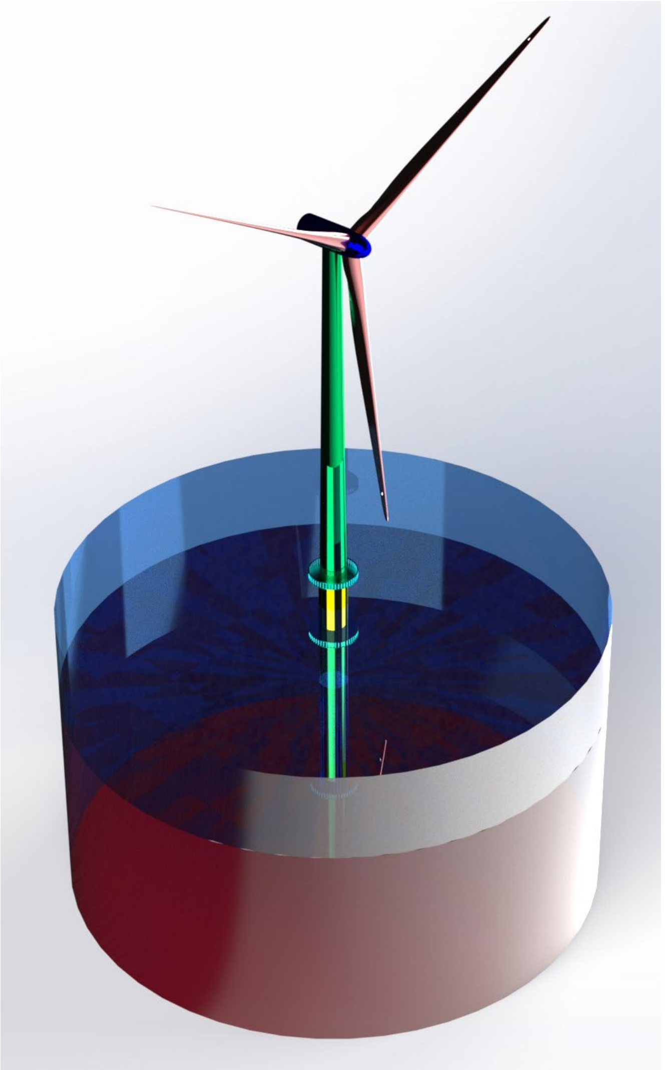
A semi-analytical approach for dynamic responses of monopile-supported OWTs subjected to accidental loads

Ahmed Hammad, Zhaolong Yu

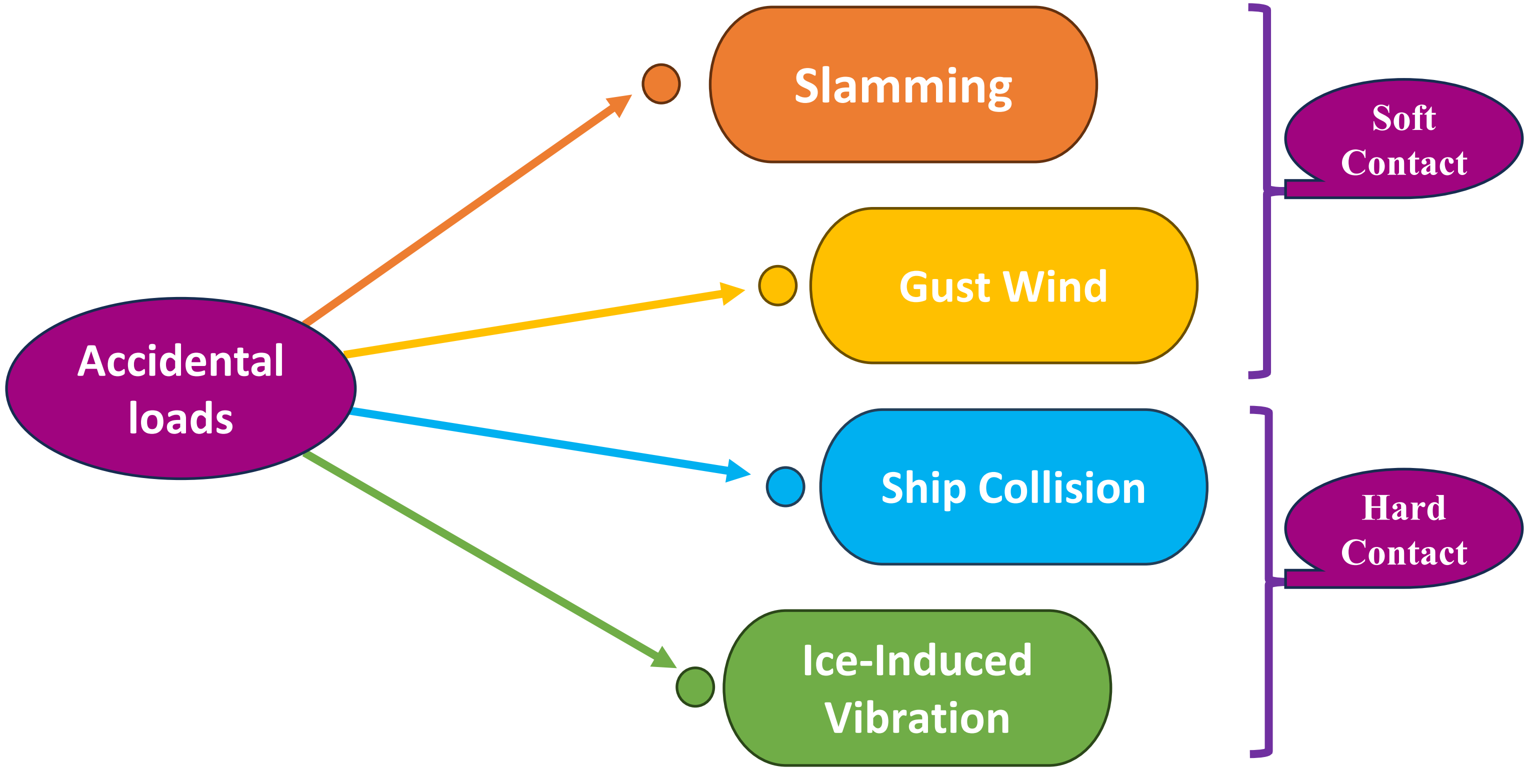
Content:

- Types of accidental loads
- Structural modelling
- Rayleigh-Ritz solution
- Modal analysis
- Slamming dynamic responses
- Ship collision dynamic responses (Contact Algorithm)

