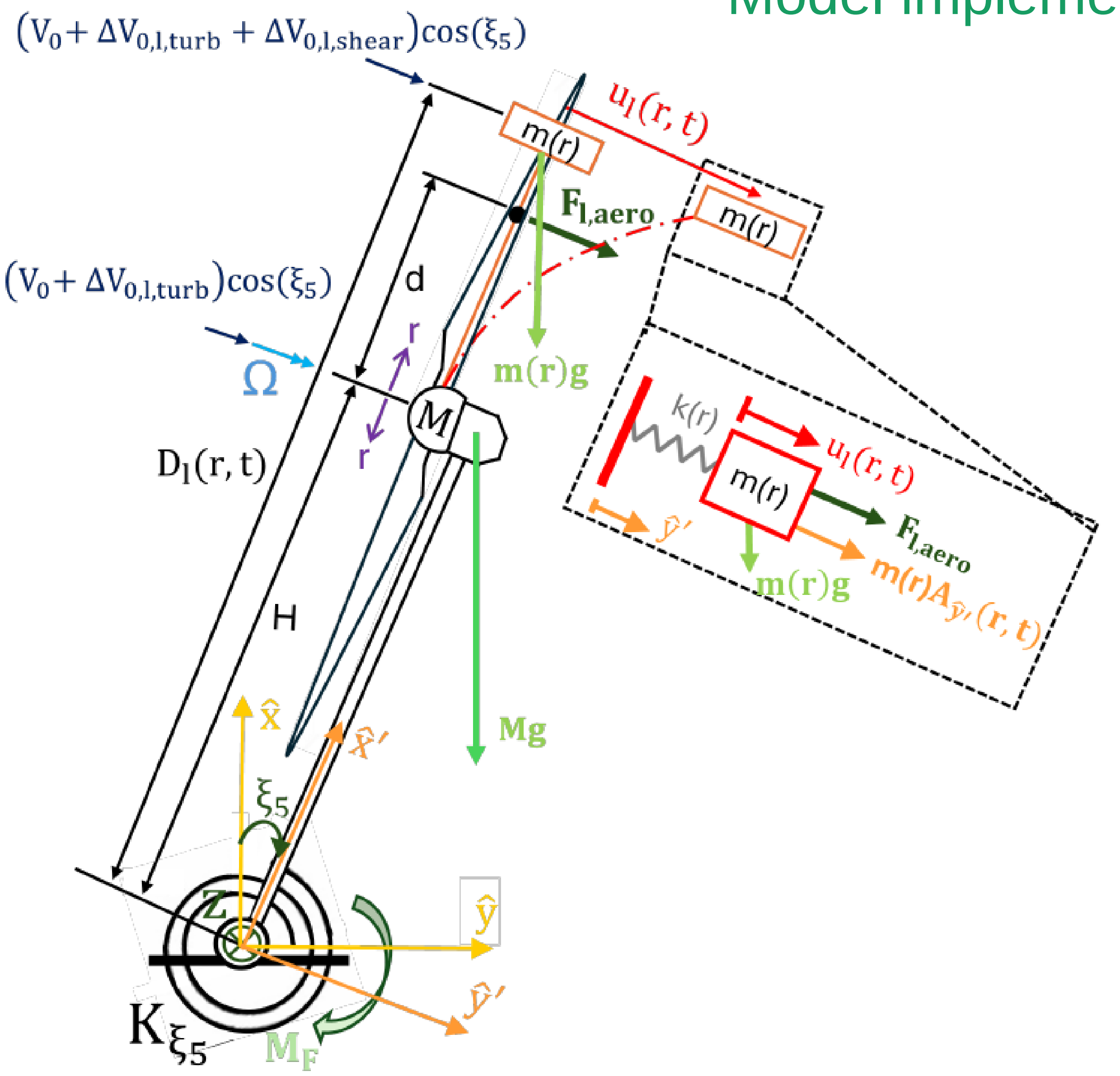
- Fast response methods are developed using Hill's method: two Fourier-based approaches and a single one is Laplace-based.

- Floating wind turbine model includes floater pitch motion, blade deflection, and dynamic stall consideration.

- Forcing exerted on the structure is influenced by the coherent turbulence velocity and sheared inflow velocity, as well as by a stochastic or harmonic floater pitch moment excitation representing the hydrodynamic moment.

- We determine the response contribution in accuracy with higher order harmonics consideration and investigate the CPU reduction benefit of fast response methods.

Model implementation



Period: $T = 2\pi/\Omega.$

Blade azimuthlal angle: $\Psi_l(t) = \frac{2\pi}{N_b}(l-1) + \Omega t,$

Blade deformation: $u_l(r,t) = \phi_{1f}(r)a_l(t)$

Moment arm distance: $D_l(d,t) = H + d\cos\Psi_l(t)$

Complete EOM assembly for the Time Domain Model (TDM)

EOM for the TDM: $\underline{\underline{M}}_S \underline{\ddot{x}} + \underline{\underline{C}}_S \underline{\dot{x}} + \underline{\underline{K}}_S \underline{x} = \underline{F}_T$

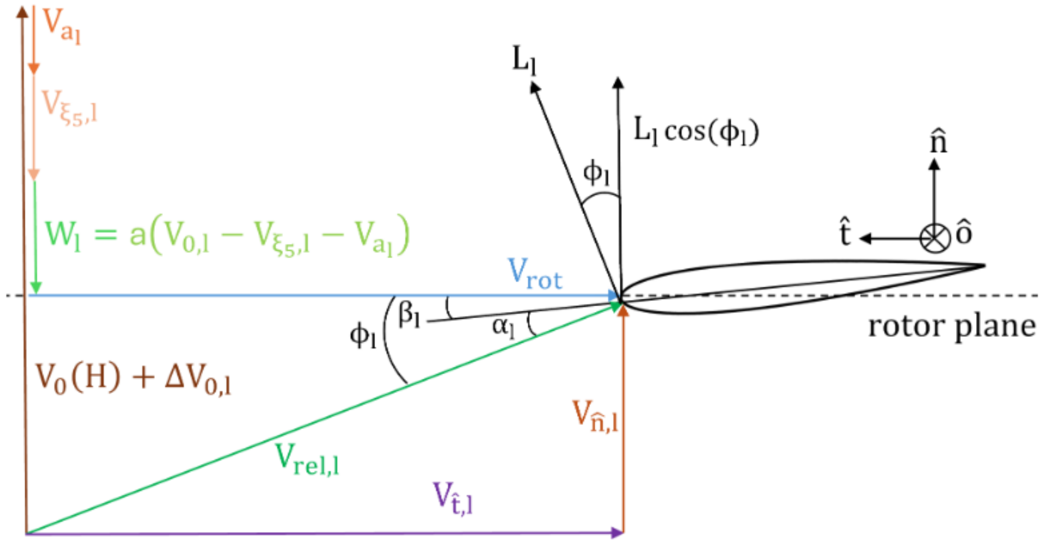
$$\underline{\underline{M}}_S = \begin{bmatrix} MH^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \int_0^{L_{blade}} m(r) \begin{bmatrix} \sum_{l=1}^{3} D_l^2(r,t) & D_1(r,t)\phi_{1f}(r) & D_2(r,t)\phi_{1f}(r) & D_3(r,t)\phi_{1f}(r) \\ D_1(r,t)\phi_{1f}(r) & (\phi_{1f}(r))^2 & 0 & 0 \\ D_2(r,t)\phi_{1f}(r) & 0 & (\phi_{1f}(r))^2 & 0 \\ D_3(r,t)\phi_{1f}(r) & 0 & 0 & (\phi_{1f}(r))^2 \end{bmatrix} dr$$

