Optimal Experimental Design for System Identification of Hydrodynamic Models

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Parameter Estimation

This work presents a robust approach for designing cyber-physical experiments for estimating the hydrodynamic properties of a floating structure. The work is an extension of the work presented by Sauder [1]. Without loss of generality, the motions of the system are assumed to be decoupled resulting in a set of scalar linear equations that take the following form:

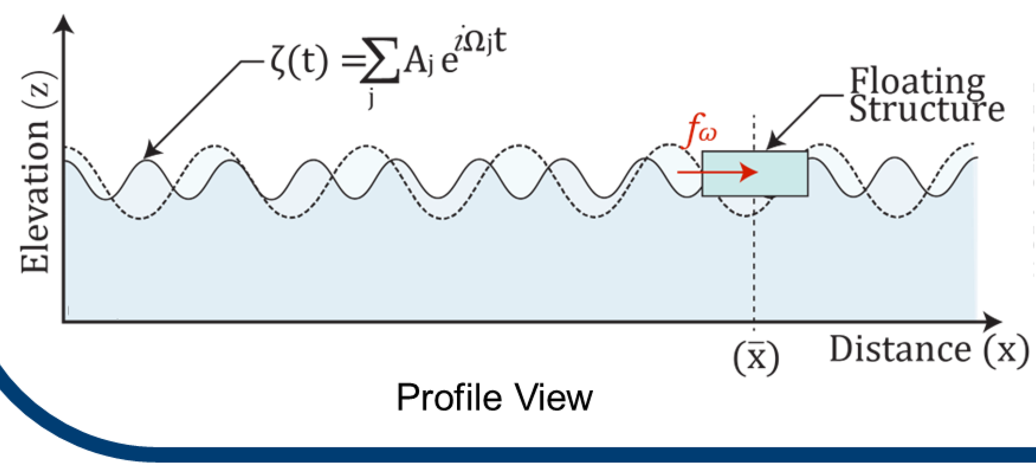
To be identified · Controlled · Measured

$$(m+\hat{a})\ddot{\eta}(t)+\left(\hat{d}_h+d_m\right)\dot{\eta}(t)+k_m\eta(t)=f_\omega(t) \quad (1)$$

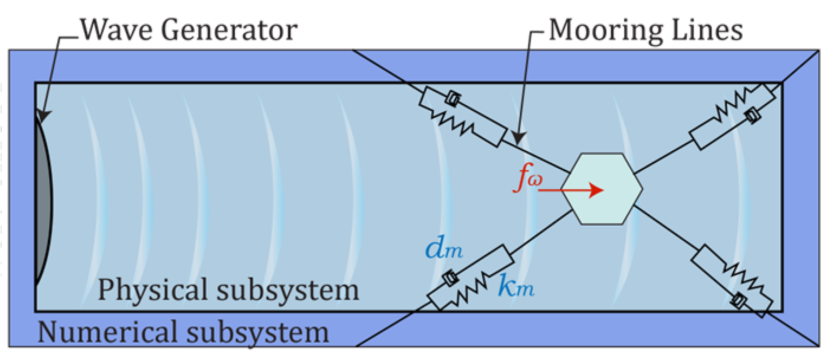
where,

- $m, \hat{a}$ – mass (structural + hydrodynamic)
- $d_m, \hat{d}_h$ – damping (mooring + hydrodynamic)
- k_m – stiffness (mooring)
- η – structural response
- f_ω – low frequency hydrodynamic force

The problem is schematically illustrated in the figure below.



Profile View



Plan View

The first step in the experimental design is to estimate the low frequency added mass and damping parameters from a set of experiments $i=\{1,\dots,N\}$ with identical wave elevation profiles but different sets of mooring stiffness and damping values (k_m, d_m):

$$(m+\hat{a})\ddot{\eta}^{(i)}(t)+\left(\hat{d}_h+d_m^{(i)}\right)\dot{\eta}^{(i)}(t)+k_m^{(i)}\eta^{(i)}(t)=f_\omega^{(i)}(t) \quad (2)$$

The key idea behind this approach is the invariance of the wave excitation $f_\omega(t)$ between experiments. Whereas the quantities $a\ddot{\eta}$ and $d_h\dot{\eta}$ are response-dependent and will vary between experiments. This means that for the correct value of $\hat{a}$ and $\hat{d}_h$ the low-frequency force time-history $f_\omega^{(i)}(t)$ will be identical between experiments. Hence, the added mass and damping can be found by minimizing a cost function proportional to the variance of the force time-history:

$$Q(a,d_h;\mathbf{k}_m,\mathbf{d}_m)=\int_0^T \mathrm{Var}_i\left[f_\omega^{(i)}(t)\right]dt \quad (3)$$

