Data-Driven Modeling of Hydrodynamic Loading using NARX and Harmonic Probing

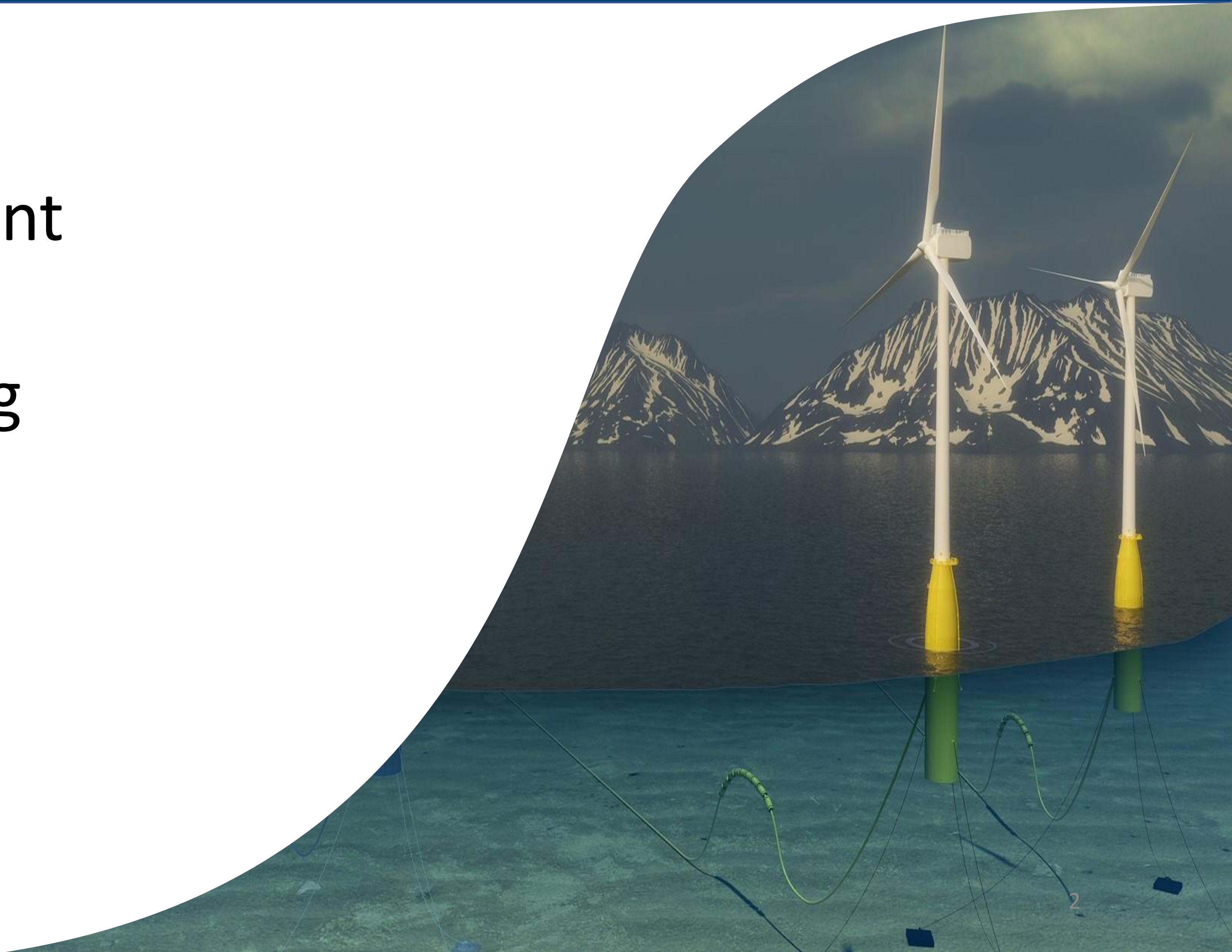
David Stamenov
Department of Civil and Architectural Engineering
University of Aarhus
Aarhus, Denmark
Email: stamenovd@cae.au.dk

Giuseppe Abbiati
Department of Civil and Architectural Engineering
University of Aarhus
Aarhus, Denmark
Email: abbiati@cae.au.dk

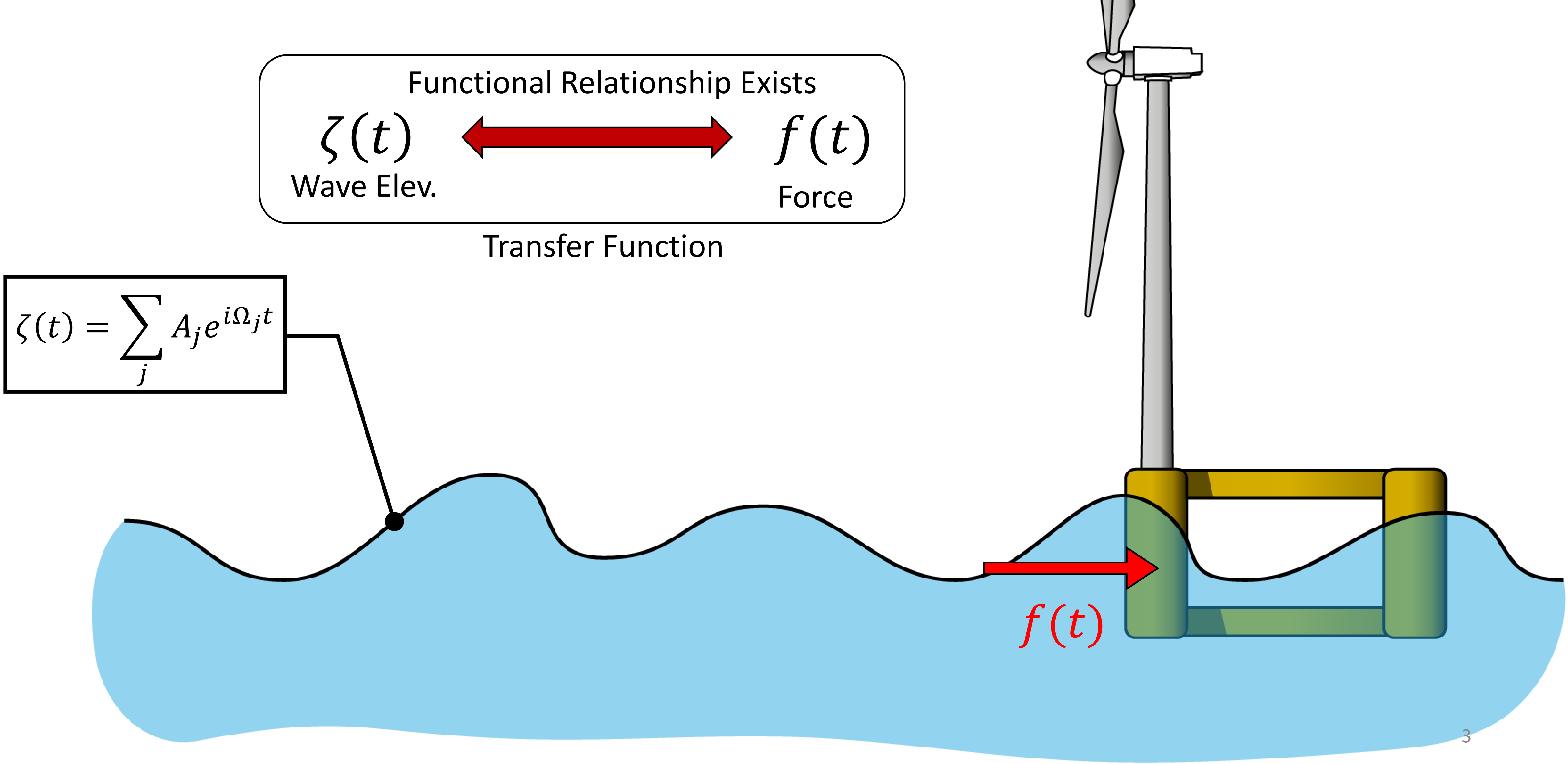
Thomas Sauder
SINTEF Ocean AS
Norwegian University of Science and Technology
Trondheim, Norway
Email: thomas.sauder@sintef.no