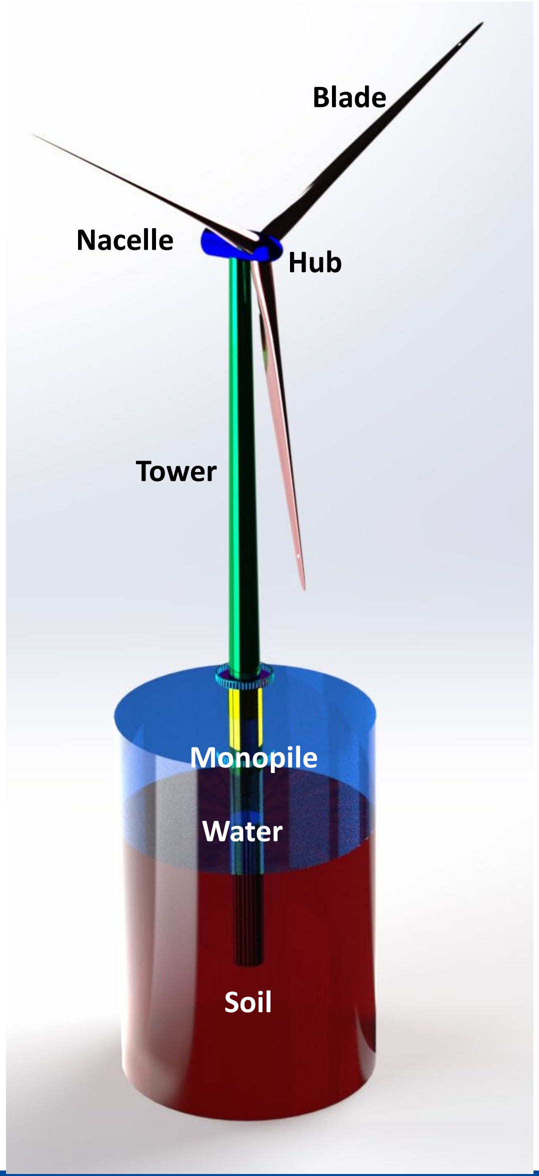
Types of accidental loads

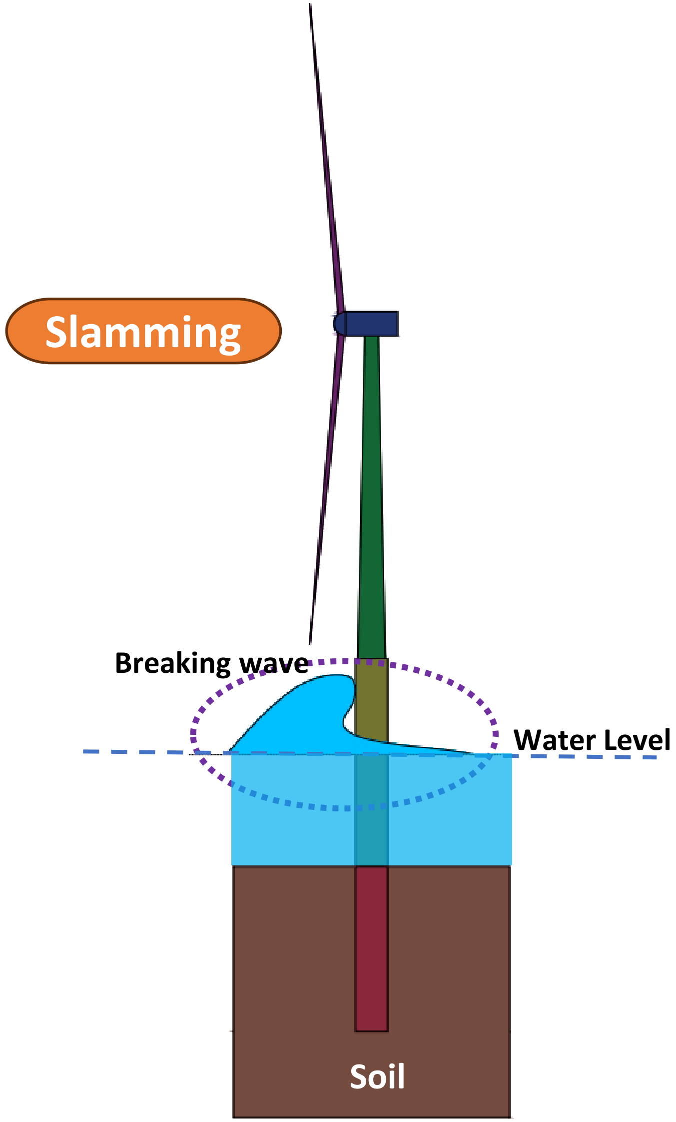


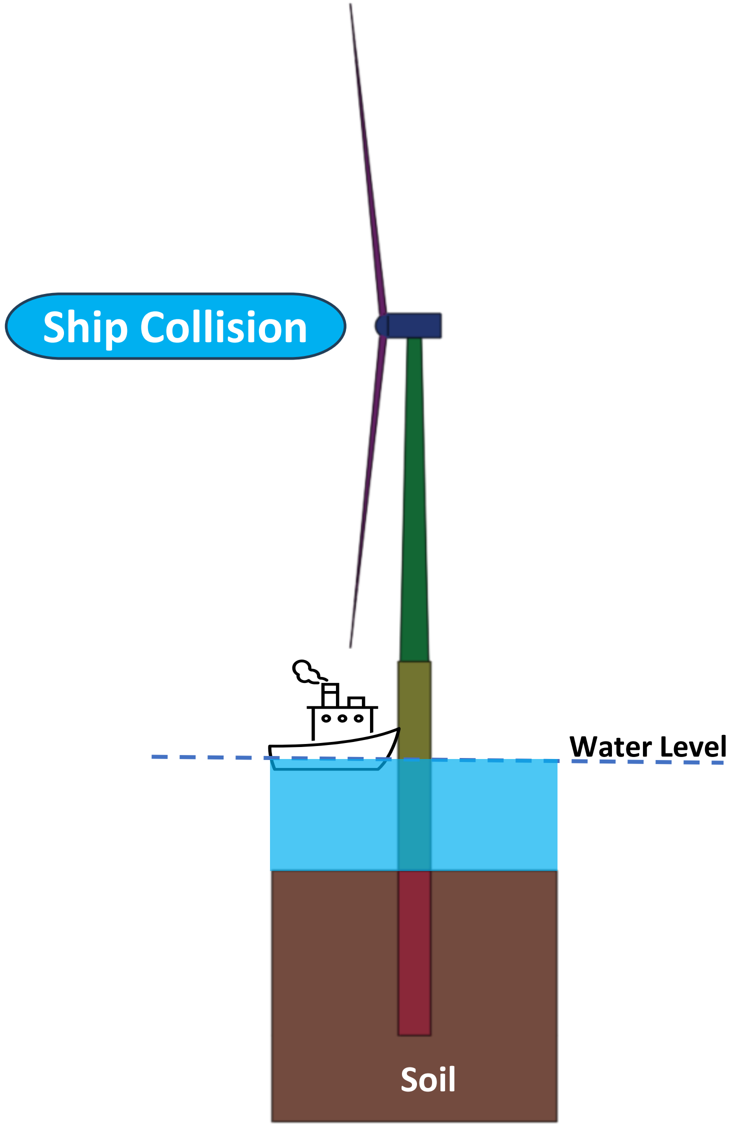
- **Soft contact**: the force is defined / generated as a **predefined force-time history** curve which applied on the structure directly.
- **Hard contact**: the force is defined / **generated step by step** considering the contact between the applied load and the structure by defining a **force-displacement relationship**.



Types of accidental loads

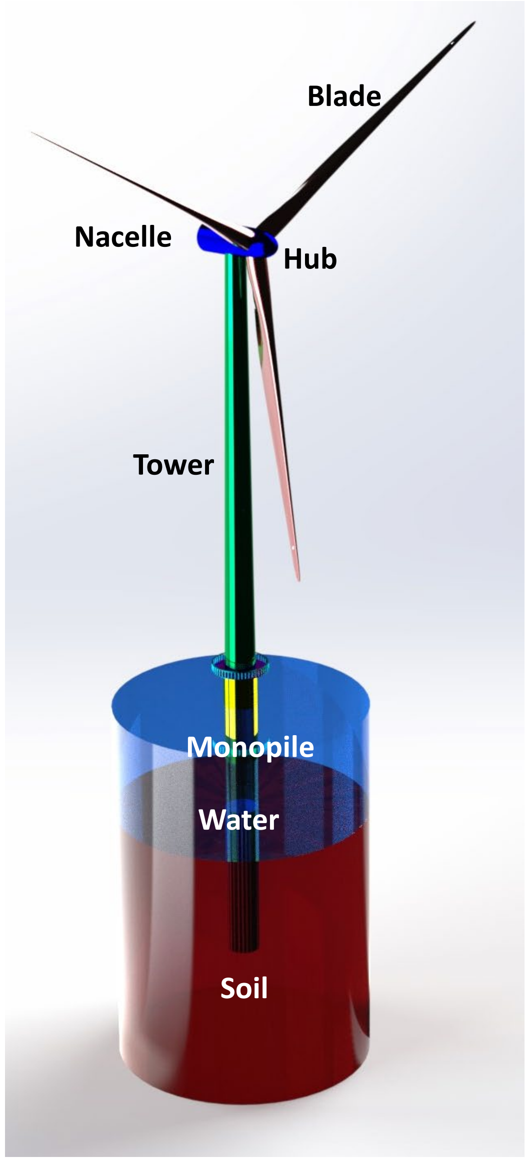


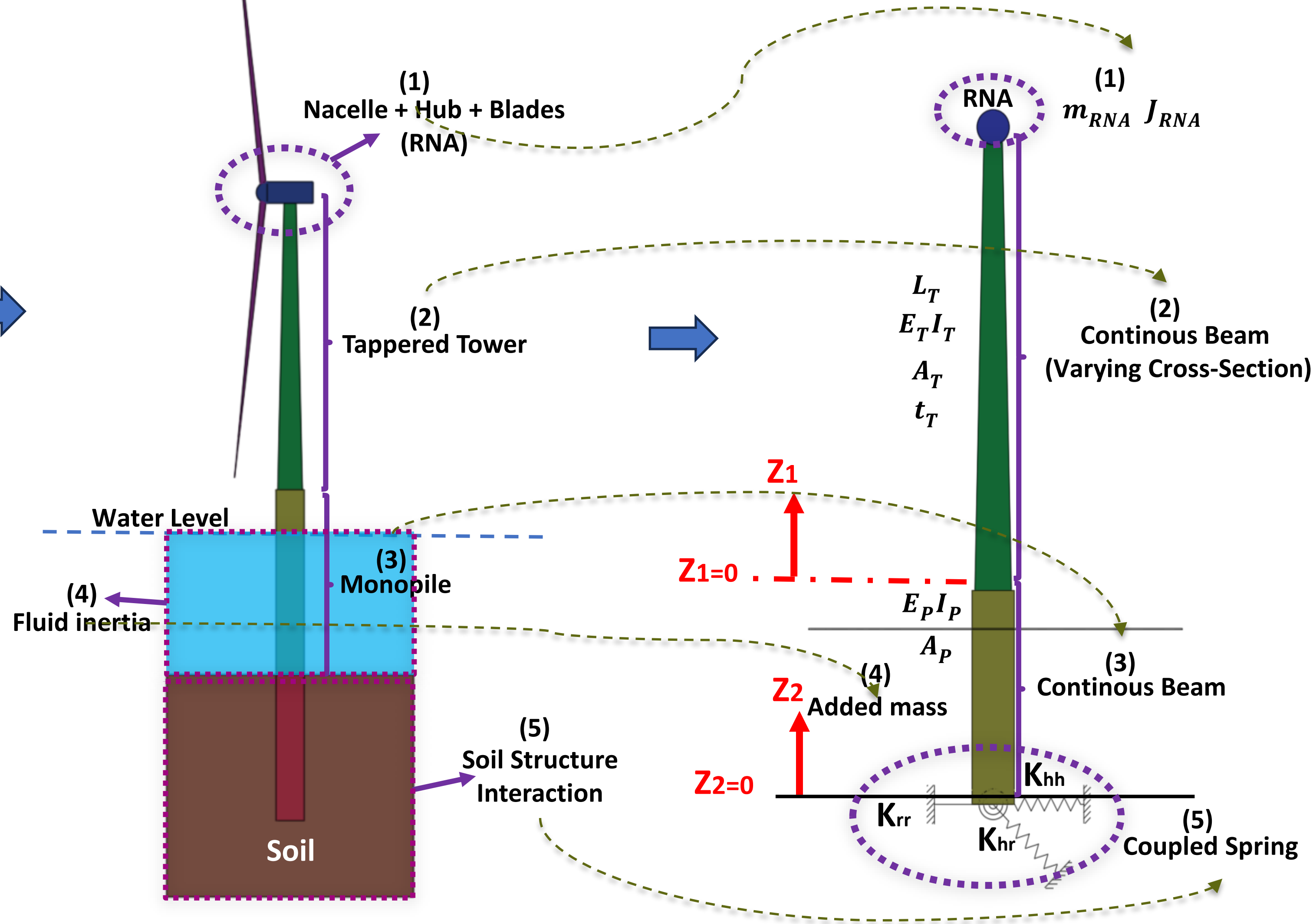




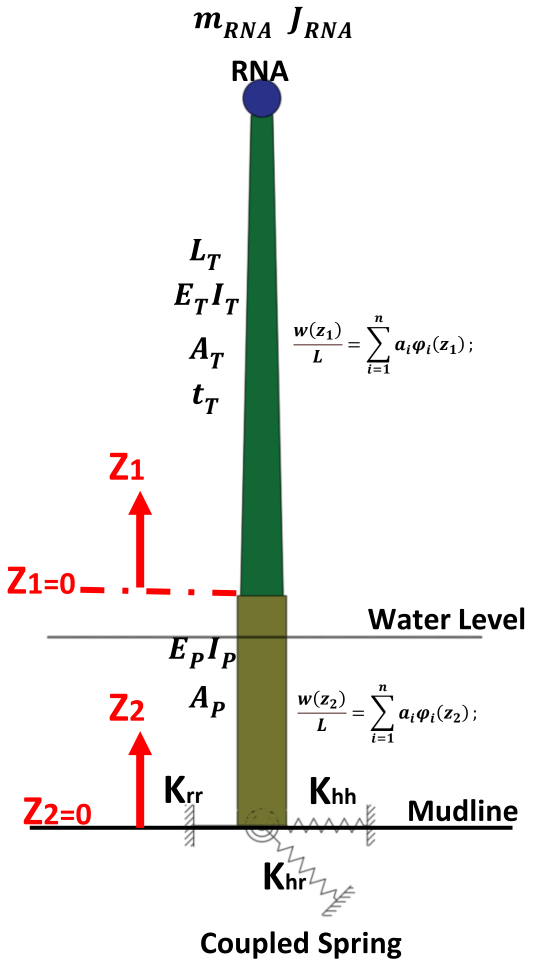
A Rayleigh-Ritz solution for high order natural frequencies

➢ The methodology employed in this study to obtain the equivalent system for analyzing the monopile-supported offshore wind turbine responses to accidental loads is founded on the Rayleigh-Ritz principle.





A Rayleigh-Ritz solution for high order natural frequencies



- The Rayleigh-Ritz approach, which assumes certain modes utilizing global elements, and overlays a finite number of assumed mode shapes to replicate the vibrations of the dynamical system.
  - The transverse vibration of the beam is:

$$W(z,t) = w(z)\,u(t);$$

  - Rayleigh ritz quotient

$$R(a_1, a_2, \cdots a_n) = \omega^2 = \frac{U_{\max}(a_1, a_2, \cdots a_n)}{T_0(a_1, a_2, \cdots a_n)};$$

  - The eigenvalue equation

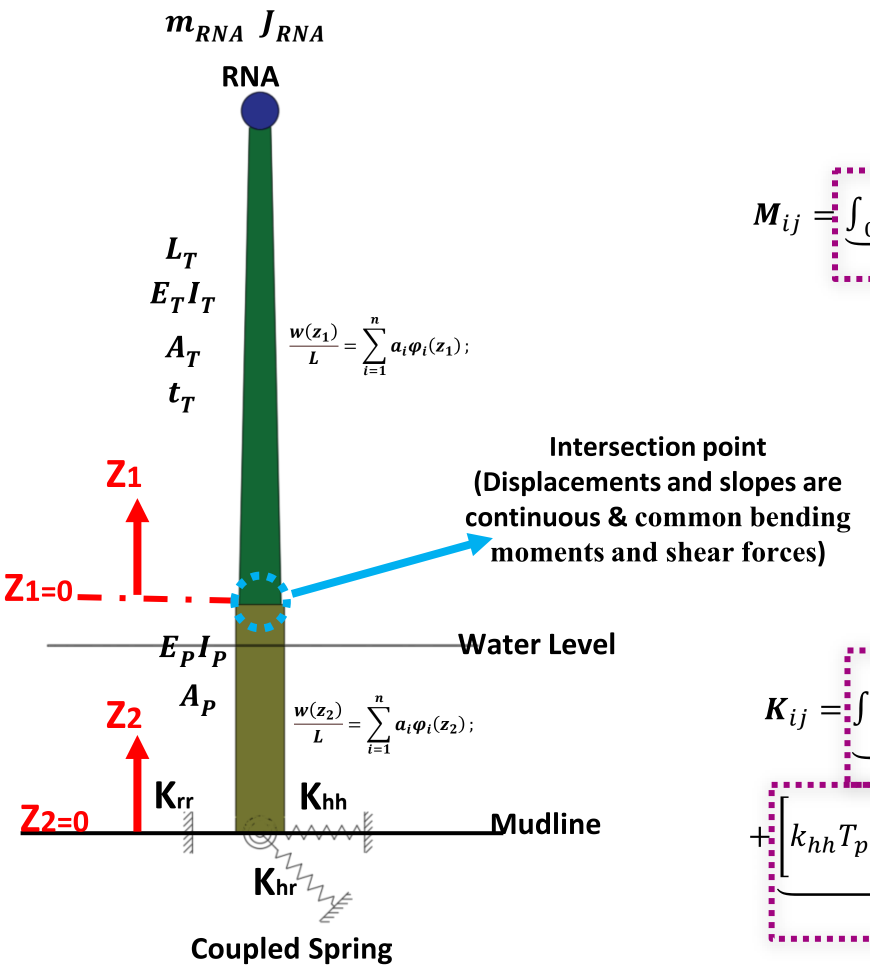
$$\frac{\partial U_{\max}}{\partial a_i} - \omega^2 \frac{\partial T_0}{\partial a_i} = 0 \quad i = 1,2,\cdots n \qquad (\mathbf{K} - \omega^2 \mathbf{M})\mathbf{a} = 0;$$

