Data-driven Modeling of Higher-Order Transfer Functions

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CYBERLAB KPN

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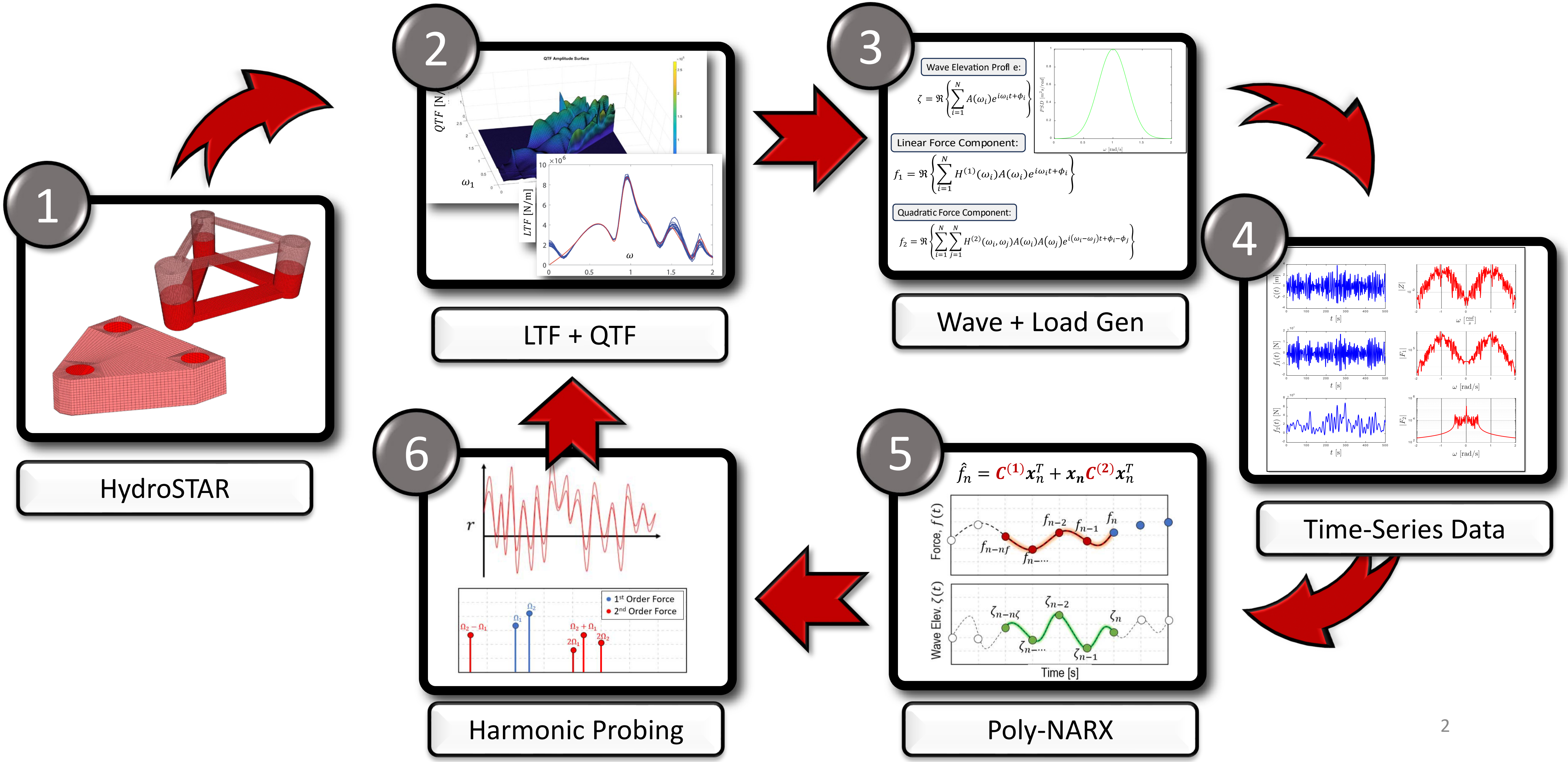
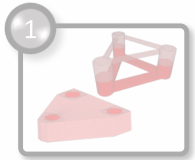
Validation Cycle



Validation Cycle



HydroSTAR

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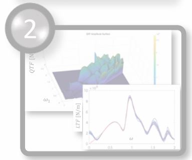
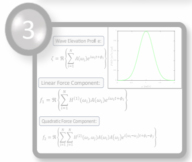


LTF + QTF

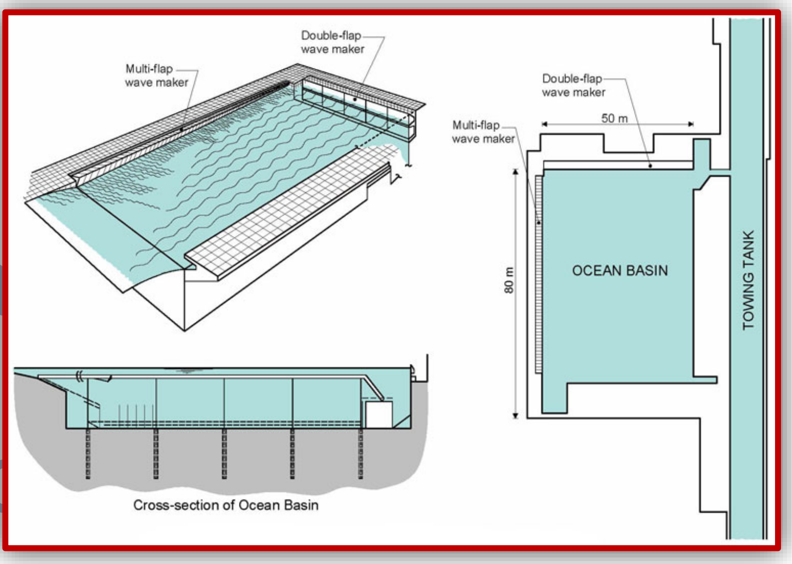
Linear and Quadratic Tra

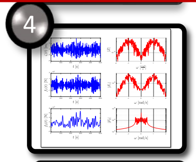


Wave + Load Gen

Synthetic Data Generati

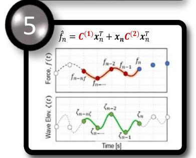
Experimental





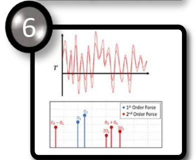
Time-Series Data

Time-Series Sets



Poly-NARX

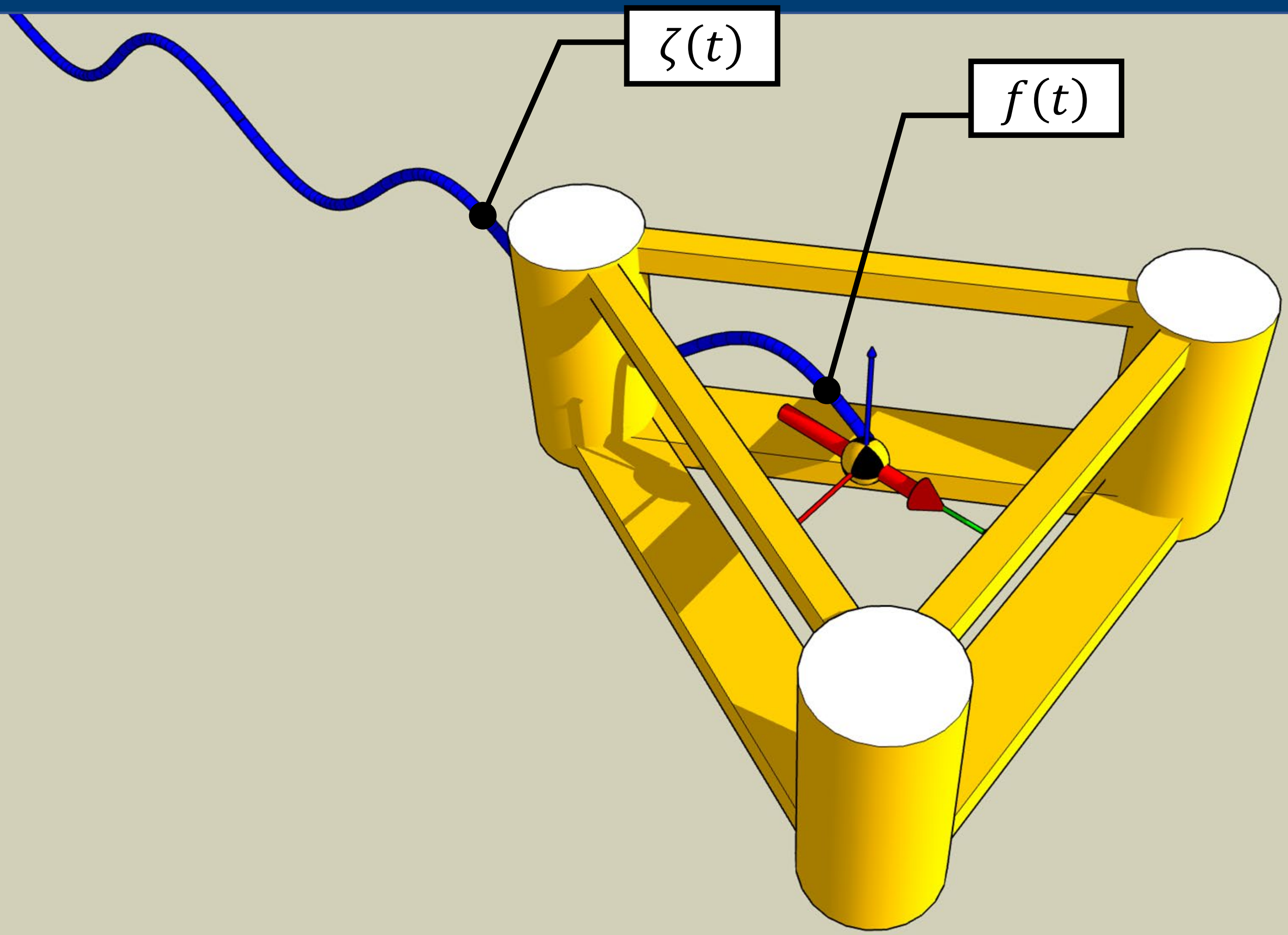
Polynomial-NARX



Harmonic Probing

Novel Harmonic Probing

3 Synthetic Data Generation



3 Synthetic Data Generation

Wave Elevation Profile:

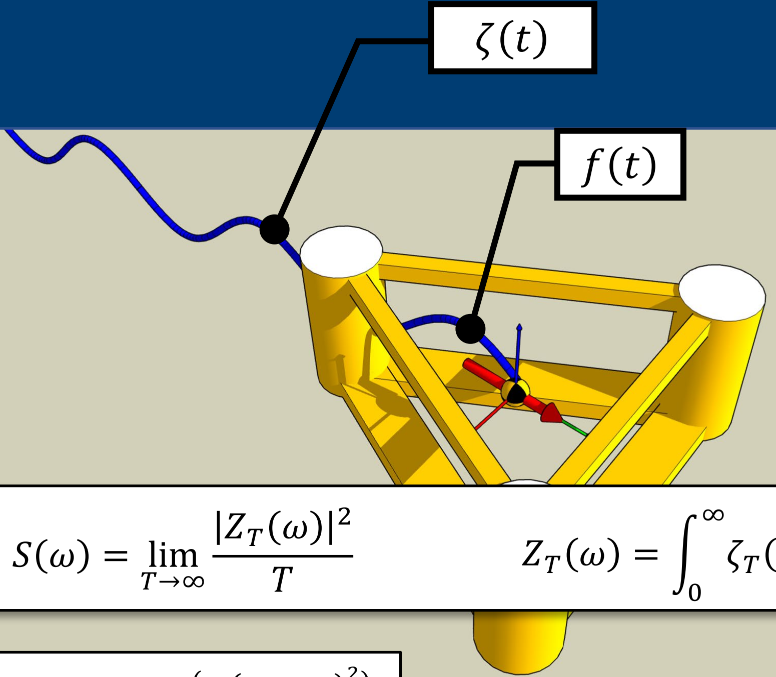
$$\zeta(t) = \Re\left\{\sum_{i=1}^{N} A(\omega_i)e^{i\omega_i t+\phi_i}\right\} \qquad A(\omega_i) = \sqrt{2S(\omega_i)dw}$$

Linear Force Component:

$$f_1(t) = \Re\left\{\sum_{i=1}^{N} H^{(1)}(\omega_i)A(\omega_i)e^{i\omega_i t+\phi_i}\right\}$$

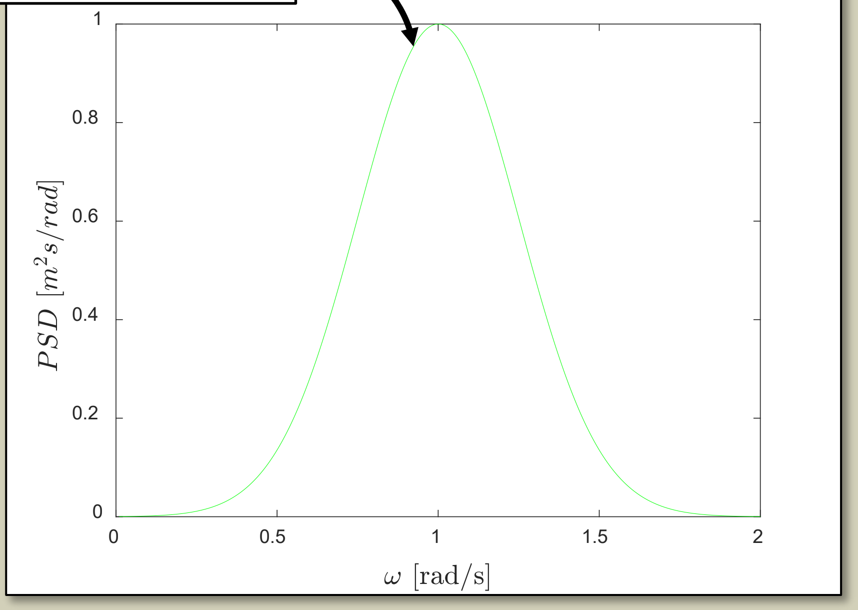
Quadratic Force Component:

$$f_2(t) = \Re\left\{\sum_{i=1}^{N}\sum_{j=1}^{N} H^{(2)}(\omega_i,\omega_j)A(\omega_i)A(\omega_j)e^{i(\omega_i-\omega_j)t+\phi_i-\phi_j}\right\}$$



$$S(\omega) = \lim_{T\to\infty}\frac{|Z_T(\omega)|^2}{T} \qquad Z_T(\omega) = \int_0^\infty \zeta_T(t)e^{-i\omega t}dt$$

$$S(\omega) = \frac{1}{\sigma_b\sqrt{2\pi}}exp\left\{-\frac{(\omega-\omega_\mu)^2}{2\sigma_b^2}\right\}$$