Outline

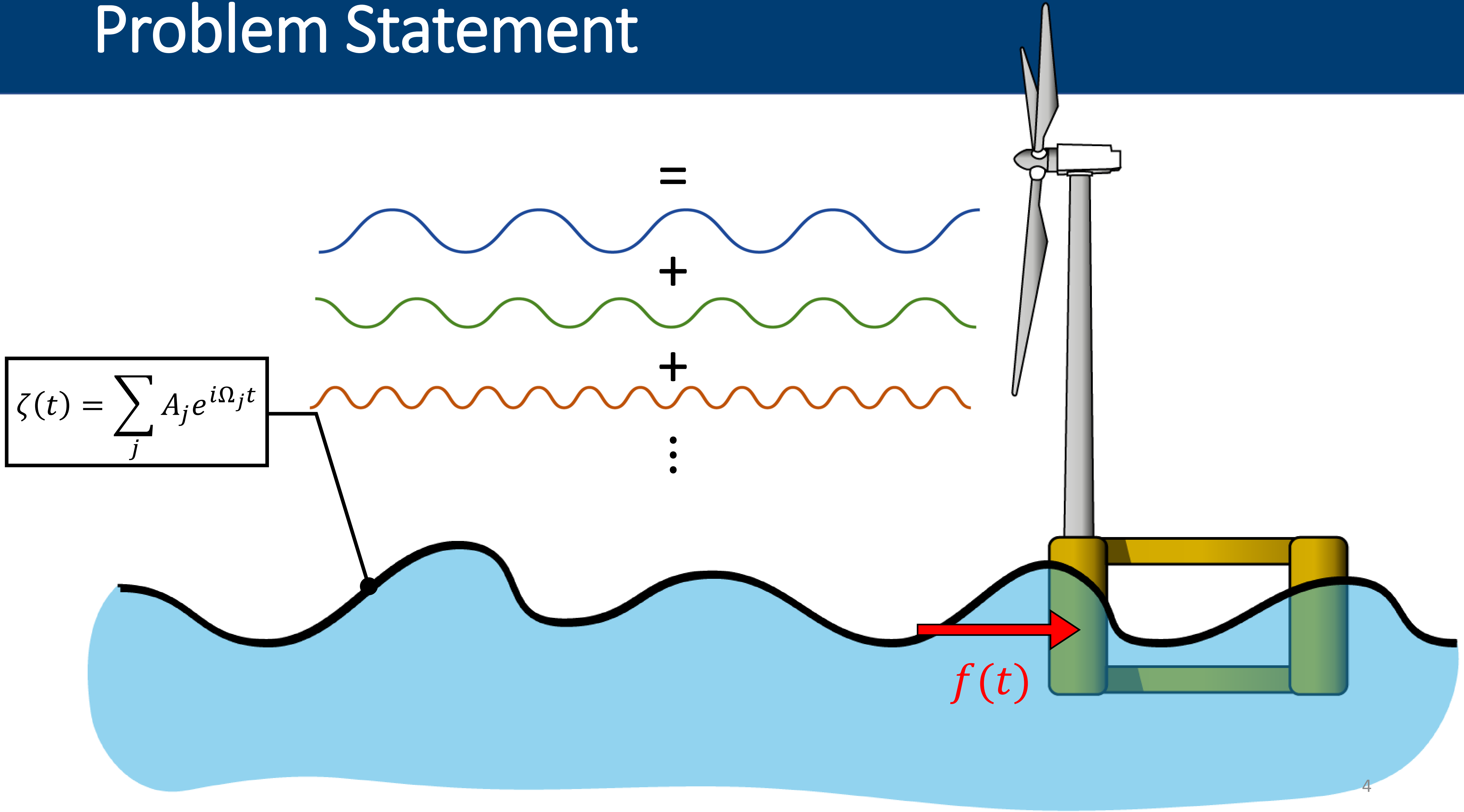
1. Problem Statement
2. Kriging-NARX
3. Harmonic Probing



Problem Statement



Problem Statement



Problem Statement

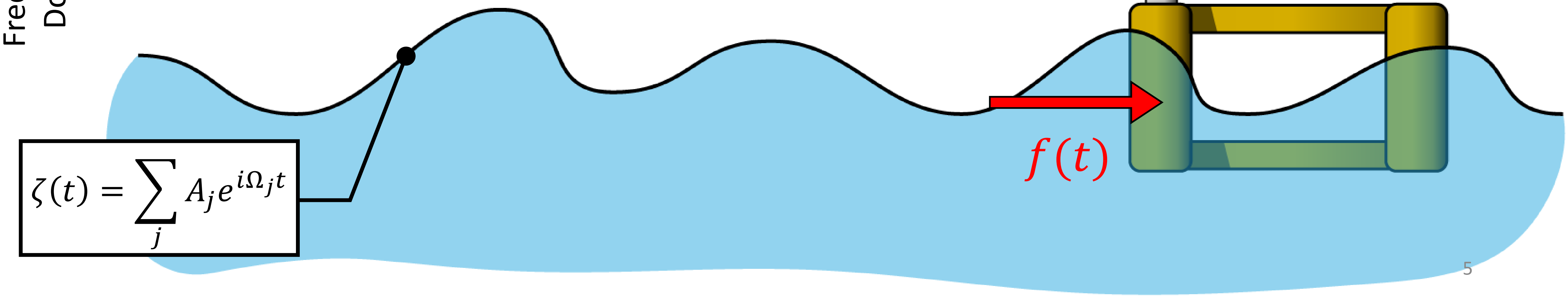
Time Domain

$$f_1(t) = \int_{-\infty}^{+\infty} h_1(\tau_1)\zeta(t-\tau_1)d\tau_1$$

1st Order Impulse Response Function

Frequency Domain

$$F_1(\omega) = H_1(\omega)Z(\omega)$$



Problem Statement

Time Domain

$$f(t) = f_1(t) + f_2(t) + \cdots + f_n(t)$$

$$f_1(t) = \int_{-\infty}^{+\infty} h_1(\tau_1)\zeta(t-\tau_1)d\tau_1$$

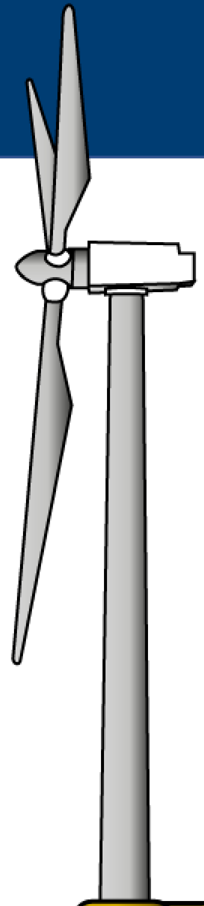
$$f_2(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h_2(\tau_1,\tau_2)\zeta(t-\tau_1)\zeta(t-\tau_2)d\tau_1 d\tau_2$$

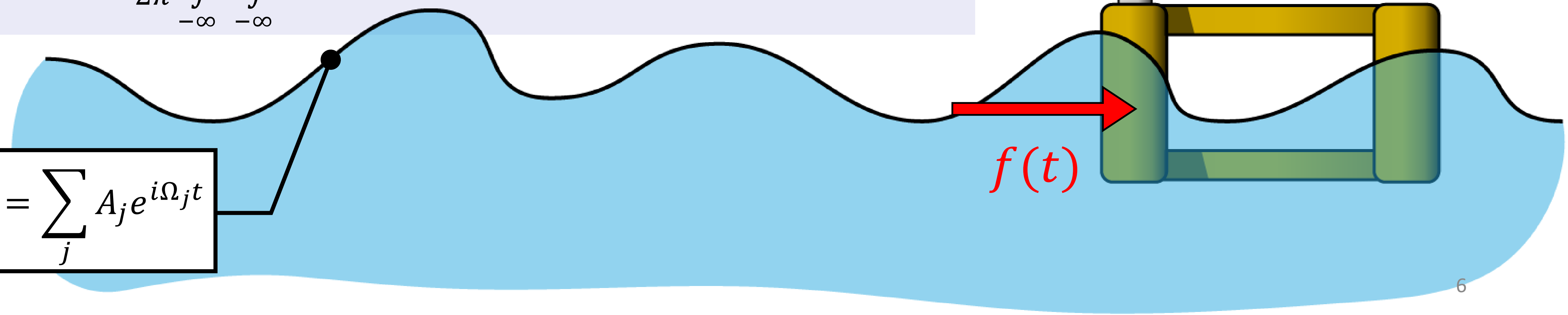
2nd Order Impulse Response Function

Frequency Domain

$$F_1(\omega) = H_1(\omega)Z(\omega)$$

$$F_2(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \delta(\omega-\omega_1-\omega_2)H_2(\omega_1,\omega_2)Z(\omega_1)Z(\omega_2)d\omega_1 d\omega_2$$