  - The basis function representing the tower is 6^{th} order polynomial, while the basis function representing the monopile is 4^{th} order polynomial.

$$\frac{w(z)}{L} = \sum_{i=1}^{n} a_i \varphi_i(z);$$

    - Explicitly presenting area and 2^{nd} moment of area of **tower**

A Rayleigh-Ritz solution for high order natural frequencies



- Constructing Mass and Stifness matrix of the system & Solving Eigenvalue problem

$$T_0 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij} L_T^2 a_i a_j;$$

$$\boldsymbol{M}_{ij} = \underbrace{\int_0^{\gamma L_T} (\rho_P A_P + \rho_W A_{CS})\, T_{pi}\varphi_{p-1}(z_2)\varphi_{q-1}(z_2) T_{qj}\, dz_2}_{\text{Monopile}} + \underbrace{\int_0^{L_T} \rho_T A_T(z_1)\varphi_{i-1}(z_1)\varphi_{j-1}(z_1)\, dz_1}_{\text{Tower}} + \underbrace{m_{RNA}\varphi_{i-1}(L_T)\varphi_{j-1}(L_T)}_{RNA\ \text{mass}} + \underbrace{\left[J_{RNA}\frac{d\varphi_{i-1}(z_1)}{dz_1}\frac{d\varphi_{j-1}(z_1)}{dz_1}\right]_{z_1=L_T}}_{RNA\ \text{rotational inertia}};$$

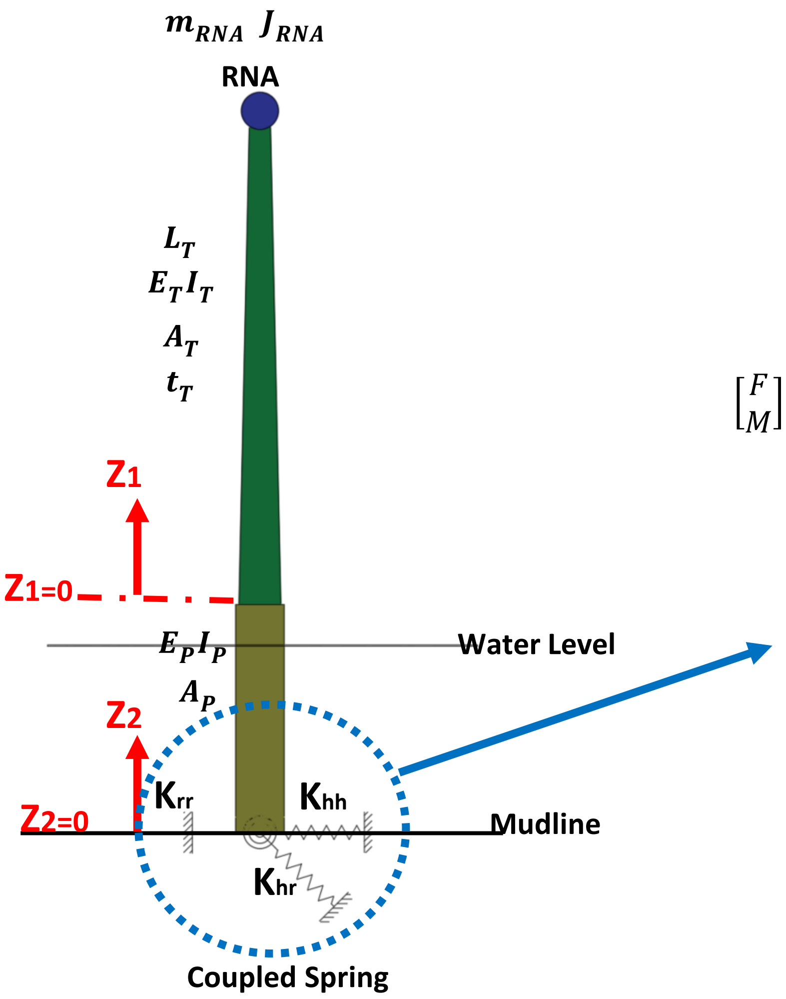
$$i,j \in [1,2,3,\cdots,7],\ p,q \in [1,2,3,4,5];$$

$$U_{max} = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} K_{ij} L_T^2 a_i a_j;$$

$$\boldsymbol{K}_{ij} = \underbrace{\int_0^{\gamma L_T} E_P I_P T_{pi}\frac{d^2\varphi_{p-1}(z_2)}{dz_2^2}\frac{d^2\varphi_{q-1}(z_2)}{dz_2^2} T_{qj}\, dz}_{\text{Monopile}} + \underbrace{\int_0^{L_T} (E_T I_T(z) + m_{RNA}\, g)\frac{d^2\varphi_{i-1}(z_1)}{dz_1^2}\frac{d^2\varphi_{j-1}(z_1)}{{dz_1}^2}\, dz_1}_{\text{Tower}}$$
$$+ \underbrace{\left[k_{hh}T_{pi}\varphi_{p-1}(z_2)\varphi_{q-1}(z_2)T_{qj} + k_{rr}T_{pi}\frac{d\varphi_{p-1}(z_2)}{dz_2}\frac{d\varphi_{q-1}(z_2)}{dz_2}T_{qj} + 2k_{hr}T_{pi}\varphi_{p-1}(z_2)\frac{d\varphi_{q-1}(z_2)}{dz_2}T_{qj}\right]_{z_2=0}}_{\text{Foundation springs}};$$

$$i,j \in [1,2,3,\cdots,7],\ p,q \in [1,2,3,4,5];$$

Soil structure interaction modelling (coupled spring model)



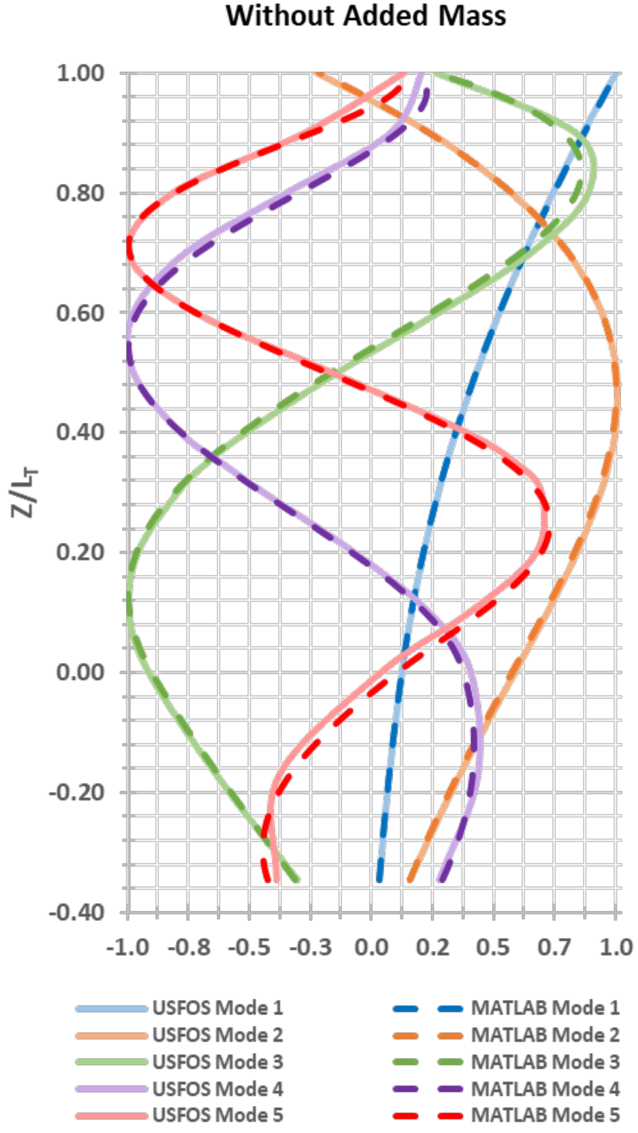
- Yu & Amdahl (2023) proved that the couple spring model by Psaroudakis (2021) can provide a good agreement with the USFOF soil modelling with the appropriate selection of soil stiffness.

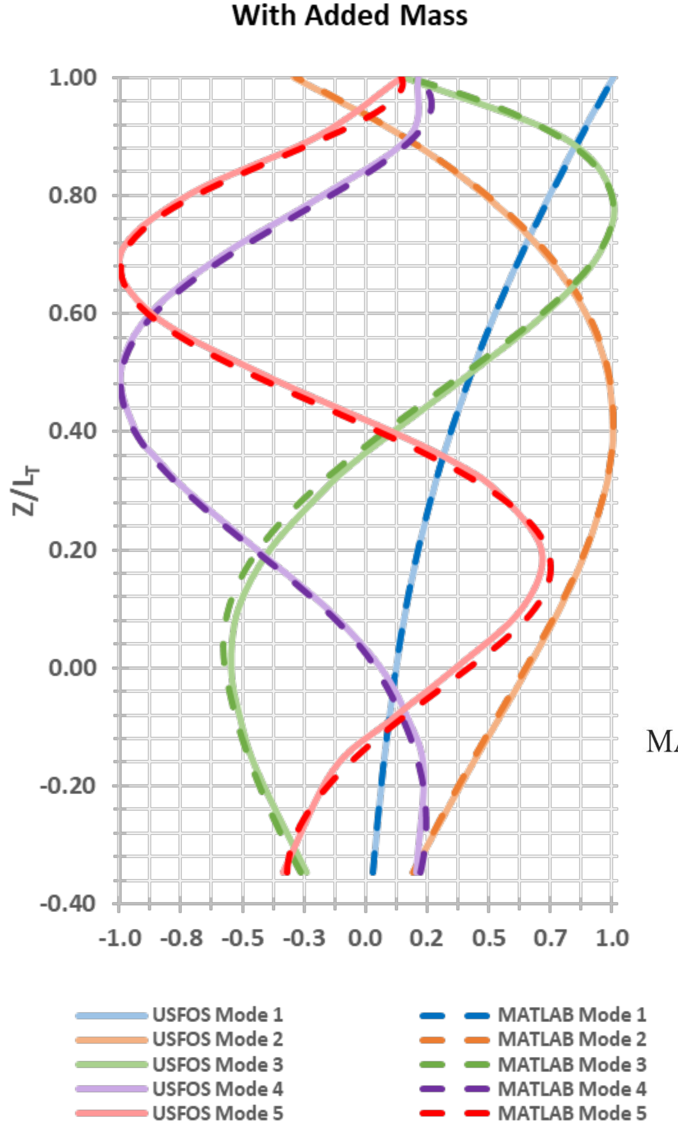
$$\begin{bmatrix} F \\ M \end{bmatrix} = \begin{bmatrix} K_{hh} K_{hr} \\ K_{rh} K_{rr} \end{bmatrix} \begin{bmatrix} \rho \\ \theta \end{bmatrix}$$

$$\begin{bmatrix} F \\ M \end{bmatrix} = \begin{bmatrix} 4E_p I_P \beta_{\text{pile}}^3 \dfrac{\sin(2\beta_{\text{pil}} L_D) + \sinh(2\beta_{\text{pil}} L_D)}{2 + \cos(2\beta_{\text{pil}} L_D) + \cosh(2\beta_{\text{pile}} L_D)} & -2E_p I_P \beta_{\text{pil}}^2 \dfrac{-\cos(2\beta_{\text{pile}} L_D) + \cosh(2\beta_{\text{pile}} L_D)}{2 + \cos(2\beta_{\text{pile}} L_D) + \cosh(2\beta_{\text{pile}} L_D)} \\ -2E_p I_P \beta_{\text{pile}}^2 \dfrac{-\cos(2\beta_{\text{pile}} L_D) + \cosh(2\beta_{\text{pile}} L_D)}{2 + \cos(2\beta_{\text{pile}} L_D) + \cosh(2\beta_{\text{pile}} L_D)} & 2E_p I_P \beta_{\text{pile}} \dfrac{-\sin(2\beta_{\text{pile}} L_D) + \sinh(2\beta_{\text{pile}} L_D)}{2 + \cos(2\beta_{\text{pile}} L_D) + \cosh(2\beta_{\text{pile}} L_D)} \end{bmatrix} \begin{bmatrix} \rho \\ \theta \end{bmatrix}$$