The minimum of this function yields the correct added mass and damping:

$$\hat{a},\hat{d}_h=\underset{a,d_h}{\arg\min}\int_0^T \mathrm{Var}_i\left[f_\omega^{(i)}(t)\right]dt \quad (4)$$

Experimental Design

The accuracy of the method described in the block above depends, to a large extent, on the shape of the cost function defined in (3). To facilitate faster and more accurate convergence to the global minimum, we would like to design an optimal cost function, $Q(a,d_h;\hat{\mathbf{k}}_m,\hat{\mathbf{d}}_m)$ [2]. We start with the Fisher information matrix which is defined as the hessian of the logarithm of the cost function (3) with respect to the unknown parameters:

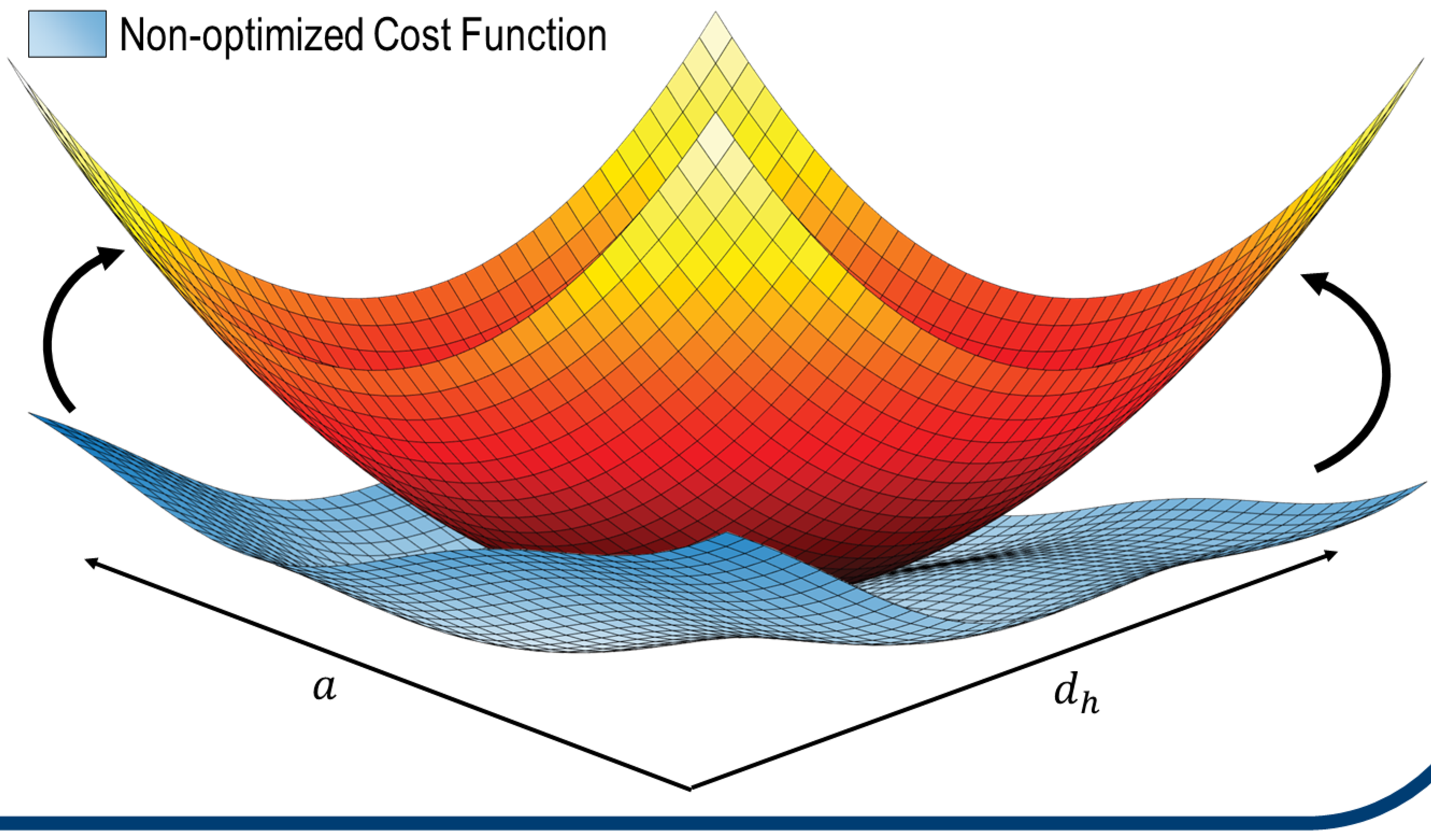
$$\mathcal{I}(a,d_h;\mathbf{k}_m,\mathbf{d}_m)=\begin{bmatrix}\dfrac{\partial^2\log(Q)}{\partial a^2} & \dfrac{\partial^2\log(Q)}{\partial d_h\partial a}\\ \dfrac{\partial^2\log(Q)}{\partial a\partial d_h} & \dfrac{\partial^2\log(Q)}{\partial d_h^2}\end{bmatrix}_{a,\,d_h} \quad (5)$$