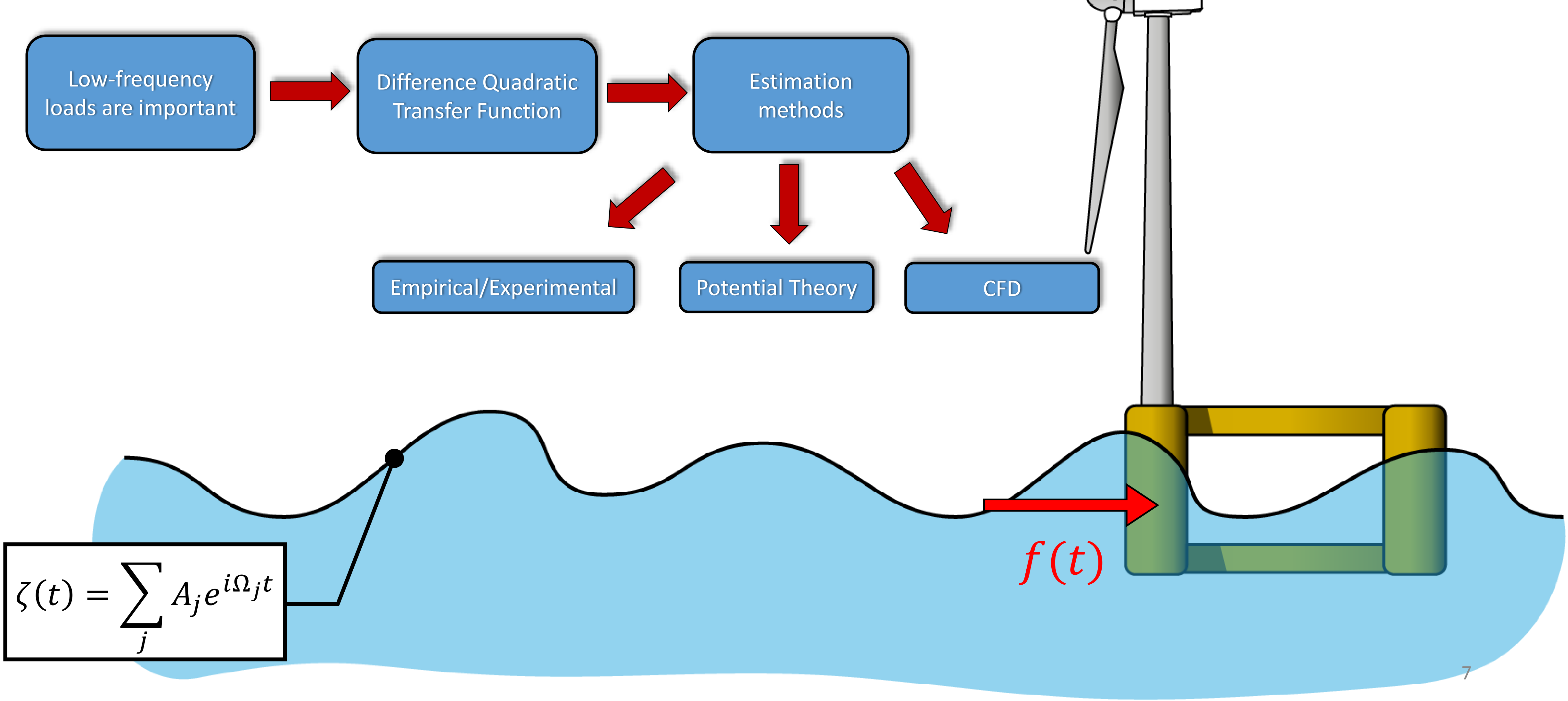




$$\zeta(t) = \sum_j A_j e^{i\Omega_j t}$$

Problem Statement

For moored structures:



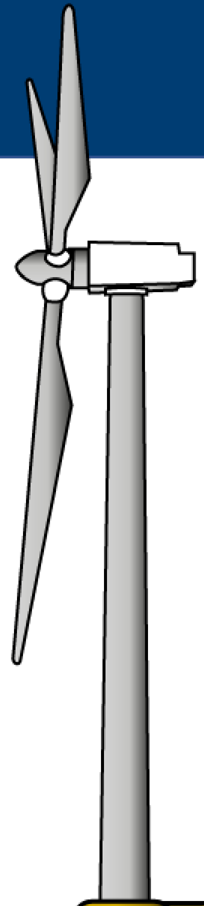
Problem Statement

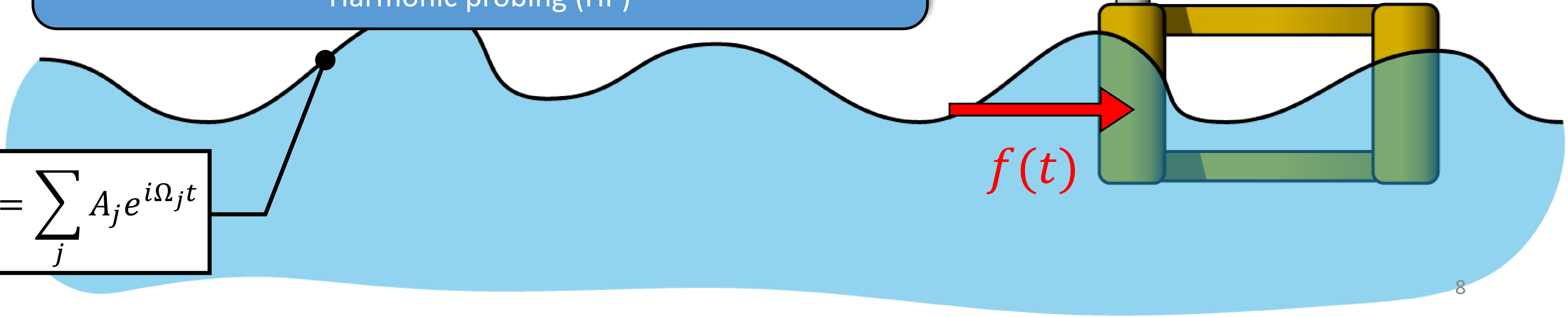
Goal of the study:

- Learn the functional relationship between the force and wave elevation profile via machine learning.
- Extract the transfer functions

Model:

Nonlinear auto-regressive model with exogenous input (NARX)
+
Harmonic probing (HP)



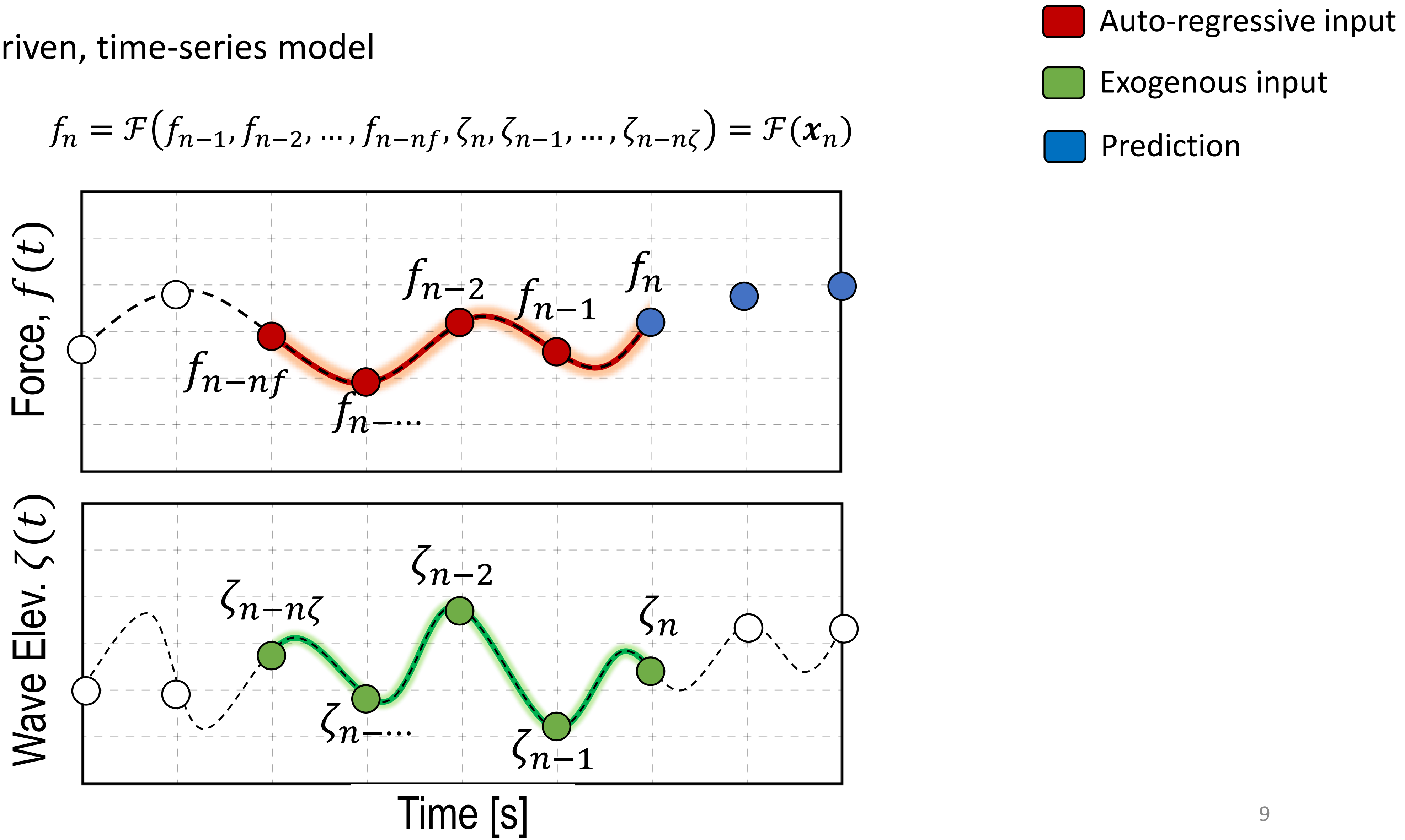


$$\zeta(t) = \sum_{j} A_j e^{i\Omega_j t}$$

Kriging-NARX

- Forecasting, data-driven, time-series model
- What it does:

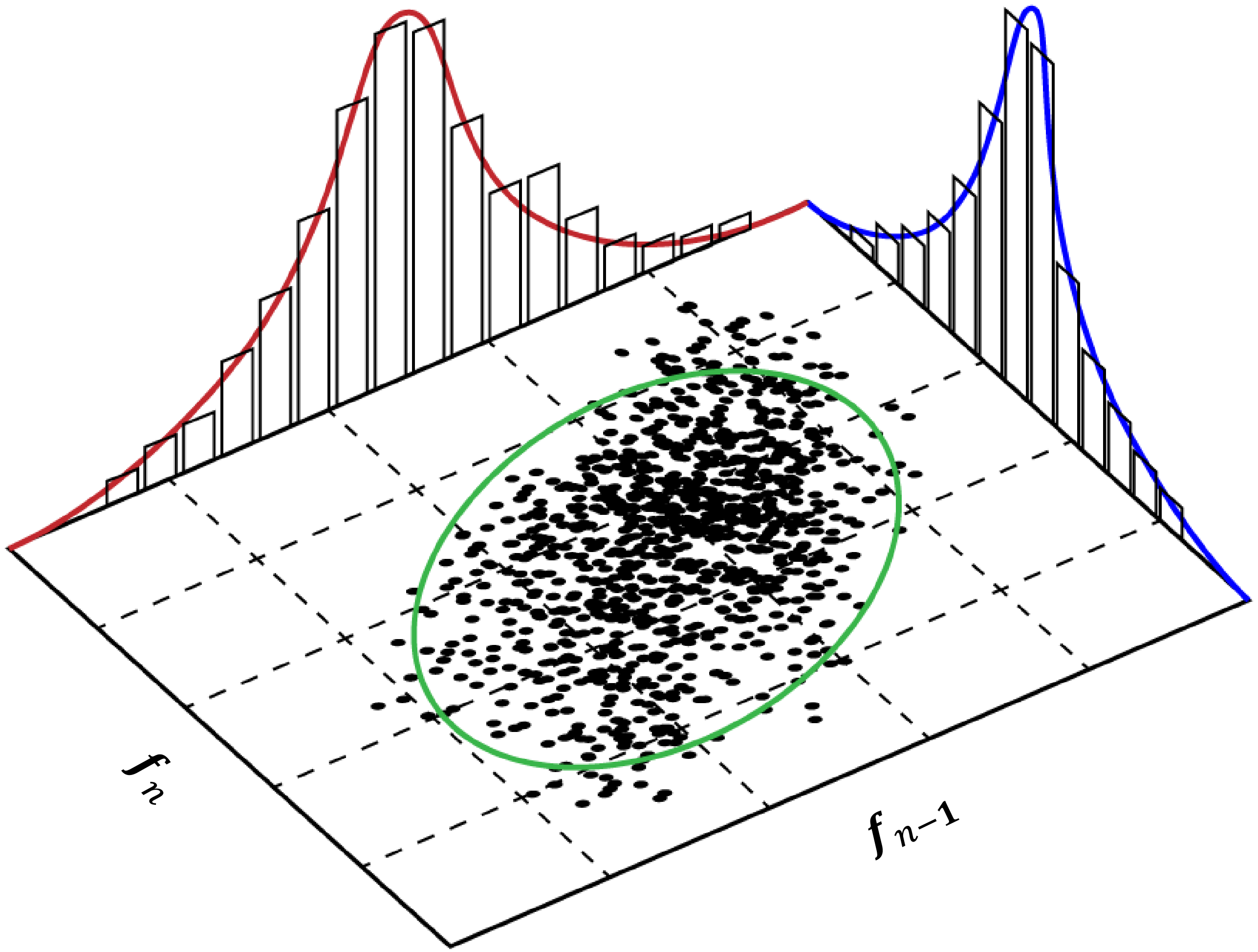
$$f_n = \mathcal{F}\left(f_{n-1}, f_{n-2}, \dots, f_{n-nf}, \zeta_n, \zeta_{n-1}, \dots, \zeta_{n-n\zeta}\right) = \mathcal{F}(\boldsymbol{x}_n)$$



Kriging-NARX

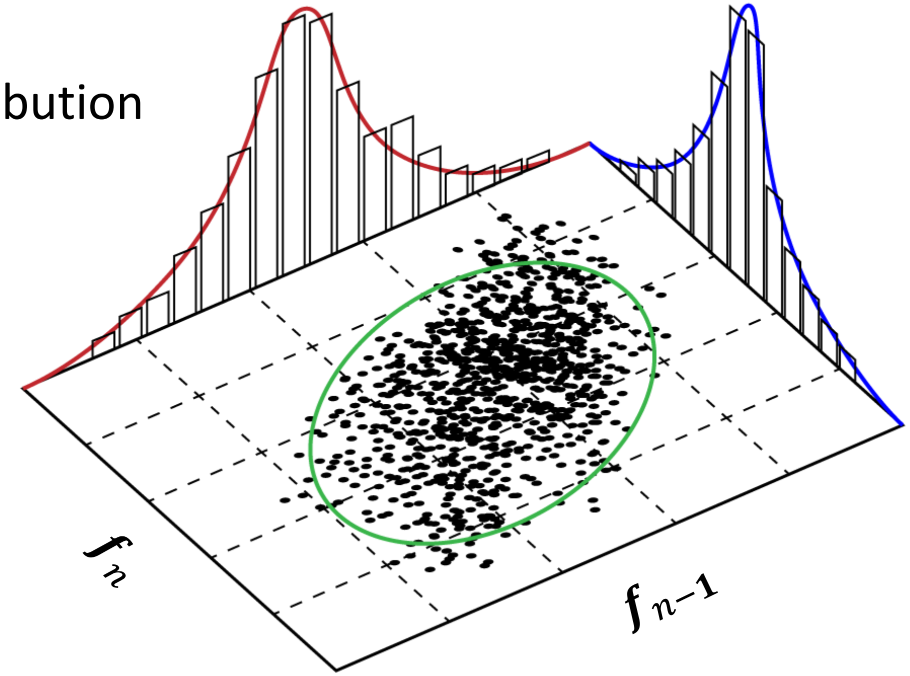
- How does it work?
- Assumes a Joint Normal Distribution

$$f_n = \mathcal{F}\left(f_{n-1}, f_{n-2}, \dots, f_{n-nf}, \zeta_n, \zeta_{n-1}, \dots, \zeta_{n-n\zeta}\right) = \mathcal{F}(\boldsymbol{x}_n)$$



Kriging-NARX

- How does it work?
- Assumes a Joint Normal Distribution



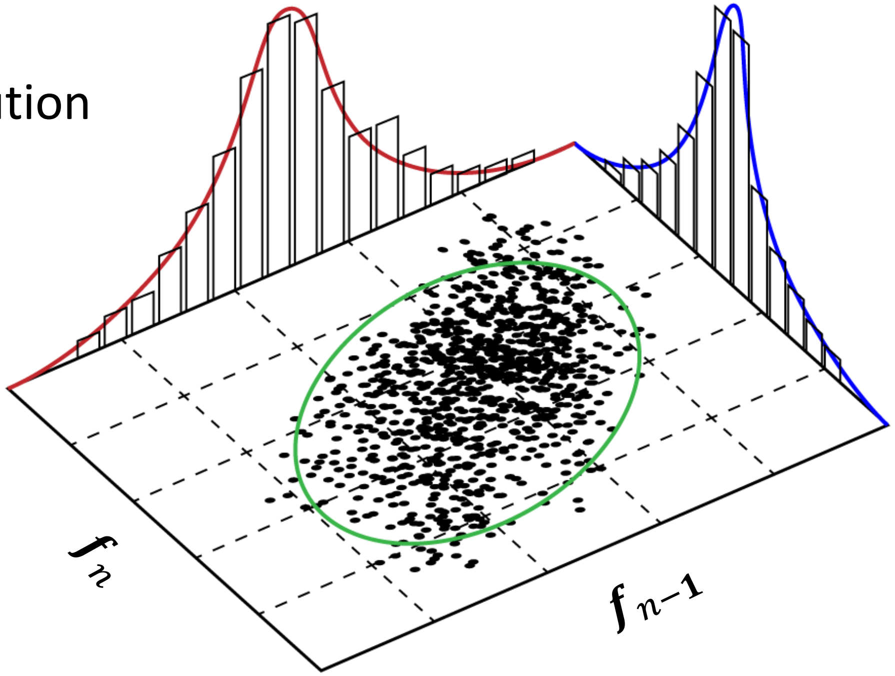
θ: length scale
σ_f: noise parameter
σ_e: nugget for num. stability

$$\begin{Bmatrix} f_n \\ f_{n-1} \\ f_{n-2} \\ \vdots \end{Bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} k(\boldsymbol{x}_n, \boldsymbol{x}_n) + \sigma_e^2 & k(\boldsymbol{x}_n, \boldsymbol{x}_{n-1}) & k(\boldsymbol{x}_n, \boldsymbol{x}_{n-2}) & \cdots \\ k(\boldsymbol{x}_{n-1}, \boldsymbol{x}_n) & k(\boldsymbol{x}_{n-1}, \boldsymbol{x}_{n-1}) + \sigma_e^2 & k(\boldsymbol{x}_{n-1}, \boldsymbol{x}_{n-2}) + \sigma_e^2 & \cdots \\ k(\boldsymbol{x}_{n-2}, \boldsymbol{x}_n) & k(\boldsymbol{x}_{n-2}, \boldsymbol{x}_{n-1}) + \sigma_e^2 & k(\boldsymbol{x}_{n-2}, \boldsymbol{x}_{n-2}) + \sigma_e^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \right)$$

$$k(\boldsymbol{x}_n, \boldsymbol{x}_{n-m}) = \sigma_f^2 \exp\left\{ \left(-\frac{1}{2\boldsymbol{\theta}^2} \| \boldsymbol{x}_n - \boldsymbol{x}_{n-m} \|^2 \right) \right\}$$