$$\underline{\underline{K}}_S = \begin{bmatrix} K_{\xi_5} + M\,g\,H & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \int_0^{L_{blade}} m(r) \begin{bmatrix} \Sigma_{l=1}^{3} g\, D_l(r,t) & -\ddot{D}_1(r,t)\phi_{1f}(r) & -\ddot{D}_2(r,t)\phi_{1f}(r) & -\ddot{D}_3(r,t)\phi_{1f}(r) \\ g\,\phi_{1f}(r) & \omega_{1f}^2\phi_{1f}^2(r) & 0 & 0 \\ g\,\phi_{1f}(r) & 0 & \omega_{1f}^2\phi_{1f}^2(r) & 0 \\ g\,\phi_{1f}(r) & 0 & 0 & \omega_{1f}^2\phi_{1f}^2(r) \end{bmatrix} dr$$

$$\underline{\underline{C}}_S = \int_0^{L_{blade}} m(r) \begin{bmatrix} \sum_{l=1}^{3} 2D_l(r,t)\dot{D}_l(r,t) & 0 & 0 & 0 \\ 2\dot{D}_1(r,t)\phi_{1f}(r) & \mu_{a_1}\omega_{1f}^2(\phi_{1f}(r))^2 & 0 & 0 \\ 2\dot{D}_2(r,t)\phi_{1f}(r) & 0 & \mu_{a_2}\omega_{1f}^2(\phi_{1f}(r))^2 & 0 \\ 2\dot{D}_3(r,t)\phi_{1f}(r) & 0 & 0 & \mu_{a_3}\omega_{1f}^2(\phi_{1f}(r))^2 \end{bmatrix} dr + \begin{bmatrix} \mu_{\xi_5}K_{\xi_5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Aerodynamic load characterization :

Airfoil velocity triangle :



Geometric identities:

$$\phi_l = \alpha_l + \beta_l$$

$$\phi_l = \tan^{-1}\left(-V_{\hat{n},l}/V_{\hat{t},l}\right)$$

$$V_{0,l} = V_0(\hat{x} = H) + \Delta V_{0,l}$$

$$V_{rel,l}^2 = V_{\hat{n},l}^2 + V_{\hat{t},l}^2$$

Lift force:

$$L_l = \frac{1}{2}\rho\{cC_{L,l}V_{rel,l}^2\}_{r=d}$$

Inflow velocity:

$$V_{0,l} = V_0(H) + \left.\frac{\partial\left(V_0(H)\left(\frac{\hat{x}}{H}\right)^{\nu}\right)}{\partial z}\right|_{\hat{x}=H} \Delta_{\hat{x},l}(d) + \Delta V_{0,turb}$$

$$= V_0(H) + \underbrace{V_0(H)\left(\frac{\nu\, d\cos\Psi_l}{H}\right)}_{\Delta V_{0,l,shear}} + \Delta V_{0,turb},$$

Normal velocity component:

$$V_{\hat{n},l} = (1-a)\left(V_{0,l} - \underbrace{\dot{\xi}_5\left(H + d\cos\Psi_l\right)}_{V_{\xi_5,l}} - \underbrace{\dot{a}_l\phi_{1f}(d)}_{V_{a_l}}\right)$$

Tangential velocity component:

$$V_{rot} = -\Omega d$$

Dynamic Stall model linearization

Dynamic lift for Stig Øye model :

$$C_L(\alpha_l, f_s) = f_s C_{L,inv}(\alpha_l) + (1 - f_s) C_{L,stall}(\alpha_l)$$

Separation function ODE linearization for Stig Øye model :

$$\dot{\underline{f}}_{s,l,lin} = -\frac{f_{s,l}}{\tau} + \frac{1}{\tau}\left(f_{s,static}\Big|_{st} + \frac{\partial f_{s,static,l}}{\partial \alpha_l}\Big|_{st} \frac{\partial \phi_l}{\partial \dot{\xi}_5}\Big|_{st} \dot{\xi}_5 + \frac{\partial f_{s,static,l}}{\partial \alpha_l}\Big|_{st} \frac{\partial \phi_l}{\partial \dot{a}_l}\Big|_{st} \dot{a}_l + \frac{\partial f_{s,static,l}}{\partial \alpha_l}\Big|_{st} \frac{\partial \phi_l}{\partial \Delta V_{0,l}}\Big|_{st} \Delta V_{0,l} \right)$$

Aerodynamic load linearization for Linear Model (LM):

$$\frac{\partial \left(L_{l,lin} \cos \phi_{l,lin}\right)}{\partial \cdot} = \frac{1}{2}\rho c \left(\frac{\partial C_{L,l}}{\partial \cdot}\Big|_{st} \cos \phi_{st} V_{rel,st}^2 + C_{L,st} \frac{\partial \cos \phi_{l,lin}}{\partial \cdot}\Big|_{st} V_{rel,st}^2 + C_{L,st} \cos \phi_{st} \frac{\partial \left(V_{rel,l}^2\right)}{\partial \cdot}\Big|_{st} \right)$$

Airfoil FFA-W3-241 :

