CYBERLAB KPN

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Regression-Based Estimation of Nonlinear Hydrodynamic Loads on the INO WINDMOOR 12MW Semi-Submersible

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Main findings

Methods from classical **regression theory** is promising for estimating **slowly varying nonlinear wave loads** from **model test** data. It allows for **flexible parametrization** of the **wave force quadratic transfer function (QTF)** and provides tools for **quantifying and reducing statistical variability** as well as for finding the **optimal bias-variability trade-off**.

Background

- Slowly varying nonlinear wave forces and resulting motions are important for mooring design of FOWTs
- Theoretical/numerical wave load models are not complete/accurate enough (potential theory) or too slow (CFD)
- The industry still relies on model testing!
- Calibrating simulation models using model test data gives the best of both worlds

State of the art

- Identification of quadratic wave force transfer function (QTF) from model test data using **cross-bi-spectral analysis** of measured wave elevation and "measured" wave force
- "measured" wave force is calculated from measured platform motion and assumed 1DOF oscillator model of the platform (assume mass, damping and stiffness, and improve iteratively)
- Statistical variability of QTF estimate reduced by "smoothing" in bi-frequency plane - by moving weighted averaging
  - Strong smoothing → Low statistical variability, but large bias due to low resolution
  - Weak smoothing → Small bias/high resolution, but also higher statistical variability

Question: How to find the right bias-variability trade-off?

Statistical variability: How much the QTF-estimate vary from one random sea-state realization to the next

Objective

Investigate system-identification by multivariate regression to estimate QTF, added mass and damping of FOWT from model test data

Motivation

Classical regression theory gives us tools to,

- Quantify statistical variability → standard-error
- Reduce statistical variability → regularization
- Finding the optimal bias-variability trade-off by minimizing cross-validation error (maximize ability to predict "un-seen" data)

Allow for flexible QTF parametrization:

- QTF as a sum of 2D cubic B-splines with coefficients as regressors
- Physics-based (semi-empirical) parametrization (Work in progress)

System identification as a regression problem

Wave force residual in frequency domain:

$$\underbrace{F_D(\omega;\mathbf{x},\zeta)}_{\substack{\text{wave force}\\ \text{estimated from QTF}}} - \underbrace{\left(\underbrace{(-\omega^2 M + i\omega B + C)}_{\substack{\text{displacement-to-force}\\ \text{transfer function}}} \underbrace{\eta(\omega)}_{\substack{\text{measured}\\ \text{displacement}}}\right)}_{\text{"measured" wave force}} = \underbrace{\epsilon(\omega)}_{\substack{\text{Residual to}\\ \text{minimize}\\ \text{(in squared sense)}}}$$

Estimated in linear regression:

- $\mathbf{x}$: QTF design vector with $O(100)$ number of regressors. Wave force is quadratic in wave height, but linear in $\mathbf{x}$.

Estimated by nonlinear regression (outer loop):

- M: Total platform mass, including (unknown) LF added mass
- B: Low-frequency (LF) damping

Measured in ocean basin:

- C: Mooring stiffness
- η: Platform displacement (its Fourier transform)
- ζ: Wave elevation (its Fourier transform)

Wave force residual at discrete frequencies

Rewrite as two equations per discrete frequency (real and imag) - over-determined equation system:

$$\mathbf{Ax} - \mathbf{b} = \mathbf{E}$$

Solved for $\mathbf{x}$ in least-square sense by minimizing "wave force error" $\mathbf{E}^T\mathbf{E}$:

$$\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$$

Nonlinear regression (outer loop) with "hybrid objective"

- Optimal mass and damping parameter need to "prioritize" frequencies around resonance:
  - M and B chosen such as to minimize *displacement error* instead of force error
  - Nonlinear regression with two regressors: M and B
- Each iterate (M_i, B_i) corresponds to "measurements" $\mathbf{b}_i$
- ...and a QTF design vector $\mathbf{x}_i = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}_i$
- $\mathbf{x}_i$ must be chosen such as to minimize the *wave force error* to avoid excessive statistical variability outside the resonance band in the QTF bi-frequency plane