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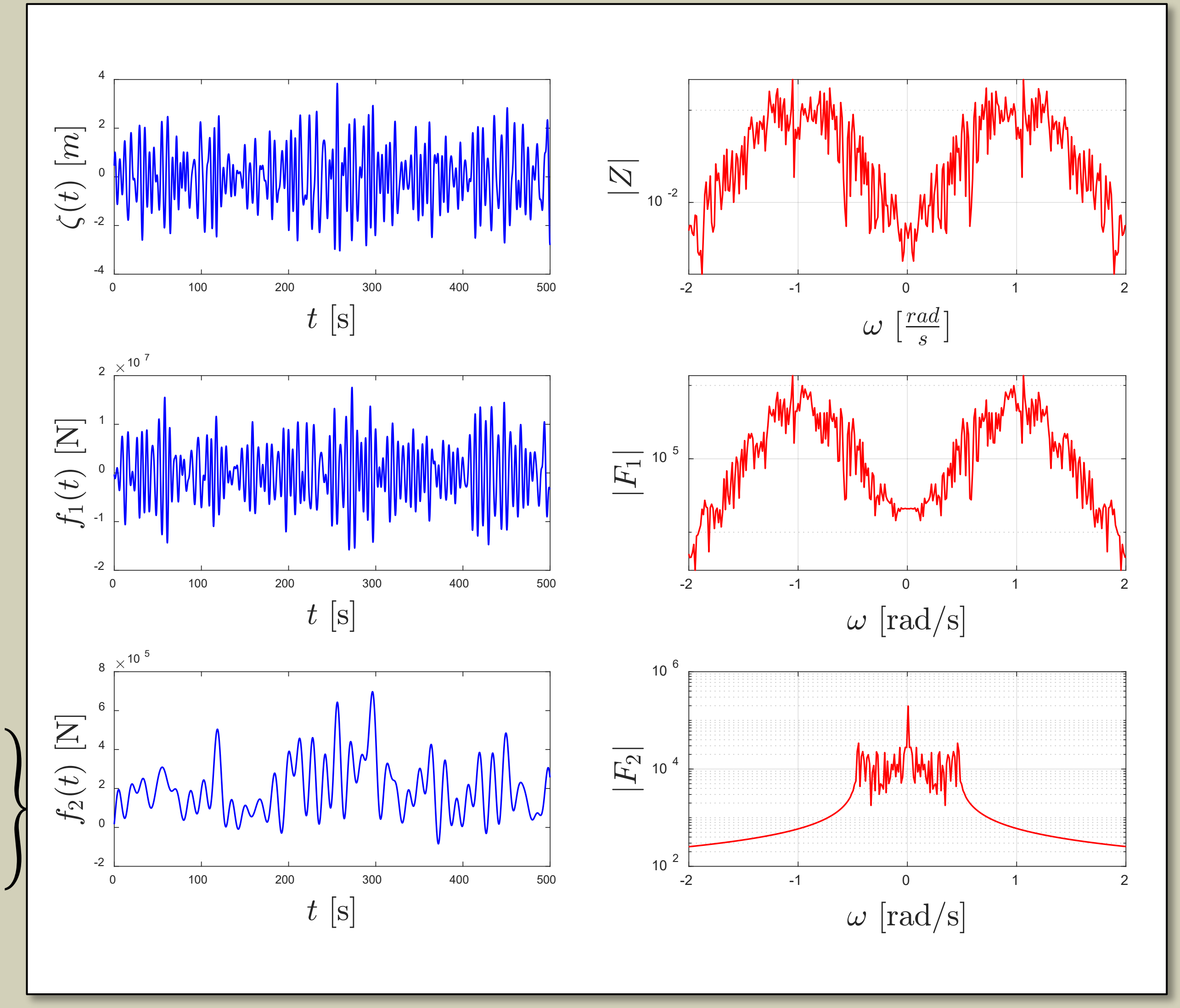
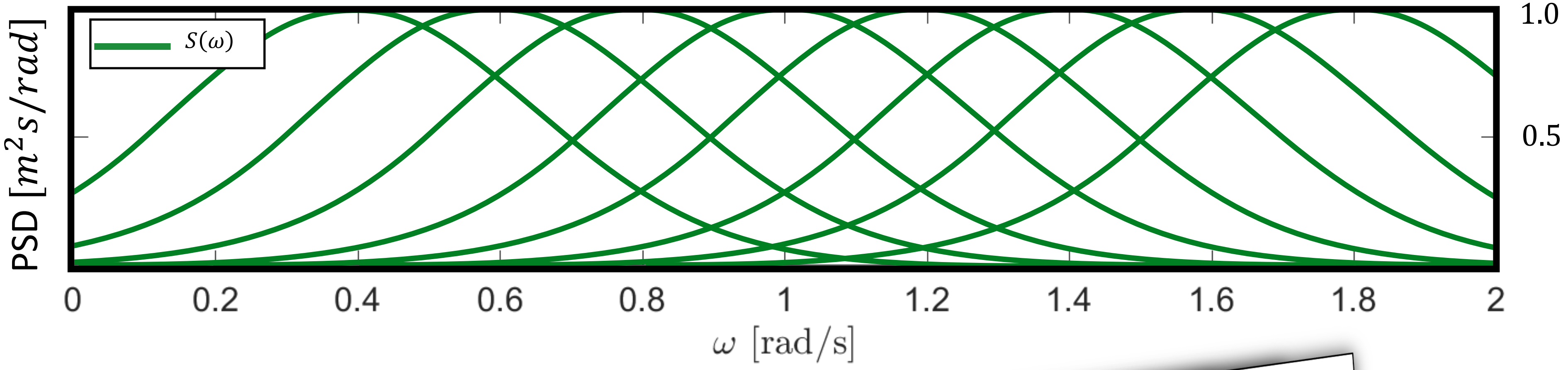
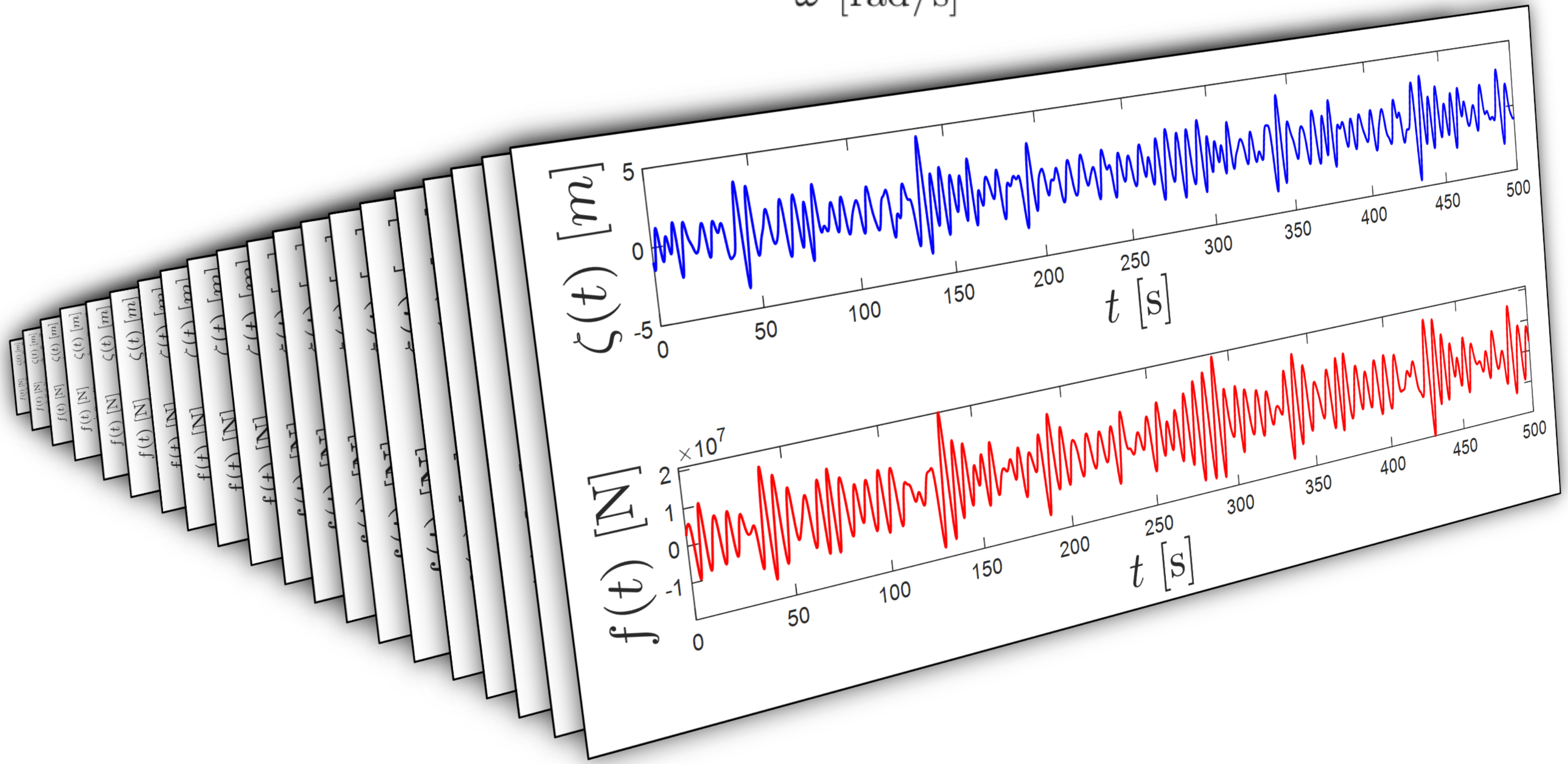
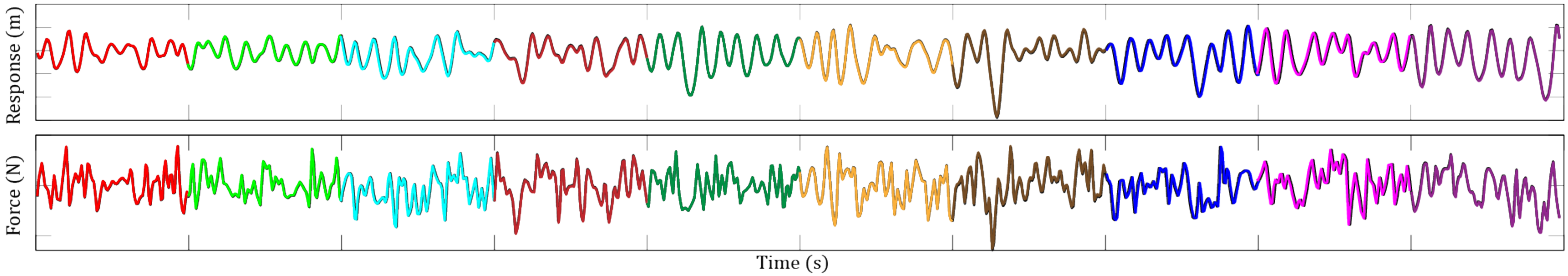