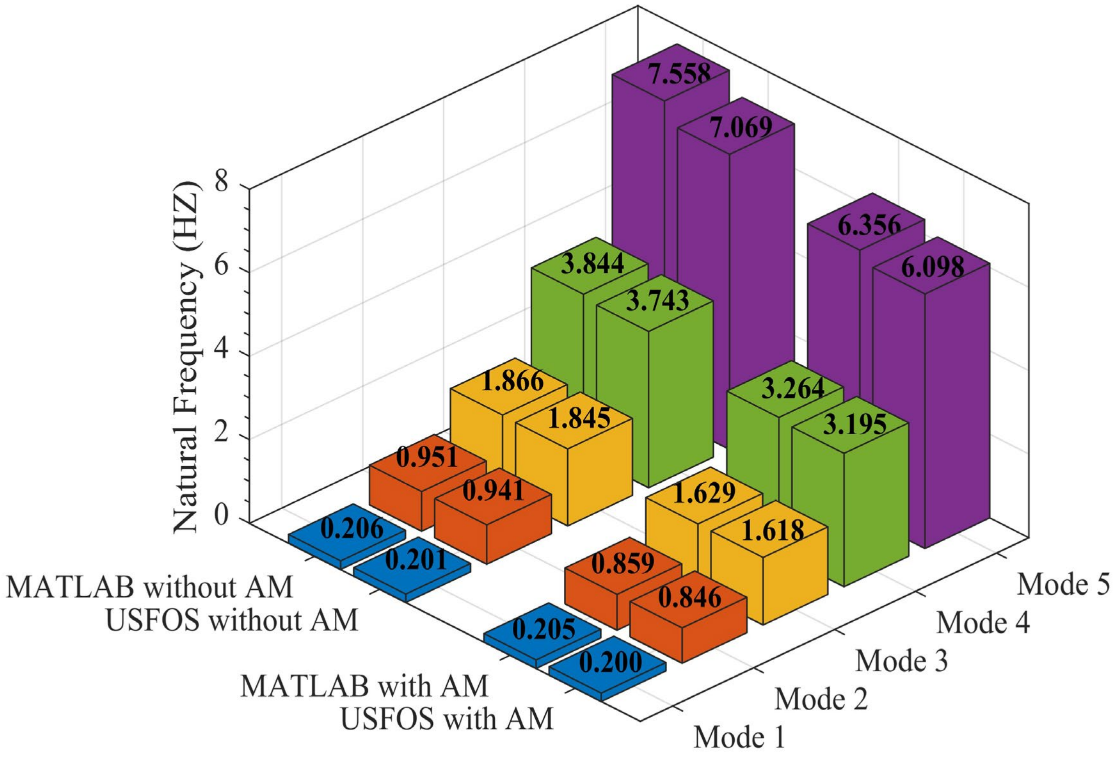
Where $\beta_{\text{pile}} = \sqrt[4]{\dfrac{k_{sub} D_p}{4 E_p L_p}}$ is the characteristic pile slenderness parameter.

Natural frequency & Eigenvectors





- **Case study:** 10 MW monopile offshore wind turbine



Linear Modal Theory

- Matrix $\boldsymbol{w}_m$ **(global mode shapes)** becomes available and can be used to assess the dynamic response of the structure to a transient collision force using Duhamel's Integral.
- The transverse displacement of the global beam assembly $w(y,t)$ can be expressed through modal expansion as follows:

$$W(z,t) = \sum_{m=1}^{\infty} w_m(z)\, u_m(t) = \boldsymbol{w}^T . \boldsymbol{u}(t);$$

$$\boldsymbol{M}\ddot{W}(z,t) + \boldsymbol{C}\dot{W}(z,t) + \boldsymbol{K}W(z,t) = \boldsymbol{P}(t);$$

$$\boldsymbol{w}_m\, \boldsymbol{M}\, \boldsymbol{w}_m^T\, \ddot{\boldsymbol{u}}_n(t) + \boldsymbol{w}_m\, \boldsymbol{C}\, \boldsymbol{w}_m^T\, \dot{\boldsymbol{u}}_n(t) + \boldsymbol{w}_m\, \boldsymbol{K}\, \boldsymbol{w}_m^T\, \boldsymbol{u}_n(t) = \boldsymbol{w}_m^T \boldsymbol{P}(t)\,;$$

Mass normalization of the mode shapes

$$\boldsymbol{w}_m\, \boldsymbol{M}\, \boldsymbol{w}_m^T = 1$$
$$\boldsymbol{w}_m\, \boldsymbol{C}\, \boldsymbol{w}_m^T = 2\xi_m\omega_m$$
$$\boldsymbol{w}_m\, \boldsymbol{K}\, \boldsymbol{w}_m^T = \omega_m^2$$

$$\ddot{\boldsymbol{u}}_m(t) + 2\xi_m\omega_m\dot{\boldsymbol{u}}_m(t) + \omega_m^2\mathbf{u}(t) = \boldsymbol{w}_m^T\boldsymbol{P}(t);$$

Duhamel's integral

$$u(t) = \frac{1}{m\omega_m^d}\int_0^t e^{-\xi\omega_m(t-\tau)}P(\tau)\sin\omega_m^d\,(t-\tau)\mathrm{d}\tau$$

Slamming analysis

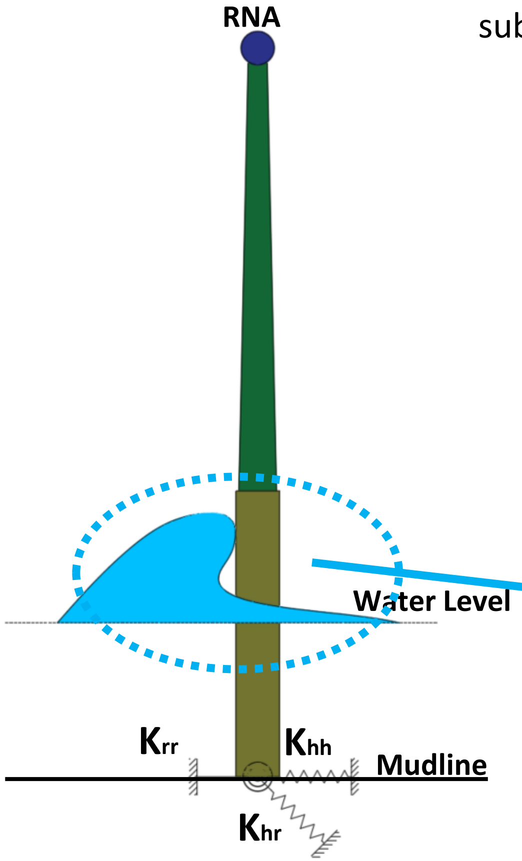
(DNV-RP-C205)

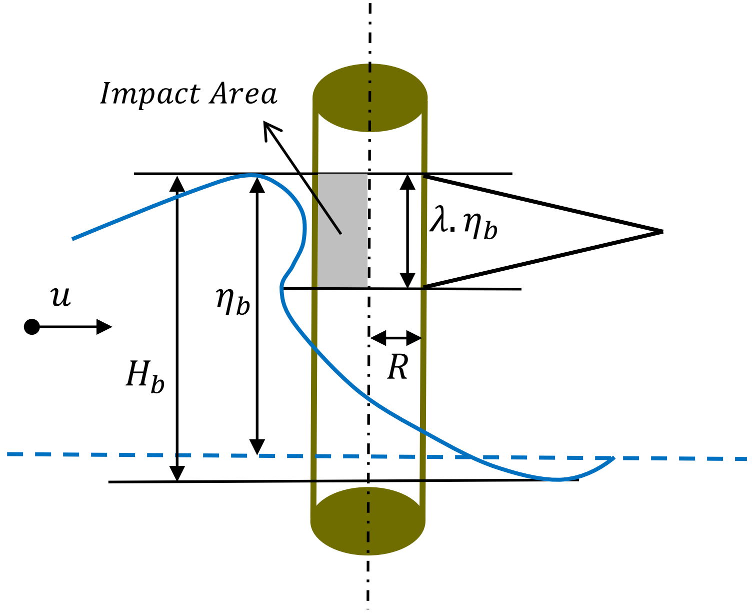
- At start of impact $\boldsymbol{C_s(0) = 5.15}$.
- The following model is a good approximation when the impacting wave is steep.
- The formula shall be applied only during penetration of the wave surface, i.e. for $\boldsymbol{0 < s < D}$. When the cylinder is fully submerged, $\boldsymbol{C_s(D) = 0.8}$.

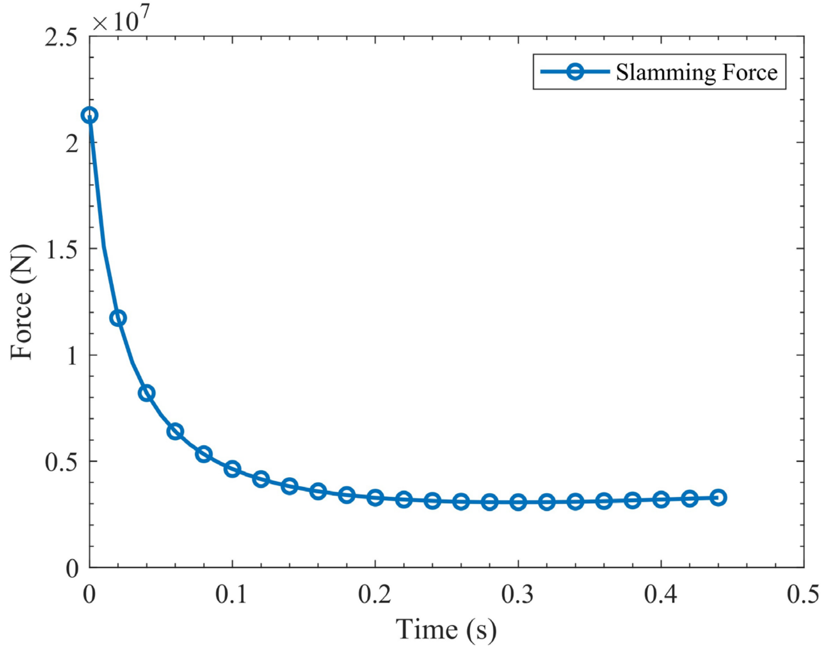
$$F_x(z, t) = \frac{1}{2}\rho C_s D u^2$$

$$C_s = 5.15\left(\frac{D}{D + 19s} + \frac{0.107s}{D}\right)$$

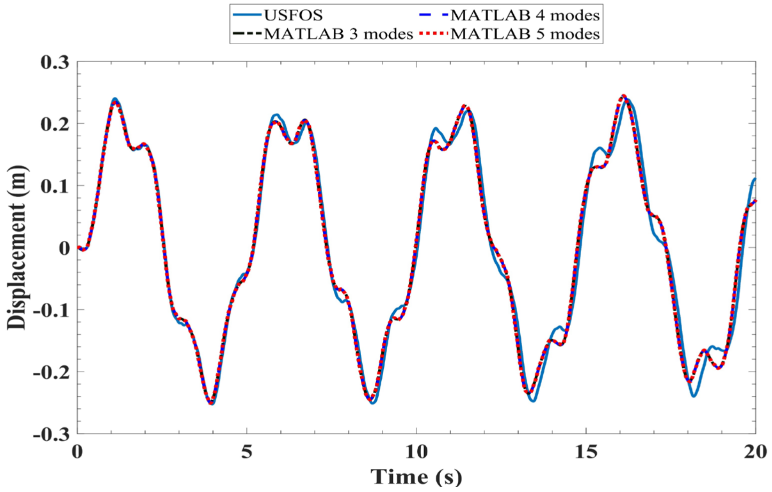
Wave period	8 sec
Wave height	12.4 m
velocity	19.6 m/s
Diameter	9 m