Quadratic transfer function (QTF) and its parametrization

From wave elevation and QTF to wave drift force

Wave drift force as a Fourier series,

$$f_D(t) = \Re \sum_q F_D(\omega_q) \exp(i\omega_q t)$$

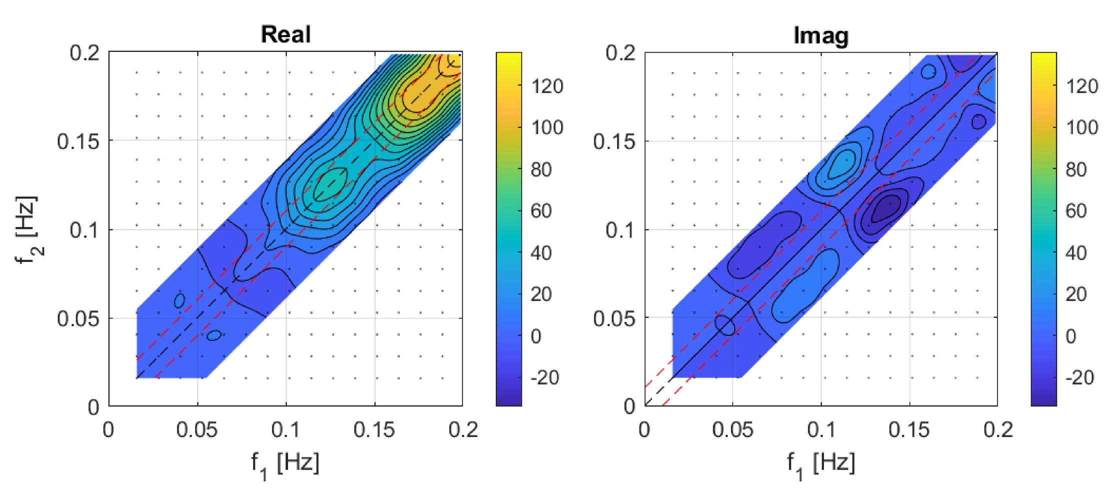
with Fourier coefficient for difference frequency ω_q:

$$F_D(\omega_q) = \sum_{q=k-r} Q(\omega_r, \omega_k)\, \zeta^*(\omega_r)\, \zeta(\omega_k)$$

Here, summation is over all wave-frequency pairs $< \omega_r, \omega_k >$ with constant difference frequency, satisfying $|\omega_k - \omega_r| = \omega_q$.

Physical interpretation: Two wave components with frequencies ω_r and ω_k have nonlinear interaction and produce a difference frequency force at $\omega_q = |\omega_k - \omega_r|$.

$Q(\omega_r, \omega_k)$: Quadratic transfer function (QTF) defined in a bi-frequency plane of interacting frequencies



QTF as a sum of cubic B-spline products

$$Q(\omega_r, \omega_k) = \sum_{i=1}^{N_\omega}\sum_{j=1}^{N_\omega} c_{ij}\mathcal{B}_i(\omega_r)\mathcal{B}_j(\omega_k)$$

- $\mathcal{B}_i(\omega_r)$ and $\mathcal{B}_j(\omega_k)$: cubic B-splines (real valued)
- $\mathbf{c} = [c_{ij}]$: Complex 2D representation of the real-values 1D QTF design vector $\mathbf{x}$.

Overlapping cubic B-splines

Regularization

Append penalty equations to "wave force residual" equation system:

$$\begin{bmatrix}\mathbf{A}_f\\ \lambda_0\mathbf{A}_0\\ \lambda_2\mathbf{A}_2\end{bmatrix}\mathbf{x} - \begin{bmatrix}\mathbf{b}_f\\ \lambda_0\mathbf{b}_0\\ 0\end{bmatrix} = \mathbf{E}$$

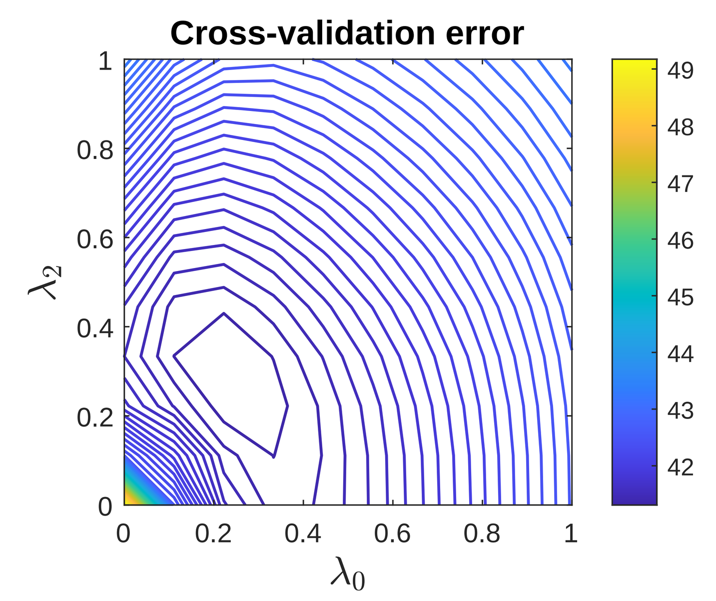
λ_0 controls closeness to reference solution (potential theory)