4 Time-Series Sets

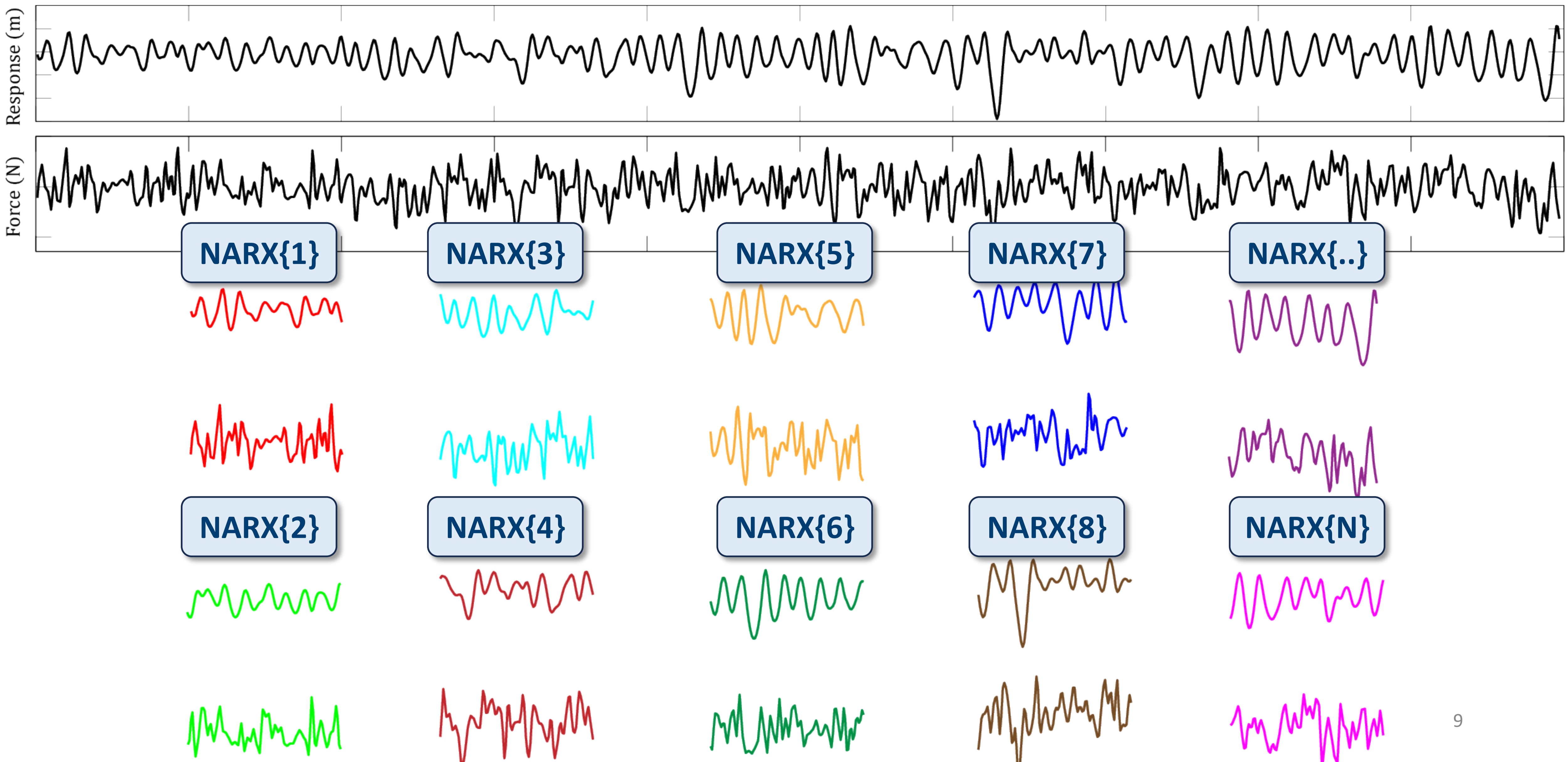




4 Time-Series Sets



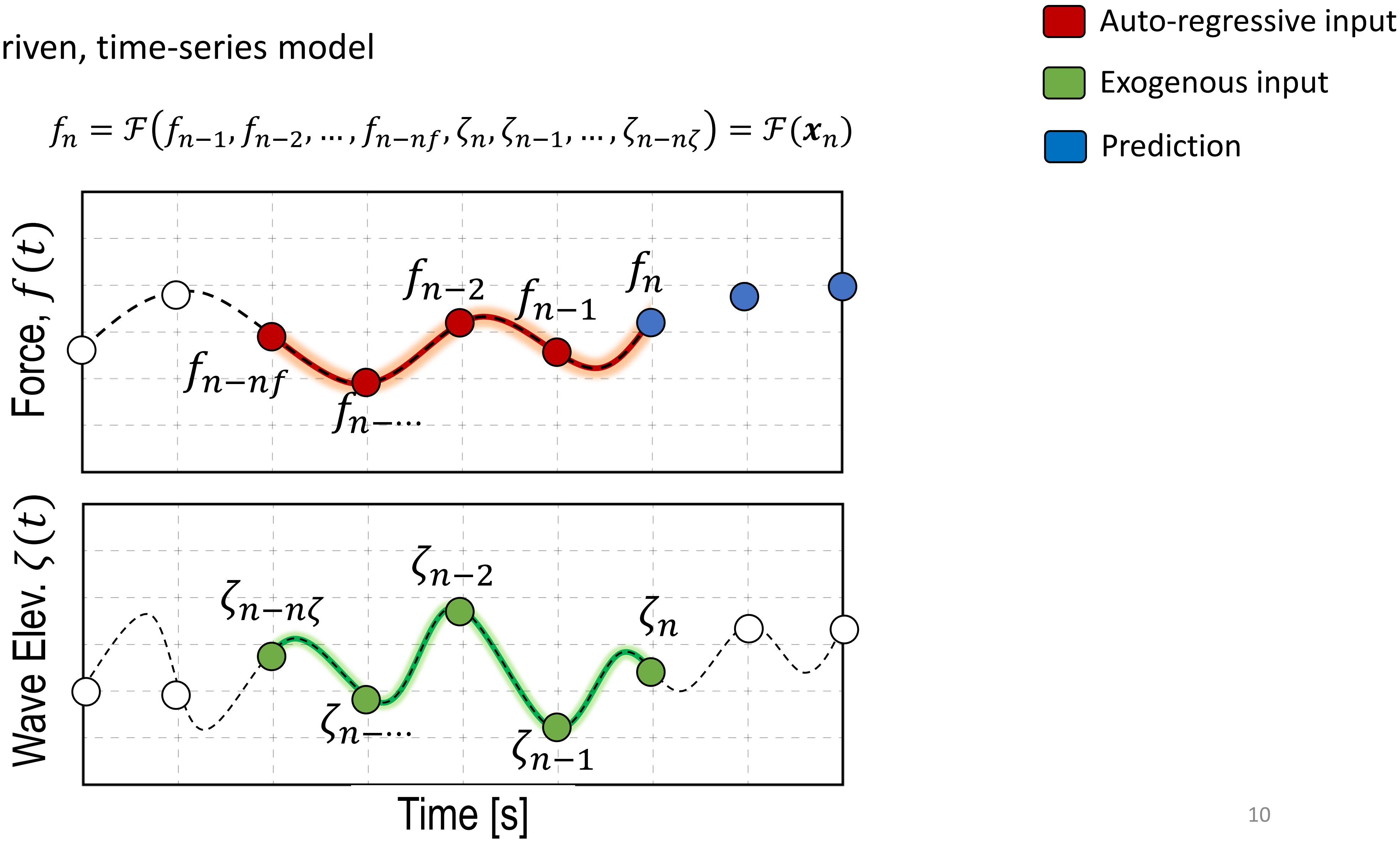
4 Time-Series Sets



5 Polynomial-NARX

- Forecasting, data-driven, time-series model
- What it does:

$$f_n = \mathcal{F}\left(f_{n-1}, f_{n-2}, \dots, f_{n-nf}, \zeta_n, \zeta_{n-1}, \dots, \zeta_{n-n\zeta}\right) = \mathcal{F}(\boldsymbol{x}_n)$$



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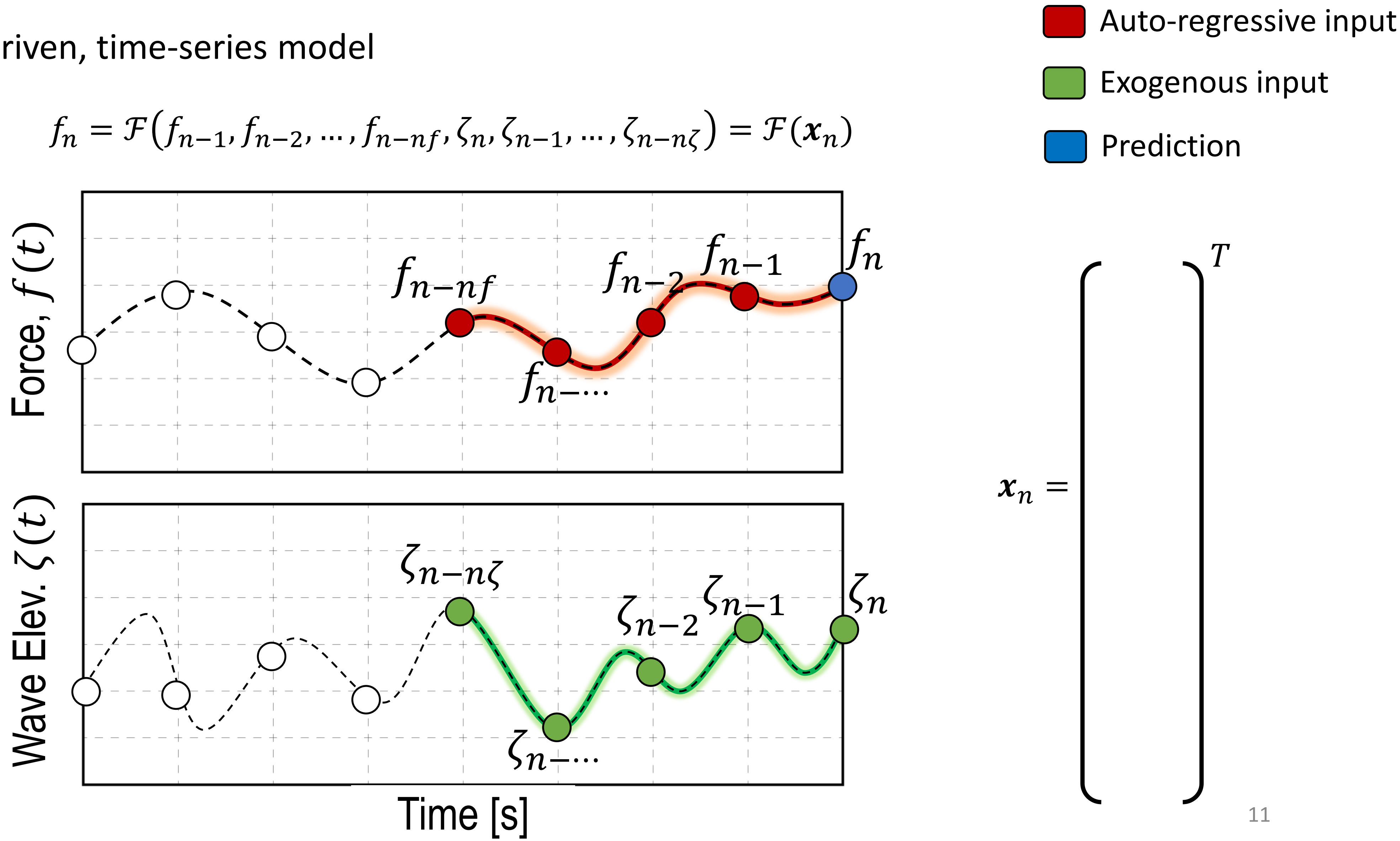


$$\boldsymbol{x}_n = \left[\quad\right]^T$$

5 Polynomial-NARX

$$\hat{f}_n = \mathcal{F}\left(f_{n-1}, f_{n-2}, \dots, f_{n-nf}, \zeta_n, \zeta_{n-1}, \dots, \zeta_{n-n\zeta}\right) = \mathcal{F}(\boldsymbol{x}_n)$$

$$p = n_f + n_\zeta + 1$$

$$\hat{f}_n = \begin{bmatrix} C_1^{(1)} & C_2^{(1)} & C_3^{(1)} & C_4^{(1)} & C_{\dots}^{(1)} & C_p^{(1)} \end{bmatrix} \begin{bmatrix} f_{n-1} \\ f_{n-\dots} \\ f_{n-nf} \\ \zeta_n \\ \zeta_{n-\dots} \\ \zeta_{n-n\zeta} \end{bmatrix} + \begin{bmatrix} f_{n-1} \\ f_{n-\dots} \\ f_{n-nf} \\ \zeta_n \\ \zeta_{n-\dots} \\ \zeta_{n-n\zeta} \end{bmatrix}^T \begin{bmatrix} C_{11}^{(2)} & C_{12}^{(2)} & C_{13}^{(2)} & C_{14}^{(2)} & C_{1\dots}^{(2)} & C_{1p}^{(2)} \\ C_{21}^{(2)} & C_{22}^{(2)} & C_{23}^{(2)} & C_{24}^{(2)} & C_{2..}^{(2)} & C_{2p}^{(2)} \\ C_{31}^{(2)} & C_{32}^{(2)} & C_{33}^{(2)} & C_{34}^{(2)} & C_{3\dots}^{(2)} & C_{3p}^{(2)} \\ C_{41}^{(2)} & C_{42}^{(2)} & C_{43}^{(2)} & C_{44}^{(2)} & C_{4\dots}^{(2)} & C_{4p}^{(2)} \\ C_{\dots 1}^{(2)} & C_{\dots 2}^{(2)} & C_{\dots 3}^{(2)} & C_{\dots 4}^{(2)} & C_{..,..}^{(2)} & C_{\dots p}^{(2)} \\ C_{p1}^{(2)} & C_{p2}^{(2)} & C_{p3}^{(2)} & C_{p4}^{(2)} & C_{p\dots}^{(2)} & C_{pp}^{(2)} \end{bmatrix} \begin{bmatrix} f_{n-1} \\ f_{n-\dots} \\ f_{n-nf} \\ \zeta_n \\ \zeta_{n-\dots} \\ \zeta_{n-n\zeta} \end{bmatrix}$$

$$\hat{f}_n = \boldsymbol{C}^{(1)} \boldsymbol{x}_n^T + \boldsymbol{x}_{\boldsymbol{n}} \boldsymbol{C}^{(2)} \boldsymbol{x}_n^T$$

5 Polynomial-NARX

$$\hat{f}_n = \mathcal{F}\left(f_{n-1}, f_{n-2}, \dots, f_{n-nf}, \zeta_n, \zeta_{n-1}, \dots, \zeta_{n-n\zeta}\right) = \mathcal{F}(\boldsymbol{x}_n)$$

$$p = n_f + n_\zeta + 1$$

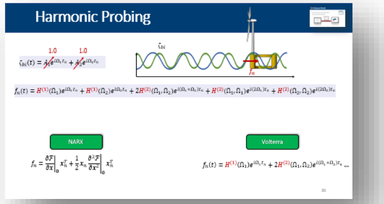
$$\hat{f}_n = \boldsymbol{C}^{(1)} \boldsymbol{x}_n^T + \boldsymbol{x}_n \boldsymbol{C}^{(2)} \boldsymbol{x}_n^T$$

$$\left(\boldsymbol{C}^{(1)}, \boldsymbol{C}^{(2)}\right) = \arg \min_{\boldsymbol{C}^{(1)}, \boldsymbol{C}^{(2)}} \left\{ \frac{1}{2N} \sum_{i=1}^{N} \left(f_i - \widehat{f}_i\left(\boldsymbol{C}^{(1)}, \boldsymbol{C}^{(2)}\right)\right)^2 + \lambda \sum_{j=1}^{p} |C_j^{(1)}| + \lambda \sum_{k=1}^{p^2} |C_k^{(2)}| \right\}$$

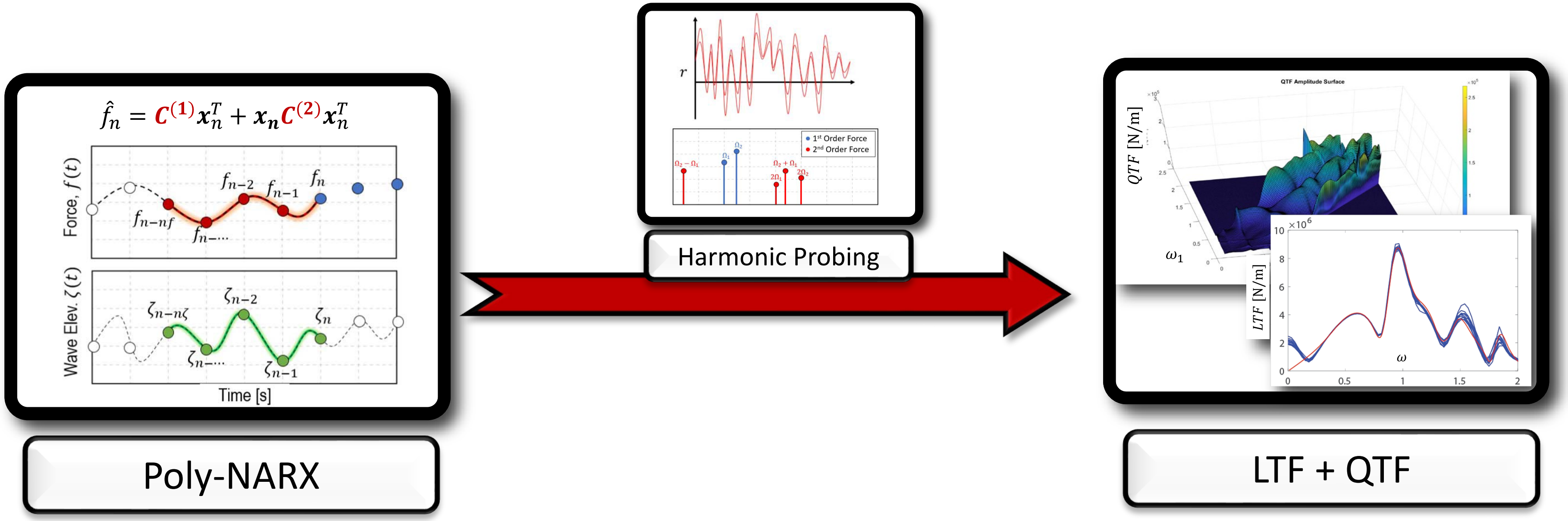
λ: nonnegative parameter

5 Probing and Results

Link to probing explanation



- Extracts Transfer Functions out of the learned coefficients.
- Improved the original method substantially.