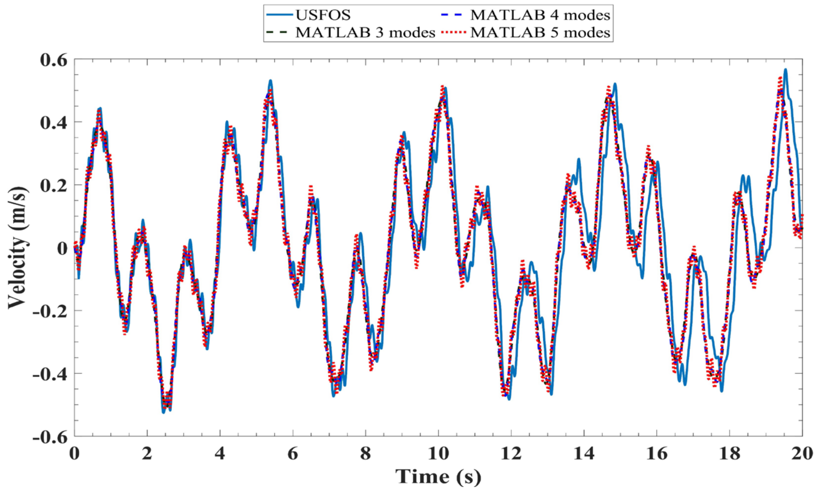


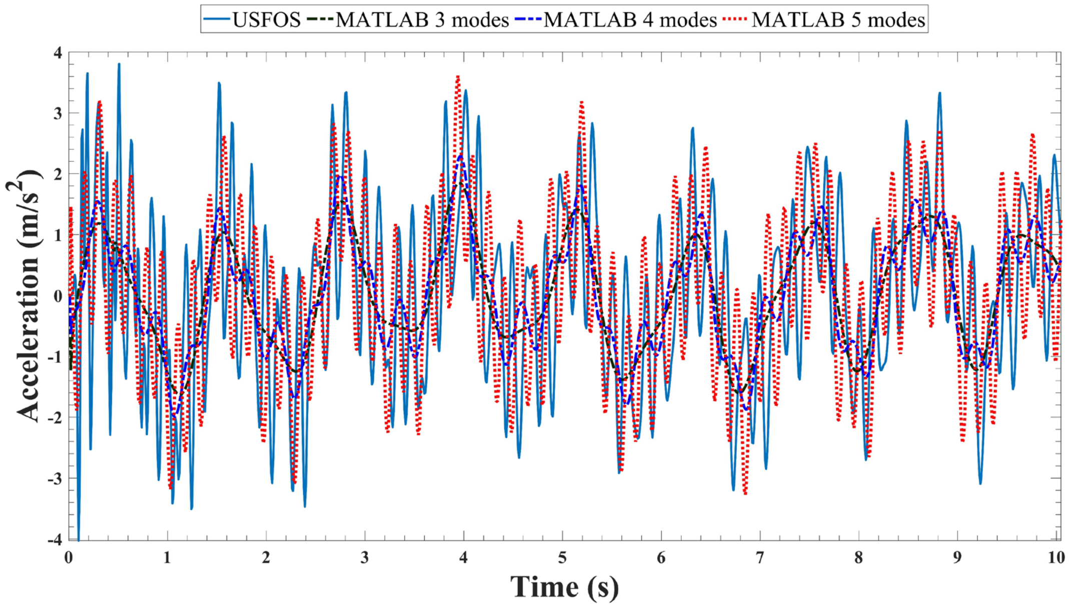




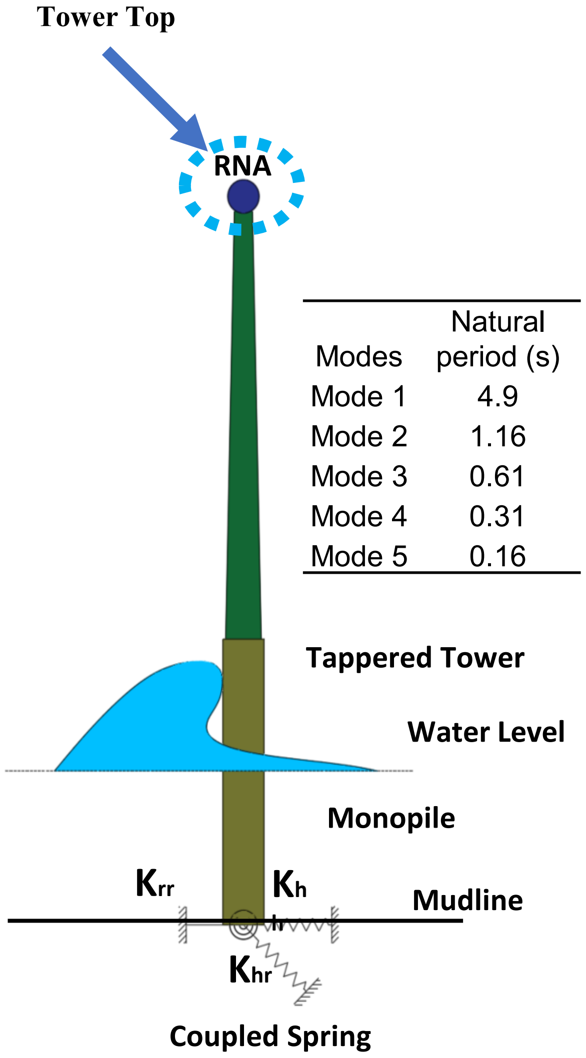
Tower top responses







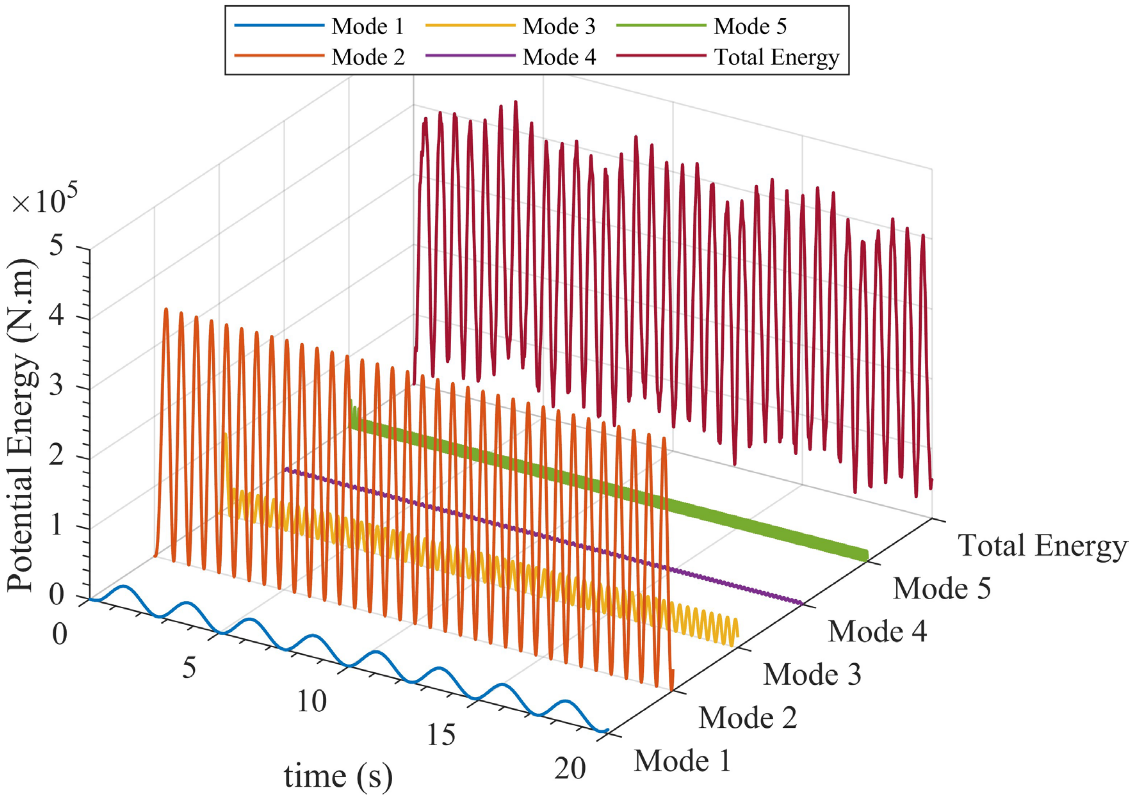
ACC	MATLAB	USFOS	Error %
3 modes	1.84	4.02	54.2
4 modes	2.28	4.02	43.2
5 modes	3.7	4.02	5.1

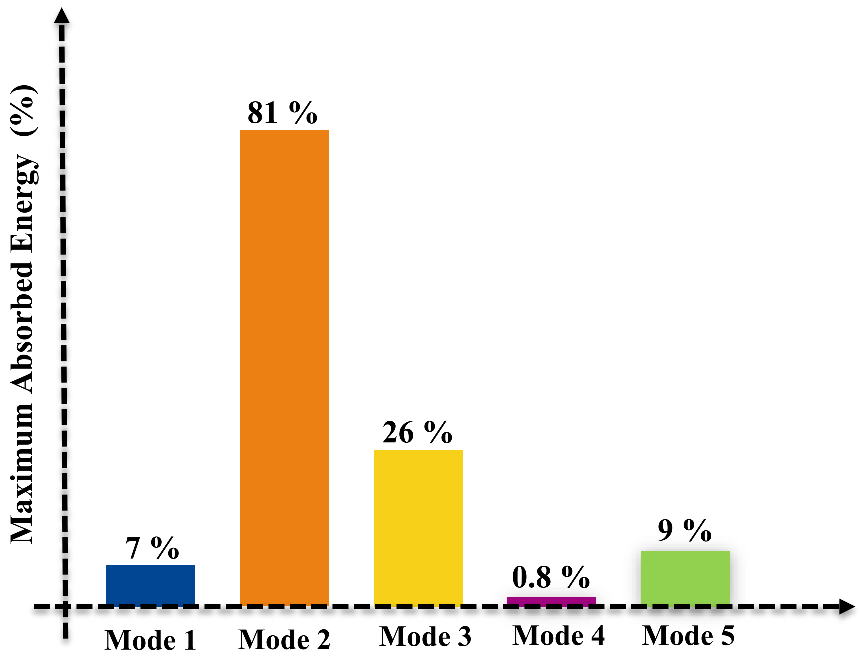


Modes	Natural period (s)
Mode 1	4.9
Mode 2	1.16
Mode 3	0.61
Mode 4	0.31
Mode 5	0.16

Energy absorbed by each mode

- Due to the phase change of the absorbed energy of each mode (the modes do not reach the maximum energy at the same time), the accumulated total maximum energy is lower than the summation of the maximum absorbed energy of all modes.