Kriging-NARX

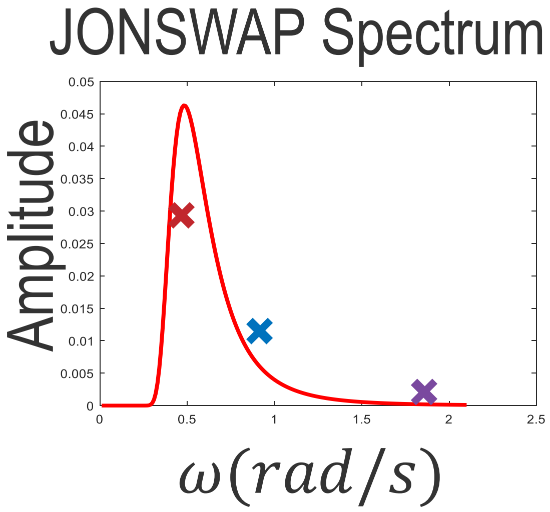
- How does it work?
- Assumes Joint Normal Distribution

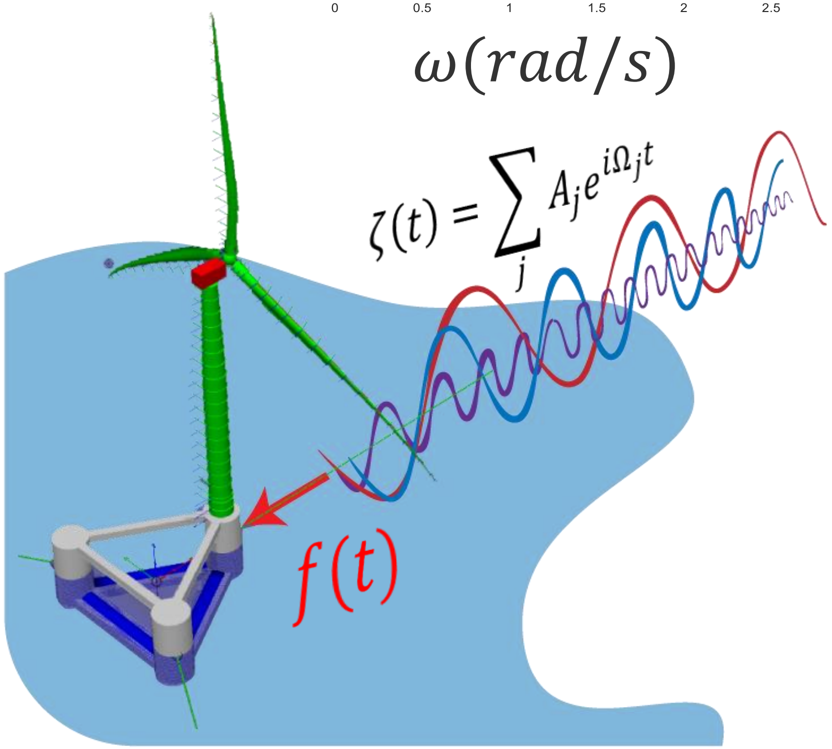


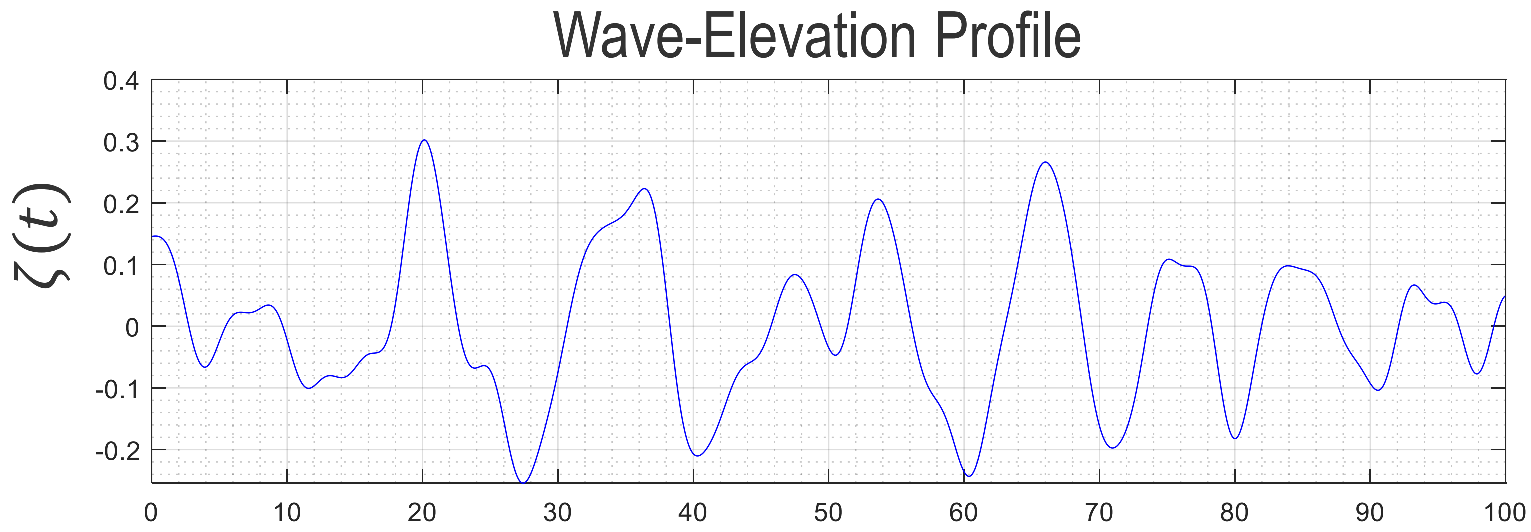
$$\begin{Bmatrix} f_n \\ \boldsymbol{F} \end{Bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ \boldsymbol{0} \end{bmatrix}, \begin{bmatrix} K(\boldsymbol{x}_n, \boldsymbol{x}_n) + \sigma_e^2 & \boldsymbol{K}(\boldsymbol{X}, \boldsymbol{x}_n) \\ \boldsymbol{K}(\boldsymbol{x}_n, \boldsymbol{X}) & \boldsymbol{K}(\boldsymbol{X}, \boldsymbol{X}) + \sigma_e^2 \boldsymbol{I} \end{bmatrix} \right)$$

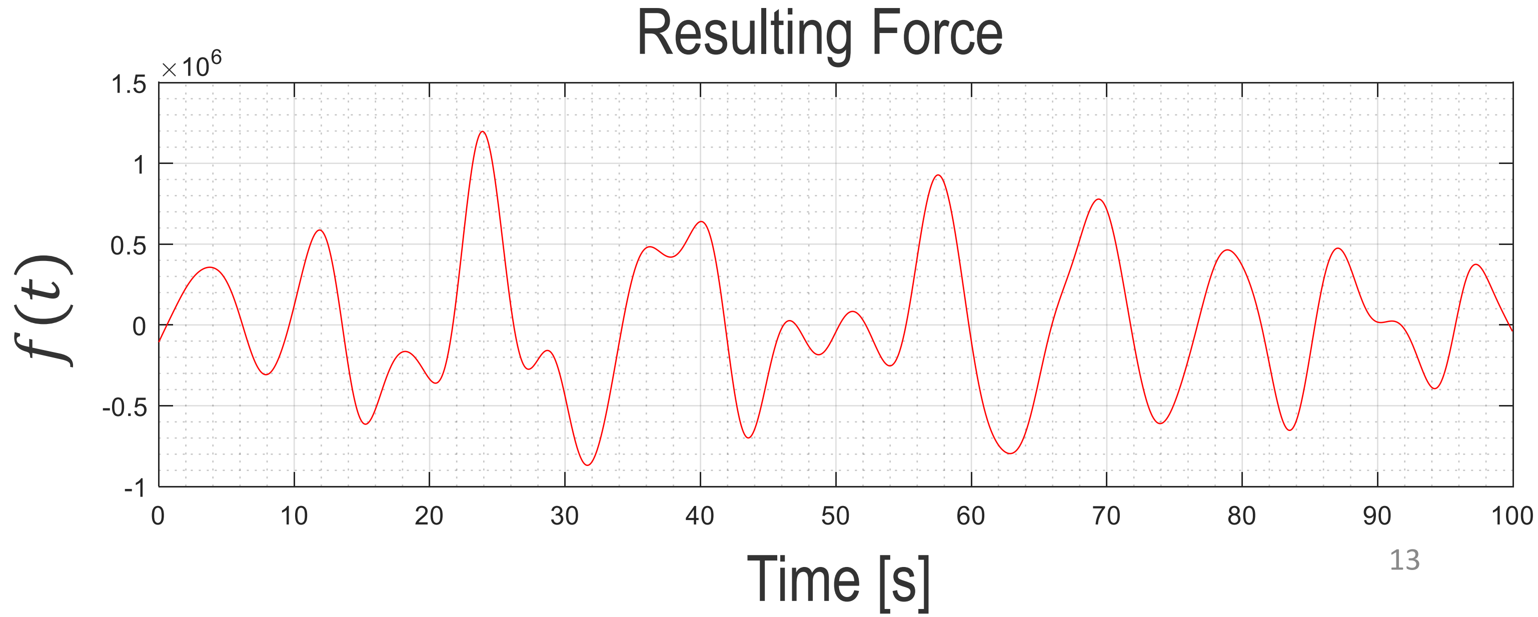
$$E[f_n | \boldsymbol{F}, \boldsymbol{X}] = \mathcal{F}(\boldsymbol{x}_n | \boldsymbol{X}) = \boldsymbol{K}(\boldsymbol{x}_n, \boldsymbol{X})[\boldsymbol{K}(\boldsymbol{X}, \boldsymbol{X}) + \sigma_e^2 \boldsymbol{I}]^{-1} \boldsymbol{F}$$