5 Probing and Results

NARX

$$f_n(t) = \boldsymbol{C}^{(1)} \boldsymbol{x}_n^T + \boldsymbol{x_n} \boldsymbol{C}^{(2)} \boldsymbol{x}_n^T$$

Volterra

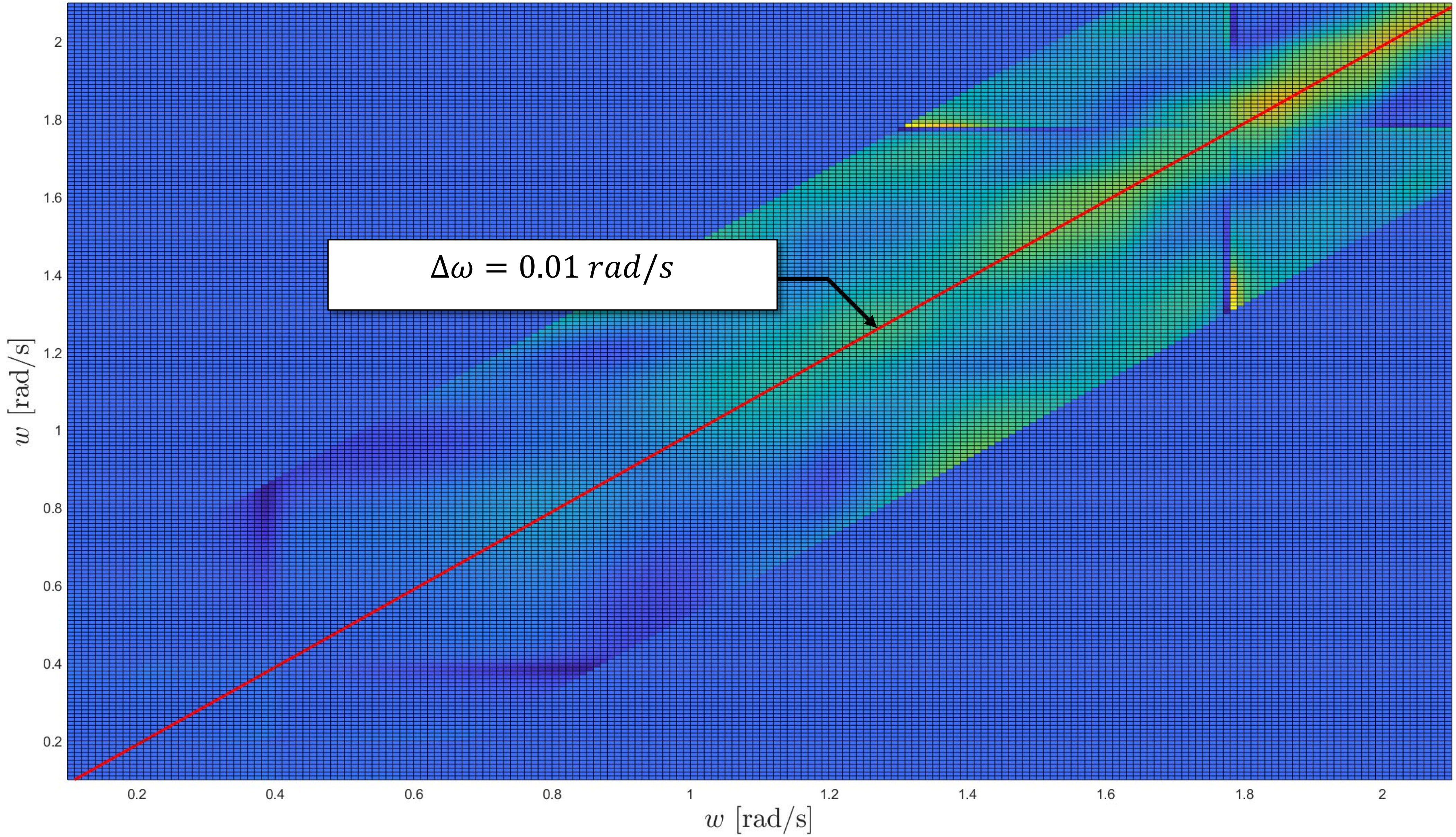
$$f_n(t) = H^{(1)}(\Omega_1) e^{i\Omega_1 t_n} + 2H^{(2)}(\Omega_1, \Omega_2) e^{i(\Omega_1+\Omega_2)t_n} \dots$$

$$Ae^{i\Omega_1 t_n} + Be^{2i\Omega_1 t_n} + Ce^{i(\Omega_1+\Omega_2)t_n} + De^{i(2\Omega_1+\Omega_2)t_n} + Ee^{i\Omega_1 t_n} + Fe^{i\Omega_2 t_n} = H^{(1)}(\Omega_1) e^{i\Omega_1 t_n} + 2H^{(2)}(\Omega_1, \Omega_2) e^{i(\Omega_1+\Omega_2)t_n}$$

$$(A + E) e^{i\Omega_1 t_n} = H^{(1)}(\Omega_1) e^{i\Omega_1 t_n}$$

$$A + E = H^{(1)}(\Omega_1)$$

5 Probing and Results



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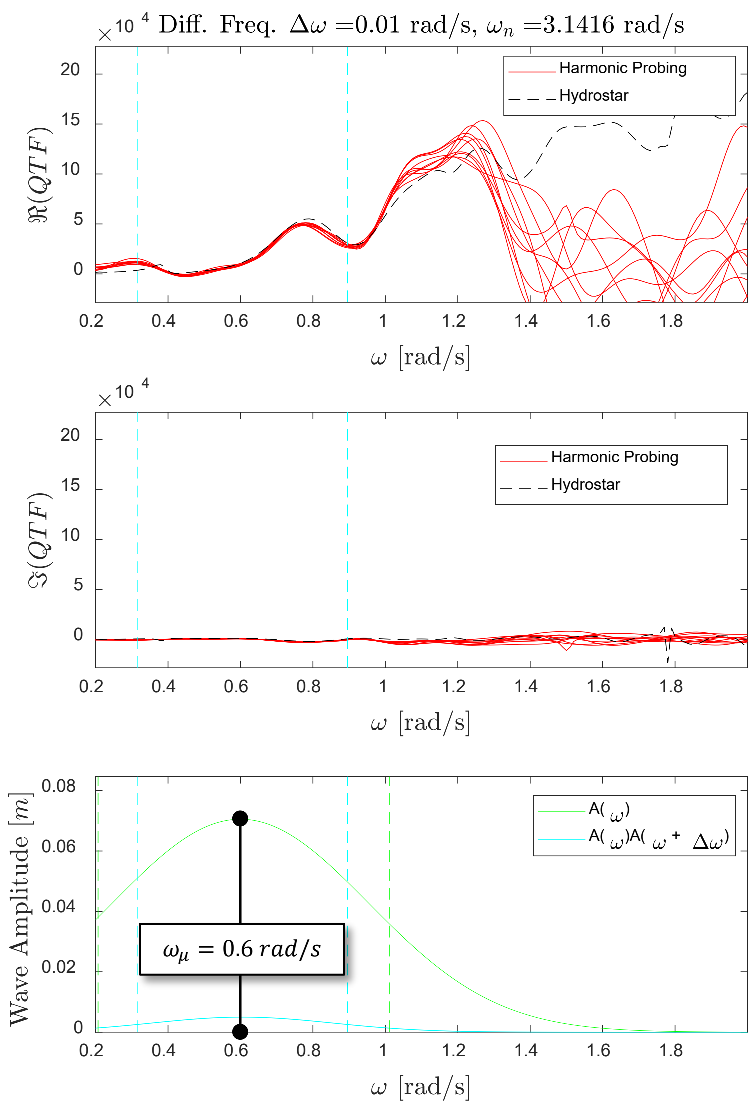

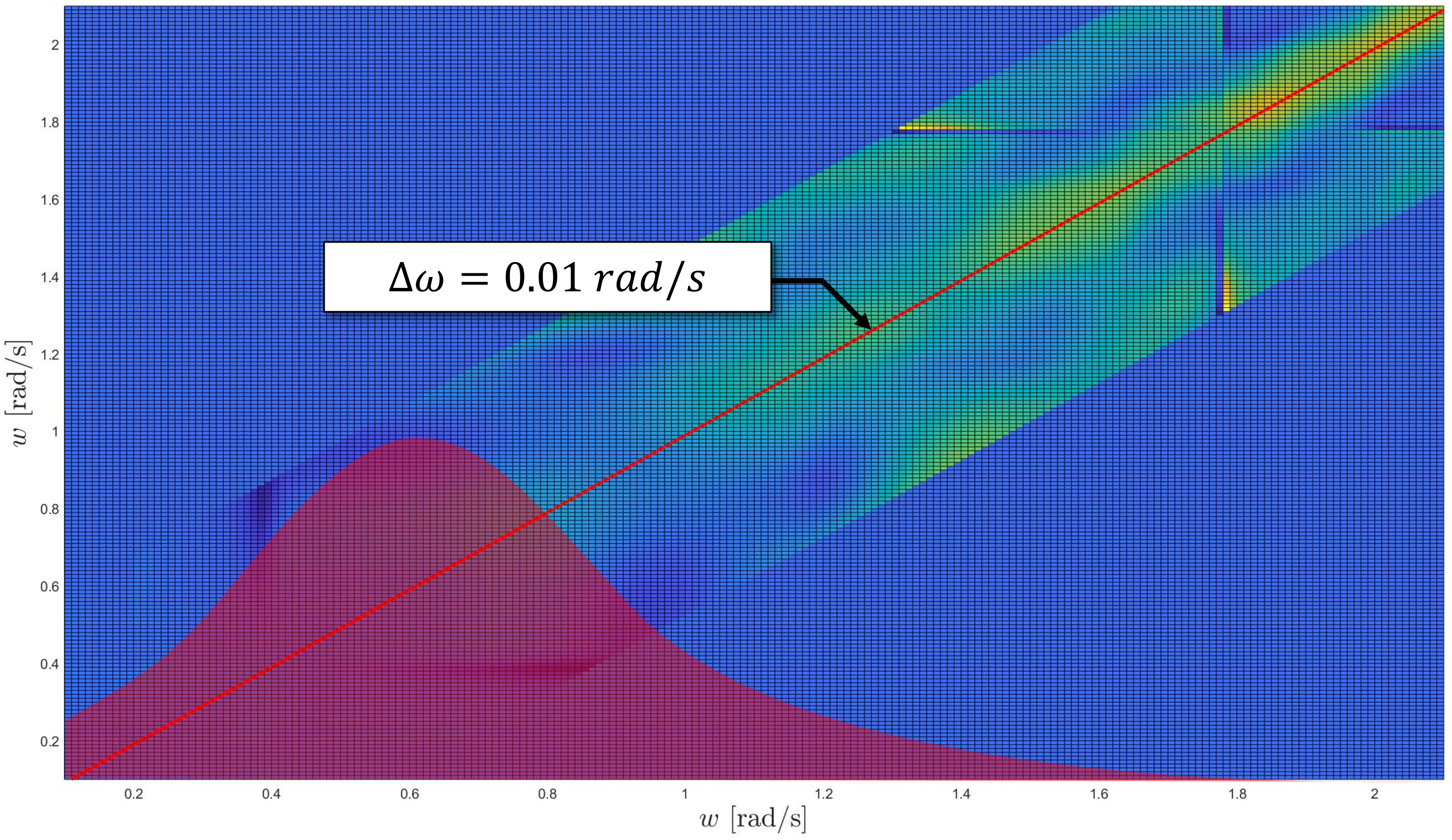

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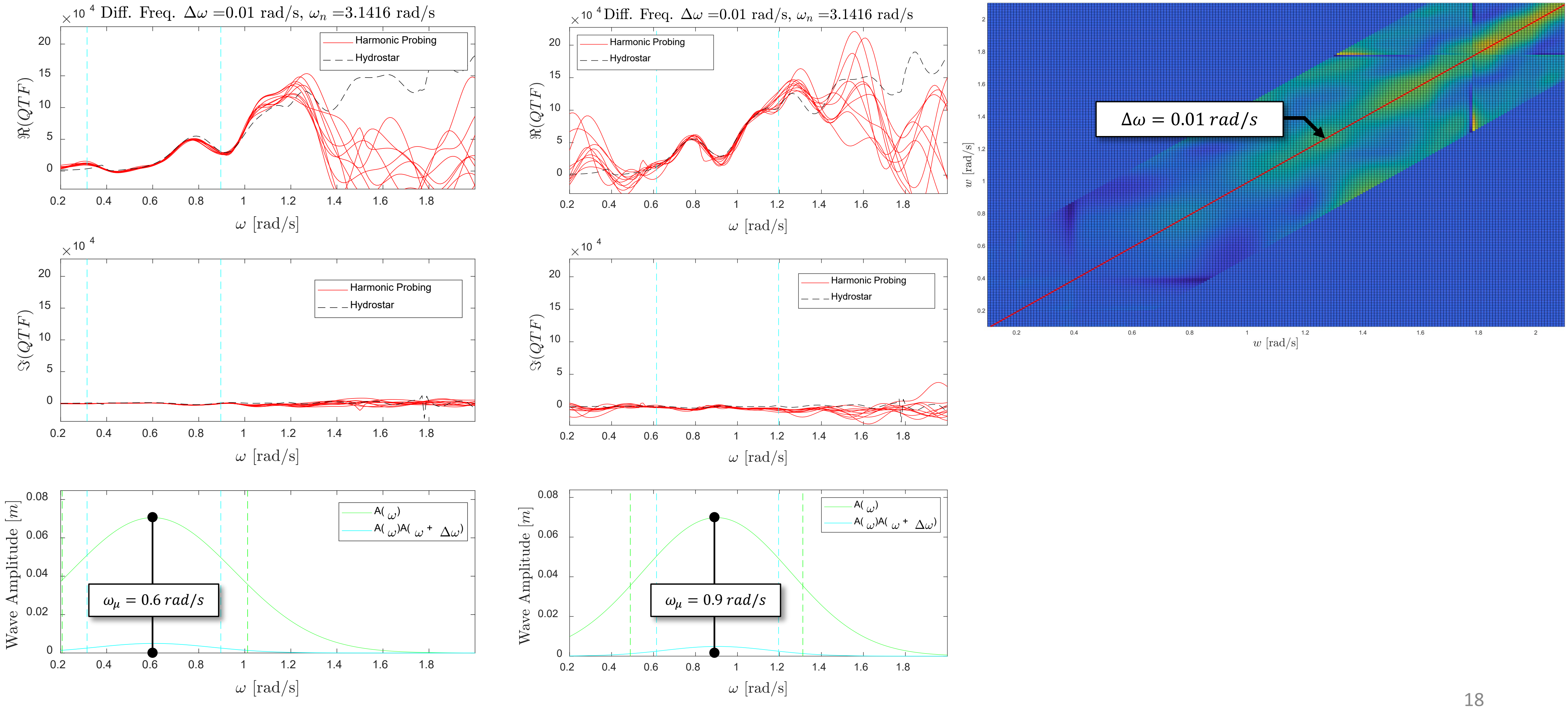