Note that in (5), for clarity, the dependence on the parameters is omitted. Since the true parameters $\hat{a},\hat{d}_h$ are unknown, we resort to optimizing the expected value of (5). This is achieved using a Monte Carlo approach sampled from a uniform distribution of a, d_h with a predefined range somewhere within which the true parameters lie. Lastly, we seek A-optimality which minimizes the expected value of the trace of the inverse of the Fisher information matrix:

$$\hat{\mathbf{k}}_m,\hat{\mathbf{d}}_m=\arg\min_{\mathbf{k}_m,\mathbf{d}_m}\mathbb{E}\left[\mathrm{tr}\left\{\left(\mathcal{I}(a,d_h;\mathbf{k}_m,\mathbf{d}_m)\right)^{-1}\right\}\right] \quad (6)$$

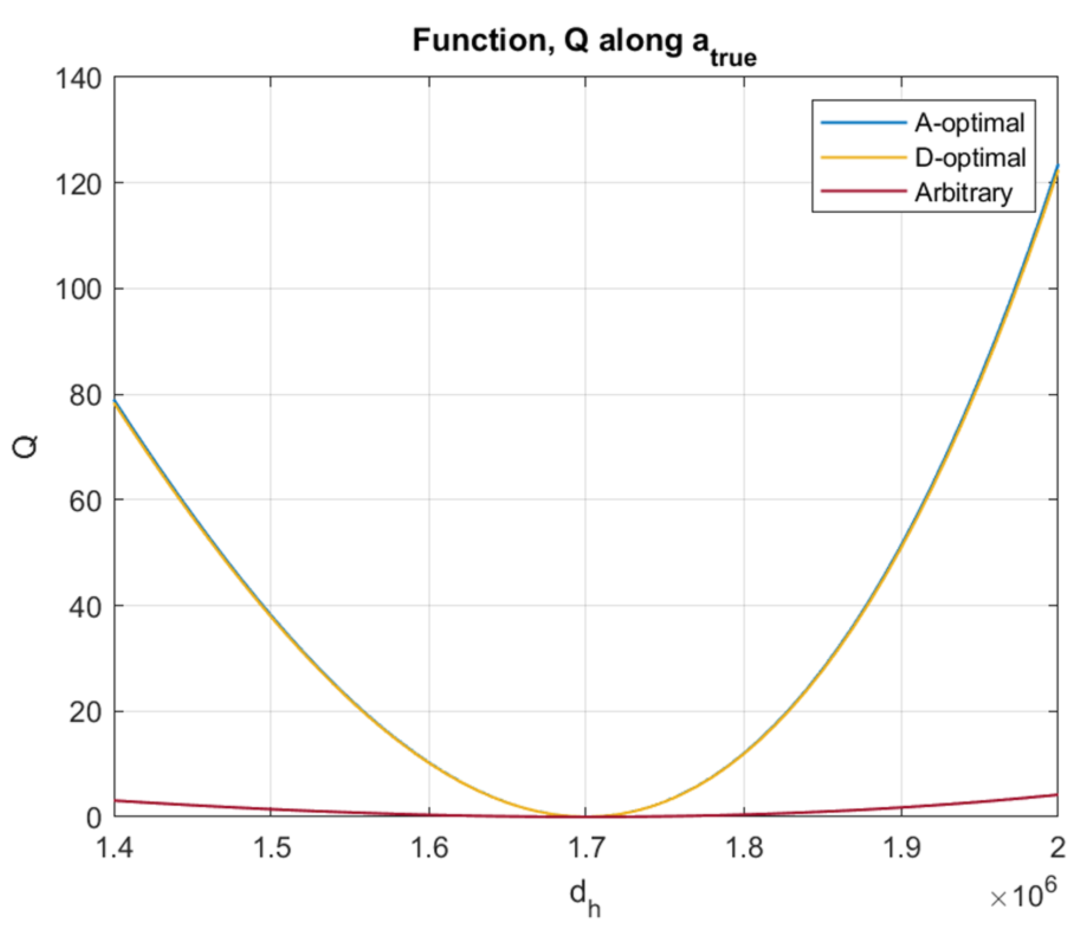
where the hat symbol denotes the set of parameters that produce an optimal cost function.