Kriging-NARX

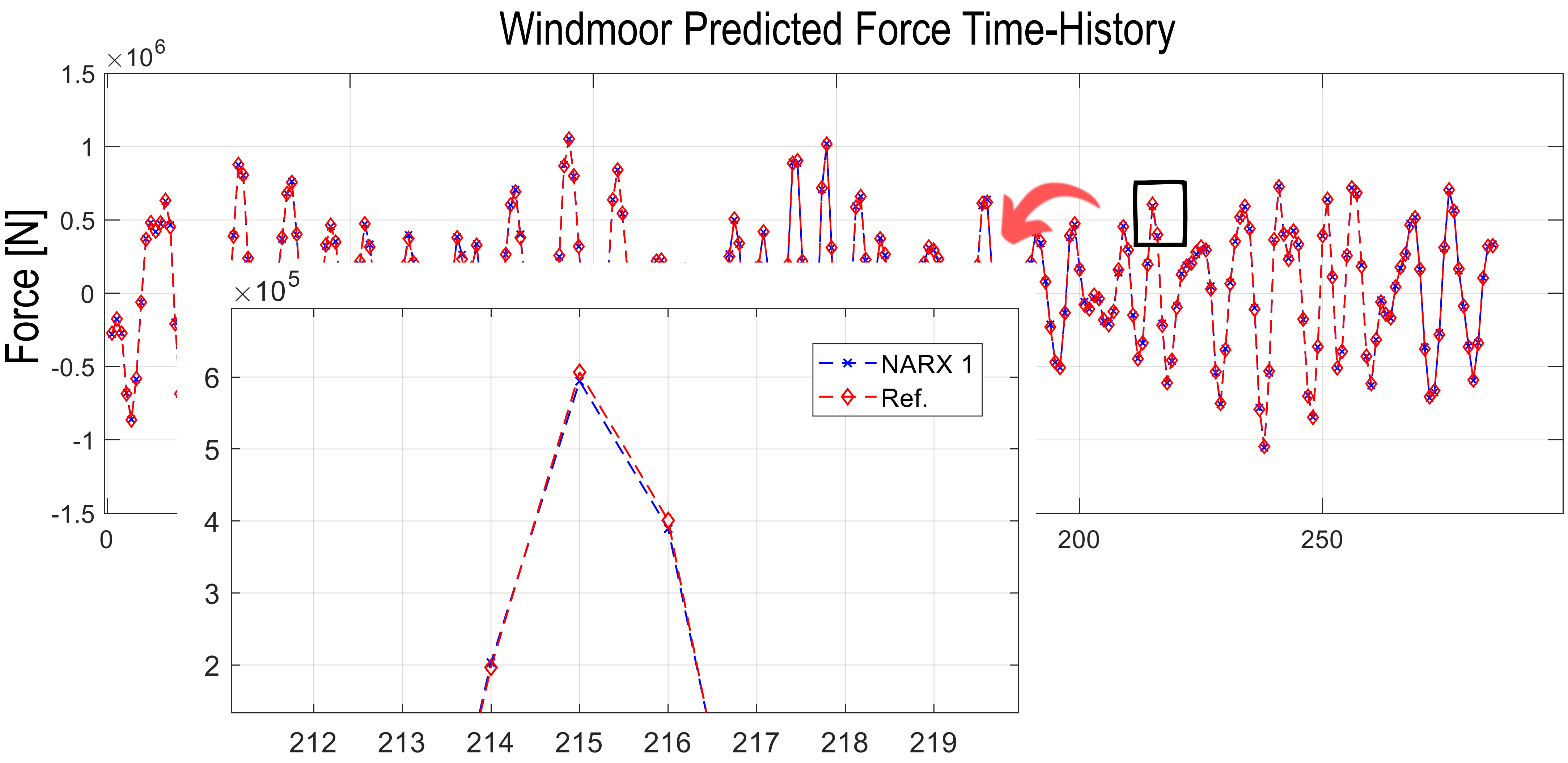








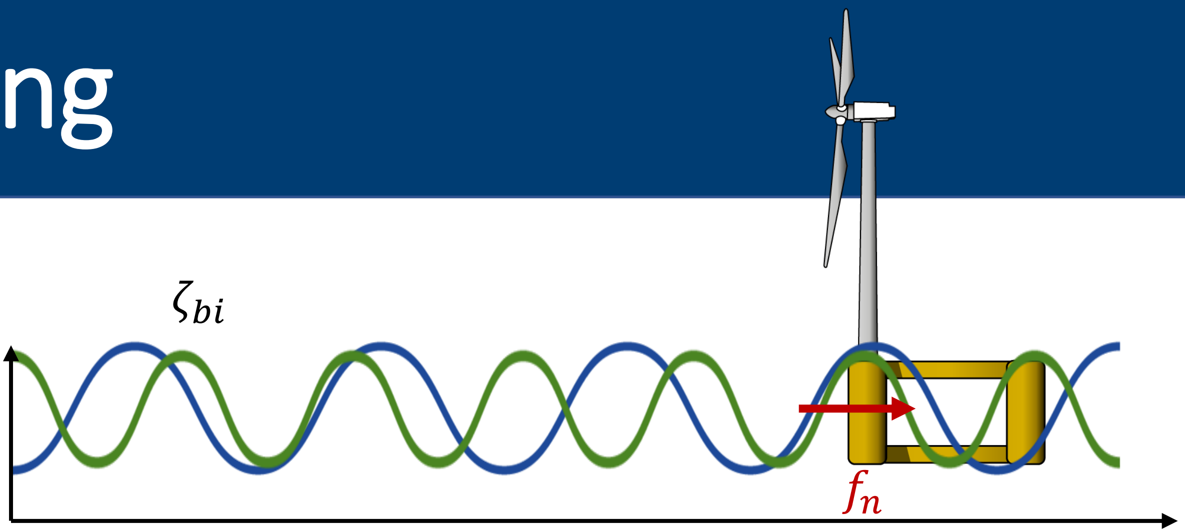
Kriging-NARX



Harmonic Probing

1.0 1.0

$$\zeta_{bi}(t) = A_1 e^{i\Omega_1 t_n} + A_2 e^{i\Omega_2 t_n}$$



$$f_n(t) = H_0 + H_1(\Omega_1)e^{i\Omega_1 t_n} + H_1(\Omega_2)e^{i\Omega_2 t_n} + 2H_2(\Omega_1,\Omega_2)e^{i(\Omega_1+\Omega_2)t_n} + H_2(\Omega_1,\Omega_1)e^{i(2\Omega_1)t_n} + H_2(\Omega_2,\Omega_2)e^{i(2\Omega_2)t_n}$$

NARX

$$E[f_n|\boldsymbol{F},\boldsymbol{X}] = \boldsymbol{K}(\boldsymbol{x}_n,\boldsymbol{X})[\boldsymbol{K}(\boldsymbol{X},\boldsymbol{X}) + \sigma_e^2\boldsymbol{I}]^{-1}\boldsymbol{F}$$

Volterra

$$f_n(t) = H_0 + H_1(\Omega_1)e^{i\Omega_1 t_n} + 2H_2(\Omega_1,\Omega_2)e^{i(\Omega_1+\Omega_2)t_n} \ldots$$

Harmonic Probing

NARX

$$E[f_n | \boldsymbol{F}, \boldsymbol{X}] = \boldsymbol{K}(\boldsymbol{x}_n, \boldsymbol{X})[\boldsymbol{K}(\boldsymbol{X}, \boldsymbol{X}) + \sigma_e^2 \boldsymbol{I}]^{-1} \boldsymbol{F}$$