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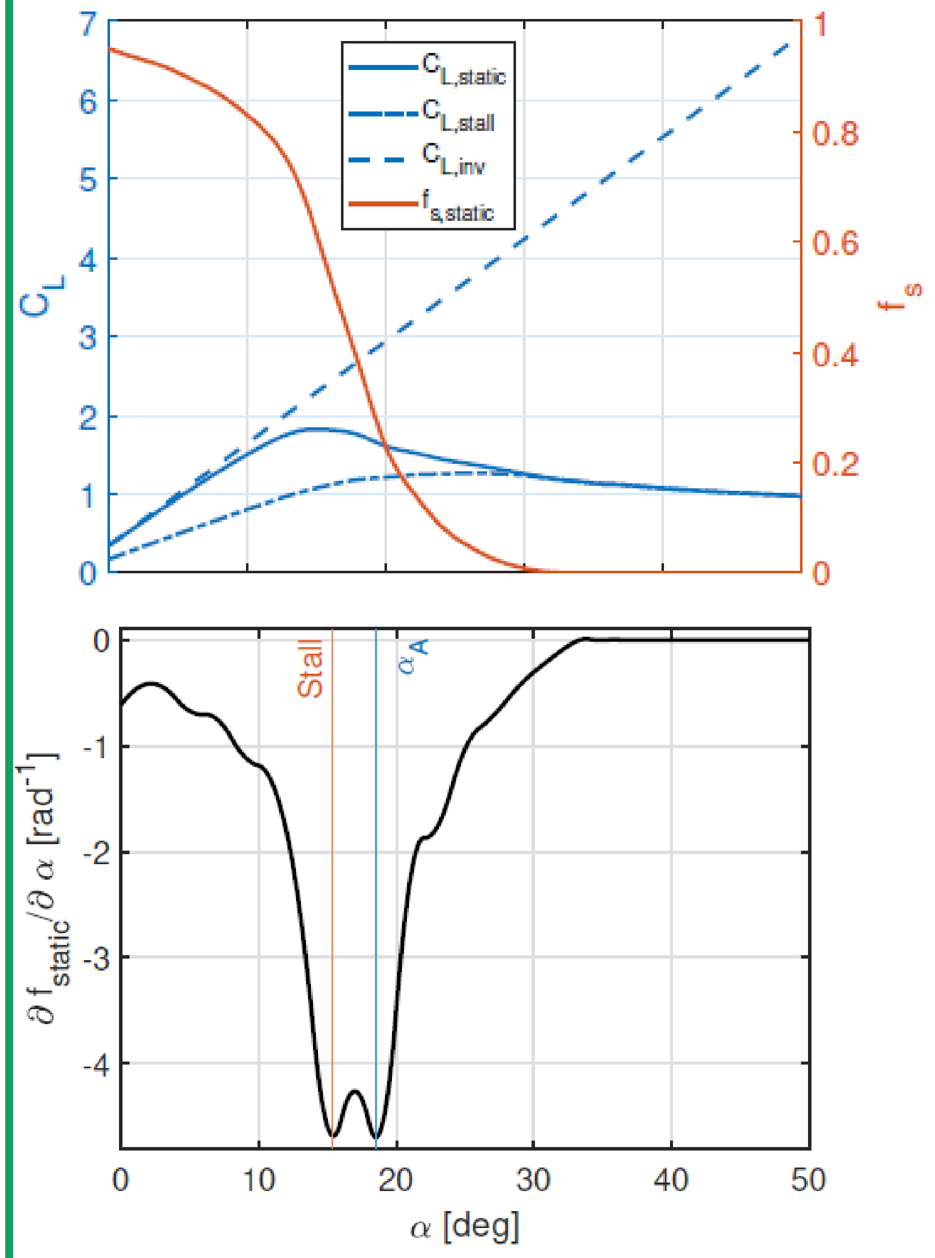


State-Space formulation for Linear Model (LM):

EOM for the LM: $$\underline{\underline{M}}_S\,\ddot{\underline{x}} + \left(\underline{\underline{C}}_S + \underline{\underline{C}}_A\right)\dot{\underline{x}} + \underline{\underline{K}}_S\,\underline{x} = \underline{F}_L$$

$$\underline{\underline{C}}_A = \begin{bmatrix} -\frac{\partial M_{aero,lin}}{\partial \dot{\xi}_5} & -\frac{\partial M_{aero,lin}}{\partial \dot{a}_1} & -\frac{\partial M_{aero,lin}}{\partial \dot{a}_2} & -\frac{\partial M_{lin}}{\partial \dot{a}_3} \\ -\frac{\partial GF_{a_1,lin}}{\partial \dot{\xi}_5} & -\frac{\partial GF_{a_1,lin}}{\partial \dot{a}_1} & 0 & 0 \\ -\frac{\partial GF_{a_2,lin}}{\partial \dot{\xi}_5} & 0 & -\frac{\partial GF_{a_2,lin}}{\partial \dot{a}_2} & 0 \\ -\frac{\partial GF_{a_3,lin}}{\partial \dot{\xi}_5} & 0 & 0 & -\frac{\partial GF_{a_3,lin}}{\partial \dot{a}_3} \end{bmatrix}_{st}$$

State-Space general form: $$\dot{\underline{q}} = \underline{\underline{A}}\,\underline{q} + \underline{F}_B$$

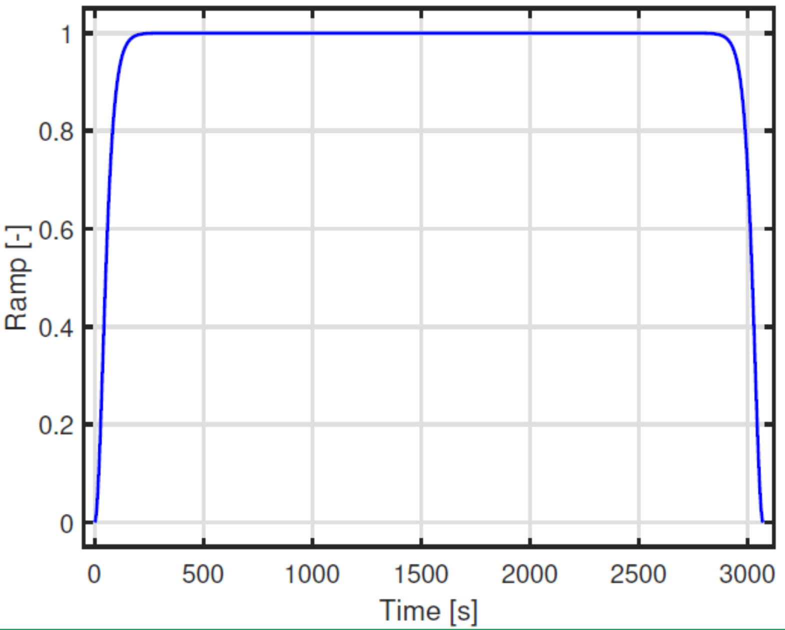
$$\underline{\underline{A}}_L = \begin{bmatrix} \left[\underline{\underline{0}}_{4\times4}\right] & \left[\underline{\underline{I}}_{4\times4}\right] & \left[\underline{\underline{0}}_{4\times3}\right] \\ \left[-\underline{\underline{M}}_S^{-1}\underline{\underline{K}}_S\right]_{4\times4} & \left[-\underline{\underline{M}}_S^{-1}\left(\underline{\underline{C}}_S + \underline{\underline{C}}_A\right)\right]_{4\times4} & \left[\underline{\underline{M}}_S^{-1}\left[\partial \underline{F}_i / \partial \underline{f}_{s,j}\right]\right]_{4\times3} \\ \left[\underline{\underline{0}}_{3\times4}\right] & \left[\partial \dot{\underline{f}}_{s,i} / \partial \dot{\underline{x}}_j\right]_{3\times4} & \left[\partial \dot{\underline{f}}_{s,i} / \partial \underline{f}_{s,j}\right]_{3\times3} \end{bmatrix}_{st}$$

$$\underline{q} = \left[\underline{x}_{4\times1}^T, \dot{\underline{x}}_{4\times1}^T, f_{s,1}, f_{s,2}, f_{s,3}\right]^T$$

Fourier Transform based Fast Response methodology

State-Space ODE general form: $\dot{\underline{\acute{q}}}(t) = \underline{\underline{\acute{A}}}\,\underline{\acute{q}} + \underline{\acute{F}}_B(t)$ $\times$

Windowing function:



Forcing input is expressed in the frequency domain:

$$\underline{\acute{F}}_B(\omega) = \mathrm{FFT}\left(\underline{\acute{F}}_B(t)\right)$$