- We "prefer" solutions close to the reference solution
- ... but λ_0 should be small enough to only effect bi-frequency regions with weak signal
- **Practical interpretation:** We fall back on potential theory gradually as we loose support from data

λ_2 controls smoothness

- We "prefer" a QTF with low curvature (2nd derivative)

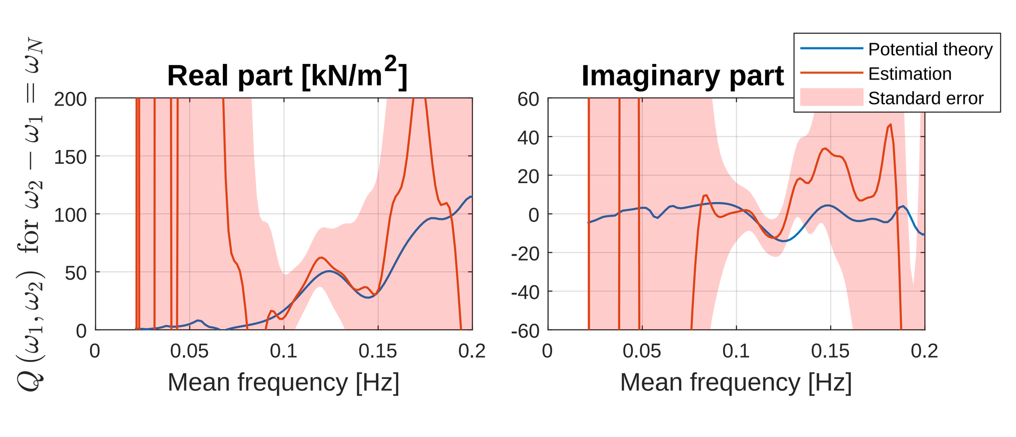
Optimal regularization



Optimal regularization parameters λ_0 and λ_2 found by minimizing cross-validation error

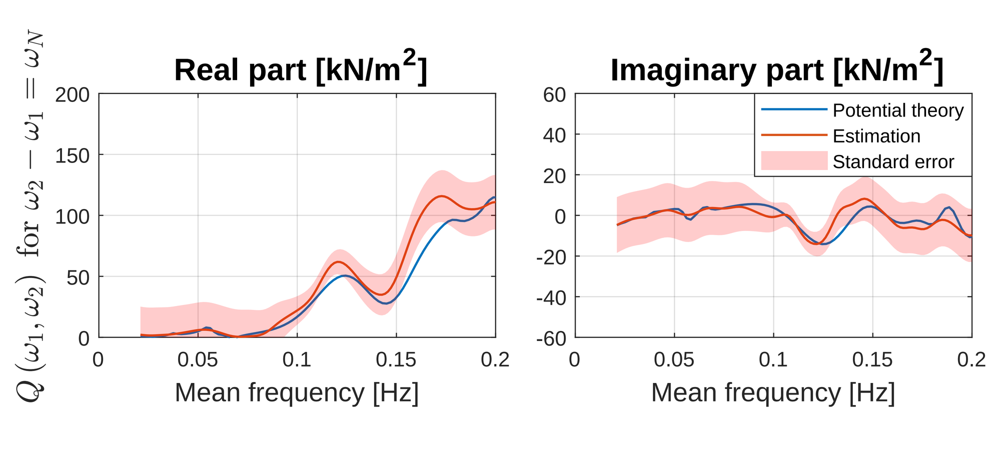
Non-regularized QTF-estimate from 3 hours of measurements

Slice through QTF surface at constant difference frequency:



Regularized QTF-estimate from 3 hours of measurements

Hs = 6.19m, Tp = 9s



Regularized QTF-estimate from 6 × 3 hours of measurements