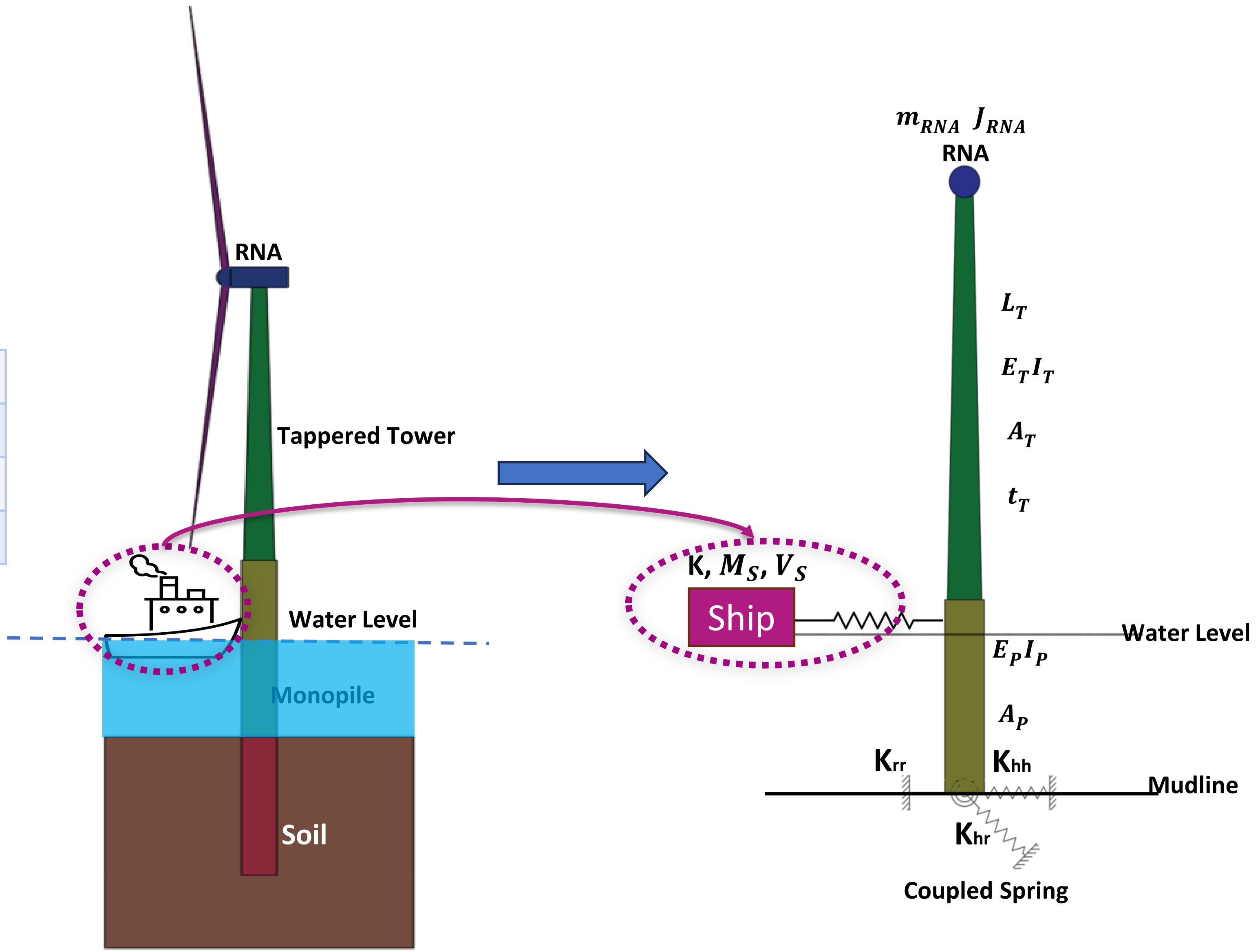


Energy	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Max. Value (N.m)	31266.66	358048.6	115730.3	3429.95	38918.74
Percentage (%)	7.07	81	26.18	0.77	8.81

Ship collision analysis

The present study focuses on the examination of beams that experience transverse impact forces caused by an external striker considering the elastic contact between the striker and the monopile.

Ship mass (M_S)	7500 ton
Ship speed (V_S)	1 m/s
Relative Stiffness (K)	30 MN
Collision point	5 m below the tower



Ship collision contact algorithm

A numerical **contact algorithm** is then developed for ship collision analysis by incorporating **a force-displacement** curve to account for the stiffness involved in the **interaction of a ship and an offshore wind turbine**.

- The forward position of the striking ship is given by:

$$x_S(\mathrm{n}) = V_S(\mathrm{n})\Delta t$$

- With a linear relation between the force and the displacement the collision force is:

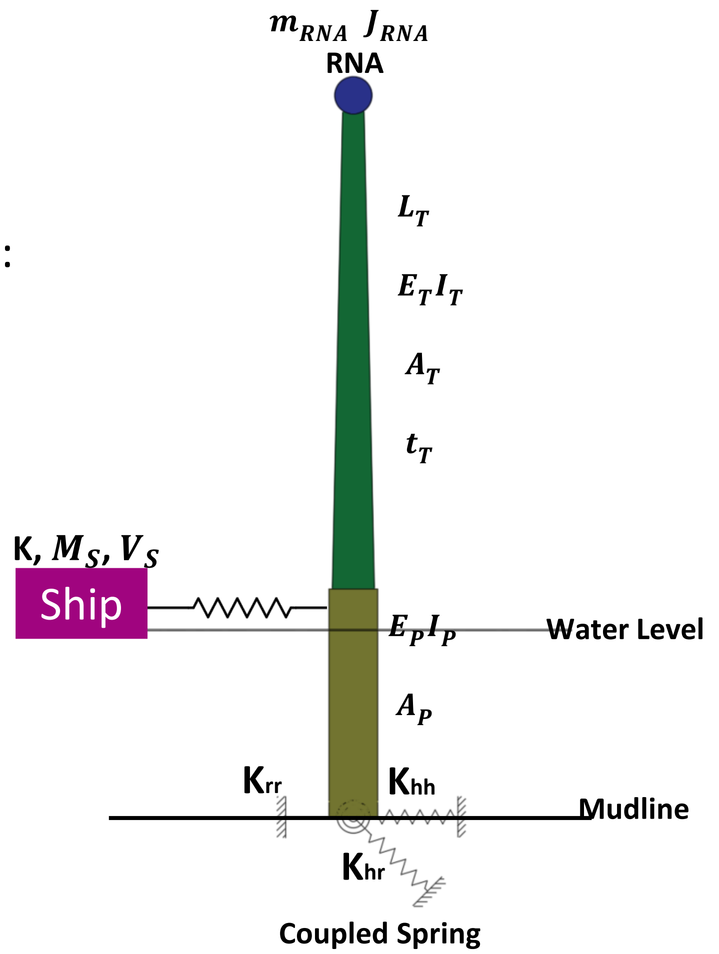
$$F(\mathrm{n}+1) = -k[x_S(\mathrm{n}+1) - W(\mathrm{n})]$$

- Where the displacement of the striking ship is given by:

$$x_S(\mathrm{n}+1) = x_S(\mathrm{n}) - (V_S(\mathrm{n})\,\Delta t) - \left(0.5\left[\frac{F(\mathrm{n})}{M_S}\right]\Delta t^2\right)$$

- Updating the velocity of the striking ship at each time step:

$$V_S(\mathrm{n}+1) = V_S(\mathrm{n}) + \left[\frac{F(\mathrm{n}+1)}{M_S}\right]\Delta t$$



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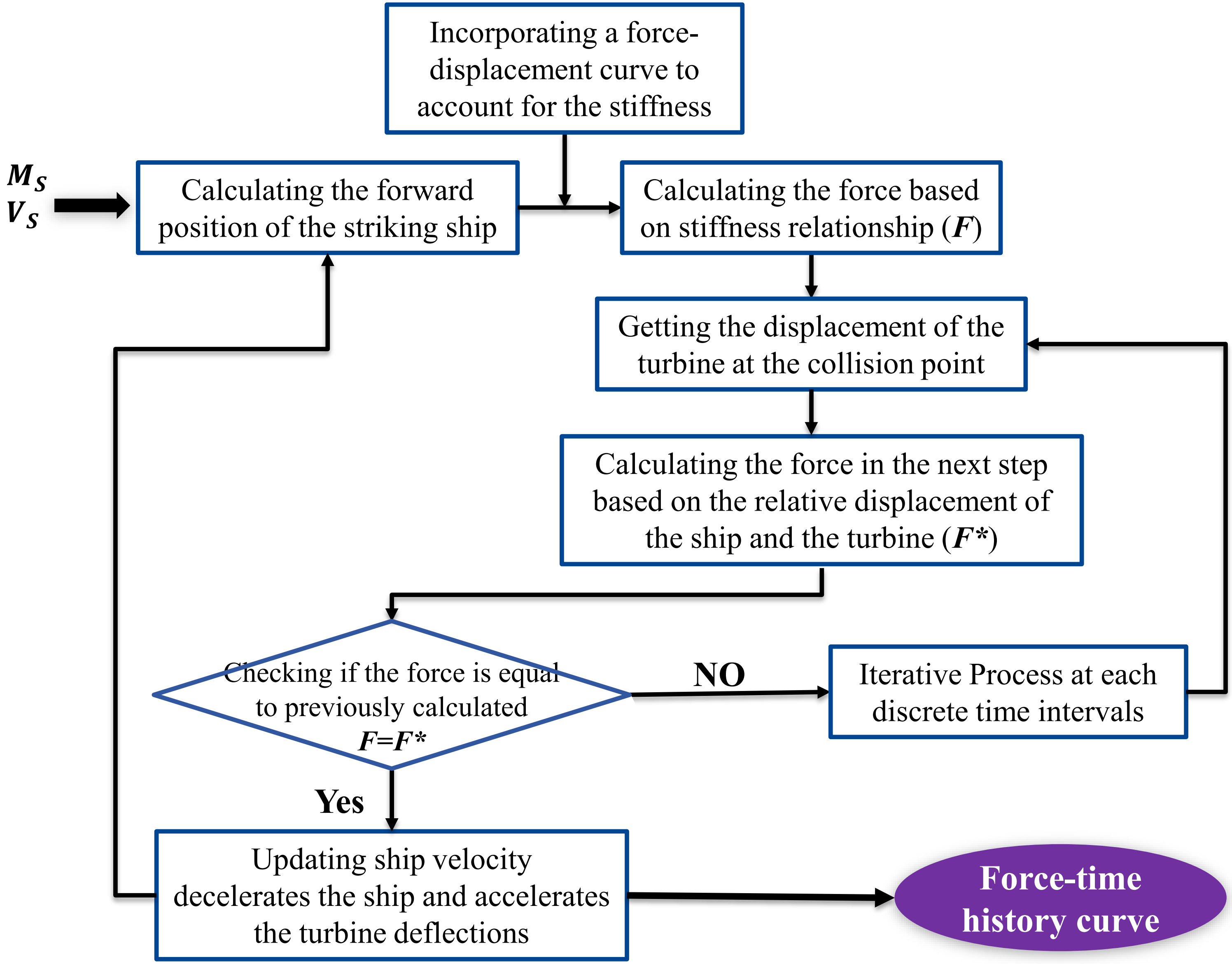