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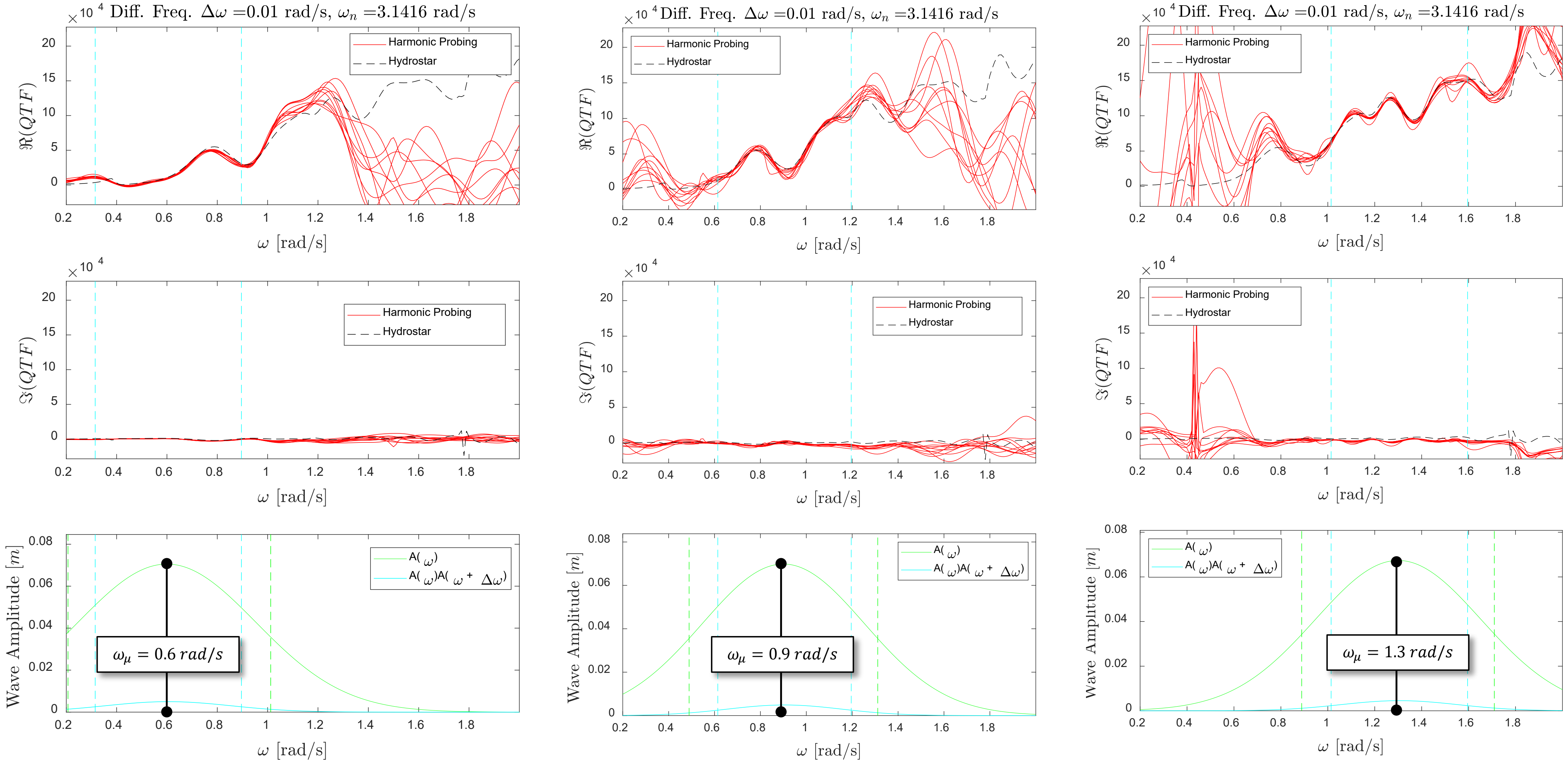

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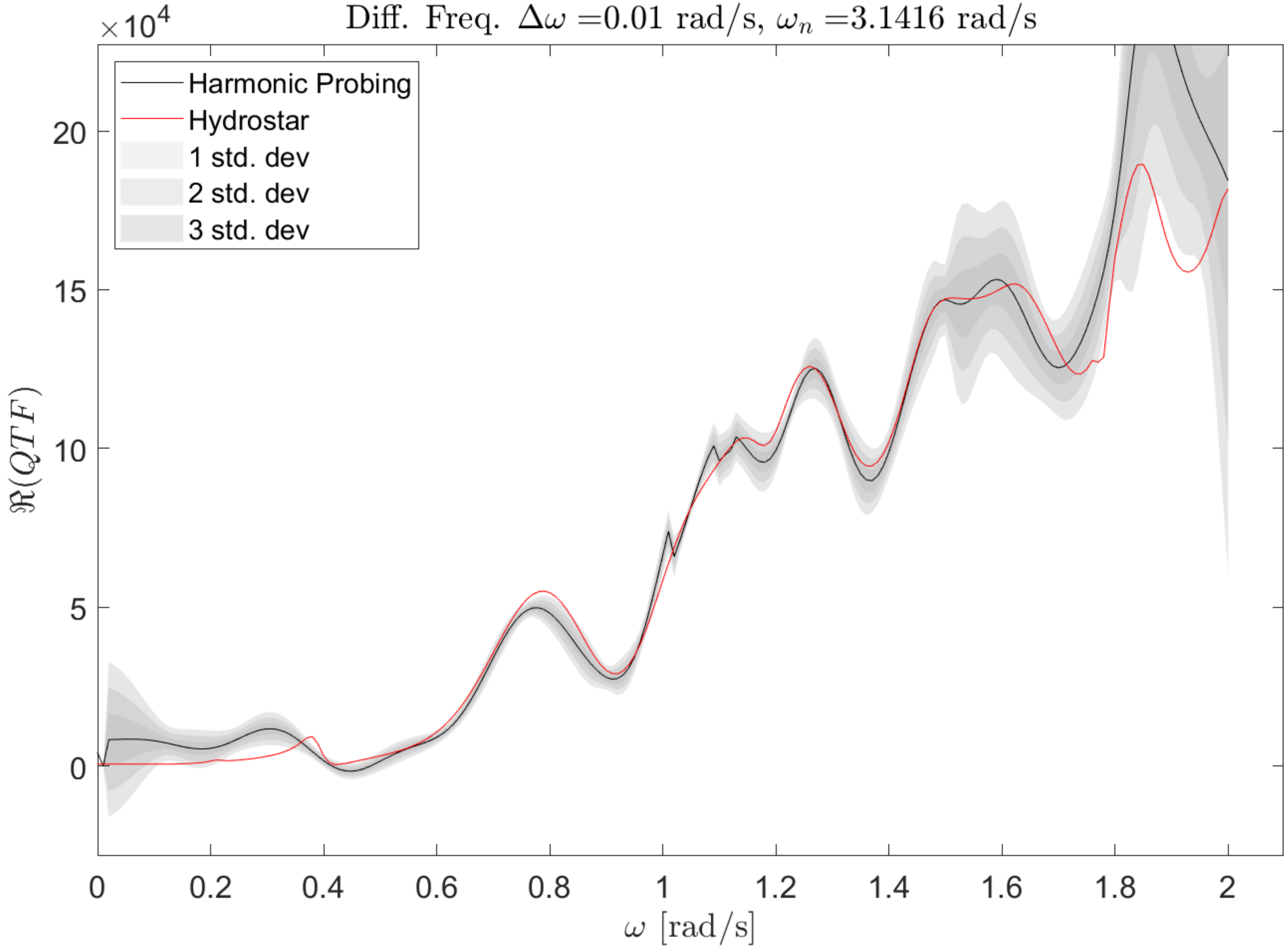

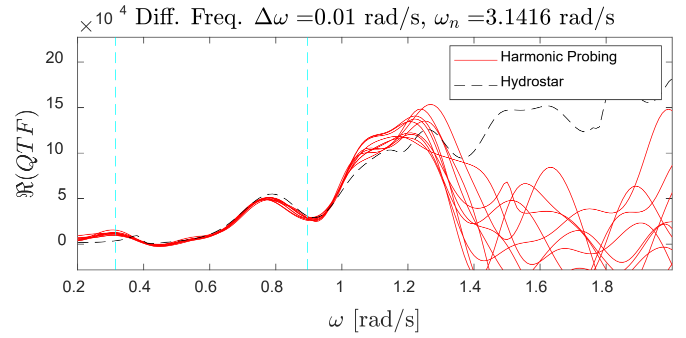

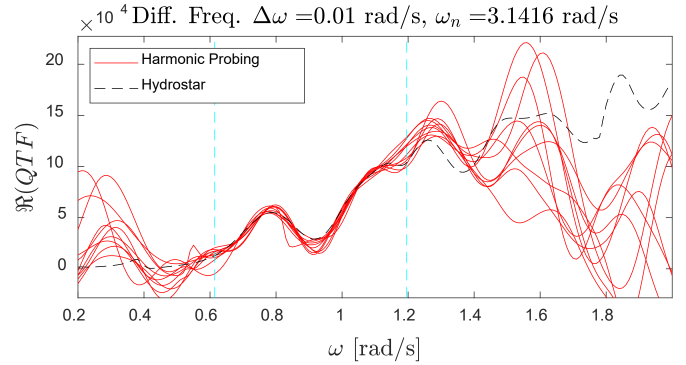

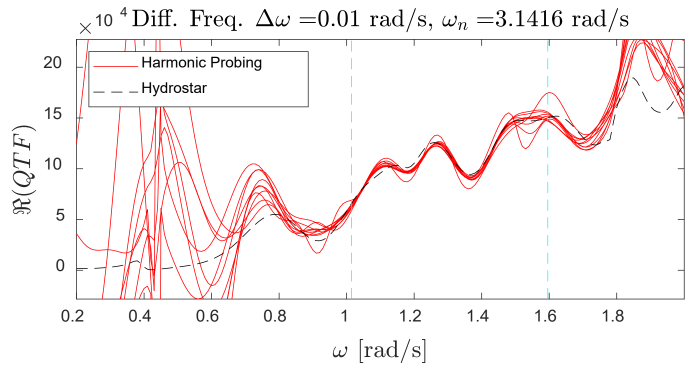

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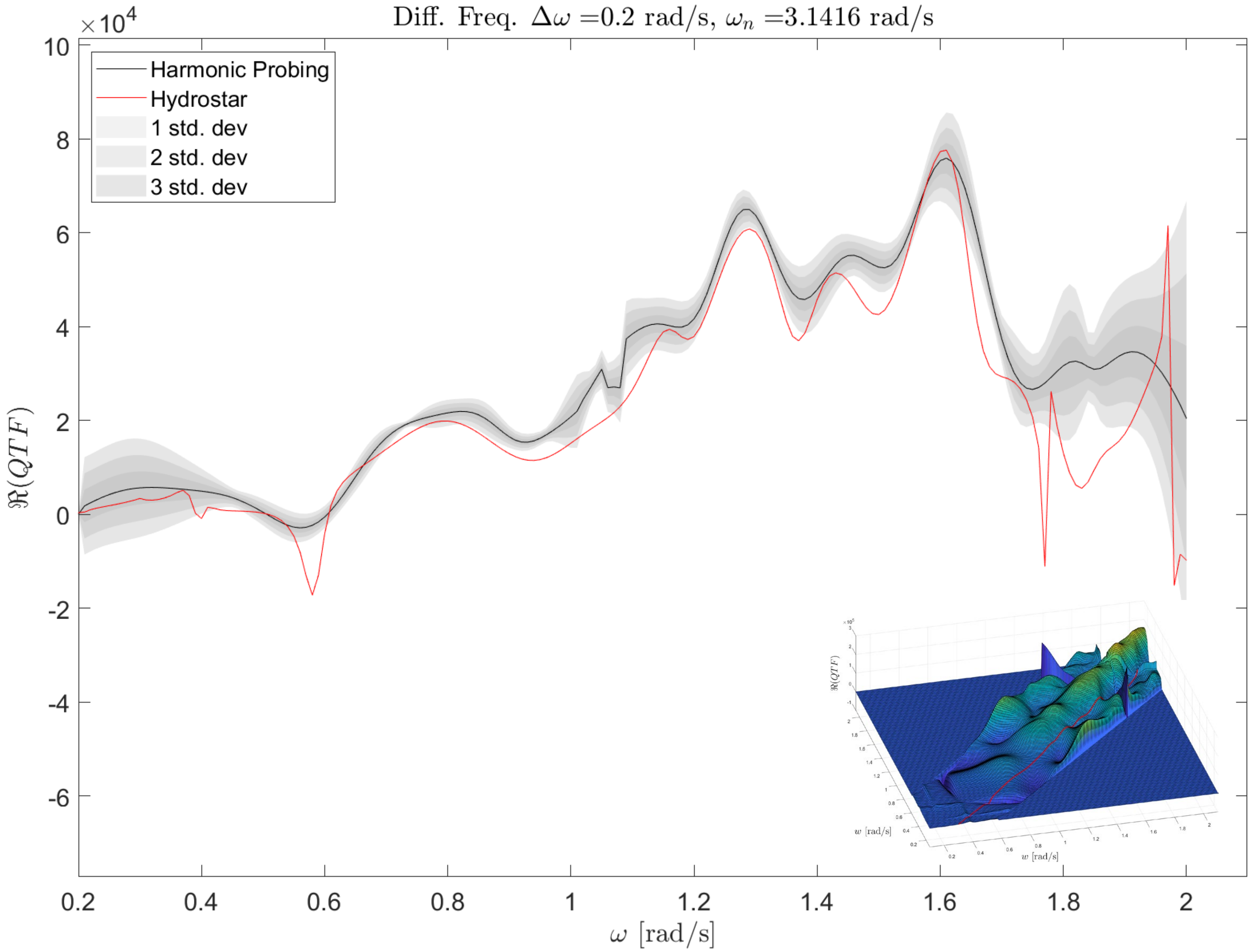