The State-Space ODE is written in the frequency domain:

$$\sum_{s=-\infty}^{\infty} \mathrm{i}\omega_s \underline{\acute{q}}(\omega_s) e^{\mathrm{i}\omega_s t} = \sum_{s=-\infty}^{\infty} \left(\underline{\underline{\acute{A}}}\,\underline{\acute{q}}(\omega_s) + \underline{\acute{F}}_B(\omega_s)\right) e^{\mathrm{i}\omega_s t}$$

Solve response in frequency domain given the transfer function and forcing input:

$$\underline{\acute{q}}(\omega) = \underbrace{\left(\mathrm{i}\omega \underline{\underline{\acute{I}}} - \underline{\underline{\acute{A}}}\right)^{-1}}_{\underline{\underline{H}}(\omega)} \underline{\acute{F}}_B(\omega)$$

Solve response in time domain through the inverse Fast Fourier Transform algorithm:

$$\underline{\acute{q}}(t) = \Re\left\{\mathrm{iFFT}\left(\underline{\acute{q}}(\omega)\right)\right\}$$

Perturbation theory

Fourier double sided truncated series (Hill's decomposition) :

$$\underline{q}(t) = \sum_{j=-N}^{N} \underline{q}_j(t) e^{\mathrm{i}j\Omega t}, \quad \underline{\dot{q}}(t) = \sum_{j=-N}^{N} \left((\mathrm{i}j\Omega)\, \underline{q}_j(t) + \underline{\dot{q}}_j(t) \right) e^{\mathrm{i}j\Omega t} \quad \text{and} \quad \underline{\underline{A}}_L(t) = \sum_{j=-N}^{N} \underline{\underline{A}}_{L,j} e^{\mathrm{i}j\Omega t}$$

$$\underline{q}_{-j} = \underline{q}_j^*, \quad \underline{\underline{A}}_{L,-j} = -\underline{\underline{A}}_{L,j}^*$$

Perturbation method applied to variables:

$$\underline{q}(t) = \delta^0 \underline{\tilde{q}}_0(t) + \sum_{n=-N}^{N} \delta^{|n|} \underline{q}_n e^{\mathrm{i}n\Omega t} = \delta^0 \underline{\tilde{q}}_0(t) + \sum_{n=1}^{N} \delta^n \underline{\tilde{q}}_n$$

$$\underline{\dot{q}}(t) = \delta^0 \underline{\dot{\tilde{q}}}_0(t) + \sum_{n=-N}^{N} \delta^{|n|} (\mathrm{i}n\Omega)\, \underline{\dot{q}}_n e^{\mathrm{i}n\Omega t} = \delta^0 \underline{\dot{\tilde{q}}}_0(t) + \sum_{n=1}^{N} \delta^n \underline{\dot{\tilde{q}}}_n$$

$$\underline{\underline{A}}(t) = \delta^0 \underline{\underline{\tilde{A}}}_0 + \sum_{n=-N}^{N} \delta^{|n|} \underline{\underline{A}}_n(t) e^{\mathrm{i}n\Omega t} = \delta^0 \underline{\underline{\tilde{A}}}_0 + \sum_{n=1}^{N} \delta^n \underline{\underline{\tilde{A}}}_n(t),$$

Double Perturbation method

System of equations:

$$\sum_{n=0}^{N} \dot{\tilde{\underline{q}}}_n \delta^n = \sum_{m=0}^{N} \sum_{l=0}^{l} \underline{\underline{\tilde{A}}}_{L,l} \, \tilde{\underline{q}}_m \delta^{(m+l)} + \underline{F}_{B,L}(t)$$

$$\delta^0 : \quad \dot{\tilde{\underline{q}}}_0(t) = \underline{\underline{\tilde{A}}}_{L,0} \tilde{\underline{q}}_0 + \underline{F}_{B,L}(t)$$

$$\delta^1 : \quad \dot{\tilde{\underline{q}}}_1(t) = \underline{\underline{\tilde{A}}}_{L,0} \tilde{\underline{q}}_1 + \underline{\underline{\tilde{A}}}_{L,1} \tilde{\underline{q}}_0$$

$$\vdots$$

$$\delta^n : \quad \dot{\tilde{\underline{q}}}_n(t) = \sum_{l=0}^{n} \underline{\underline{\tilde{A}}}_{L,l} \, \tilde{\underline{q}}_{n-l} = \underline{\underline{\tilde{A}}}_{L,0} \tilde{\underline{q}}_n + \sum_{l=1}^{n} \underline{\underline{\tilde{A}}}_{L,l} \, \tilde{\underline{q}}_{n-l}$$

Harmonics to solve:

$$\underline{q}_0(\omega) = \left(\mathrm{i}\omega \underline{\underline{\hat{I}}} - \underline{\underline{\tilde{A}}}_{L,0} \right)^{-1} \underline{F}_{B,L}(\omega)$$

$$\dot{\tilde{\underline{q}}}_1(\omega) = \left(\mathrm{i}\omega \underline{\underline{\hat{I}}} - \underline{\underline{\tilde{A}}}_{L,0} \right)^{-1} \underline{\underline{\tilde{A}}}_{L,1} \tilde{\underline{q}}_0(\omega)$$

$$\underline{q}_n(\omega) = \underbrace{\left(\mathrm{i}\omega \underline{\underline{\hat{I}}} - \underline{\underline{\tilde{A}}}_{L,0} \right)^{-1}}_{\underline{\underline{H}}(\omega)} \underbrace{\sum_{l=1}^{n} \underline{\underline{\tilde{A}}}_{l} \tilde{\underline{q}}_{n-l}(\omega)}_{\underline{\tilde{F}}_B(\omega)}$$

$$\underline{q}(t) = \sum_{n=0}^{N} \underbrace{\Re\{\mathrm{iFFT}\left(\tilde{\underline{q}}_n(\omega)\right)\}}_{\tilde{\underline{q}}_n(t)}$$

Single Perturbation

Difference in perturbation applied to $\underline{\underline{A}}_L(t)$:

$$\underline{\underline{A}}_L(t) = \underline{\underline{A}}_{L,0}(t) + \varepsilon \underline{\underline{\tilde{A}}}_L$$

Same as Double Perturbation:

$$\underline{q}(t) = \underline{\tilde{q}}_0(t) + \sum_{n=1}^{N} \varepsilon^n \underline{\tilde{q}}_n(t), \quad \underline{\dot{q}}(t) = \underline{\dot{\tilde{q}}}_0(t) + \sum_{n=1}^{N} \varepsilon^n \underline{\dot{\tilde{q}}}_n(t)$$

System of equations:

$$\begin{aligned}
\varepsilon^0 : &\quad \underline{\dot{\tilde{q}}}_0(t) = \underline{\underline{\tilde{A}}}_{L,0}\underline{\tilde{q}}_0 + \underline{F}_{B,L}(t) \\
\varepsilon^1 : &\quad \underline{\dot{\tilde{q}}}_1(t) = \underline{\underline{\tilde{A}}}_{L,0}\underline{\tilde{q}}_1 + \underline{\underline{\tilde{A}}}_L\underline{\tilde{q}}_0 \\
&\quad \vdots \\
\varepsilon^n : &\quad \underline{\dot{\tilde{q}}}_n(t) = \underline{\underline{\tilde{A}}}_{L,0}\underline{\tilde{q}}_n + \underline{\underline{\tilde{A}}}_L\,\underline{\tilde{q}}_{n-1}
\end{aligned}$$

Additional derivations are not covered in this presentation.