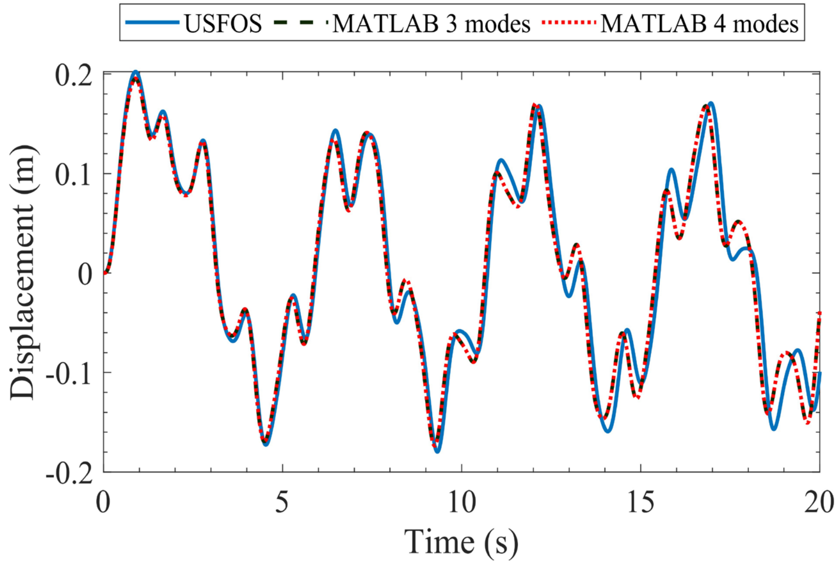

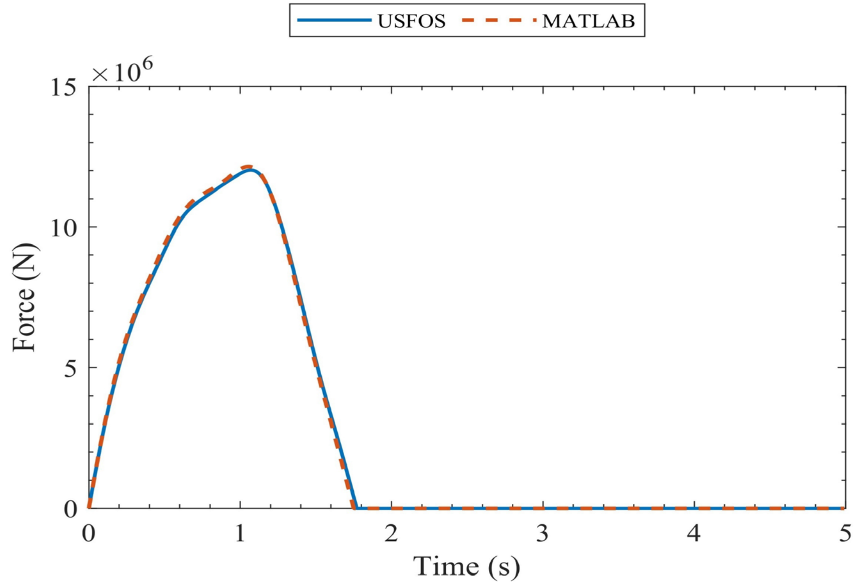

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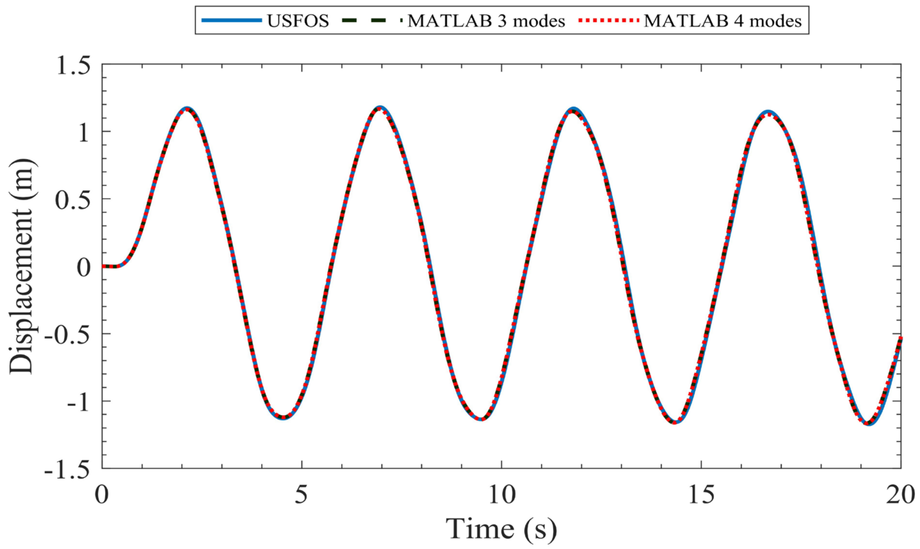
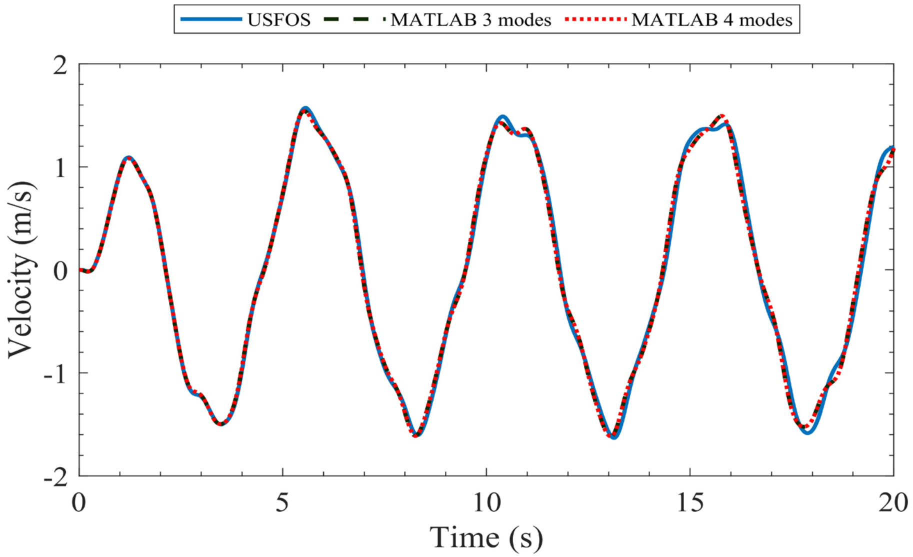
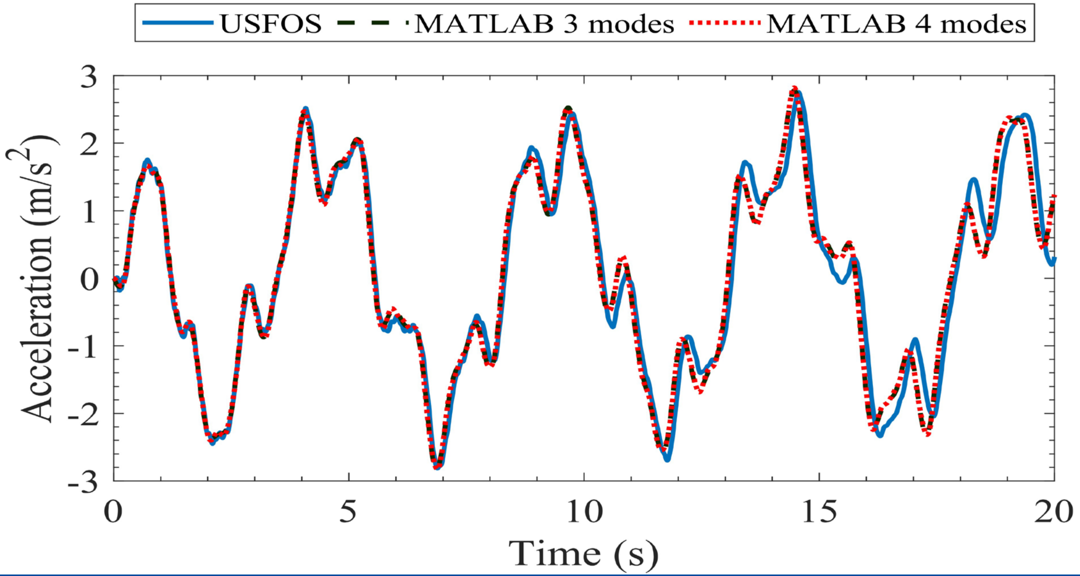
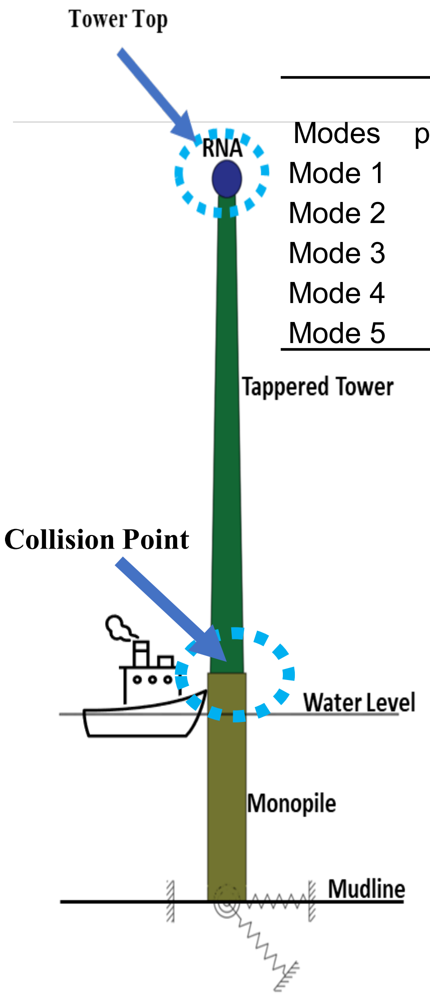
Tower top responses







ACC	MATLAB	USFOS	Error %
3 modes	2.822	2.815	0.240
4 modes	2.820	2.815	0.179



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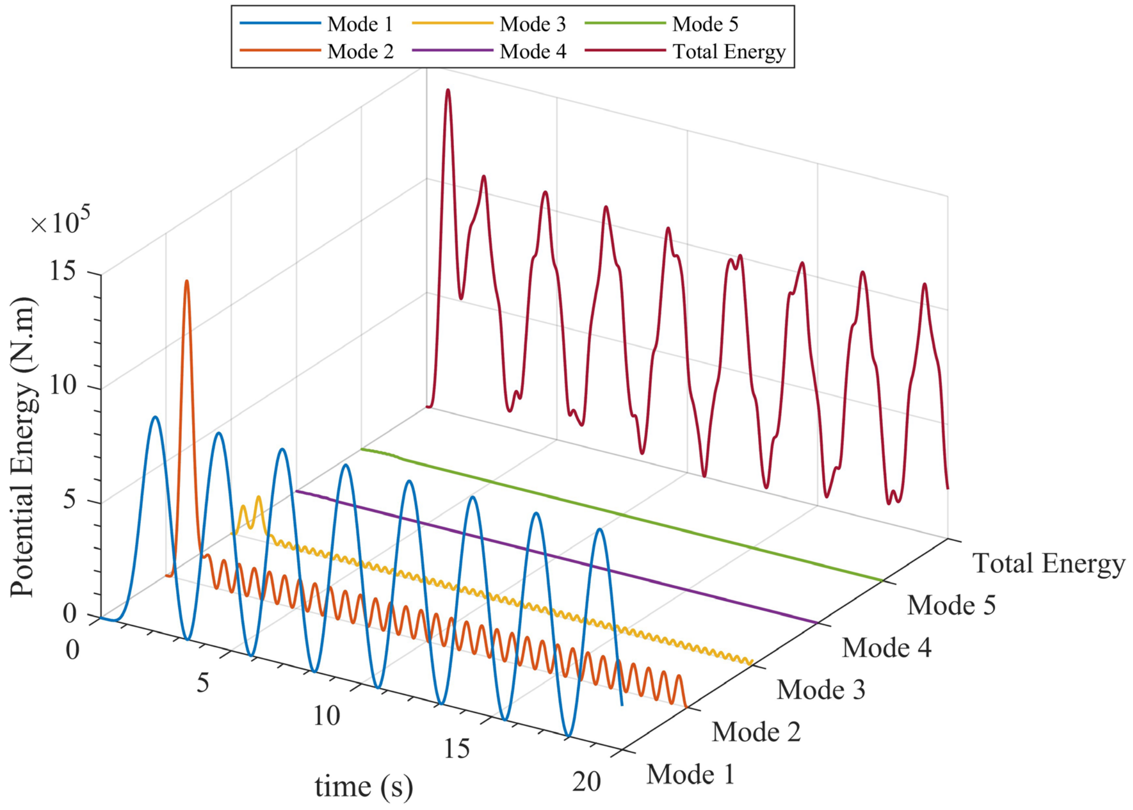
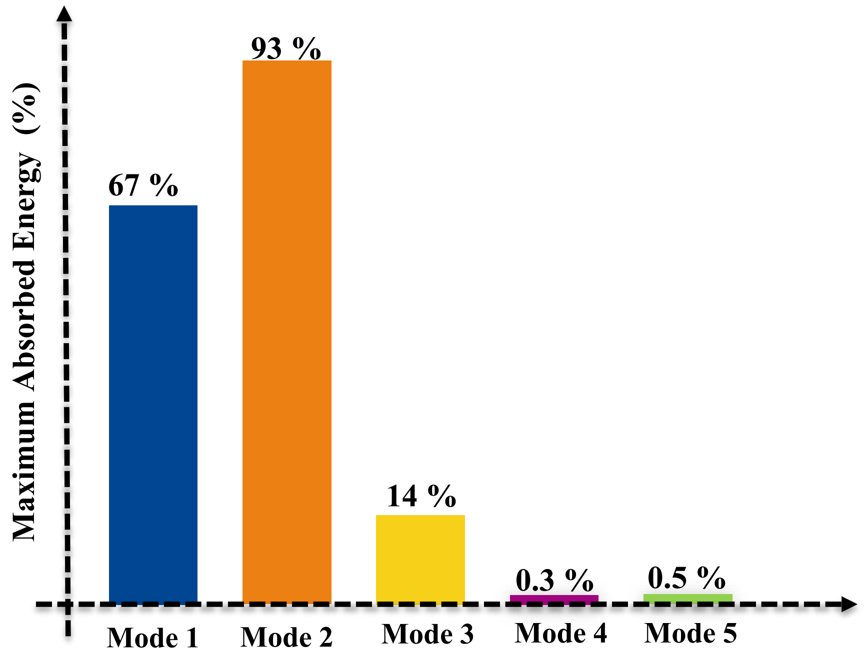
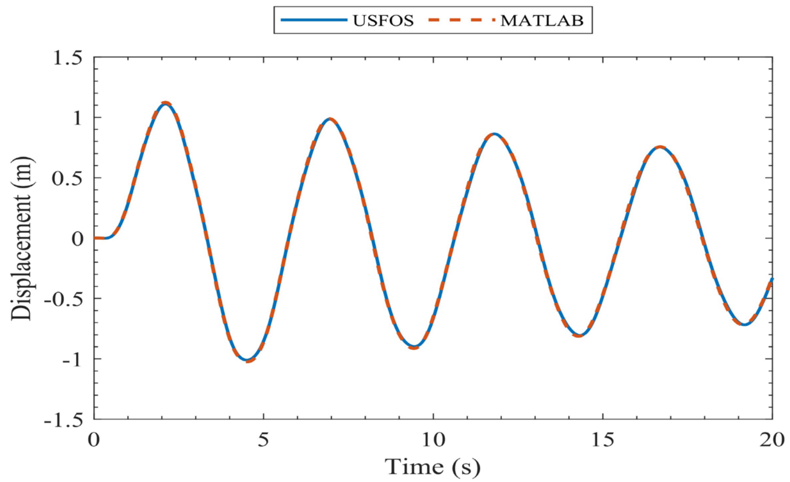