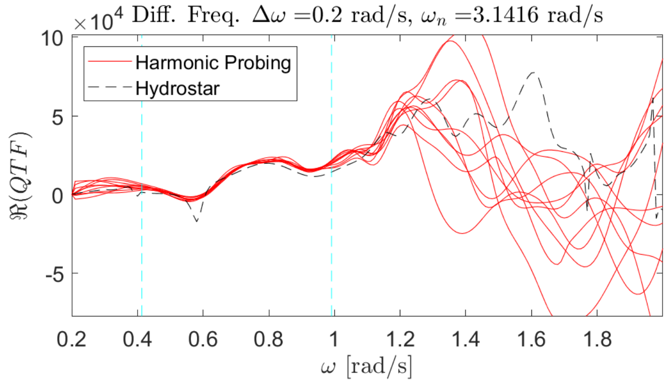

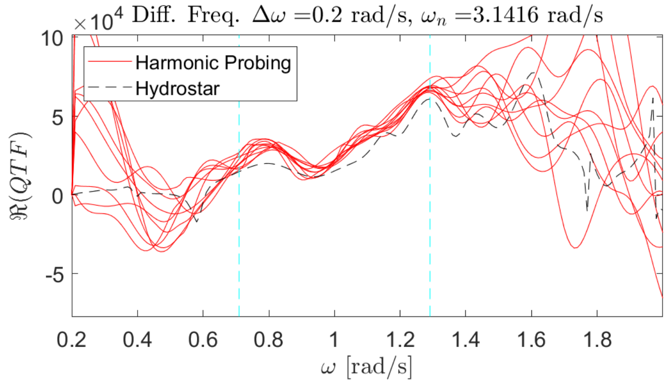

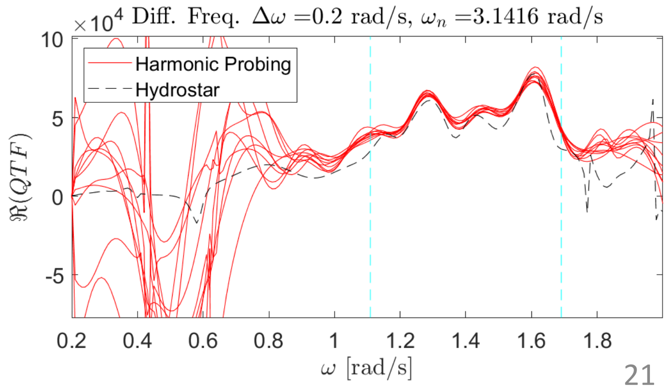

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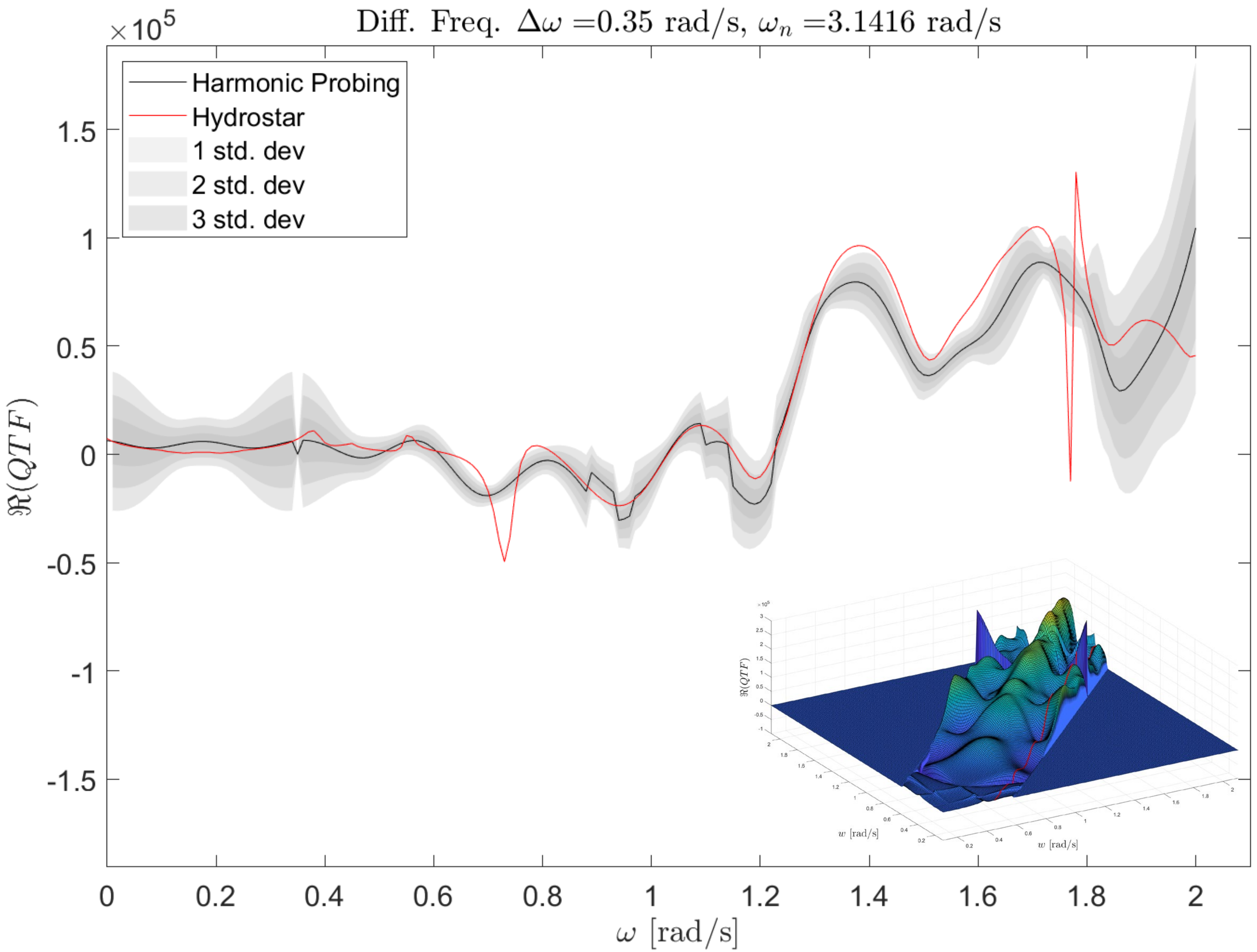
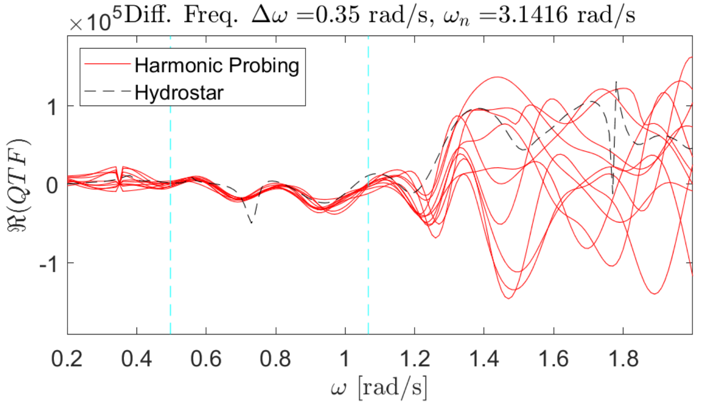
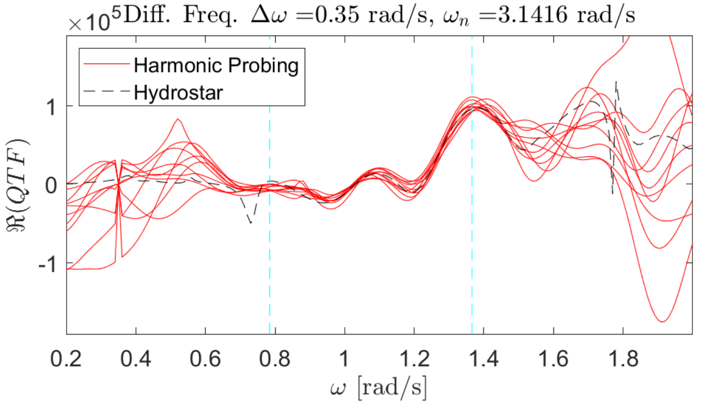
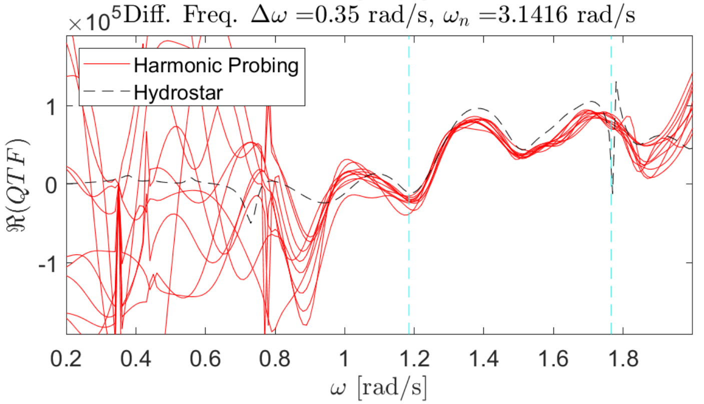
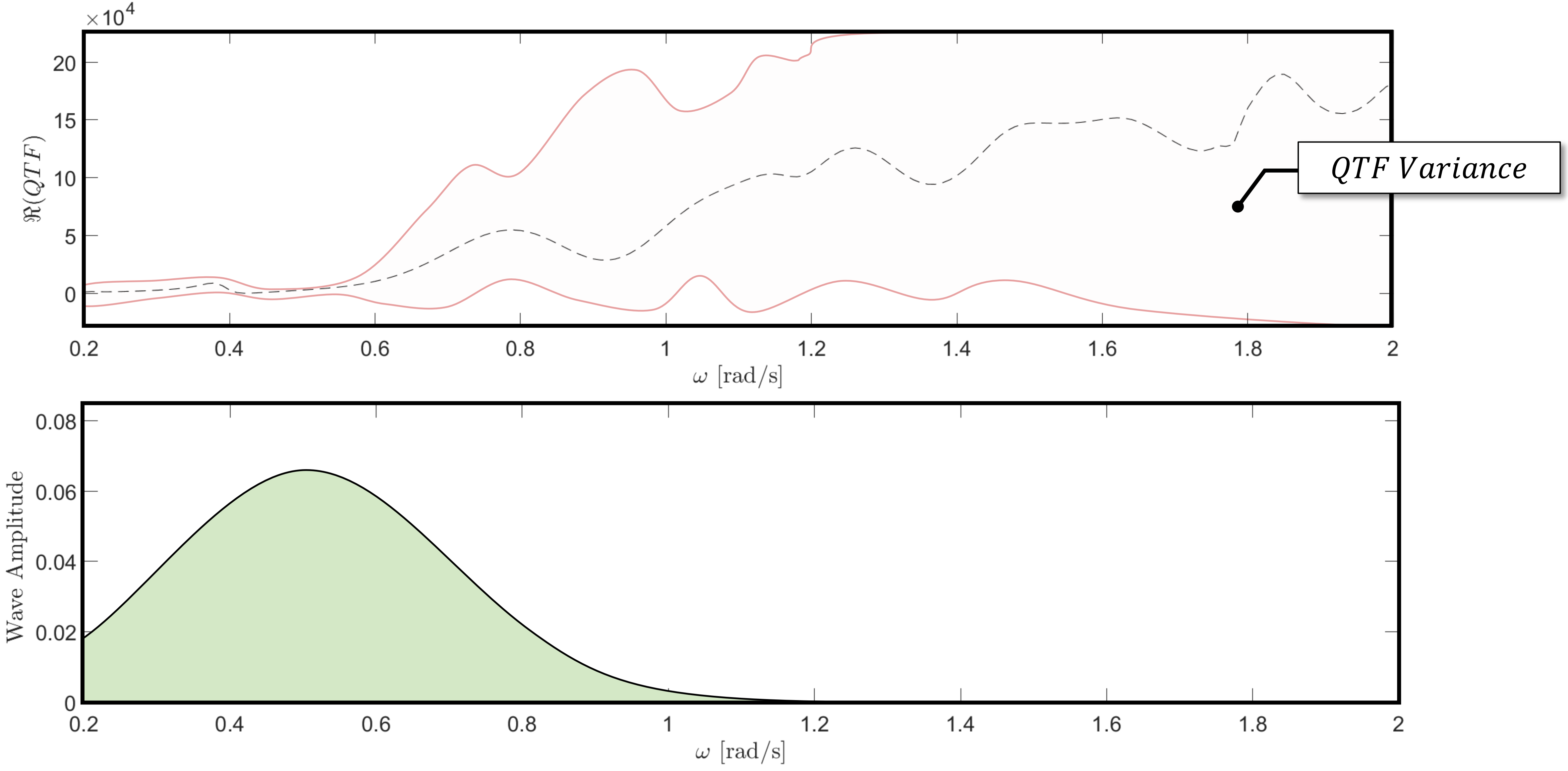