Laplace Transform and Single Perturbation approach combined

The inverse of the Laplace Transform of the equation defined in the s-domain determines the solution in time domain:

$$\mathcal{L}^{-1}\left\{\underline{q}_0(s)\right\} = \mathcal{L}^{-1}\left\{\left(s\underline{\underline{\hat{I}}} - \underline{\underline{A}}_{L,0}\right)^{-1}\left(\frac{1}{s}\underline{F}_{B,L}(t_i) + \underline{q}_0(t_{i-1})\right)\right\}$$

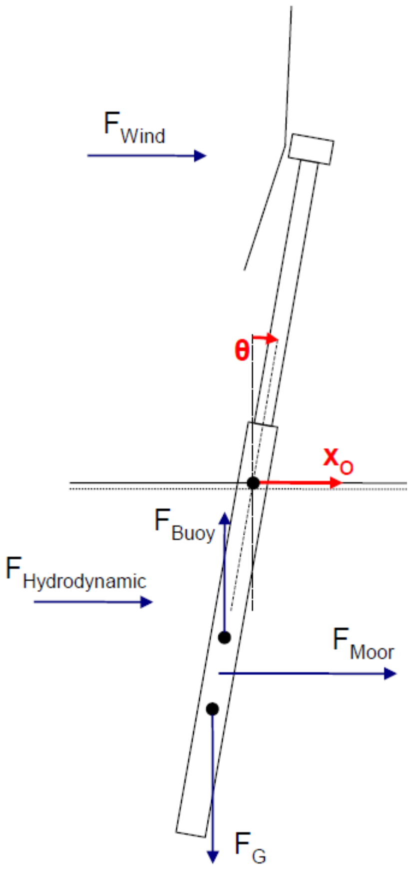
$$\mathcal{L}^{-1}\left\{\dot{\underline{\tilde{q}}}_1(s)\right\} = \mathcal{L}^{-1}\left\{\left(s\underline{\underline{\hat{I}}} - \underline{\underline{A}}_{L,0}\right)^{-1}\left(\frac{1}{s}\left(\underline{F}_{B,L}\left(t_i\right)\right)_{\mathrm{mod}} + \underline{\tilde{q}}_1(t_{i-1})\right)\right\}$$

$$\left(\underline{F}_{B,L}\left(t_i\right)\right)_{\mathrm{mod}} = \left(\sum_{n=1}^{N}\underline{\underline{\tilde{A}}}_{L,n}\right)\underline{q}_0(t_i)$$

Additional derivations are not covered in this presentation.

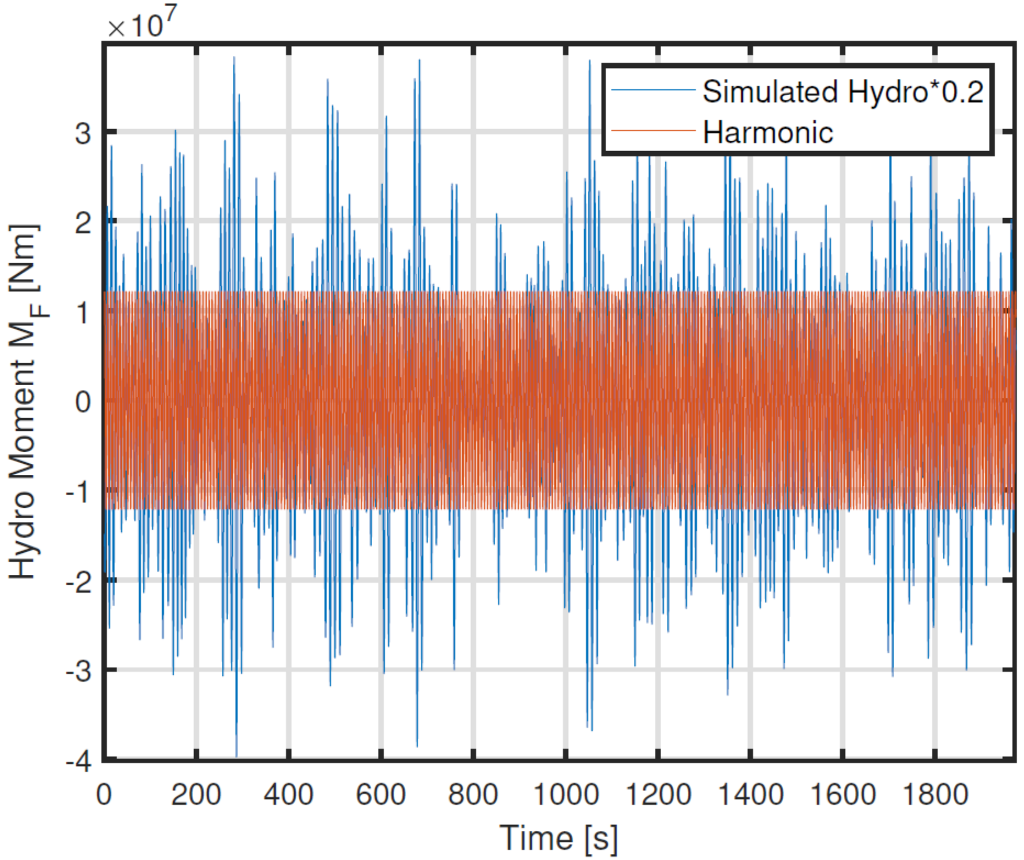
Stochastic hydrodynamic moment computed for the floater pitch moment characterization

Variable	Symbol	Value
Mooring lines horizontal stiffness	K_{Moor}	66700 N/m
Mooring lines anchoring location	$\hat{x}_{Moor}$	60 m
Floater spar bottom location	$\hat{x}_{Bot}$	-120 m
Floater spar diameter	D_{Spar}	11.2 m
Floater mass	$M_{Floater}$	$1.0897 \cdot 10^{7}$ kg
Floater center of mass	$\hat{x}_{CM,Floater}$	-105.95 m
Floater inertia at center of mass	$I_{CM,Floater}$	$1.1627 \cdot 10^{10}$ kg m^2
Tower mass	M_{Tower}	$5.4692 \cdot 10^{5}$ kg
Tower center of mass	$\hat{x}_{CM,Tower}$	56.40 m
Tower inertia at center of mass	$I_{CM,Tower}$	$4.2168 \cdot 10^{8}$ kg m^2
Rotor hub height	H	119 m
Rotor mass	M_{Rotor}	227962 kg
Nacelle mass	$M_{Nacelle}$	446036 kg



Load cases distinction

Stochastic hydrodynamic moment:



Load cases analysed:

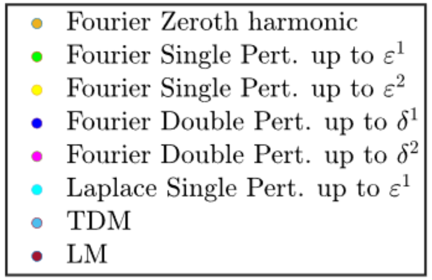
Load Case	Aero: Inflow velocity			Hydro: Floater pitching moment	
	Constant	Sheared	Turbulent	Harmonic	Stochastic
C			✓	✓	
D	✓				✓
E		✓	✓		✓

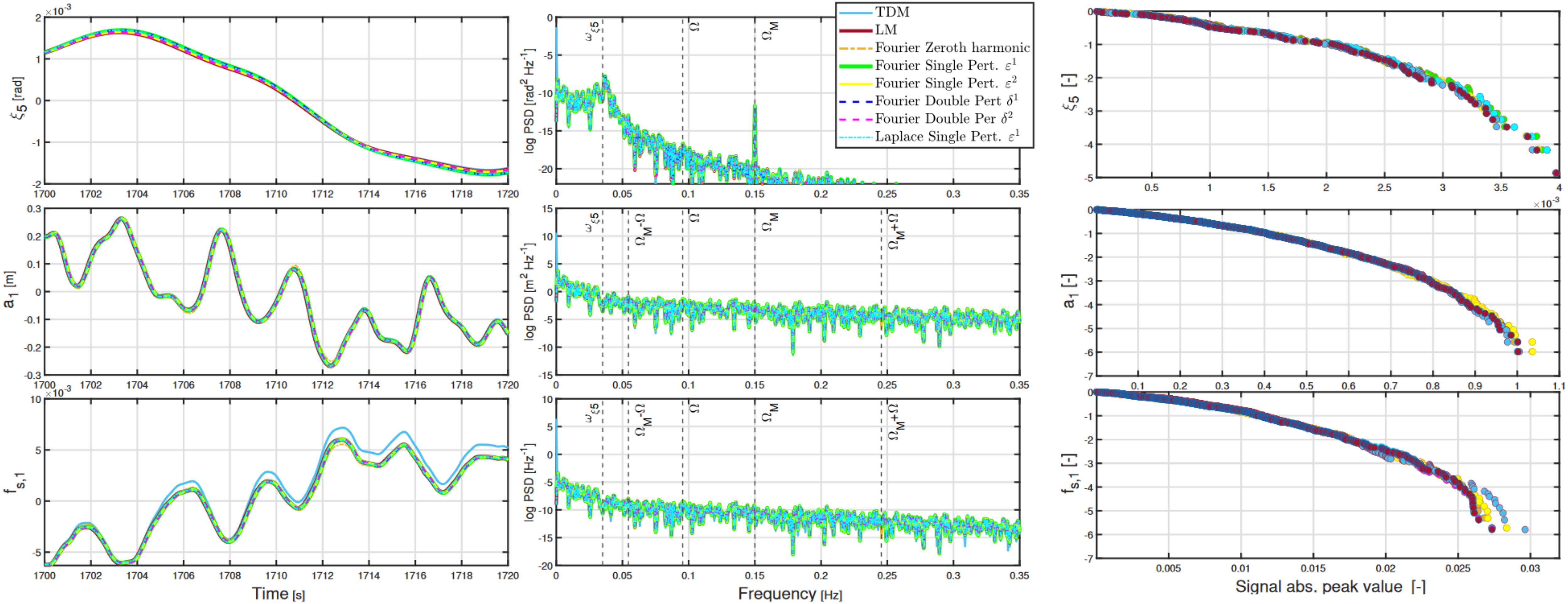
Harmonic hydrodynamic moment:

$$M_F = A_M \cos\left(\Omega_M t\right)$$

Load Case C:

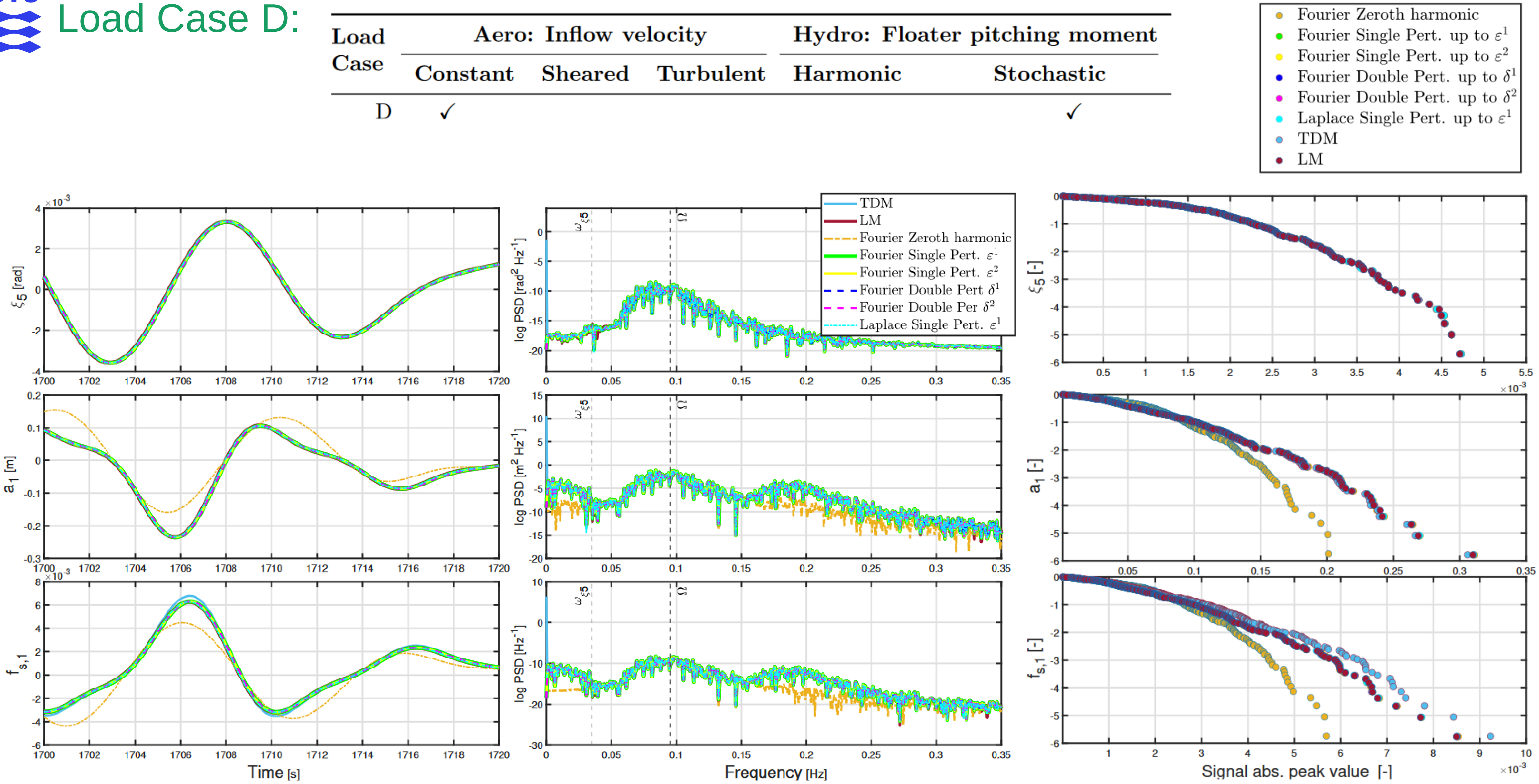
Load Case	Aero: Inflow velocity			Hydro: Floater pitching moment	
	Constant	Sheared	Turbulent	Harmonic	Stochastic
C			✓	✓	