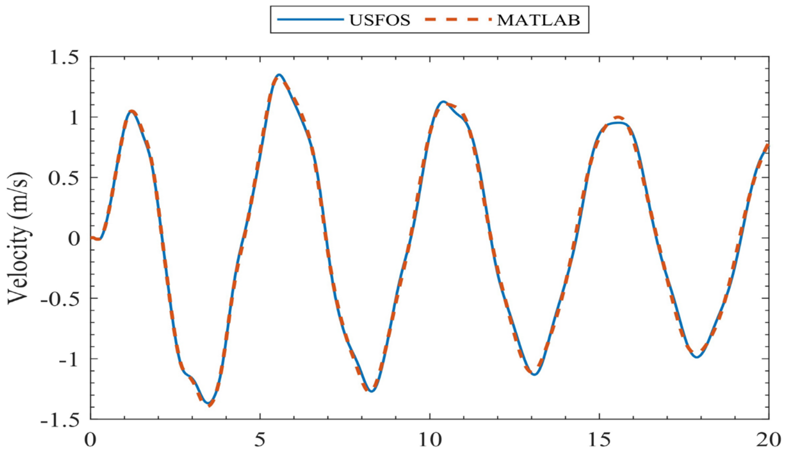


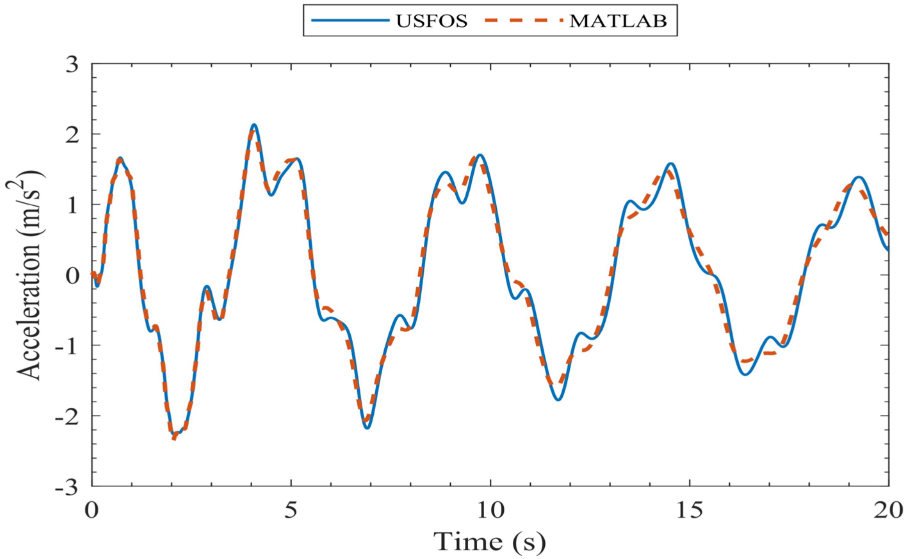
Energy	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Max. Value (N.m)	940290.44	1314832.87	190389.50	3493.02	7422.96
Percentage (%)	66.59	93.12	13.48	0.25	0.53

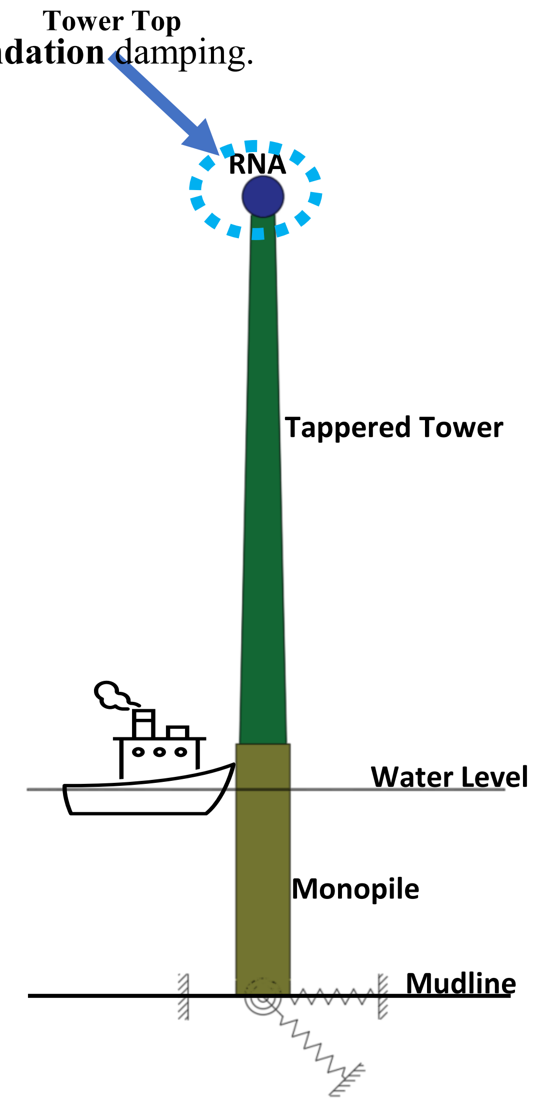
Damping assumption of 2% of critical damping

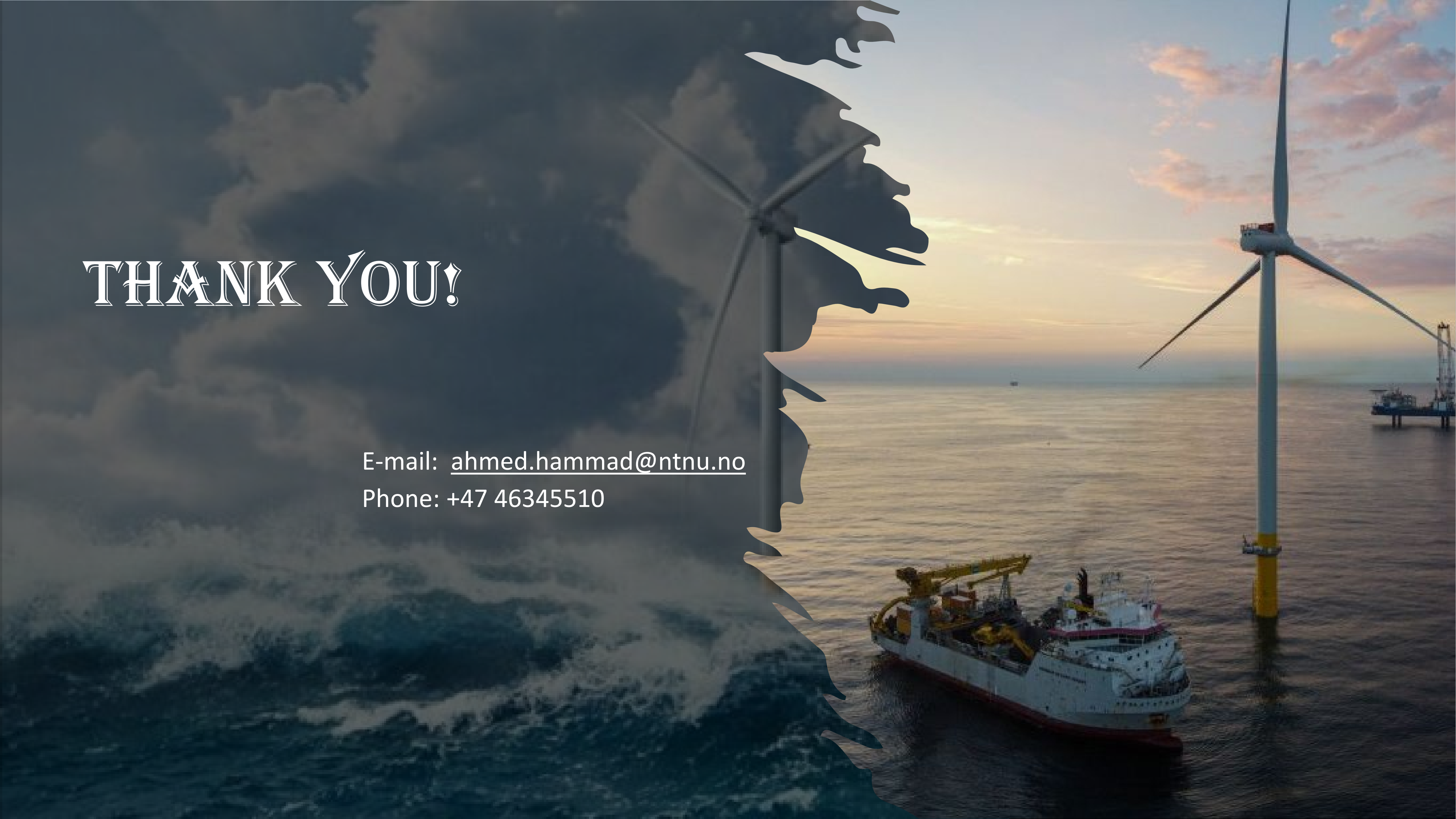
- We assumed **damping of 2%** of critical damping in case of **parked condition**.
- The previously mentioned damping value associated with damping comes from **structural, hydrodynamic and foundation** damping.
- A fixed damping ratio has been applied for all involved modes.











THANK YOU!

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