Volterra

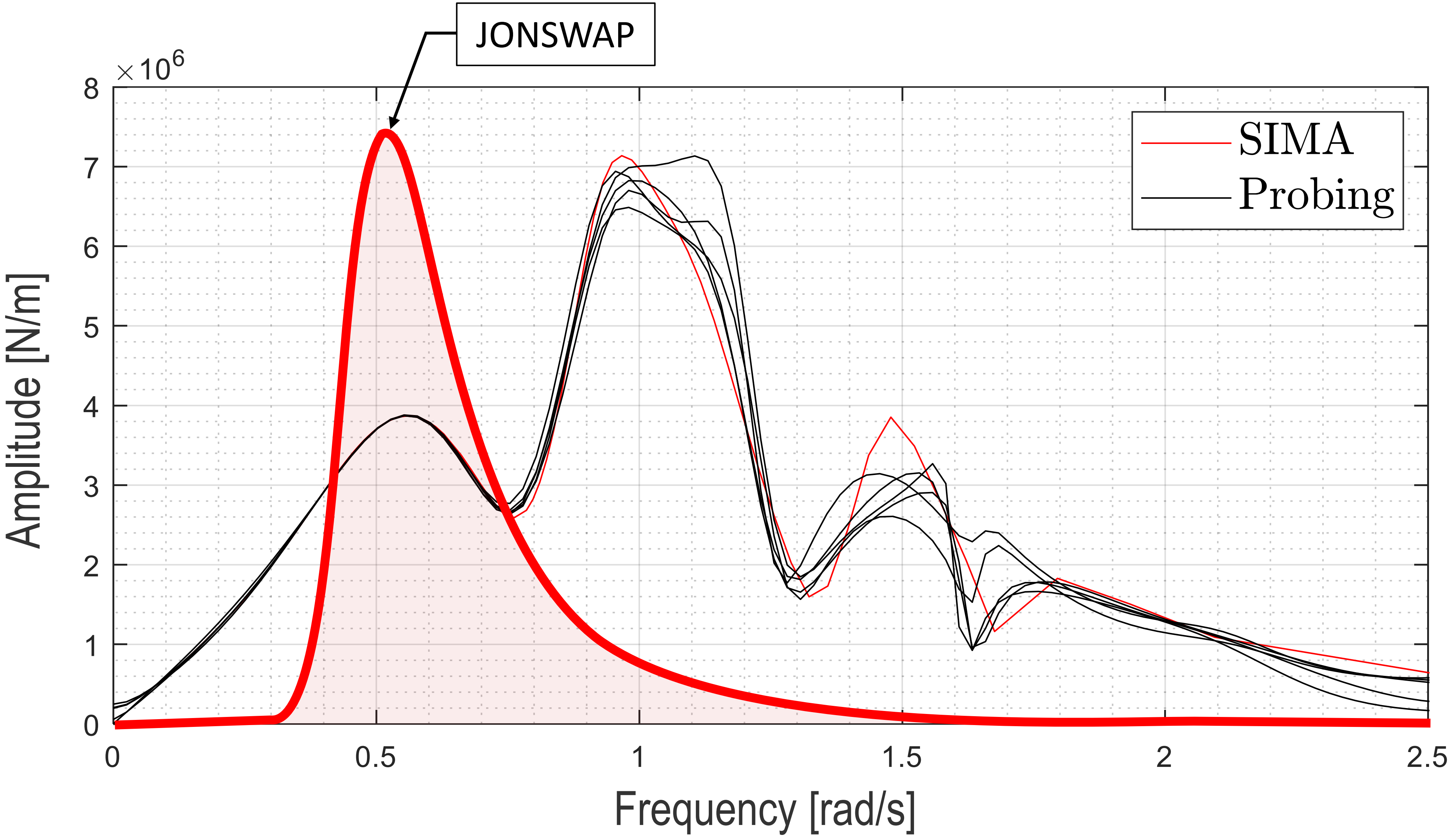
$$f_n(t) = H_0 + H_1(\Omega_1)e^{i\Omega_1 t_n} + 2H_2(\Omega_1, \Omega_2)e^{i(\Omega_1+\Omega_2)t_n} \ldots$$

$$\boldsymbol{x}_n = \begin{bmatrix} f_{n-1} \\ \zeta_n \\ \zeta_{n-1} \end{bmatrix}^T = \begin{bmatrix} \left[H_0 + H_1(\Omega_1)e^{i\Omega_1} + H_1(\Omega_2)e^{i\Omega_2} + 2H_2(\Omega_1, \Omega_2)e^{i(\Omega_1+\Omega_2)} + \cdots\right]e^{t_{n-1}} \\ e^{i\Omega_1 t_n} + e^{i\Omega_2 t_n} \\ e^{i\Omega_1 (t_{n-1})} + e^{i\Omega_2 (t_{n-1})} \end{bmatrix}^{\mathrm{T}}$$

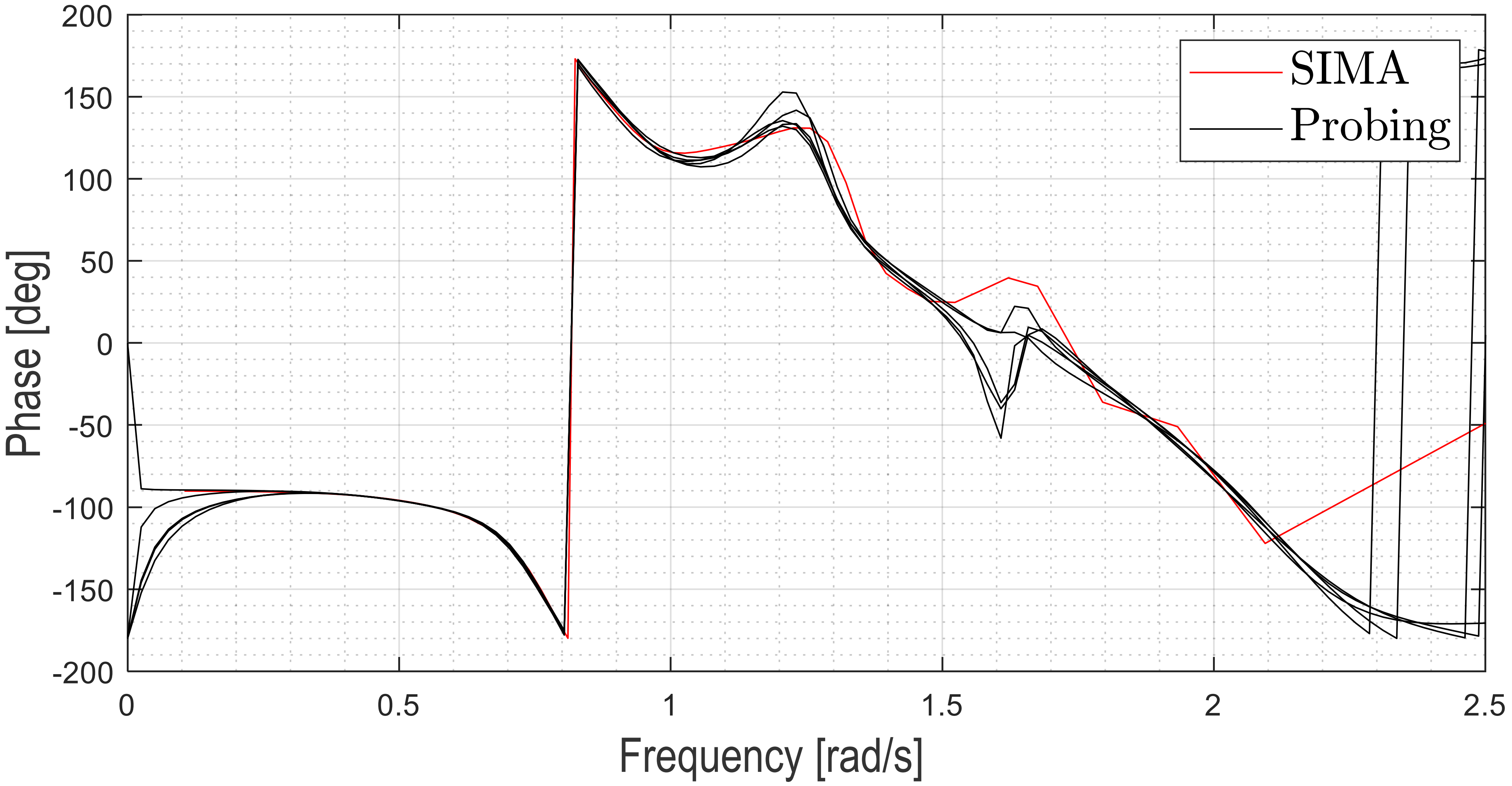
Linearize and solve for H_0, H_1, and H_2

$$f_n = \mathcal{F}(\boldsymbol{x}_n | \boldsymbol{X}, \boldsymbol{F}) \approx \mathcal{F}(\boldsymbol{0}) + \left.\frac{\partial \mathcal{F}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{0}} \boldsymbol{x}_n + \frac{1}{2}\boldsymbol{x}_n^T \left.\frac{\partial^2 \mathcal{F}}{\partial \boldsymbol{x}^2}\right|_{\boldsymbol{0}} \boldsymbol{x}_n$$