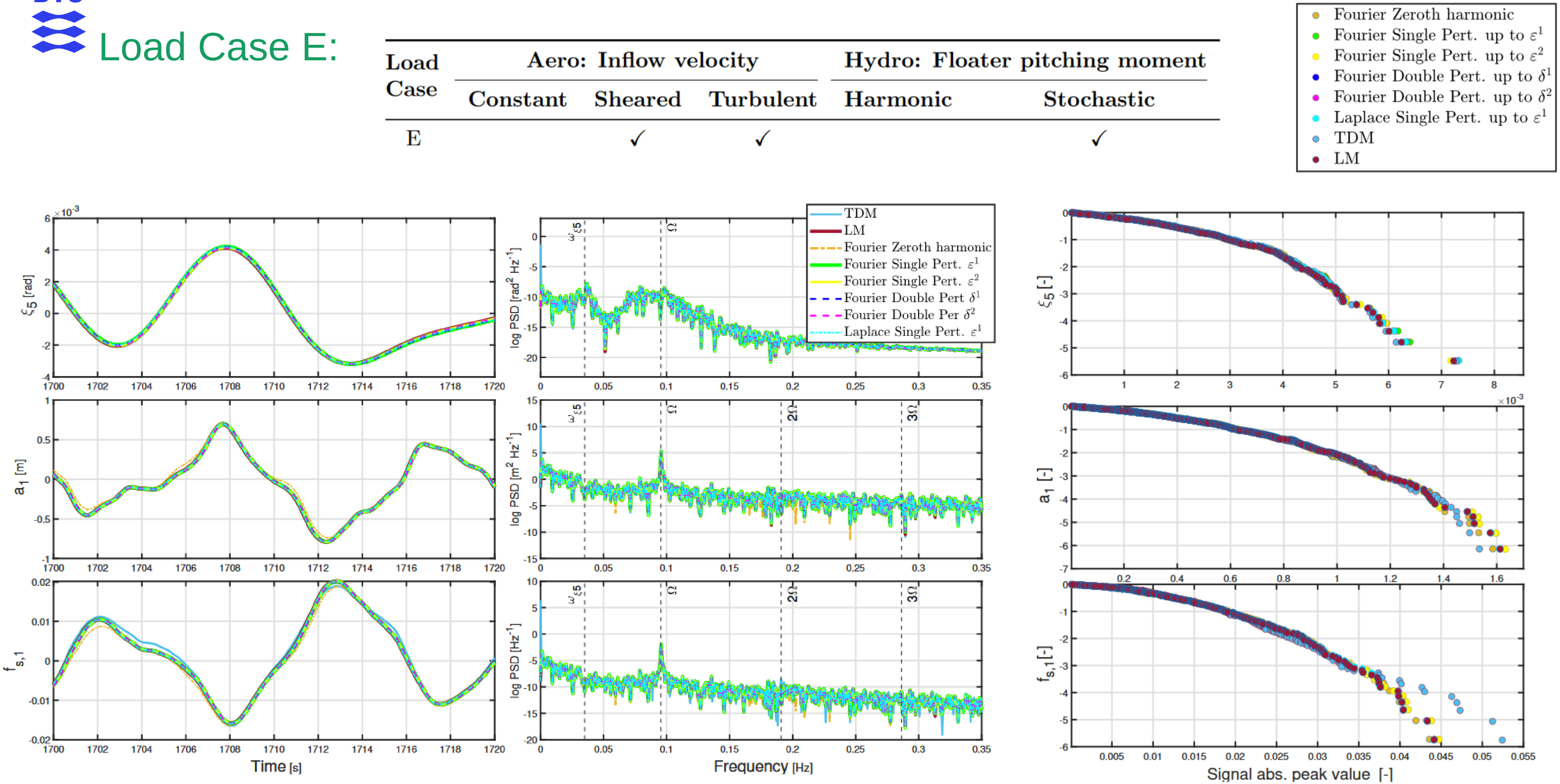
Load Case D:

Load Case	Aero: Inflow velocity			Hydro: Floater pitching moment	
	Constant	Sheared	Turbulent	Harmonic	Stochastic
D	✓				✓



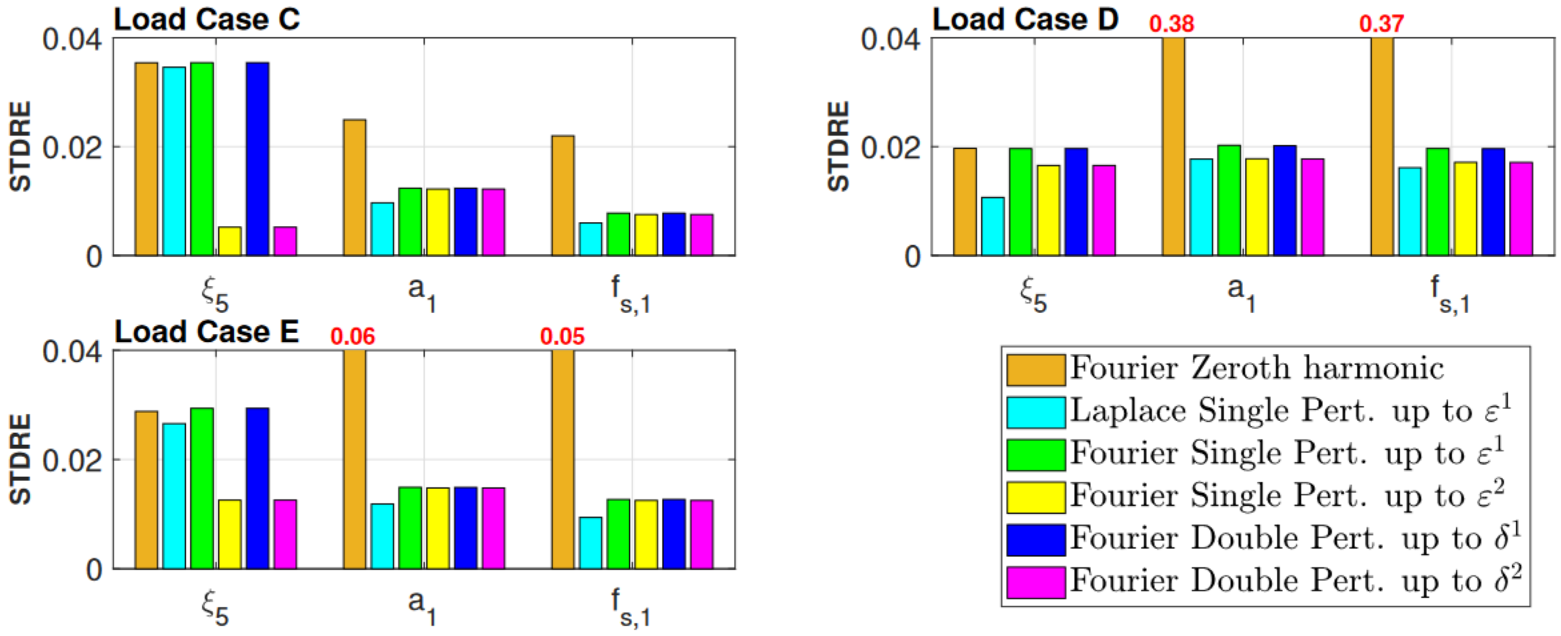
Load Case E:

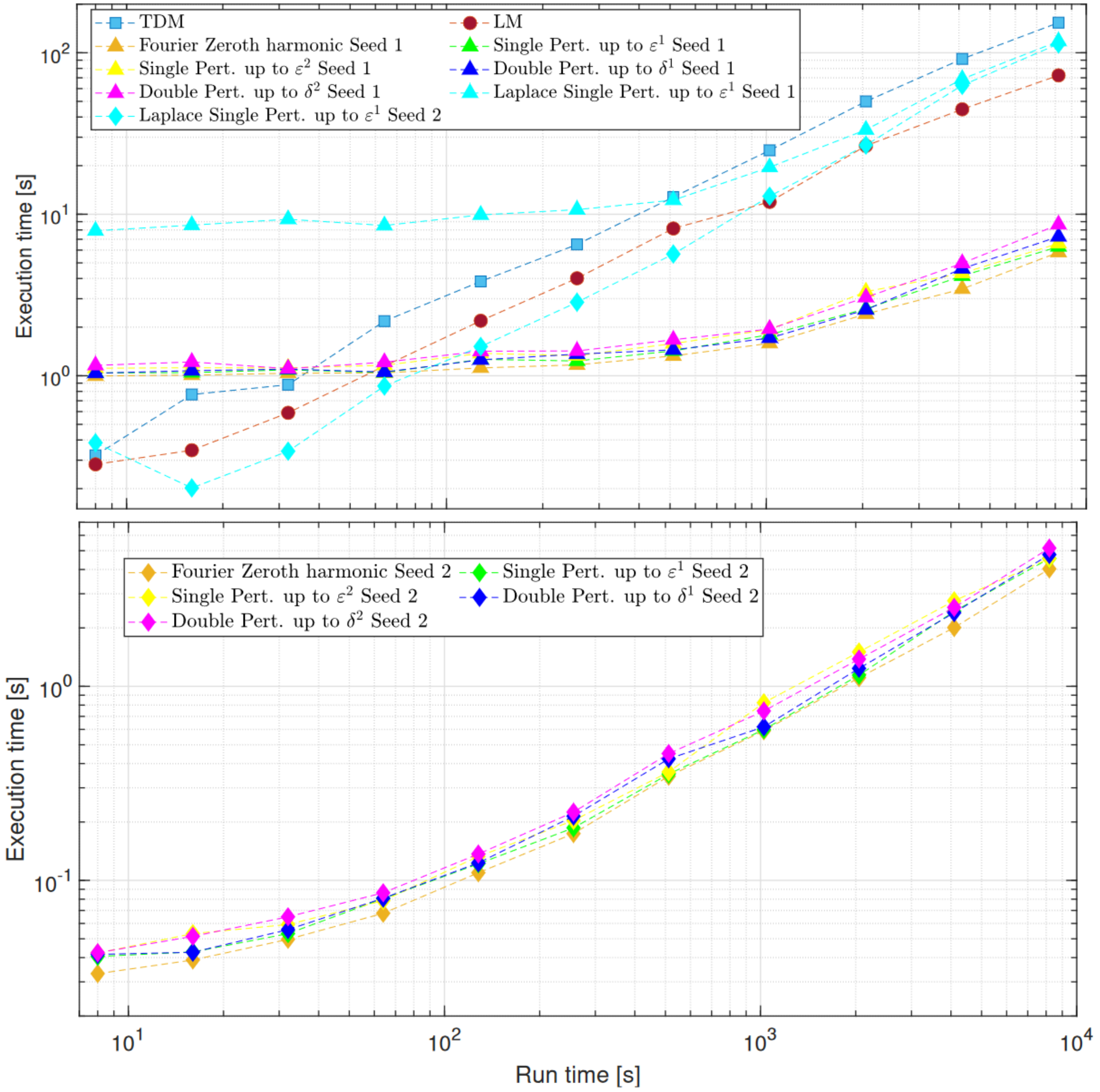
Load Case	Aero: Inflow velocity			Hydro: Floater pitching moment	
	Constant	Sheared	Turbulent	Harmonic	Stochastic
E		✓	✓		✓



Standard Deviation Relative Error (STDRE):

$$\epsilon_{STD}\left(\tilde{q}_{i,method}(t)\right) = \frac{\sigma\left(\tilde{q}_{i,method}(t) - \tilde{q}_{i,LM}(t)\right)}{\sigma\left(\tilde{q}_{i,LM}(t)\right)}$$





Fast Response methods CPU time analysis:

Seed 1

Forcing time series computation with inverted Mass matrix stored.

Zeroth harmonic : Compute $\underline{\underline{A}}_{L,0}$ through Hill's decomposition.

Single Pert. : Compute average $\underline{\underline{A}}_{L,0}$ and higher harmonics matrix $\underline{\underline{\tilde{A}}}_L(t) = \underline{\underline{A}}_L(t) - \underline{\underline{A}}_{L,0}$ for time series of period T length.

Double Pert. : Compute $\underline{\underline{A}}_0$ and higher harmonics $\underline{\underline{A}}_{L,j}$ from Hill decomposition, followed by $\underline{\underline{\tilde{A}}}_{L,j}(t)$ for time.

Laplace Single Pert. : Simplify s-domain equation symbolically and compute inverse Laplace for time solution.

Seed 2

Forcing time series computation for new Seed 2 using stored inverted Mass matrix from previous Seed 1

Zeroth harmonic : Use stored $\underline{\underline{A}}_{L,0}$

Single Pert. : Use stored matrices $\underline{\underline{A}}_{L,0}$ and $\underline{\underline{\tilde{A}}}_L(t)$

Double Pert. : Use stored matrices $\underline{\underline{A}}_{L,0}$ and $\underline{\underline{\tilde{A}}}_{L,j}(t)$

Laplace Single Pert. : Use stored time domain solution

Concluding remarks

- Single and Double Perturbation methods with the Fourier Transform both give the same accuracy when going to up to the same harmonic order consideration. The Laplace based Single Perturbation method gives a different accuracy compared to the Fourier based methods.

- For a simulation run time of 4096 seconds, the Single Perturbation method requires only 5 seconds execution time and outperforms the linear model which takes instead 40 seconds as well as the time domain model which needs 90 seconds. The Single Perturbation method usually is faster than the Double Perturbation method. The Laplace Transform based Single Perturbation approach is slower than the other methods by requiring roughly a similar execution time as the Linear Model.

- The overhead computational costs have been reduced, and time loops have been avoided where possible to improve execution time efficiency of Fast Response methods.