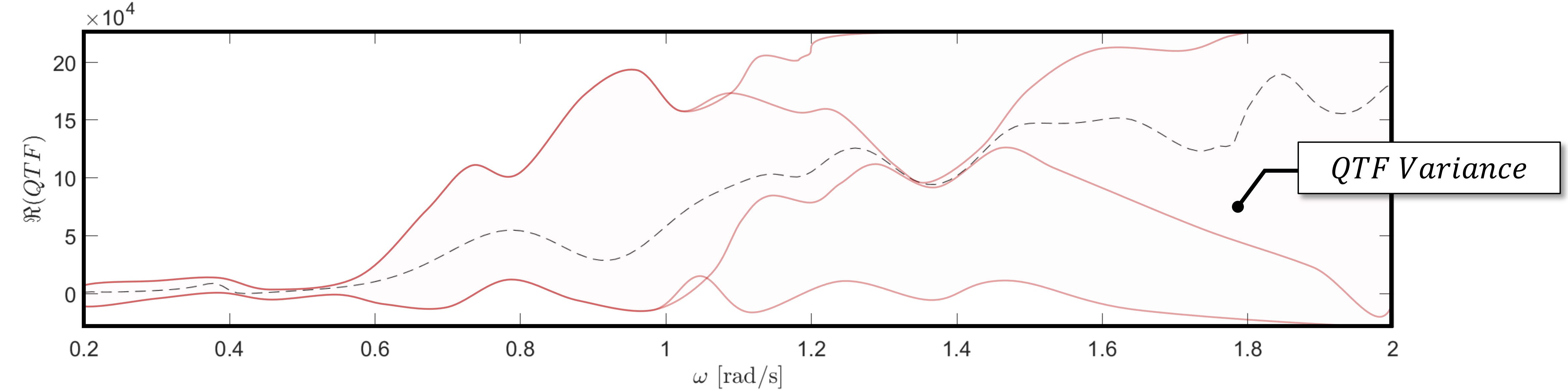


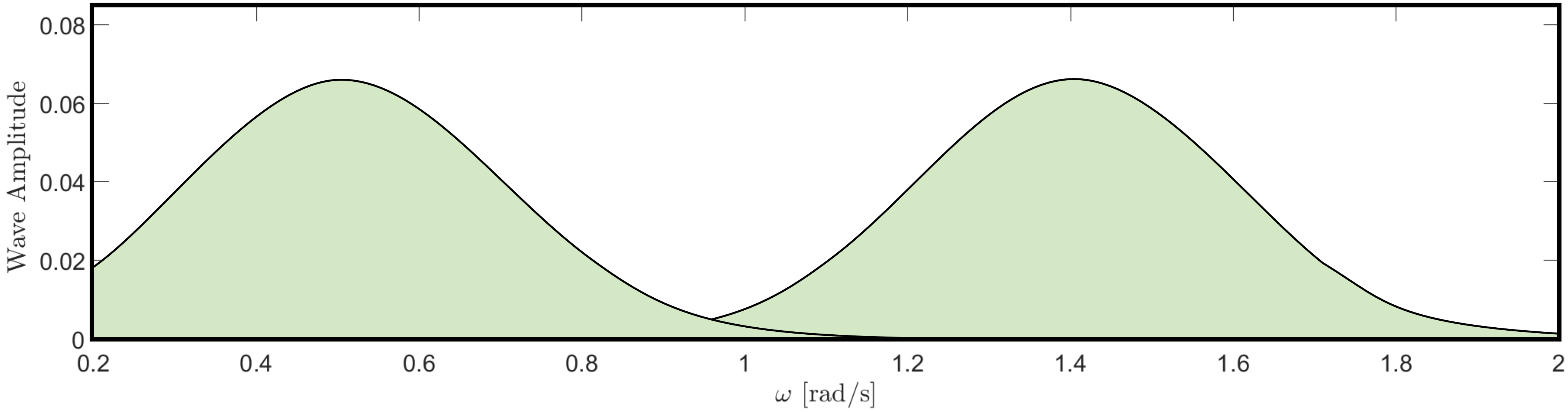


Optimal Experimental Design Perspective



Optimal Experimental Design Perspective





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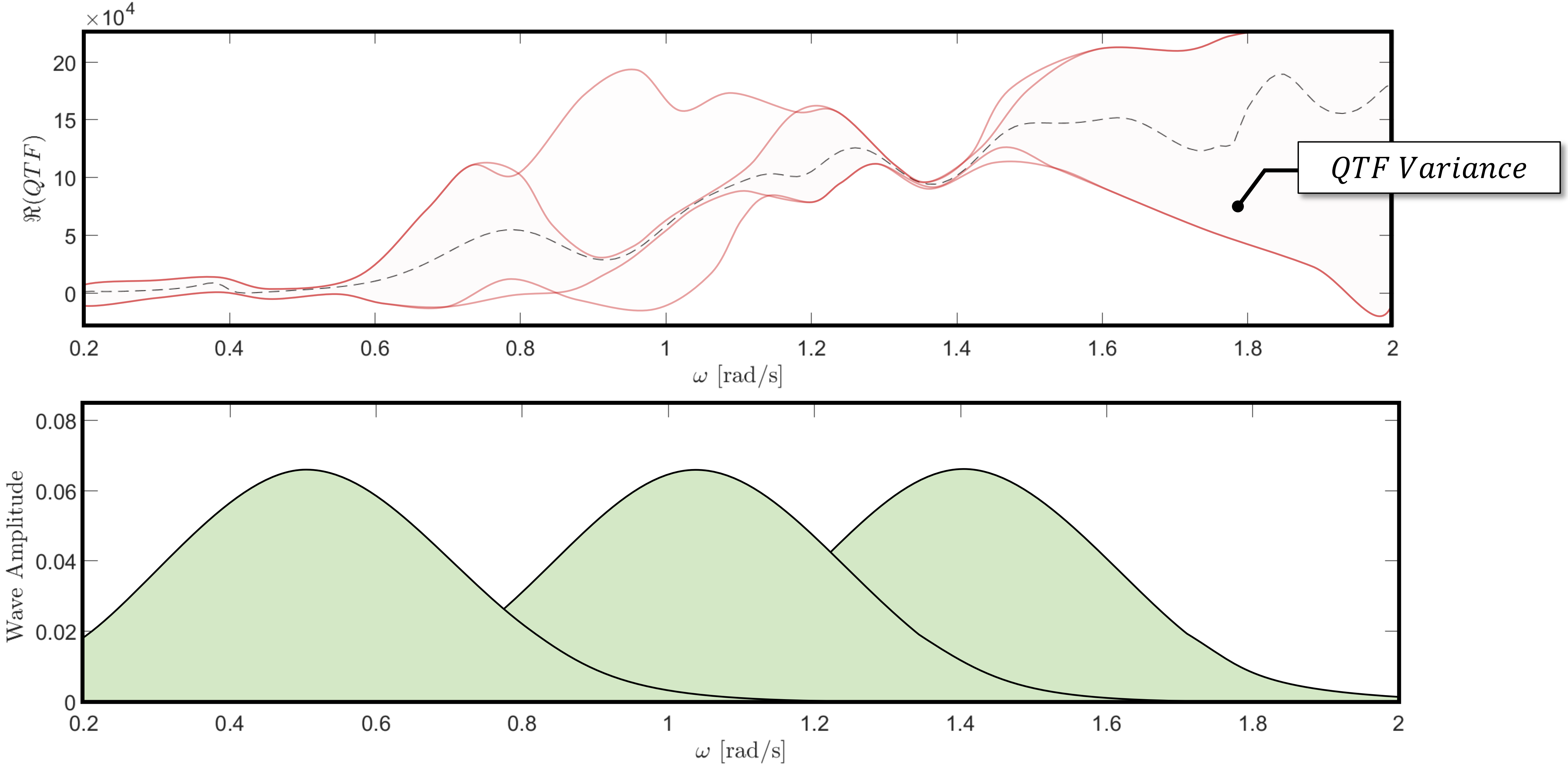

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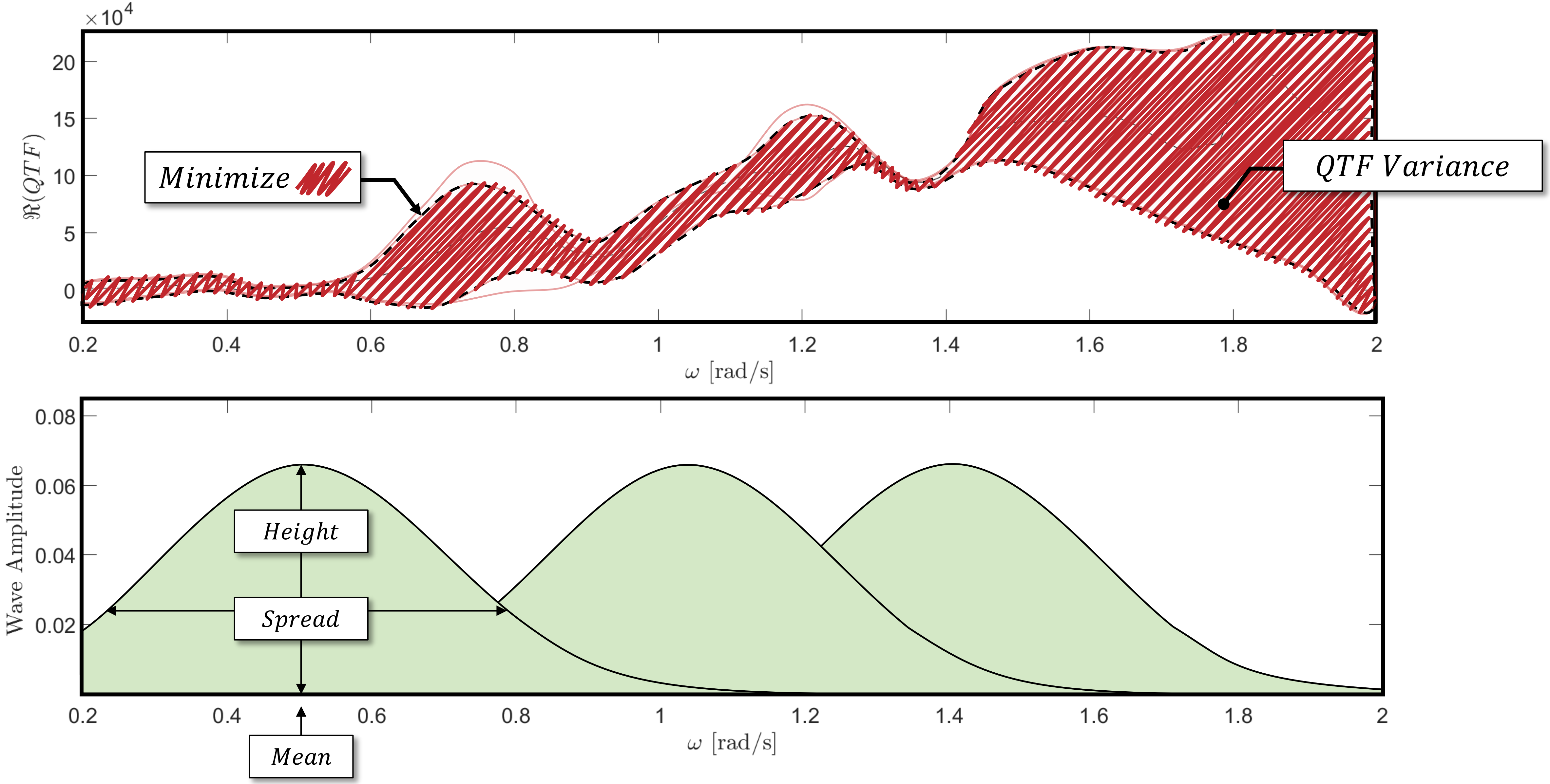

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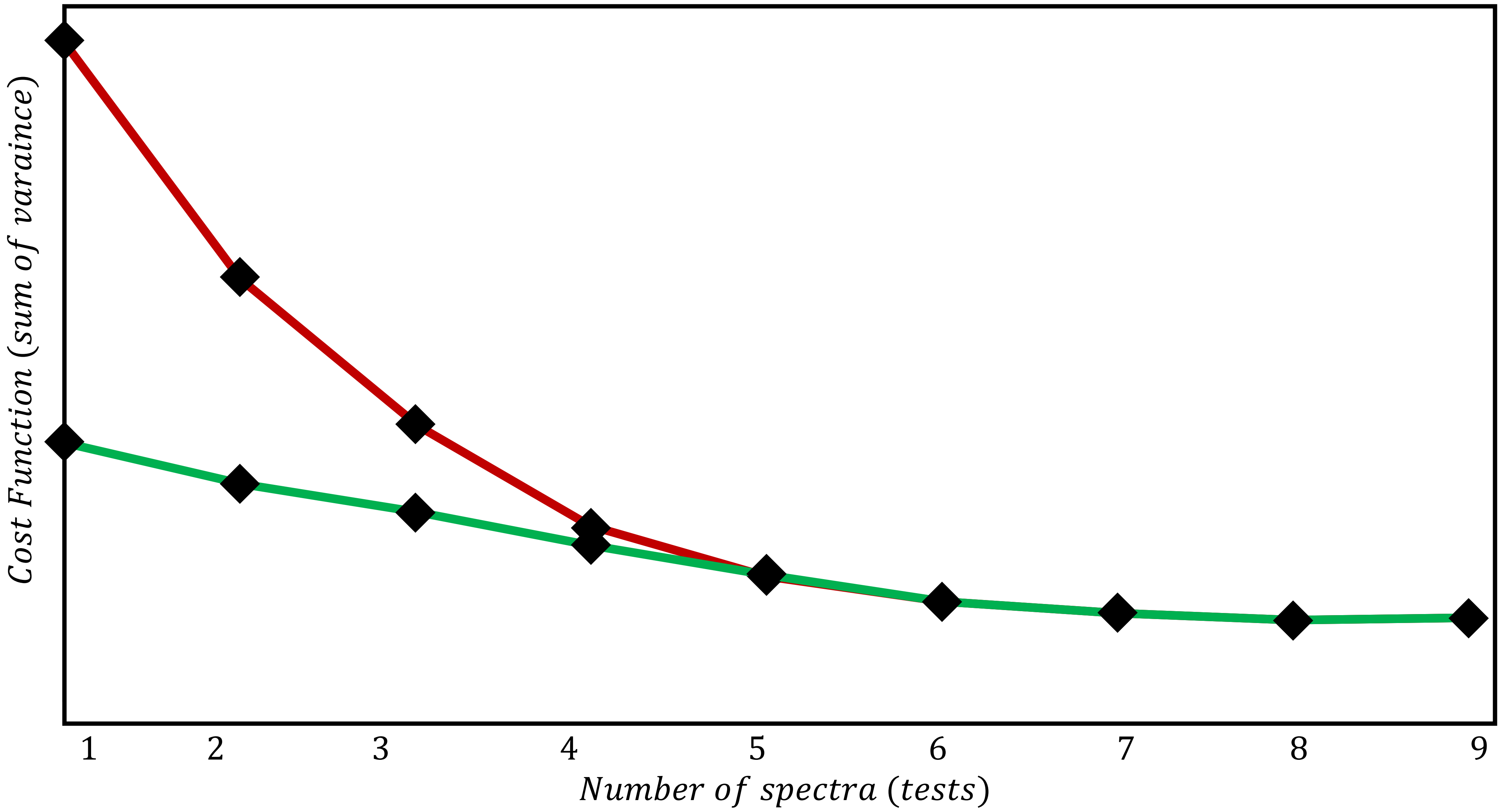
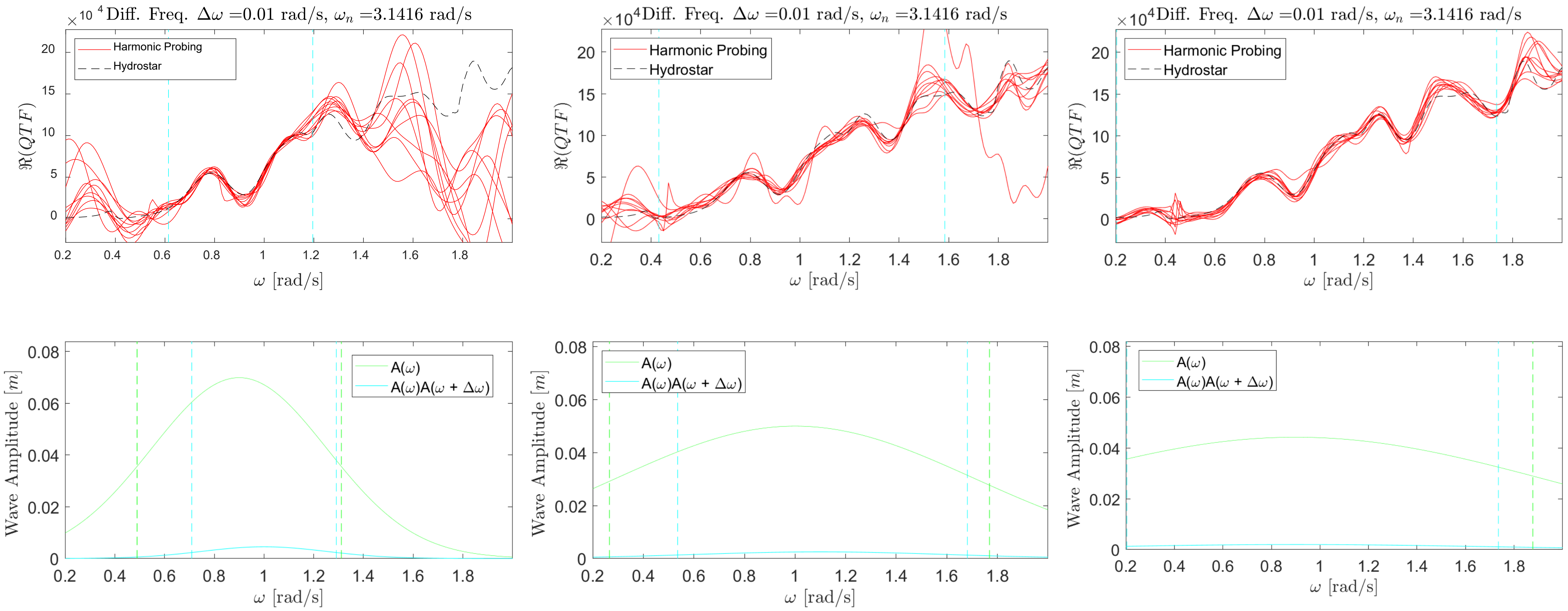