Harmonic Probing

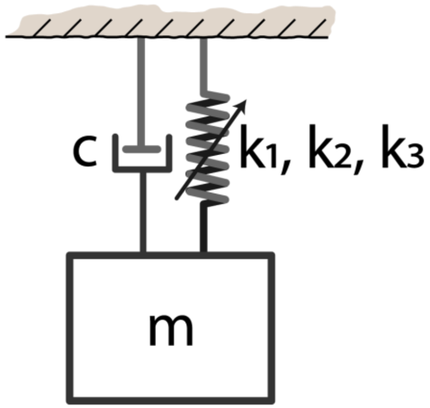


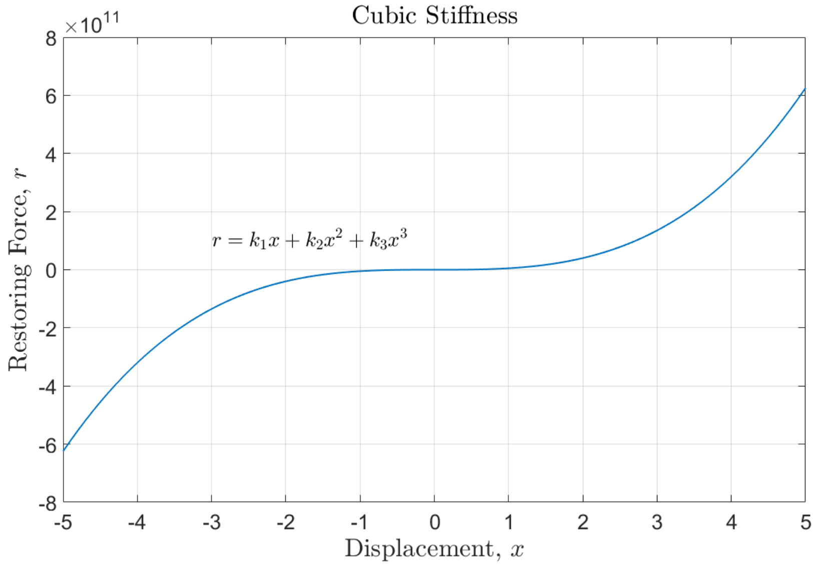
Harmonic Probing

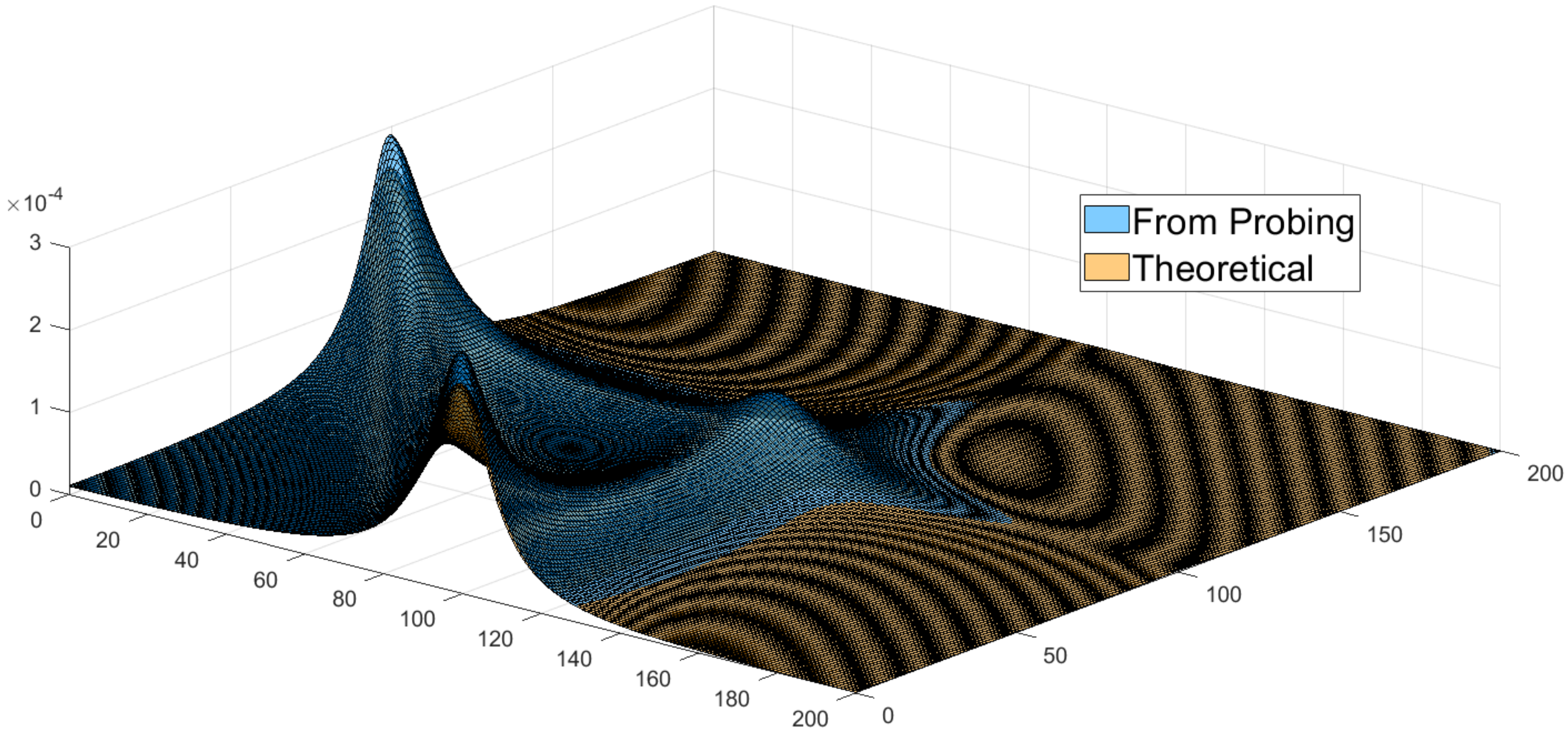


Harmonic Probing

Example QTF validated for a nonlinear duffing oscillator

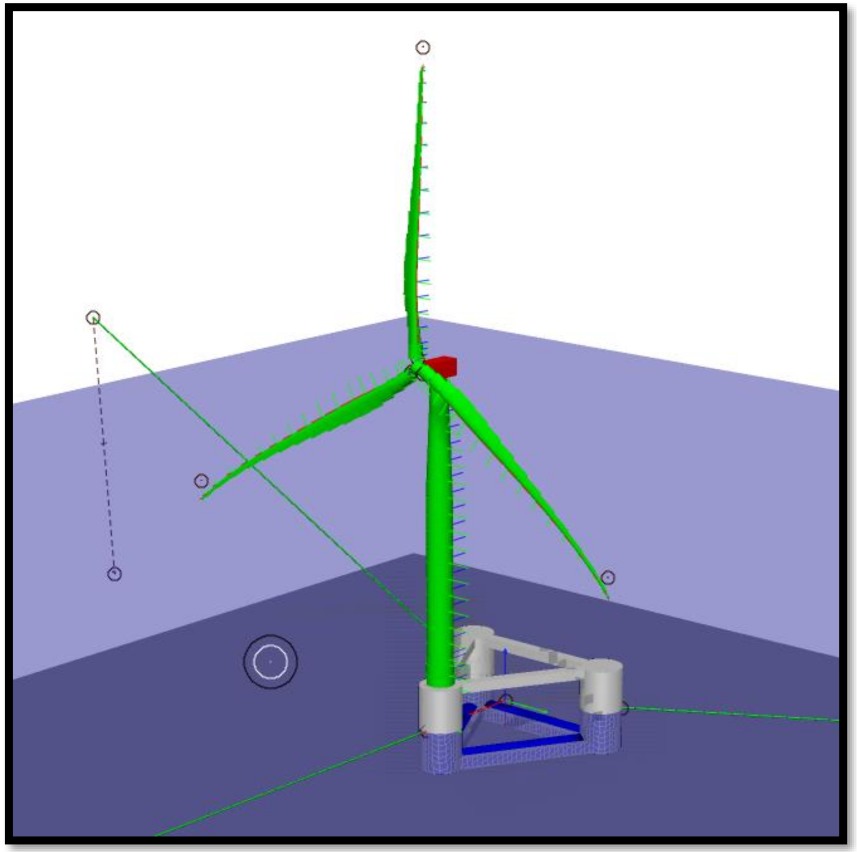


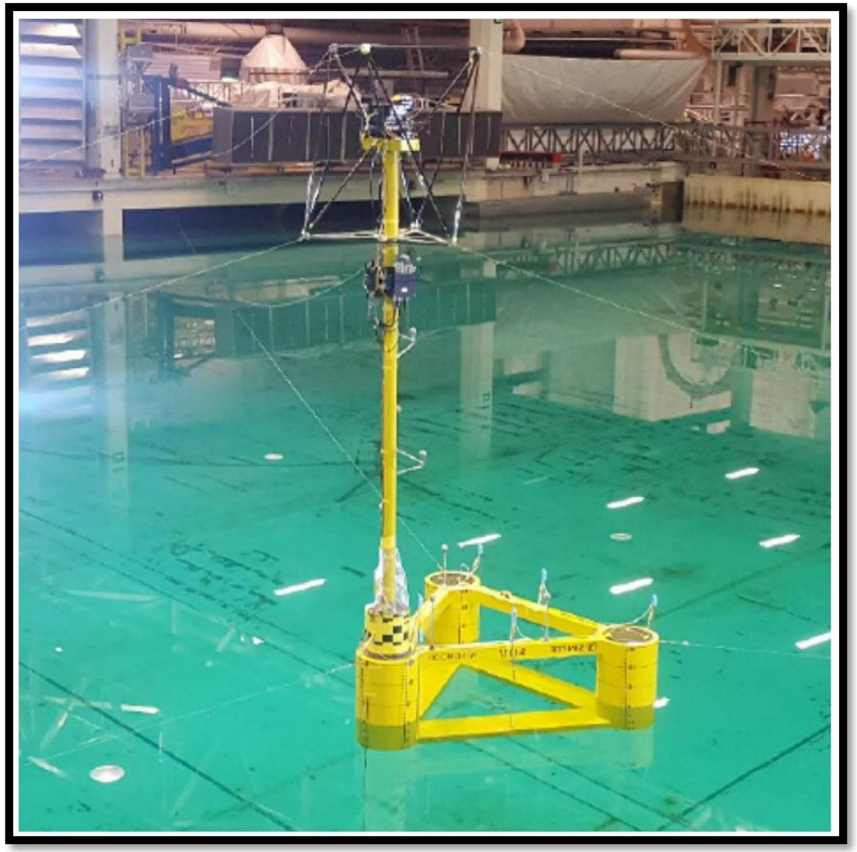




Future Outlook

- Estimate the low-frequency QTF for the Windmoor floater
- Investigate signals with different sampling rates





Q&A