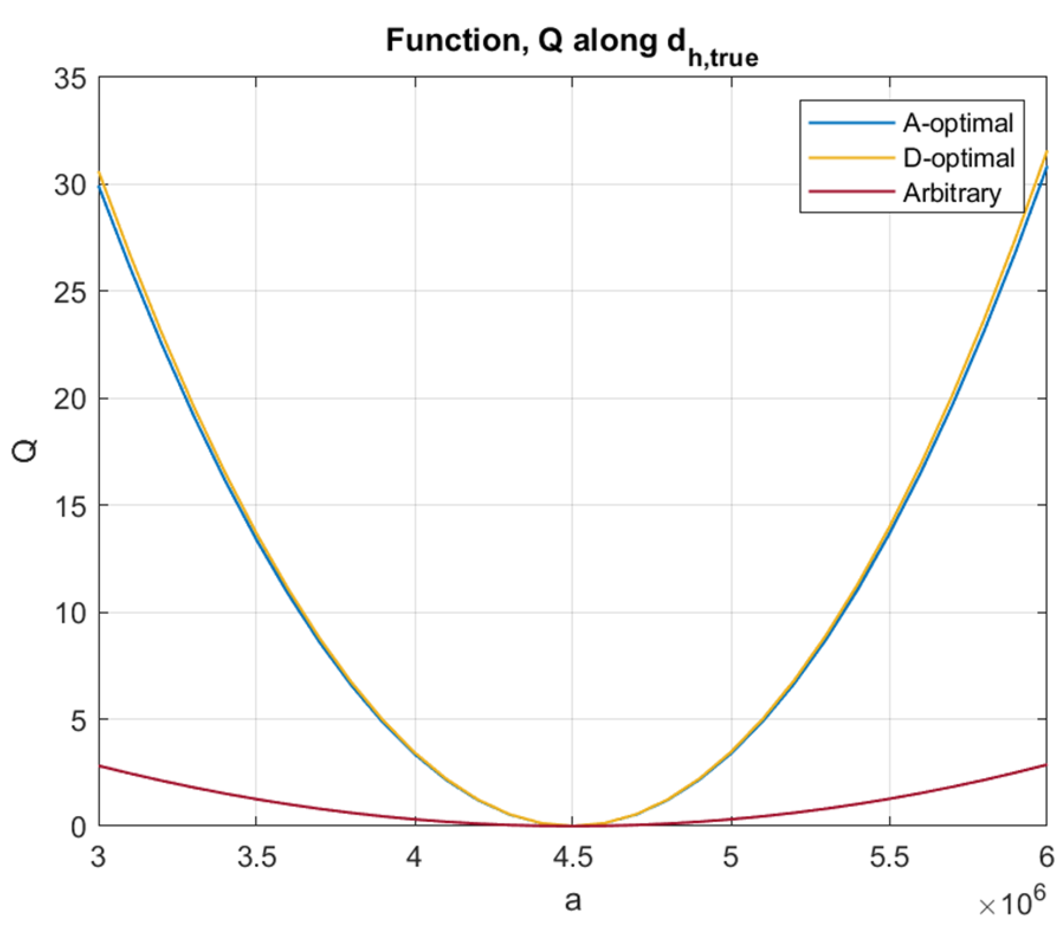
An illustrative depiction of the optimization process of the cost function $Q(a,d_h;\mathbf{k}_m,\mathbf{d}_m)$ is shown next. The optimized surface exhibits a well-defined minimum which facilitates fast and accurate convergence.



Results

The proposed procedure was validated with synthetically generated data with $\hat{d}_h = 1.7e6$ Ns/m and $\hat{a} = 4.5e6$ kg. The figures below show a comparison of a cost function with arbitrarily selected parameters $\{\mathbf{k}_m,\mathbf{d}_m\}$ versus a cost function whose parameters have been optimized with A- and D-optimality. The figures visually depict the improved curvature of the optima.





Outlook

Currently, the method is suitable for the design of experiments that identify the low-frequency added mass and damping. The next step is to extend the approach to the entire frequency bandwidth. This is slightly more challenging because of the frequency-dependence of the added mass and damping. Instead of estimating the asymptotic values of a, d_h the procedure estimates the zeros and poles of the retardation function $R(t-\tau)$ with the equation of motion now being [3]:

$$[M+M_\infty]\ddot{X}+\int_{-\infty}^{t}\dot{X}(\tau)R(t-\tau)d\tau+KX=F(t)$$

References

[1] Sauder T 2021 Empirical estimation of low-frequency nonlinear hydrodynamic loads on moored structures Applied Ocean Research 117 102895

[2] Abbiati G and Sauder T 2021 Proc. of the ASME 2021 40th Int. & Conference on Ocean, Offshore and Arctic Engineering p 1

[3] Molin B 2023 Offshore Structure Hydrodynamics (Cambridge, United Kingdom: Cambridge University Press)

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