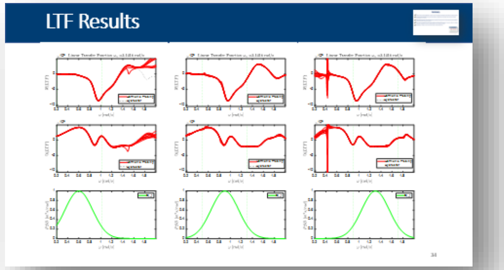
Optimal Experimental Design Perspective



$\mu = 0.90$ r/s	$\mu = 1.00$ r/s	$\mu = 0.90$ r/s
$\sigma = 0.25$ r/s	$\sigma = 0.50$ r/s	$\sigma = 0.75$ r/s

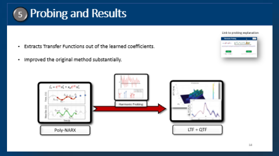
Summary

- Validated harmonic probing algorithm for both LTF and QTF estimations using synthetic data (Windmoor Hydrostar model).
- It is observed that the dispersion of LTF and QTF estimates is smaller where the spectral energy of the wave elevation profile (WEP) is concentrated.
- Identify different portion of LTF and QTF independently using WEPs characterized by Gaussian spectral energy distributions with different mean and standard deviation.
- A procedure for merging LTF and QTF estimates obtained for different WEP is proposed; the procedure allows for obtaining LTF and QTF estimates for the entire frequency range of interest with good accuracy.
- Optimization of number, mean, and standard deviation of each WEP spectrum



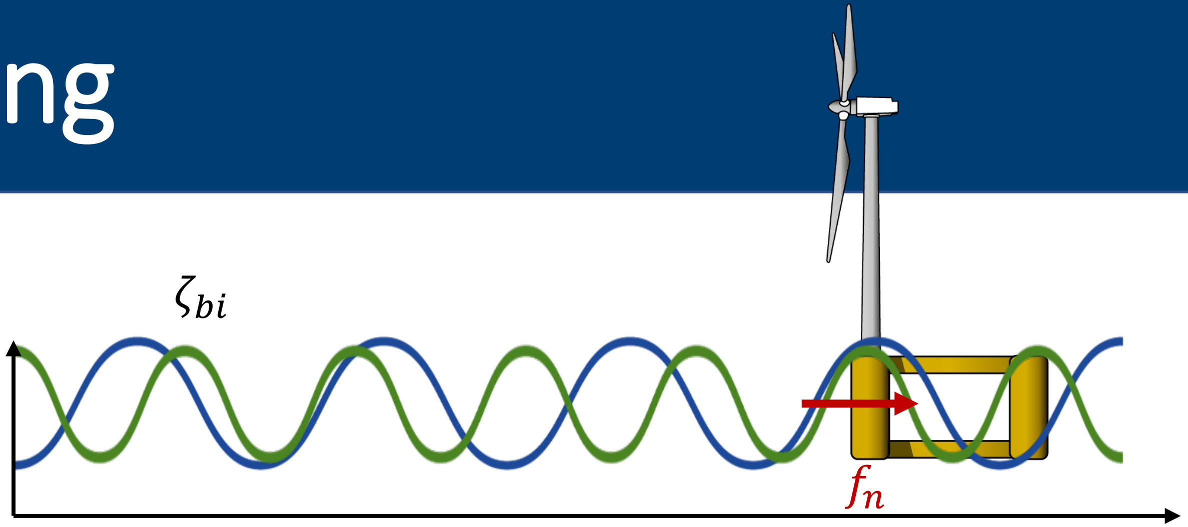
Appendix A

Harmonic Probing



1.0 1.0

$$\zeta_{bi}(t) = A_1 e^{i\Omega_1 t_n} + A_2 e^{i\Omega_2 t_n}$$



$$f_n(t) = H^{(1)}(\Omega_1)e^{i\Omega_1 t_n} + H^{(1)}(\Omega_2)e^{i\Omega_2 t_n} + 2H^{(2)}(\Omega_1,\Omega_2)e^{i(\Omega_1+\Omega_2)t_n} + H^{(2)}(\Omega_1,\Omega_1)e^{i(2\Omega_1)t_n} + H^{(2)}(\Omega_2,\Omega_2)e^{i(2\Omega_2)t_n}$$

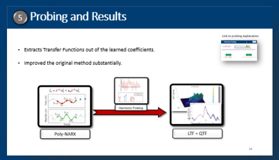
NARX

$$f_n = \left.\frac{\partial \mathcal{F}}{\partial \boldsymbol{x}}\right|_{\mathbf{0}} \boldsymbol{x}_n^T + \frac{1}{2}\boldsymbol{x}_n \left.\frac{\partial^2 \mathcal{F}}{\partial \boldsymbol{x}^2}\right|_{\mathbf{0}} \boldsymbol{x}_n^T$$

Volterra

$$f_n(t) = H^{(1)}(\Omega_1)e^{i\Omega_1 t_n} + 2H^{(2)}(\Omega_1,\Omega_2)e^{i(\Omega_1+\Omega_2)t_n} \ldots$$

Harmonic Probing



NARX

$$f_n = \left.\frac{\partial \mathcal{F}}{\partial \boldsymbol{x}}\right|_{\mathbf{0}} \boldsymbol{x}_n^T + \frac{1}{2}\boldsymbol{x}_n \left.\frac{\partial^2 \mathcal{F}}{\partial \boldsymbol{x}^2}\right|_{\mathbf{0}} \boldsymbol{x}_n^T$$

$$\boldsymbol{x}_n = (f_{n-1}, f_{n-2}, \dots, f_{n-nf}, \zeta_n, \zeta_{n-1}, \dots, \zeta_{n-n\zeta})$$

Volterra

$$f_n(t) = H^{(1)}(\Omega_1)e^{i\Omega_1 t_n} + 2H^{(2)}(\Omega_1,\Omega_2)e^{i(\Omega_1+\Omega_2)t_n} \dots$$

Present *n*: $\left[e^{i\Omega_1 t_n}\right]$

Past *n-1*: $\left[e^{i\Omega_1 t_n}\right]e^{-i\Omega_1 \Delta t}$

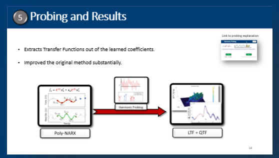
Past *n-2*: $\left[e^{i\Omega_1 t_n}\right]e^{-2i\Omega_1 \Delta t}$

$\vdots$

$$\boldsymbol{x}_n = \begin{bmatrix} f_{n-1} \\ f_{n-2} \\ \zeta_n \\ \zeta_{n-1} \end{bmatrix}^T = \begin{bmatrix} \left[H^{(1)}(\Omega_1)e^{i\Omega_1} + H^{(1)}(\Omega_2)e^{i\Omega_2} + 2H^{(2)}(\Omega_1,\Omega_2)e^{i(\Omega_1+\Omega_2)} + \cdots\right]e^{t_n-\Delta t} \\ \left[H^{(1)}(\Omega_1)e^{i\Omega_1} + H^{(1)}(\Omega_2)e^{i\Omega_2} + 2H^{(2)}(\Omega_1,\Omega_2)e^{i(\Omega_1+\Omega_2)} + \cdots\right]e^{t_n-2\Delta t} \\ e^{i\Omega_1 t_n} + e^{i\Omega_2 t_n} \\ e^{i\Omega_1 (t_{n-1})} + e^{i\Omega_2 (t_n-\Delta t)} \end{bmatrix}^{\mathbf{T}}$$

$$\left.\frac{\partial \mathcal{F}}{\partial \boldsymbol{x}}\right|_{\mathbf{0}} \boldsymbol{x}_n^T + \frac{1}{2}\boldsymbol{x}_n \left.\frac{\partial^2 \mathcal{F}}{\partial \boldsymbol{x}^2}\right|_{\mathbf{0}} \boldsymbol{x}_n^T = H^{(1)}(\Omega_1)e^{i\Omega_1 t_n} + 2H^{(2)}(\Omega_1,\Omega_2)e^{i(\Omega_1+\Omega_2)t_n}$$

Harmonic Probing



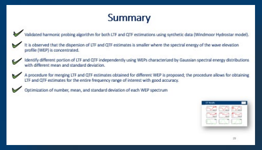
$$\left.\frac{\partial \mathcal{F}}{\partial \boldsymbol{x}}\right|_{\mathbf{0}} \boldsymbol{x}_n^T + \frac{1}{2} \boldsymbol{x}_n \left.\frac{\partial^2 \mathcal{F}}{\partial \boldsymbol{x}^2}\right|_{\mathbf{0}} \boldsymbol{x}_n^T = H^{(1)}(\Omega_1) e^{i\Omega_1 t_n} + 2H^{(2)}(\Omega_1, \Omega_2) e^{i(\Omega_1+\Omega_2)t_n}$$

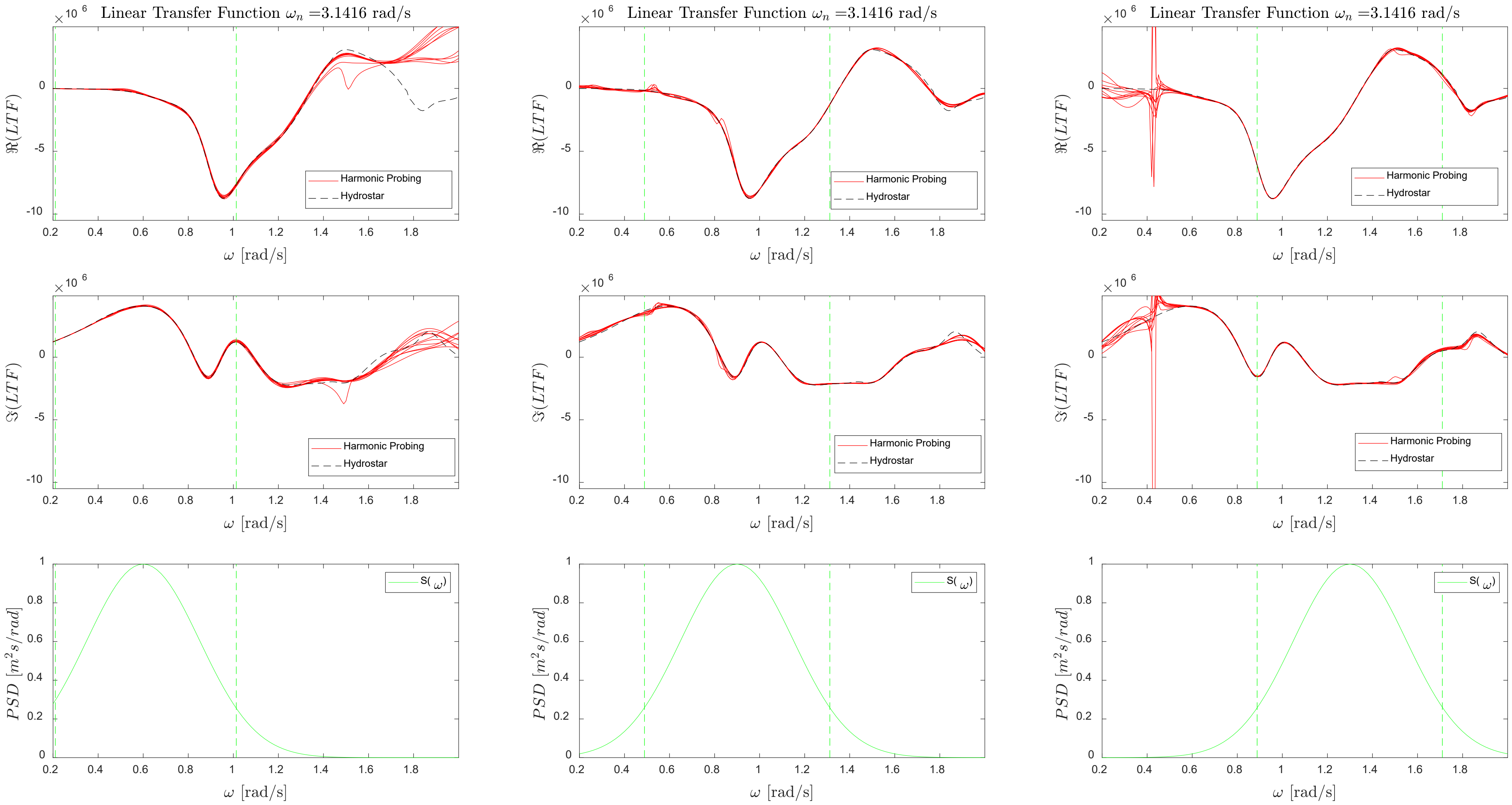
$$A e^{i\Omega_1 t_n} + B e^{2i\Omega_1 t_n} + C e^{i(\Omega_1+\Omega_2)t_n} + D e^{i(2\Omega_1+\Omega_2)t_n} + E e^{i\Omega_1 t_n} + F e^{i\Omega_2 t_n} = H^{(1)}(\Omega_1) e^{i\Omega_1 t_n} + 2H^{(2)}(\Omega_1, \Omega_2) e^{i(\Omega_1+\Omega_2)t_n}$$

$$(A + E) e^{i\Omega_1 t_n} = H^{(1)}(\Omega_1) e^{i\Omega_1 t_n}$$

$$A + E = H^{(1)}(\Omega_1)$$

LTF Results


Summary


Linear Transfer Function ω_n =3.1416 rad/s
Linear Transfer Function ω_n =3.1416 rad/s
Linear Transfer Function ω_n =3.1416 rad/s
$\times 10^6$
$\Re(LTF)$
$\Im(LTF)$
$PSD\ [m^2 s/rad]$
ω [rad/s]
Harmonic Probing
Hydrostar
S(ω)